Armadale Academy
Higher Mathematics

Assessment
Revision Booklet

R4 I can calculate the gradient of perpendicular lines.

Perpendicular Gradients

1. Write down the gradient of the line perpendicular to the gradient given

(a) $m = 3$ **(b)** $m = -2$ **(c)** $m = 6$

(d) $m = \frac{1}{3}$ **(e)** $m = -\frac{1}{4}$ **(f)** $m = \frac{1}{5}$

(g) $m = -\frac{2}{3}$ **(h)** $m = \frac{5}{4}$ **(i)** $m = -\frac{3}{5}$

2. Write down the gradient of the line perpendicular to the given line

(a) $y = 5x + 2$ **(b)** $y = \frac{2}{3}x - 7$

(c) $y = 2 - 3x$ **(d)** $y = 4 - \frac{1}{2}x$

(e) $y = 3x - 3$ **(f)** $y = x + 9$

(g) $y - 4x + 12 = 0$ **(h)** $3x - y - 8 = 0$

(i) $3x - 2y + 7 = 0$ **(j)** $8y + 4x - 2 = 0$

R5 I can find the point of intersection of straight lines.

Finding points of intersection

1. Find the point of intersection between each pair of lines

(a) $3x + 4y = -7$; and $2x + y = -3$

(b) $y = -x + 12$; and $y = x - 4$

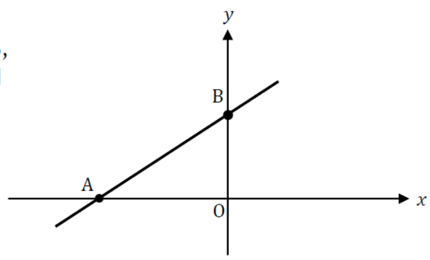
(c) $y = -x$; and $4x + 3y = 3$

(d) $2x - 5y = 1$; and $4x - 3y = 9$

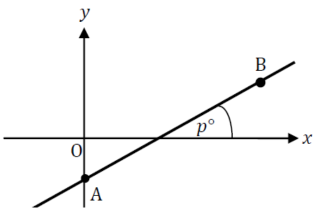
NR1 I can apply $m = tan\theta$ in the context of a problem.

$m = tan\theta$

1. Find the equation of the line AB, where A is the point $(-3, 0)$ and the angle BAO is $30°$.



2. Find the size of the angle $p°$ that the line joining the points $A(0, -2)$ and $B(4\sqrt{3},\ 2)$ makes with the positive direction of the x-axis.



NR2 I can solve straight line problems involving parallel and perpendicular lines.

Finding the equation of a straight line

1. Find the equation of the straight line through the point $(-1, 5)$ which is parallel to the line with equation $3x - y + 1 = 0$.

2. Find the equation of the straight line which passes through the point $(-1, 4)$ and is perpendicular to the line with equation $4x + y - 3 = 0$.

3. Find the equation of the straight line which is parallel to the line with equation $2x + 3y = 6$ and which passes through the point $(2, -1)$.

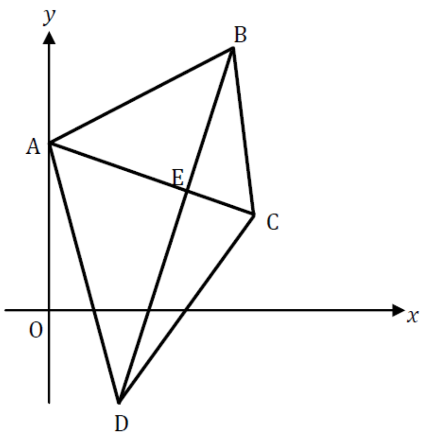
NR3 I know the properties of: midpoints; altitudes; medians; perpendicular bisectors and can apply these in problems (including points of intersection).

Equation of an altitude

Equation of a median

Points of intersection

1. A quadrilateral has vertices A$(-2, 8)$, B$(6, 12)$, C$(7, 5)$ and D$(1, -3)$ as shown in the diagram.



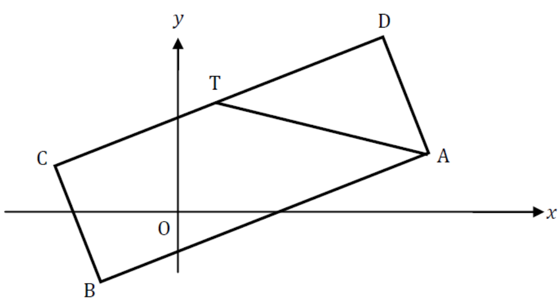
(a) Find the equation of diagonal BD.

