Essential Skills
National 5 Maths

Content:

Original by G Rennie (CDR edit - Mar 22)

NATIONAL 5 FORMULAE LIST

The roots of $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$ or $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

Area of a triangle: $A = \dfrac{1}{2} ab \sin C$

Volume of a sphere: $V = \dfrac{4}{3} \pi r^3$

Volume of a cone: $V = \dfrac{1}{3} \pi r^2 h$

Volume of a pyramid: $V = \dfrac{1}{3} Ah$

Standard deviation: $s = \sqrt{\dfrac{\sum(x - \bar{x})^2}{n-1}} = \sqrt{\dfrac{\sum x^2 - (\sum x)^2 / n}{n-1}}$, where n is the sample size.

Essential Skills 1

The questions in this series of worksheets appear frequently.

These are the GIFTS you must take to succeed

Completing the Square   (Non-Calculator)

Write the following in the form $(x + a)^2 + b$ and state the coordinates of the turning point.

1. $x^2 + 8x - 3$

2. $x^2 - 6x - 1$

3. $x^2 + 12x + 20$

4. $x^2 - 18x$

5. $x^2 - 2x + 7$

6. $x^2 + 10x + 13$

7. $x^2 + 4x - 9$

8. $x^2 - 6x + 6$
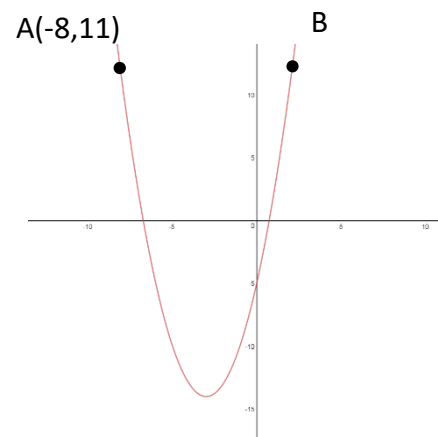
9. $x^2 + 14x - 25$

10. $x^2 - 4x + 1$

APPLYING QUESTION

The curve $y = x^2 + 6x - 5$ is shown.

(a)   Determine the coordinates of the turning point and the y-intercept,

(b)   Given that A is (-8, 11) write down the coordinates of B.

A(-8,11)   B

Essential Skills 2

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Simplifying Surds   (Non-Calculator)

Simplify:

1. $\sqrt{20} + \sqrt{45} - \sqrt{5}$

2. $2\sqrt{3} - \sqrt{108} + \sqrt{75}$

3. $7\sqrt{2} + \sqrt{18} - \sqrt{128}$

4. $\sqrt{6} - \sqrt{54} - \sqrt{24}$

5. $\sqrt{160} + 2\sqrt{10} - \sqrt{90}$

6. $\sqrt{63} - \sqrt{28} - \sqrt{7}$

7. $\sqrt{44} - \sqrt{99} + 4\sqrt{11}$

8. $3\sqrt{5} + \sqrt{320} - \sqrt{180}$

9. $4\sqrt{2} + \sqrt{8} - \sqrt{98}$

10. $\sqrt{27} - 2\sqrt{12} + \sqrt{3}$

APPLYING QUESTION

The Rectangle shown has a perimeter of $\sqrt{72}$ and breadth of $\sqrt{2}$.

Calculate its length.

Essential Skills 3

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<u>Algebraic Fractions</u>   (Non-Calculator)

Write as a fraction in its simplest form:

1. $\dfrac{3}{x+4} + \dfrac{2}{x+1}$

2. $\dfrac{4}{x-5} + \dfrac{3}{x+2}$

3. $\dfrac{1}{x+2} - \dfrac{3}{x+7}$

4. $\dfrac{6}{2x-1} - \dfrac{2}{x-1}$

5. $\dfrac{2}{x+3} - \dfrac{2}{3x+1}$

6. $\dfrac{x-3}{5} + \dfrac{x+2}{2}$

7. $\dfrac{2b+3}{3} - \dfrac{b}{5}$

8. $\dfrac{1}{p-1} + \dfrac{3}{3p+5}$

9. $\dfrac{3x-1}{3} - \dfrac{2x-3}{2}$

10. $\dfrac{1}{x} + \dfrac{3}{x^2}$

APPLYING QUESTION

A cyclist cycling on difficult terrain was able to cover x km at 4 km/h

(a) Write an expression in terms of x for time of his journey.
On the return leg they took a more favourable route with 3 additional km. They were able to cycle at a speed of 6km/h

(b) Work out, as a single fraction in terms of x, the **total** time for the whole journey.

