

Chapter 11

Fractions

What is a Fraction ? Revision

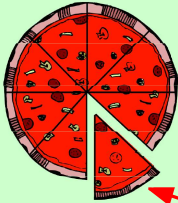
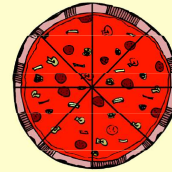
Be able to identify a "basic" fraction

Shown is a pizza cut into 8 equal slices.

Each bit is "1 out of the 8" bits.

We can write this as a fraction as "one eighth" or :-

$$\frac{1}{8}$$



1 of the 8 slices was removed

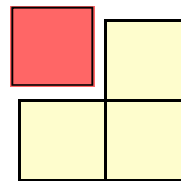
this tells you how many slices the shape was cut into

$$\frac{1}{8}$$

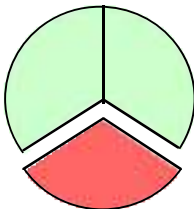
this indicates how many slices you want

Exercise 1 This exercise may be done orally.

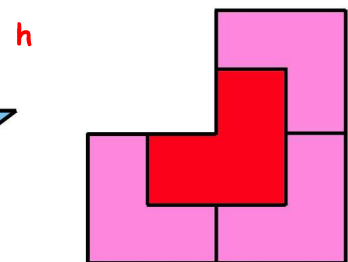
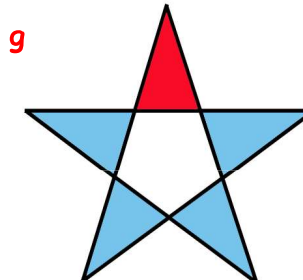
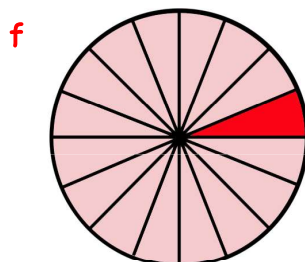
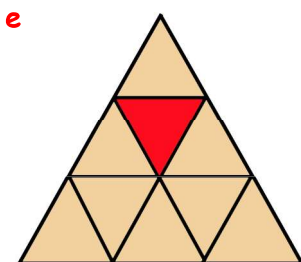
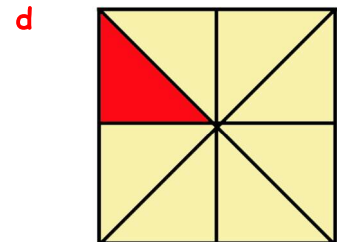
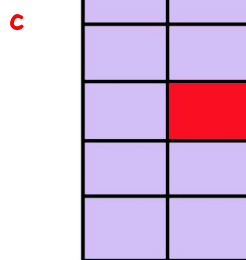
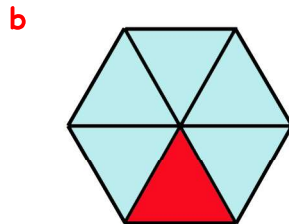
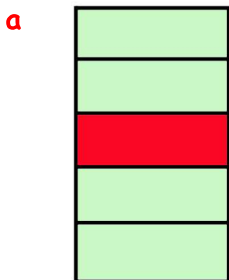
1. What fraction of this shape is coloured red ?



2. What fraction of this circle is coloured red ?



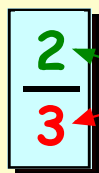
3. What fraction of each shape is the red bit ?



Identifying Fractions

Determine what fraction of a shape has been chosen

A fraction consists of 2 parts :-



the **DENOMINATOR**

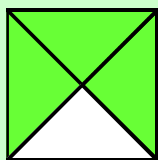
tells you the type of fraction you are dealing with (**thirds** here).

the **NUMERATOR**

tells you the number or "how many" of the thirds (in this case **two**).

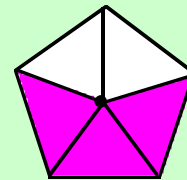
Examples :-

This shape shows 3 out of 4 equal parts are **green**.