(b) The equation of diagonal AC is $x + 3y = 22$. Find the coordinates of E, the point of intersection of the diagonals.

(c) **(i)** Find the equation of the perpendicular bisector of AB.

(ii) Show that this line passes through E.

2. The diagram shows rectangle ABCD with A$(7, 1)$ and D$(5, 5)$.



(a) Find the equation of AD.

(b) The line from A with equation $x + 3y = 10$ intersects with CD at T.

Find the coordinates of T.

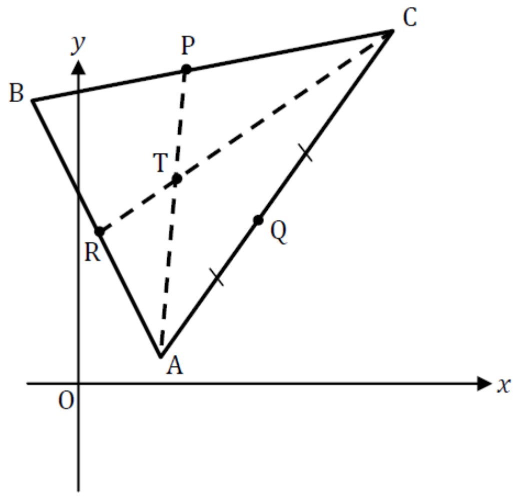
(c) Given that T is the midpoint of CD, find the coordinates of C and B.

3. Triangle ABC has vertices A$(4, 1)$, B$(-4, 17)$ and C$(18, 21)$.

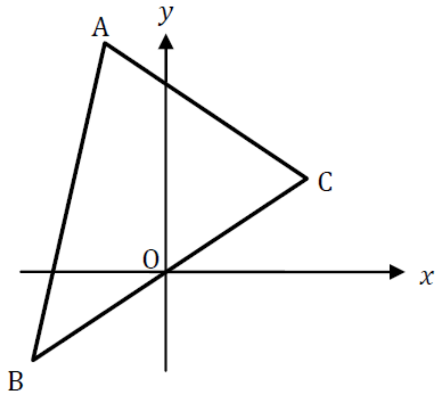
Medians AP and CR intersect at the point T$(6, 13)$.

(a) Find the equation of median BQ.

(b) Verify that T lies on BQ.



4. Triangle ABC has vertices A$(-2, 6)$, B$(-4, -2)$ and C$(4, 2)$ as shown. Find



(a) the equation of the line p, the median from C of triangle ABC.

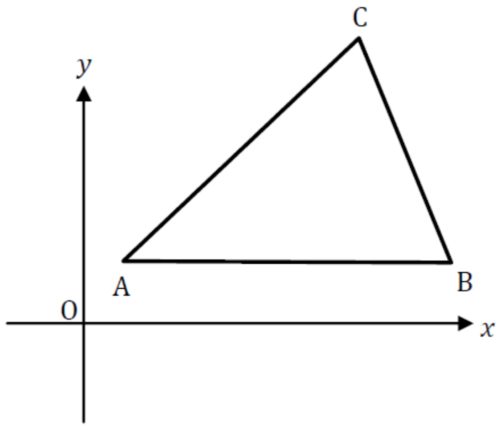
(b) the equation of the line q, the perpendicular bisector of BC.

(c) the coordinates of the point of intersection of the lines p and q.

5. Triangle ABC has vertices A$(1, 2)$, B$(11, 2)$ and C$(7, 6)$ as shown.

(a) Write down the equation of l_1, the perpendicular bisector of AB.

(b) Find the equation of l_2, the perpendicular bisector of AC.



(c) Find the point of intersection of the lines l_1 and l_2.

Differentiation

R1 I can differentiate algebraic functions which can be simplified to an expression in powers of x, including terms expressed as surds.