Essential Skills 4

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Changing the subject of a Formula   (Non-Calculator)

Change the subject to the indicated letter:

1. $A = bc^2 + d$ (c) 2. $V = \pi r^2 h$ (r)

3. $H = \sqrt{ft}$ (t) 4. $W = \dfrac{d^2}{p}$ (p)

5. $g = (vip)^2$ (v) 6. $A = \dfrac{1}{2} ab\sin C$ (a)

7. $gh^3 - d = w$ (h) 8. $P = \dfrac{5hs}{t}$ (h)

9. $D = \dfrac{3(a+b)}{f}$ (a) 10. $T = \sqrt[3]{6t - 3}$ (t)

APPLYING QUESTION   (Calculator)

A cosmetics company aim to reduce the volume of a spherical bath bomb by 20%

(a)   If it originally had a volume of $480cm^3$, what will its new volume be?

(b)   Calculate the radius of the resized bath bomb.

Essential Skills 5

The questions in this series of worksheets appear frequently.

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Factorising Difference of Two Squares   (Non-Calculator)

Factorise the following:

1. $a^2 - b^2$

2. $x^2 - 9$

3. $4a^2 - d^2$

4. $9f^2 - 64$

5. $p^2 - 25$

6. $4p^2 - 81$

7. $g^2 - 100h^2$

8. $9c^2 - 49d^2$

9. $x^2 - 121$

10. $8a^2 - 18t^2$ *careful

APPLYING QUESTION

(a)   Factorise $3j^2 - 3k^2$

(b)   Hence, evaluate when $j = 2.3$ & $k = 0.7$

Essential Skills 6

The questions in this series of worksheets appear frequently.

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Multiplying out Brackets   (Non-Calculator)

Multiply out and simplify:

    1. $3(x - 3) + 2(x - 5)$ 2. $-7(2t - 3w) - 11(t - 1)$

    3. $(x + 4)(x + 6)$ 4. $(x - 8)(x - 7)$

    5. $(3x + 4)(2x - 1)$ 6. $(5x - 3)(x - 2)$

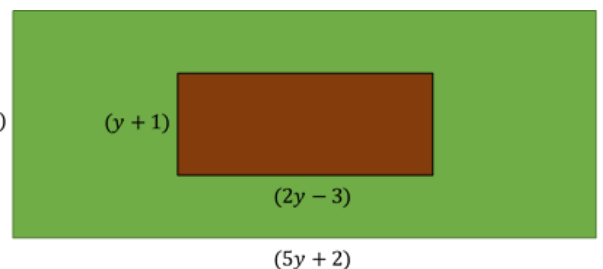   7. $(4x + 1)(3x - 2)$ 8. $(x + 4)^2$

    9. $(2x - 1)^2$ 10. $(3s - 4t)^2$

APPLYING QUESTION

A garden has length 5y+2 and breadth 2y + 1.
A rectangular flower bed of length 2y – 3 and
breadth y + 1 is cut out the grass.
Find an expression in terms of y for the area
of grass remaining

$(2y + 1)$ $(y + 1)$

$(2y - 3)$

$(5y + 2)$

The questions in this series of worksheets appear frequently.

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<u>Multiplying out Brackets</u>   (Non-Calculator)

Multiply out and simplify:

1. $(x + 3)(x^2 + 2x + 1)$

2. $(x + 2)(3x^2 + 5x - 1)$

3. $(2x + 1)(x^2 - 3x + 4)$

4. $(x - 2)(x^2 + 5x + 2)$

5. $(x - 5)(x^2 - 3x - 10)$

6. $(2x + 3)(x^2 - 4x + 3)$

7. $(3x - 1)(2x^2 + 4x - 1)$

8. $(x - 1)(x^2 - 7x + 6)$

9. $(x + 8)(3x^2 + x - 4)$

10. $(x - 4)(2x^2 - 2x + 1)$

<u>APPLYING QUESTIONS</u>

Multiply out and simplify:

(a) $(x + 2)(x - 3)(x + 1)$

(b) $(x + 2)(x - 1)^2$

(c) $(2x - 3)^3$

Essential Skills 8

The questions in this series of worksheets appear frequently.