$\frac{3}{4}$ of this shape is **green**.

This shape shows 3 out of 5 equal parts are **purple**.



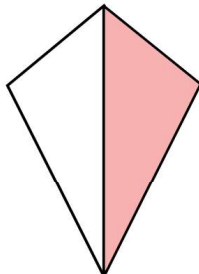
$\frac{3}{5}$ of this shape is **purple**.

$\frac{2}{5}$ of this shape is **not** purple.

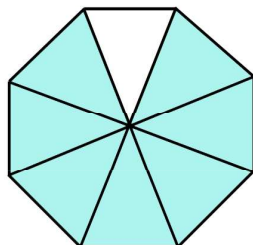
Exercise 2

1. For each of the following, write the fraction that is coloured :-

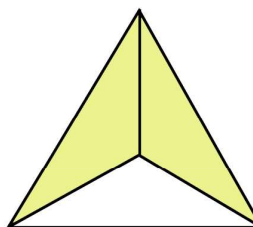
a



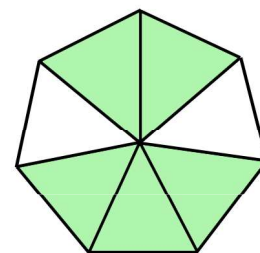
b



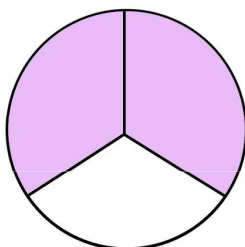
c



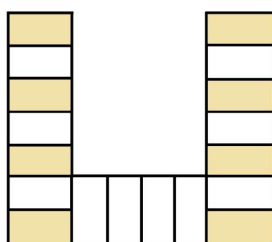
d



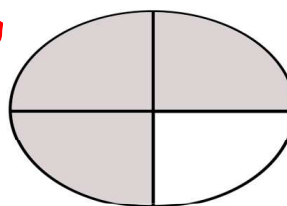
e



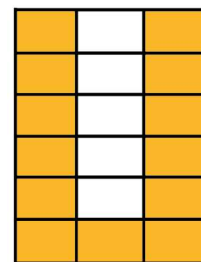
f



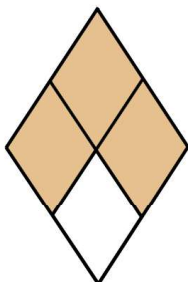
g



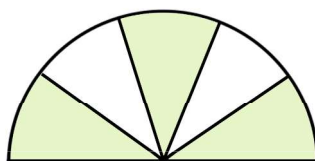
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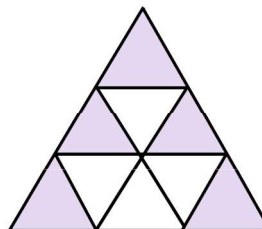
i