Introduction

Fractions and Roots

More complex differentiation

Find the derivative of the following

(a) $y = x^3 + 3x^2 + 5x$ **(b)** $y = 3x^5 + 2x^4 - x$ **(c)** $y = x^2 + 6x - 1$

(d) $f(x) = x^{\frac{2}{3}} + 4x^2$ **(e)** $f(x) = 3x^{\frac{1}{2}} - 2x^{-5}$ **(f)** $y = 5x^{-2} - 3x^{\frac{1}{2}}$

(g) $f(x) = \frac{1}{2\sqrt[3]{x}} + x^2$ **(h)** $y = 3x^7 - \frac{1}{5\sqrt[4]{x^3}}$ **(i)** $y = \frac{3}{5\sqrt[2]{x^5}} + 5$

(j) $y = \frac{2}{3\sqrt[4]{x^3}} + 2x^2 + x$ **(k)** $y = 5x^2 - \frac{1}{\sqrt[3]{x^2}}$ **(l)** $y = 4x^{-1} - 4x^{\frac{2}{3}}$

(m) $f(x) = 5x^3 - 6x^{-\frac{1}{2}}$ **(n)** $f(x) = 4x^2 + \frac{6}{\sqrt[3]{x}}$ **(o)** $y = x^2 - 5 - \frac{1}{x^2}$

Find the derivative of the following

(a) $y = (x+1)(x+2)$ **(b)** $y = (x+2)(x-3)$ **(c)** $y = (x+2)^2$

(d) $y = (x-3)(x+4)$ **(e)** $y = (x-5)(2x-2)$ **(f)** $y = x(x-4)$

(g) $y = (2x-3)(x+4)$ **(h)** $y = x^2(x-2)$ **(i)** $y = \frac{1}{x^2}(x^3 + 2x)$

(j) $y = \frac{1}{x}(x^2 + x)$ **(k)** $y = (\frac{1}{x} + 1)^2$ **(l)** $y = \frac{1}{\sqrt{x}}(\frac{1}{\sqrt{x}} - 1)$

Find the derivative of the following

(a) $y = \frac{x^2+3x+5}{x}$ **(b)** $y = \frac{2x^3+x^2+x}{x}$ **(c)** $y = \frac{x^4-6x+x^3}{x^2}$

(d) $y = \frac{x+5}{x}$ **(e)** $y = \frac{3+x^3}{x^2}$ **(f)** $y = \frac{x+2}{\sqrt{x}}$

(g) $y = \frac{(x+1)(x+2)}{x}$ **(h)** $y = \frac{(x-1)(x+3)}{x^2}$ **(i)** $y = \frac{3x^2+5x+1}{2x^2}$

NR1 I can determine the equation of a tangent to a curve, at a given point by differentiation.

Equations of tangents

1. Find the equation of the tangent to the curve at the given point

(a) $y = x^2$ at (1,4) **(b)** $y = x^2 + 2x$ at (0,2)

(c) $y = x^3 + 3$ at (2,8) **(d)** $y = x^2 - 6x + 5$ at x = 2

(e) $y = \sqrt{x} + 1$ at x = 1 **(f)** $y = x^2 + 2x - 3$ at (0,2)

(g) $f(x) = 5x^2 - 6x^{\frac{1}{2}}$ at x = 1 **(h)** $y = (x+4)^2$ at (2,-2)

(i) $y = \frac{x^2+5x+10}{x^2}$ at x = 1 **(j)** $y = \frac{1}{\sqrt{(x+1)}}$ at (3,3)

2. A curve has equation $y = 3x^2 - 4x$. At the point T the tangent to this curve has a gradient of 2, find the coordinates of T and hence the equation of the tangent.

3. A curve has equation $f(x) = 2x^2 + 8x - 3$. A tangent to this curve has a gradient of -4, find the equation of this tangent.

4. A tangent to the equation $y = \frac{2}{\sqrt{x}}$ has a gradient of -1, find the equation of this tangent.

5. A curve has equation $y = x^3 - 6x$. There are two tangents with a gradient 6. Find the equation of both tangents

NR2 I can determine use the stationary points of a curve and state their nature.

Stationary points

1. Find the coordinates of the Stationary points and determine their nature

(a) $y = x^3 - 6x^2 + 9x$ **(b)** $y = x^3 - 3x^2 - 9x + 12$

(c) $y = x^4 - 4x^3$ **(d)** $y = x^3 - 3x + 2$

(e) $y = x^3 - 12x + 2$ **(f)** $y = 2x^3 - 7x^2 + 4x + 1$

(g) $y = 3x^4 + 16x^3$ **(h)** $y = 8x^3 - x^2 + 11$

(i) $y = \frac{x^3 + 4x + 16}{x}$ **(j)** $y = (x-2)(x^2+1)$

NR3 I can sketch the graph of an algebraic function by determining stationary points and where it cuts the axes.