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Factorising Trinomials when a=1   (Non-Calculator)

Factorise the following:

    1. $a^2 + 6a + 8$ 2. $b^2 + 11b + 30$

    3. $c^2 - 8c + 12$ 4. $d^2 - 13d + 40$

    5. $e^2 + e - 56$ 6. $f^2 - 3f - 54$

    7. $g^2 + 15g + 54$ 8. $h^2 + 13h - 30$

    9. $j^2 - 6j - 55$ 10. $3k^2 + 6k - 189$ *careful

APPLYING QUESTION

    (a)   Factorise $x^2 - 16$

    (b)   Hence, simplify $\dfrac{x^2 - 3x - 28}{x^2 - 16}$

Essential Skills 9

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Calculations involving Fractions (Non-Calculator)

Calculate the following:

1. $2\frac{1}{3} - 1\frac{2}{5}$

2. $6\frac{2}{5} \div 3\frac{1}{2}$

3. $3\frac{2}{3} \times 1\frac{10}{11}$

4. $1\frac{1}{6} + 2\frac{3}{8}$

5. $2\frac{3}{4} \div 1\frac{1}{5}$

6. $2\frac{2}{9} \times 4\frac{3}{5}$

7. $5\frac{1}{2} - 2\frac{5}{6}$

8. $\frac{1}{7}(2\frac{1}{4} + 1\frac{3}{5})$

9. $\frac{5}{6}of\ \frac{2}{3} + 1\frac{1}{6}$

10. $2\frac{1}{2} \times (2\frac{1}{8} - 1\frac{2}{5})$

APPLYING QUESTION

Calculate:

$$3\frac{1}{15} \div (2\frac{1}{5} + 1\frac{2}{3})$$

Essential Skills 10

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Standard Deviation   (Calculator)

Calculate the mean and the standard deviation of each:

1.   14, 17, 15, 23, 20, 19

2.   8, 13, 7, 6, 8, 9, 5

3.   1.8, 3.7, 4, 2.6, 5.9

4.   102, 108, 112, 109, 110, 107

5.   47, 56, 61, 52, 59

6.   1, 2, 4, 1, 3, 2, 1

7.   9, 14, 11, 13, 8, 11

8.   33, 39, 40, 38, 35

9.   1305, 1301, 1298, 1300, 1295, 1307

10.   41, 35, 33, 46, 38

APPLYING QUESTION

The prices, in pence, at five petrol stations around Airdrie for a litre of unleaded are:

121   119   120   117   118

(a)   Calculate the mean and standard deviation.

(b)   Why do you think the standard deviation must be so low?

(c)   If each petrol station had to put their price up by 4 pence what effect would it have on the mean and standard deviation?

Essential Skills 11

The questions in this series of worksheets appear frequently.

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Quadratic Formula   (Calculator)

Solve the following to 1 decimal place:

1. $x^2 + 6x + 2 = 0$ 2. $3x^2 + 4x - 1 = 0$

3. $5x^2 - x - 3 = 0$ 4. $4x^2 - 7x + 1 = 0$

5. $x^2 + 4x - 2 = 0$ 6. $4 - 4x - x^2 = 0$ *Careful

7. $9x^2 - 8x + 1 = 0$ 8. $2x^2 + 3x - 5 = 0$

9. $5x^2 - 9x + 2 = 0$ 10. $2x^2 = 3x + 3$ *Careful

APPLYING QUESTION

The curve $f(x) = 2x^2 - 4x - 3$ is shown.

Determine the coordinates of A and B

Give your answers to 3 significant figures

A   B
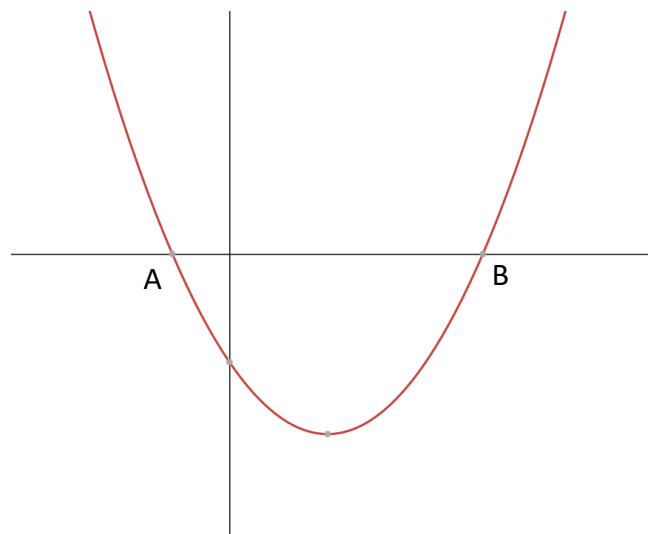
Essential Skills 12

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Trigonometric Equations   (Calculator)