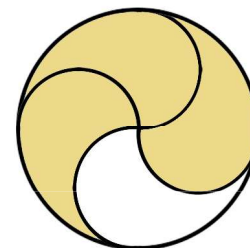
j



k

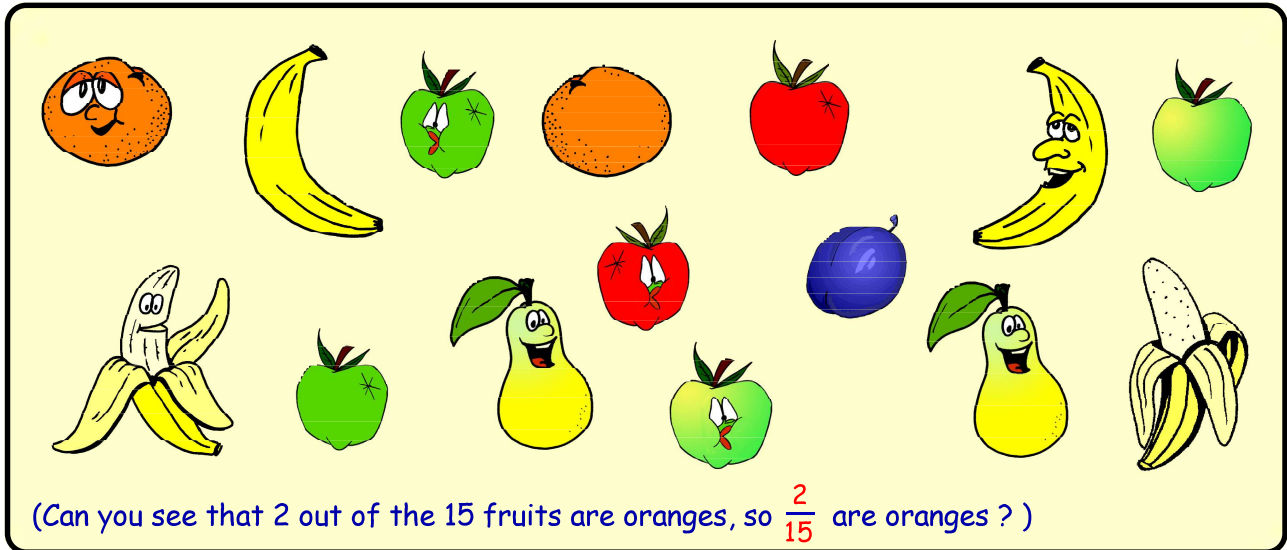


l



2. For each shape in question 1, write what fraction is **not** coloured.

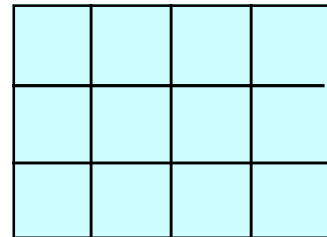
The picture shows 15 pieces of fruit.



3. Write down what fraction of the fruits are :-

- a bananas ($\frac{?}{15}$) b apples c plums d pears.

4. a Use a ruler to draw this rectangle measuring 4 boxes by 3 boxes. Shade in any 5 boxes.
Can you see that $\frac{5}{12}$ of the rectangle is shaded ?


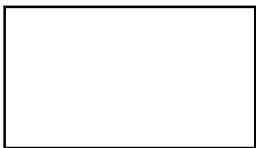
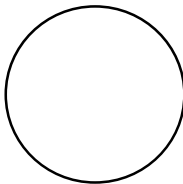
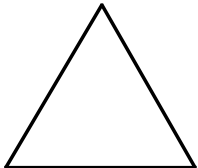
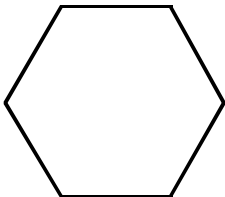
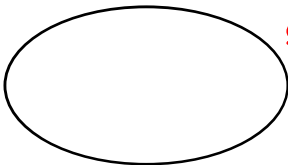
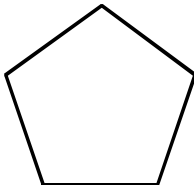



b Draw the same box again.
This time shade or colour in $\frac{7}{12}$ of the shape.

c Draw the same box again. This time shade or colour in $\frac{1}{6}$ of the shape.
(Hint : for every 6 equal parts shade in 1 part).

5. Trace or copy the following shapes.

Neatly and carefully show how to colour in the fraction of the shape asked for in red.