Curve Sketching

1. Sketch the graph of the following functions, stating clearly where it cuts the x & y axis

(a) $y = x^2 - 6x$ **(b)** $y = x^2 + 5x + 6$ **(c)** $y = x^2 - 5x$

(d) $y = 2x^3 - 3x^2 - 36x$ **(e)** $y = (x-1)^2(x+2)$ **(f)** $y = 12 - 4x - x^2$

NR4 I can determine where a function is strictly increasing/decreasing.

Increasing /Decreasing functions

1. State whether the function is increasing or decreasing

(a) $y = x^2 - 4x$ at x = 3 **(b)** $y = x^3 - 3x + 2$ at x = -1

(a) $y = x^2 - 10x + 4$ at x = -2 **(b)** $y = \frac{1}{3}x^3 - \frac{1}{2}x^2 + 2$ at x = 4

(a) $y = 4x^2 + 5x + 7$ at x = 1 **(b)** $y = 2x^4 - 4x^2 + 12$ at x = -1

2. For each function state the intervals in which it is increasing AND decreasing

(a) $y = x^2 - 5x + 12$ **(b)** $y = 2x^2 + x + 3$

(a) $y = 8 + 2x - x^2$ **(b)** $y = x^3$

(a) $y = 3x - x^3$ **(b)** $y = x^2(3 - 2x)$

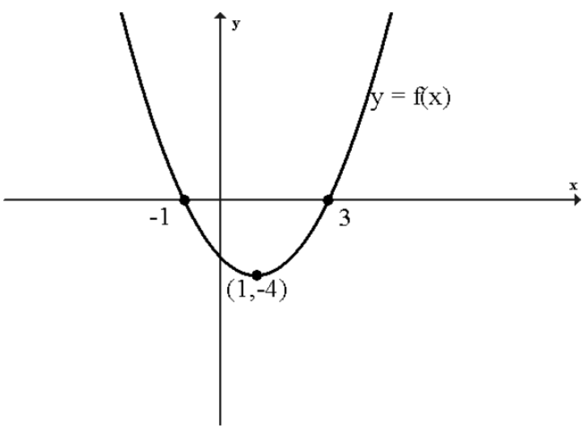
3. Show that the function $y = x^3 - x^2 + x$ is never decreasing

NR5 I can sketch graphs of derivatives.

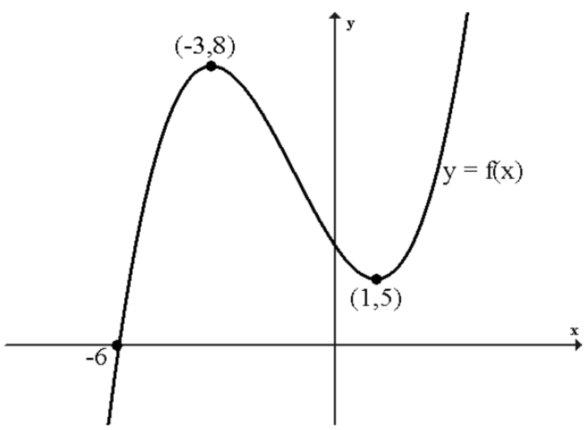
Graphs of Derivatives

1. For each graph of f(x) sketch f'(x)

(a)



(b)



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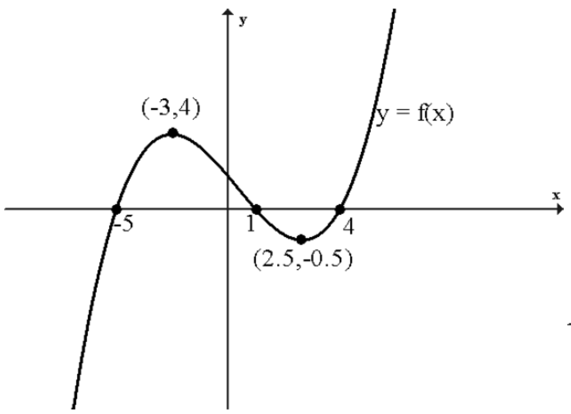