Solve the following: $(0 \leq x \leq 360)$

1. $2sinx - 1 = 0$

2. $2cosx - \sqrt{3} = 0$

3. $5tanx - 1 = 2$

4. $6sinx + 2 = 3$

5. $3cosx + 1 = 3$

6. $2tanx + 11 = 20$

7. $5sinx - 1 = -3$

8. $4cosx + 7 = 5$

9. $2tanx + 3 = 1$

10. $20sinx + 17 = 25$

APPLYING QUESTION

The curve $y = 4sinx - 1$ is shown.

The line $y = -3$ intersects at A and B

Determine the coordinates of A and B
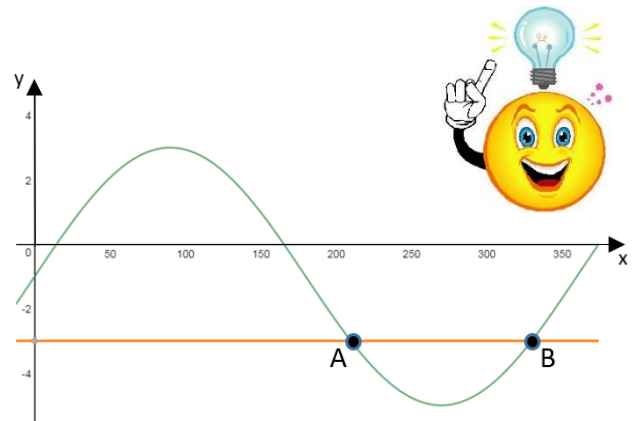
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<u>Functional Notation</u>   (Non-Calculator)

1.	If	$f(x) = 3x - 4$	Evaluate:	(a) $f(2)$		(b) $f(-1)$
2.	If	$f(x) = x^2 - 1$	Evaluate:	(a) $f(4)$		(b) $f(-2)$
3.	If	$f(x) = 2x^3 + 3$	Evaluate:	(a) $f(3)$		(b) $f(-1)$
4.	If	$f(x) = 3x^2$	Evaluate:	(a) $f(5)$		(b) $f(-4)$
5.	If	$f(x) = 3x^2 - 1$	Evaluate:	(a) $f(4)$		(b) $f(-2)$
6.	If	$f(x) = 7 - x$	Evaluate:	(a) $f(3)$		(b) $f(-7)$
7.	If	$f(x) = 5 - x^2$	Evaluate:	(a) $f(2)$		(b) $f(-3)$
8.	If	$f(x) = -x^3$	Evaluate:	(a) $f(1)$		(b) $f(-4)$
9.	If	$f(x) = 4 + x^2$	Evaluate:	(a) $f(5)$		(b) $f(-3)$
10.	If	$f(x) = 3 + 2x - x^3$	Evaluate:	(a) $f(2)$		(b) $f(-1)$

<u>APPLYING QUESTION</u>

A function is defined as $h(x) = 24 - 5x$

(a)   Evaluate $h(-3)$

(b)   Express $h(p - 4)$ in its simplest form.

(c)   Given that $h(t) = 59$, find the value of t.

(d)   Solve $3x + 9 = 2h(x)$

Essential Skills 14

The questions in this series of worksheets appear frequently.