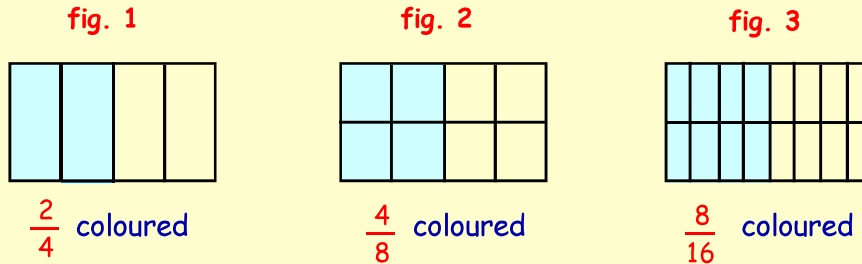
a		b		c		d	
	$\frac{3}{8}$		$\frac{7}{10}$		$\frac{5}{8}$		$\frac{2}{3}$
e		f		g		h	
	$\frac{5}{6}$		$\frac{3}{4}$		$\frac{2}{5}$		$\frac{1}{6}$

Equivalent Fractions

Recognise and be able to find equivalent fractions

Two fractions might look different because they have different **numerators** and different **denominators** but they might still represent the same number.

Look at these diagrams representing fractions :-



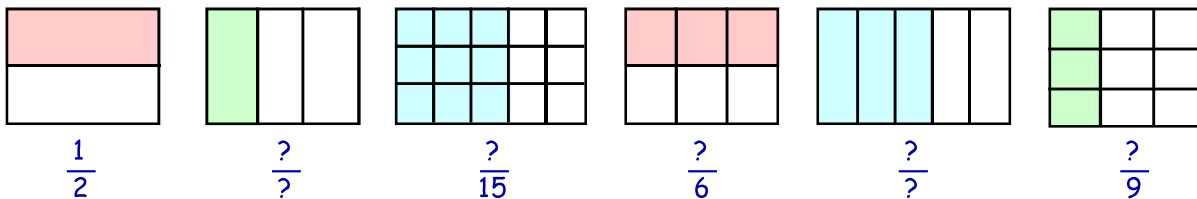
In each shape $\frac{1}{2}$ has been coloured.

This means $\frac{1}{2} = \frac{2}{4} = \frac{4}{8} = \frac{8}{16}$.

These are called **equivalent** fractions. (**Equivalent** means the **same as**).

Exercise 3

1. Copy the following and write down underneath each figure what fraction is shaded.



- From the pictures you can see another fraction equal to $\frac{1}{2}$. ($\frac{1}{2} = \frac{?}{?}$).
- The second and last diagrams show that $\frac{1}{3}$ is the same as $\frac{?}{?}$.
- The third and the fifth diagram show that $\frac{?}{15}$ is the same as $\frac{?}{?}$.

It is possible to find a fraction **equivalent** to $\frac{1}{4}$ by simply "multiplying the numerator and the denominator by the same number" :-

$$\Rightarrow \frac{1}{4} \text{ becomes } \frac{1 \times 5}{4 \times 5} = \frac{5}{20} \quad \begin{array}{l} \text{numerator} \times 5 \\ \text{denominator} \times 5 \end{array}$$

- Multiply the top and the bottom of $\frac{1}{5}$ by **2** to create a new fraction. What is it?
 - Multiply the top and the bottom of $\frac{1}{5}$ by **3** to create a new fraction. What is it?
 - Multiply the top and the bottom of $\frac{1}{5}$ by **10** to create a new fraction. What is it?

3. a Multiply the top and the bottom of $\frac{2}{3}$ by 2 to create a new fraction. What is it?
 b Multiply the top and the bottom of $\frac{2}{3}$ by 3 to form a new fraction. What is it?
 c Find **four** more fractions equivalent to $\frac{2}{3}$.

4. Multiply the top and bottom of each fraction by 2 to create a new fraction **equivalent** to the one given :-

a $\frac{1}{2}$ b $\frac{2}{5}$ c $\frac{3}{7}$ d $\frac{5}{8}$ e $\frac{9}{10}$ f $\frac{11}{15}$.

5. Repeat question 4, but multiply the top and bottom of each fraction by 3.

6. Multiply the top and bottom of each fraction by a number of your own choice to create a new fraction **equivalent** to the one given :-

a $\frac{2}{3}$ b $\frac{5}{7}$ c $\frac{7}{9}$ d $\frac{3}{8}$ e $\frac{7}{10}$ f $\frac{5}{12}$.