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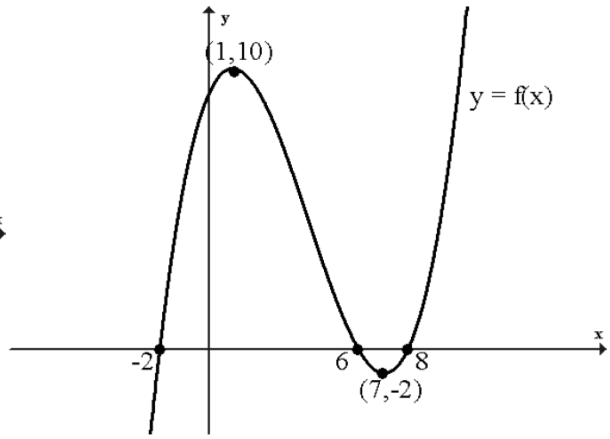
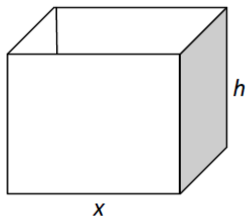


NR7 I can solve optimization problems in context using differentiation.

Optimisation

1. A plastic box with a square base and an open top is being designed. It must have a volume of 108 cm^3 .

   The length of the base is x cm and the height is h cm.



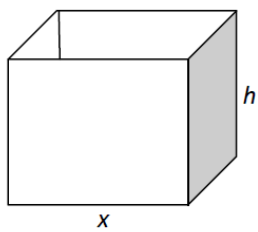
   (a) Show that the total surface area A is given by

$$A(x) = x^2 + \frac{432}{x}$$

   (b) Find the dimensions of the tray using the least amount of plastic

2. An open tank is to be designed in the shape of a cuboid with a square base. It must have a surface area of 100 cm^2 .

   The length of the base is x cm.



   (a) Show that the volume V is given by

$$A(x) = \frac{x}{4(100-x^2)}$$

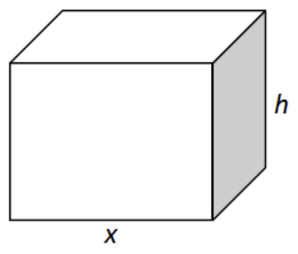
   (b) Find the length of the base which gives the tank a maximum volume.

3. The height h m of a ball thrown upwards is given by the formula $h(x) = 20t - 5t^2$ where t is the time in seconds from when the ball is thrown.

   (a) When does the ball reach its maximum height?

   (b) Calculate the maximum height of the ball.

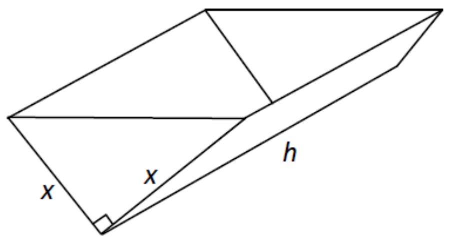
4. A Cuboid measures ***x*** by ***x*** by ***h*** units. The volume is 125 units^2.



(a) Show that the surface area of this Cuboid is given by $A(x) = 2x^2 + \frac{500}{x}$

(b) Find the value of ***x*** such that the surface area is minimised.

5. An open trough is in the shape of a triangular prism, the trough has a capacity of 256 000 cm^3.



(a) Show that the surface area of this trough is given by

$$A(x) = x^2 + \frac{1024\,000}{x}$$

(b) Find the value of ***x*** such that the surface area is minimised.

R1 I can evaluate the definite integral of a polynomial functions with integer limits.

Definite integrals

1. Find

(a) $\int_0^1 (x^2 - 3x + 4)\ dx$ **(b)** $\int_0^1 (4x^2 + 3x)\ dx$ **(c)** $\int_0^1 (x^3 + 2x^2 - 1)\ dx$

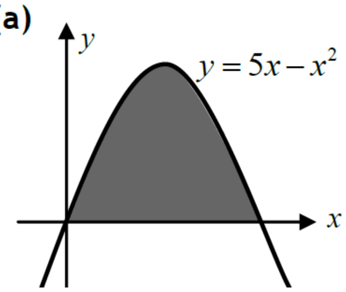
(d) $\int_0^2 (2x - 1)(x + 2)\, dx$ **(e)** $\int_{-1}^1 2x^2\,(2x + 1)\, dx$ **(f)** $\int_{-2}^1 (2x^3 - x^2 + 3x)\ dx$

NR2 I can evaluate the area enclosed between a function and the x axis.