These are the GIFTS you must take to succeed

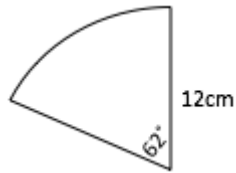<u>Straight Lines</u>   (Non-Calculator)

Find the equation of the line connecting the points:

1. $A\,(\,2,5\,)$ & $B\,(\,8,23\,)$

2. $C\,(\,0,7\,)$ & $D\,(\,5,17\,)$

3. $E\,(-3,2\,)$ & $F\,(\,2,7\,)$

4. $G\,(-1,-4\,)$ & $H\,(\,3,4\,)$

5. $J\,(-4,7\,)$ & $K\,(\,1,2\,)$

6. $L\,(-5,0\,)$ & $M\,(\,0,-10\,)$

7. $P\,(-4,0\,)$ & $Q\,(\,0,5\,)$

8. $R\,(\,0,-3\,)$ & $S\,(\,4,7\,)$

9. $T\,(\,3,1\,)$ & $U\,(\,7,7\,)$

10. $V\,(-2,5\,)$ & $W\,(\,4,-3\,)$

Essential Skills 15

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<u>Simultaneous Equations</u>   (Calculator may be used on Applying Q2)

Solve the system of equations:

1. $3x + 2y = 12$ 2. $4x + 3y = 19$

          $2x + y = 7$ $5x - y = 0$

3. $2x + 7y = 18$ 4. $5x + 2y = 3$

          $3x + 5y = 16$ $4x + 3y = 1$

5. $7x - 3y = 6$ 6. $2x - 5y = 18$

          $4x - 2y = 2$ $3x + 3y = 6$

7. $x - 3y = 1$ 8. $5x - 3y = -12$

          $2x + y = -12$ $4x + y = 4$

9. $7x - 3y = -19$ 10. $12x + y = 31$

          $6x - 2y = -14$ $4x - 2y = -6$

APPLYING QUESTIONS

1.   Find the point of intersection of lines $3x + 2y = 33$ and $4x - y = 22$

2.   An Excelsior stadium concert has room for x standing spectators and y seated spectators.

(a)   If the capacity is 12000 tickets, make an equation in terms of x and y.

(b)   A standing ticket costs £28·50 and a seated ticket is £41.

        Make an equation in terms of x and y given that the takings for a sold-out concert were £472, 500.

(c)   How many of each ticket were sold?

Essential Skills 16

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Arcs Length   (Calculator)

Calculate the length of arc in each:

1.
8cm
120°

2.
12cm
62°

3.
3m
225°

4.
7cm
100°

5.
3.2cm
58°

6.
1.8 m
32°

7.
6cm
162°

8.
18.6cm
212°

9.
2.3cm
81°

10.
43mm
135°
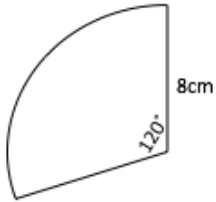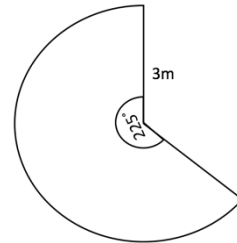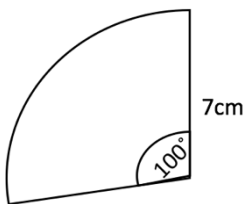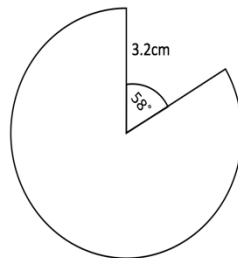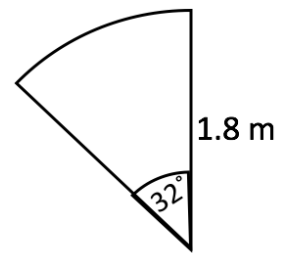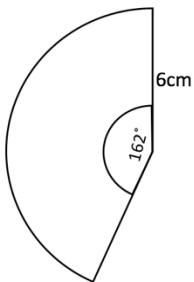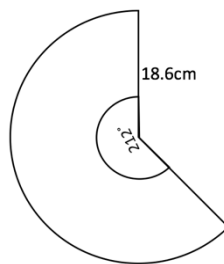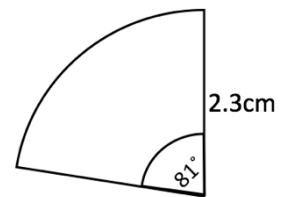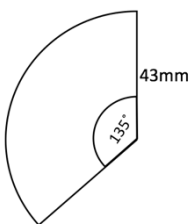
APPLYING QUESTIONS

1.   Calculate the perimeter of the shaded section:

120°
8.1cm
3.7cm
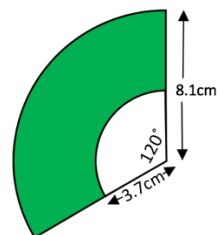
Essential Skills 17

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Algebraic Fractions 2   (Non-Calculator)