We can **SIMPLIFY** fractions (like $\frac{15}{18}$) by "dividing" top and bottom by a number.

=> $\frac{15}{18}$ becomes $\frac{15 \div 3}{18 \div 3} = \frac{5}{6}$ (This cannot be **simplified** any further).

7. Divide the top line and bottom line of each fraction by 2, to simplify each one :-

a $\frac{6}{8}$ b $\frac{2}{12}$ c $\frac{10}{14}$ d $\frac{6}{16}$ e $\frac{18}{20}$ f $\frac{86}{100}$.

8. Divide the top line and bottom line of each fraction by 3, to simplify each one :-

a $\frac{3}{6}$ b $\frac{6}{9}$ c $\frac{9}{12}$ d $\frac{9}{15}$ e $\frac{18}{21}$ f $\frac{15}{27}$.

9. For each of the following fractions, divide the numerator and the denominator by a number to **simplify** the fraction :- (Check that your answer can't be simplified further).

a $\frac{15}{18} \div 3$ b $\frac{4}{10}$ c $\frac{6}{18}$ d $\frac{7}{14}$ e $\frac{10}{25}$ f $\frac{20}{30}$
 g $\frac{12}{16}$ h $\frac{10}{15}$ i $\frac{9}{21}$ j $\frac{20}{24}$ k $\frac{12}{15}$ l $\frac{18}{20}$
 m $\frac{30}{40}$ n $\frac{10}{100}$ o $\frac{40}{60}$ p $\frac{25}{100}$ q $\frac{6}{60}$ r $\frac{5}{100}$.

10. Simplify as far as possible :-

a $\frac{22}{33}$ b $\frac{26}{39}$ c $\frac{30}{45}$ d $\frac{360}{480}$ e $\frac{63}{99}$ f $\frac{34}{51}$.

Fractions of a Quantity (basic)

Be able to find a basic fraction of a number

To find $\frac{1}{4}$ of something, you **divide** by 4.

To find $\frac{1}{6}$ **divide** by 6

To find $\frac{1}{10}$ **divide** by 10.

Examples :-

Find: a $\frac{1}{4}$ of 20

$$\begin{array}{r} \text{a} \\ 20 \div 4 \\ = 5 \end{array}$$

b $\frac{1}{6}$ of 18

$$\begin{array}{r} \text{b} \\ 18 \div 6 \\ = 3 \end{array}$$

c $\frac{1}{10}$ of 80

$$\begin{array}{r} \text{c} \\ 80 \div 10 \\ = 8 \end{array}$$

Exercise 4

1. Find :-

a $\frac{1}{3}$ of 15

b $\frac{1}{2}$ of 40

c $\frac{1}{4}$ of 24

d $\frac{1}{5}$ of 30

e $\frac{1}{3}$ of 21

f $\frac{1}{5}$ of 60

g $\frac{1}{4}$ of 60

h $\frac{1}{3}$ of 48

i $\frac{1}{4}$ of 100

j $\frac{1}{3}$ of 120

k $\frac{1}{5}$ of 300

l $\frac{1}{2}$ of 150.

2. Find :-

a $\frac{1}{7}$ of 21

b $\frac{1}{8}$ of 48

c $\frac{1}{9}$ of 72

d $\frac{1}{8}$ of 72

e $\frac{1}{6}$ of 84

f $\frac{1}{10}$ of 340

g $\frac{1}{8}$ of 400

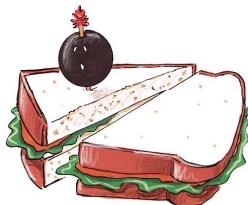
h $\frac{1}{9}$ of 810

i $\frac{1}{7}$ of 140.

3. a There are 36 desks in a classroom. $\frac{1}{4}$ of them are in need of repair. How many desks need repaired ?