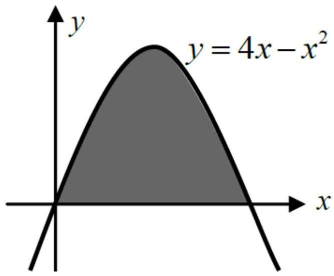
Area under a curve

1. Find the shaded area in the following diagrams

(a)



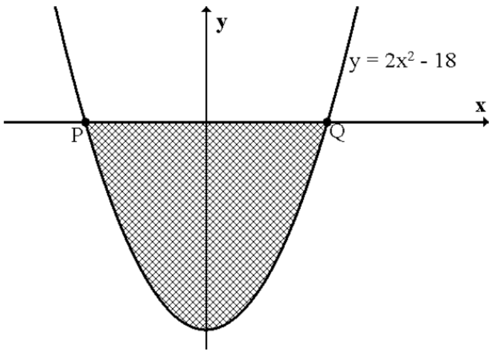
(b)



2. The diagram shows part of the graph of $y = 2x^2 - 18$.

(a) Find the coordinates of P and Q.

(b) Find the shaded area.

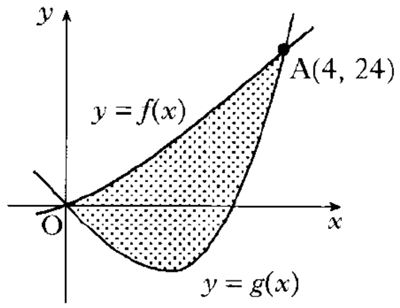


NR3 I can evaluate the area enclosed between two functions.

Area between curves

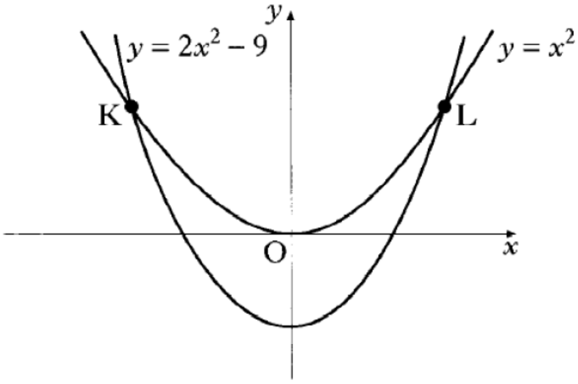
1. The incomplete graphs of $f(x) = x^2 + 2x$ and $g(x) = x^3 - x^2 - 6x$ are shown in the diagram. The graphs intersect at A(4, 24) and the origin.

Find the shaded area enclosed between the curves.

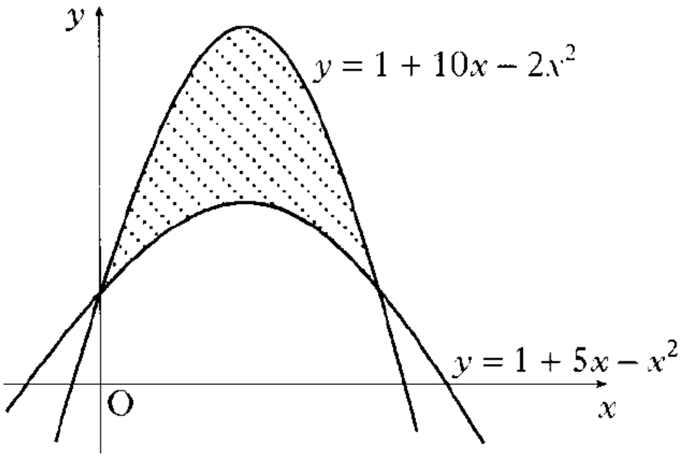


2. The curves with equations $y = x^2$ and $y = 2x^2 - 9$ intersect at K and L as shown.

Calculate the area enclosed between the curves.



3. Calculate the shaded area enclosed between the parabolas with equations $y = 1 + 10x - 2x^2$ and $y = 1 + 5x - x^2$.



NR4	**I can solve differential equations of the form $\frac{dy}{dx} = f(x)$ and give a particular solution.**

Differential Equations

1. Given the gradient $\frac{dy}{dx}$ of the curve at the point (x, y) and a point on the curve, find the equation of each curve:

a) $\frac{dy}{dx} = 3x^2 - 6x + 1$ (3,4)

b) $\frac{dy}{dx} = 4x^3 - 6x^2$ (1,9)

2. Find the solution to the following differential equations:

a) $\frac{dy}{dx} = 4x^3 + \frac{2}{x^3}$ and $y = 0$ *when* $x = 1$

b) $\frac{dy}{du} = \frac{u^2+1}{u^2}$ and $y = 4$ *when* $u = 2$