By factorising numerators and denominators, simplify:

1. $\dfrac{x^2+5x+6}{3x+6}$

2. $\dfrac{x^2+4x-21}{2x+14}$

3. $\dfrac{x^2+8x+12}{x^2+6x}$

4. $\dfrac{x^2-9x+14}{x^2+3x-10}$

5. $\dfrac{2x^2-5x-3}{x^2-9}$

6. $\dfrac{3x-2}{3x^2+13x-10}$

7. $\dfrac{2x^2-50}{4x^2-19x-5}$

8. $\dfrac{4x^2-4x-3}{2x^2-5x+3}$

9. $\dfrac{2x^2+7x+3}{3x^2+8x-3}$

10. $\dfrac{x^2+x-56}{2x^2+11x-40}$

APPLYING QUESTION

(a)   Factorise $x^2 - 3x - 54$

(b)   Hence, simplify $\dfrac{x^2-3x-54}{3x^2+17x-6}$

Essential Skills 18

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Percentages: Reversing the Change   (Calculator)
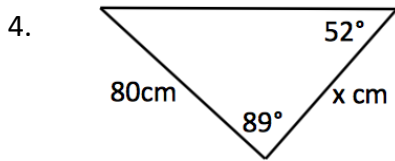
Find the original value:

1.   £212 after having been increased by 6%

2.   105g after having been decreased by 30%

3.   £12750 after a 2% rise

4.   €6750 in a 10% off sale

5.   448ml after an increase of 12%

6.   £96 after an increase of 20%

7. $79.20 after having been decreased by 20%

8.   £36750 after a 5% wage rise

9.   £48 after a 40% discount

10.   €7.82 after a 15% increase in price

APPLYING QUESTION

The salary of an employee in a firm has increased by 5%.

Their new salary is £26880.

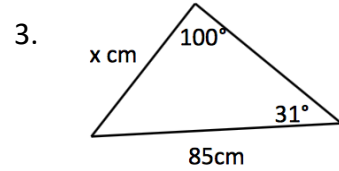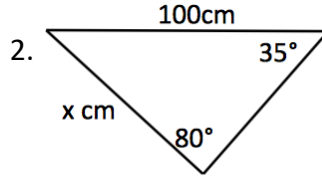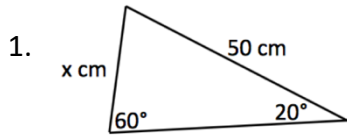How much more is this than their previous salary?

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Indices   (Non-Calculator)

Simplify, leaving your answers with positive indices:

1. $\dfrac{x^5 \times x^6}{x^3}$

2. $\dfrac{x^7 \times x^{-4}}{x^2}$

3. $\dfrac{3x^2 \times x^4}{x^{-5}}$

4. $\dfrac{5x^3 \times 4x^2}{2x^3}$

5. $\dfrac{8x^5 \times 3x}{12x^2}$

6. $\dfrac{3x^2 \times 2x^{-1}}{7x}$

7. $\dfrac{2x^3 \times 5x}{15x^{-6}}$

8. $\dfrac{x^8 \times 3x^{-6}}{x^5}$

9. $\dfrac{2x^2 y^3 \times 6x^2 y}{4xy^2}$

10. $\dfrac{3x^2 y^{\frac{1}{3}} \times 6x^{-1} y^{\frac{8}{3}}}{9x^3 y^2}$

APPLYING QUESTION

(a)   Simplify, $\dfrac{x^5 y^3 \times 2x^{-1} y}{3x^2 y^5}$,

leaving your answer with positive indices.

(b)   Hence, evaluate $\dfrac{x^5 y^3 \times 2x^{-1} y}{3x^2 y^5}$, when $x = -3$ and $y = 2$.