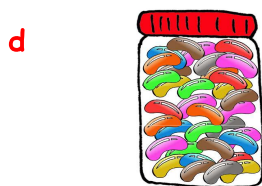


b



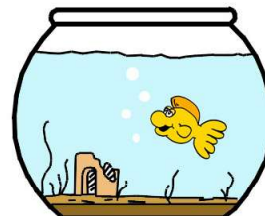
- A cafe served 42 packs of sandwiches one lunchtime. $\frac{1}{3}$ of the packs were lettuce and tomato. How many lettuce and tomato sandwiches were sold ?

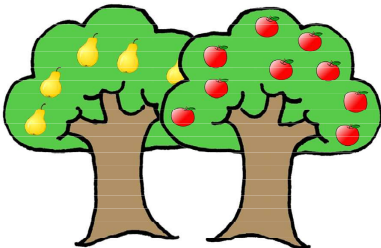
3. c Last week, Citroëult produced 60 new cars.
 $\frac{1}{5}$ of the cars were exported to France.
 How many cars went to France ?



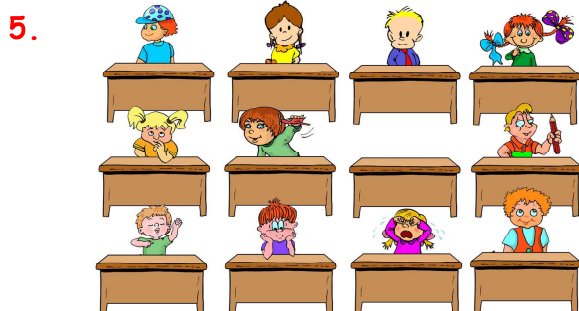
- d There are 48 jelly beans in a jar.
 $\frac{1}{6}$ of them are red.
 How many red jelly beans are there ?

- e A tank in a pet shop held 40 tropical fish.
 $\frac{1}{8}$ of them were goldfish.
 (i) How many goldfish were there in the tank ?
 (ii) How many of fish were **not** goldfish ?



- f  $\frac{1}{10}$ of the trees in an orchard were pear trees
 and $\frac{1}{5}$ of them were apple.
 If there were 60 trees altogether in the orchard,
 (i) how many pear trees were there
 (ii) how many apples trees ?

4. Jerry was on a 60 kilometre cycling trip for charity.
 He got a puncture **one third** of the way into the trip.
 a How far had Jerry cycled before the puncture ?
 b How many kilometres had he still to go ?



5. There are 36 children in Primary 5.
- a **third** of them have dark brown hair.
 - a **quarter** of them have light brown hair.
 - a **sixth** of them have black hair.
 - a **ninth** of them are blondes.
- The rest have red hair.

How many of the class have hair that is :-

- a dark brown b light brown c black
 d blonde e red f brown ?

6. I'm thinking of a number. I can find a $\frac{1}{2}$ of it with no remainder. I can find a $\frac{1}{3}$ of it with no remainder. I can find a $\frac{1}{5}$ of it with no remainder. I can find a $\frac{1}{6}$ of it with no remainder. When I try to find a $\frac{1}{7}$ of the number I'm left with a remainder of 4. Can you find the number I am thinking of ? (It is smaller than 100 !).



Fractions of a Quantity

Be able to find any fraction of a quantity

To find $\frac{2}{3}$ of a number (like 15), you do it **using 2 steps**.

Step 1 :- Find $\frac{1}{3}$ of 15 first ($\div 3$) $\Rightarrow \frac{1}{3}$ of 15 = $15 \div 3 = 5$

Step 2 :- Now find $\frac{2}{3}$ of 15 by ($\times 2$) $\Rightarrow \frac{2}{3}$ of 15 = $5 \times 2 = 10$

Set the working down as follows :-

$$\frac{3}{5} \text{ of } 25 \Rightarrow (25 \div 5) \Rightarrow 5 \times 3 = 15.$$

$$\frac{2}{7} \text{ of } 35 \Rightarrow (35 \div 7) \Rightarrow 5 \times 2 = 10.$$

$$\frac{7}{10} \text{ of } 60 \Rightarrow (60 \div 10) \Rightarrow 6 \times 7 = 42.$$

Rule :-

To find a fraction, like $\frac{5}{8}$ of something,

\Rightarrow "divide by the denominator" (8)

\Rightarrow then "multiply by the numerator" (5)

Exercise 5

1. Do the following :-

a $\frac{2}{5}$ of 30 = $(30 \div 5) \Rightarrow$ then $6 \times 2 = \dots$

b $\frac{3}{4}$ of 24 = $(24 \div \dots) \Rightarrow$ then $\dots \times 3 = \dots$

c $\frac{5}{6}$ of 18

d $\frac{4}{5}$ of 20

e $\frac{3}{8}$ of 40

f $\frac{7}{10}$ of 100

g $\frac{2}{3}$ of 66

h $\frac{2}{9}$ of 27

i $\frac{4}{9}$ of 63

j $\frac{3}{11}$ of 44

k $\frac{9}{10}$ of 80

l $\frac{2}{5}$ of 35

m $\frac{2}{7}$ of 21

n $\frac{7}{8}$ of 56

o $\frac{3}{4}$ of 400

p $\frac{3}{10}$ of 1000

q $\frac{2}{15}$ of 30

r $\frac{4}{7}$ of 35

s $\frac{7}{10}$ of 60

t $\frac{5}{9}$ of 63

u $\frac{5}{8}$ of 32

v $\frac{3}{16}$ of 32

w $\frac{9}{10}$ of 200

x $\frac{7}{100}$ of 300

y $\frac{7}{10}$ of 80

z $\frac{9}{20}$ of 60.

2. Do the following :-

a $\frac{2}{17}$ of 1700 grams

b $\frac{2}{15}$ of £15 000

c $\frac{9}{11}$ of €330

d $\frac{18}{19}$ of 19 kg

e $\frac{7}{20}$ of 60 ml

f $\frac{3}{19}$ of 38 kg

g $\frac{3}{50}$ of \$100

h $\frac{4}{15}$ of 150 metres

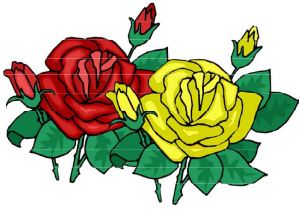
i $\frac{8}{12}$ of 6 litres.



3. a The ticket inspector on a train counted 36 passengers.
 $\frac{3}{4}$ of the passengers were adults.
 How many adults were on the train ?



b



A gardener has 30 rose bushes in his garden.
 $\frac{2}{5}$ of them are red, $\frac{3}{10}$ are yellow and the rest are white.

- (i) How many of the bushes are red ?
 (ii) How many are white ?

You may use a calculator for the rest of this exercise.



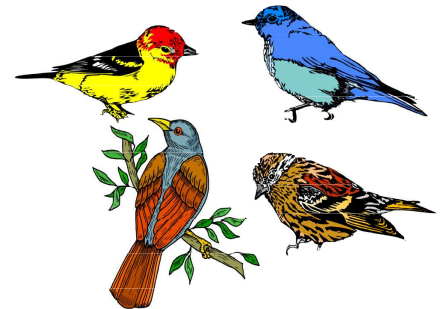
4. a Which would you prefer to have :-

- a $\frac{4}{5}$ share in prize money winnings of £5500 or
- a $\frac{5}{7}$ share in a lottery win of £6300 ?



- b There are 365 days in a year. It rained on $\frac{2}{5}$ of them.
 (i) On how many days did it rain ? (ii) How many dry days were there ?

- c A group of bird watchers spent a weekend on an island.
 They counted 1800 birds, of which $\frac{1}{6}$ were from Europe,
 $\frac{2}{9}$ were from South Africa, $\frac{5}{12}$ were from South America
 and the rest were local British birds.



List how many of the 1800 birds came from each area.

- d Mandy started the day with £200. She spent $\frac{1}{4}$ of her money on the rail fare to London.
 She spent $\frac{1}{5}$ of **what she had left** on her lunch. She then spent $\frac{3}{8}$ of **what was remaining** on a new pair of shoes. How much did Mandy then have left ?

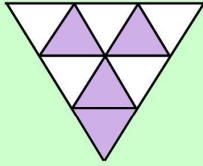
5. a By finding $\frac{2}{3}$ of 12 first, go on to find $\frac{1}{2}$ of ($\frac{2}{3}$ of 12). Now find $\frac{1}{3}$ of 12 and compare.
 b By finding $\frac{3}{4}$ of 16 first, go on to find $\frac{1}{2}$ of ($\frac{2}{3}$ of ($\frac{3}{4}$ of 16)). Now find $\frac{1}{4}$ of 16. Compare.
 c Find $\frac{4}{5}$ of 30 first. Now find $\frac{1}{2}$ of ($\frac{2}{3}$ of ($\frac{3}{4}$ of ($\frac{4}{5}$ of 30))). Now find $\frac{1}{5}$ of 30. Compare.
 Can you see what is happening ?
 d Find $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{6}{7}$ of $\frac{7}{8}$ of $\frac{8}{9}$ of $\frac{9}{10}$ of 200 in 10 without a calculator.
6. a **Hard** Here is a list of fractions :- $\frac{2}{3}$, $\frac{8}{9}$, $\frac{14}{15}$, $\frac{11}{12}$, $\frac{3}{4}$, $\frac{7}{8}$, $\frac{9}{10}$ and $\frac{4}{5}$.
 List them **in order** starting with the **largest** fraction.
- b **Harder**. Here is another list of fractions :- $\frac{2}{3}$, $\frac{7}{9}$, $\frac{8}{11}$, $\frac{13}{16}$, $\frac{3}{4}$, $\frac{11}{15}$, $\frac{3}{5}$ and $\frac{5}{7}$.
 Find a way of deciding which is the **largest** fraction and try to list the fractions **in order**.

The 3 я's

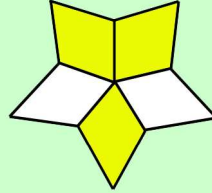
Revisit - Review - Revise

1. For each shape, say what fraction has been coloured :-

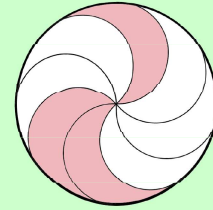
a



b



c



2. For each shape in question 1, write down the fraction **not** coloured.

3. Copy and complete :-

a $\frac{1}{3} = \frac{?}{9}$

b $\frac{2}{5} = \frac{8}{?}$

c $\frac{11}{?} = \frac{33}{39}$

4. Write down **two** fractions equivalent to :-

a $\frac{1}{5}$

b $\frac{3}{8}$

c $\frac{7}{10}$

5. Write each of these fractions in their **simplest** form :-

a $\frac{4}{12}$

b $\frac{18}{27}$

c $\frac{36}{48}$

6. Find :-

a $\frac{1}{3}$ of £24

b $\frac{1}{5}$ of 40 grams

c $\frac{1}{9}$ of 81 euros.

7. David chose 36 cards from a pack of cards. He found a **quarter** were hearts, a **third** were spades a **sixth** were diamonds and the rest were clubs.

a How many hearts did David have ?

b How many diamonds ?

c How many clubs ?



8. Find :-

a $\frac{4}{5}$ of 45 ml

b $\frac{3}{7}$ of \$280

c $\frac{7}{8}$ of £2400.

9. A vase of flowers, containing 350 ml of water, was left in the sun.

After a day, $\frac{1}{7}$ of the water had evaporated.

During the 2nd day, $\frac{3}{5}$ of the **remaining** water evaporated.

How much water was left in the vase after 2 days ?