Essential Skills 20

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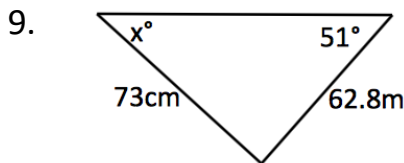
These are the GIFTS you must take to succeed

<u>The Sine Rule</u>   (Calculator)
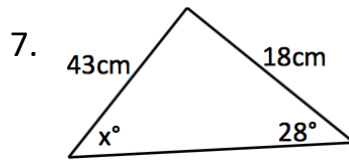
Calculate side, x in each:

1.
50 cm
x cm
60°   20°

2.
100cm
35°
x cm
80°

3.
100°
x cm
31°
85cm

4.
52°
80cm
x cm
89°

5.
J
82°
x m
34°
K   L
12.6m

Calculate angle, x in each:

6.
13.6m
9.5m
A 59°   x°

7.
43cm   18cm
x°   28°

8.
53m
x°
48m
101°

9.
x°   51°
73cm   62.8m

10.
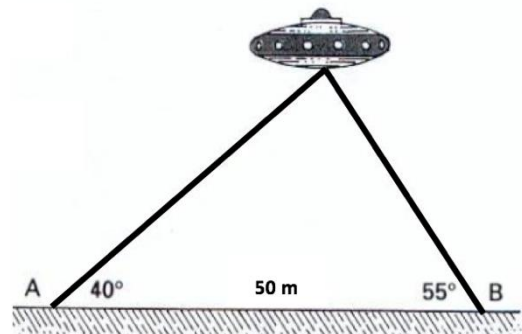x°
8mm
39°
11 mm
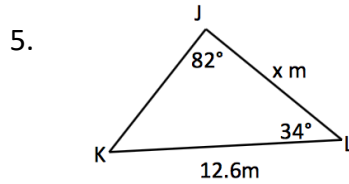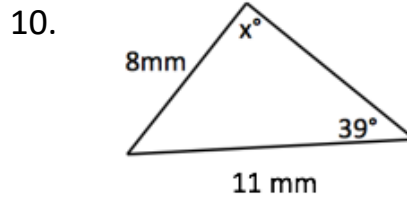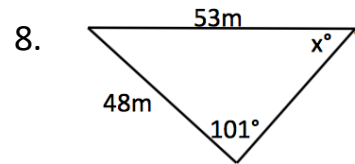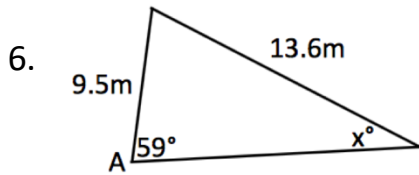
<u>APPLYING QUESTION</u>

Aaron and Brandon spot a UFO above Brannock High.

Aaron measures elevation at 40˚ from his viewpoint

Brandon measures 55˚ from his.

They are standing 50 metres apart.
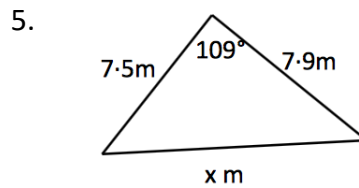
What height is the UFO above the ground?

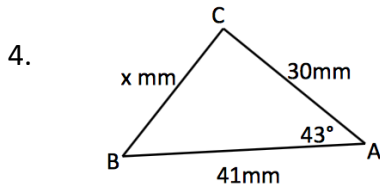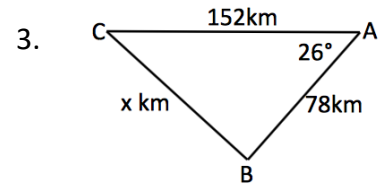A   40°   50 m   55°   B
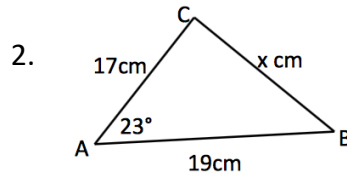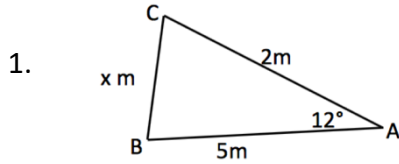
Essential Skills 21

The questions in this series of worksheets appear frequently.

These are the GIFTS you must take to succeed

The Cosine Rule   (Calculator)

Calculate side, x in each:

1.
C, 2m, x m, 12°, A, B, 5m

2.
C, 17cm, x cm, 23°, A, 19cm, B

3.
152km, C, 26°, A, x km, 78km, B

4.
C, x mm, 30mm, 43°, A, B, 41mm

5.
7·5m, 109°, 7·9m, x m

Calculate angle, x in each:

6.
5·2cm, 9·7cm, 7·4cm, x°

7.
12cm, 8cm, x°, 13cm

8.
33cm, x°, 24cm, 45cm

9.
5cm, 7cm, x°, 11cm

10.
8·4cm, 7·1cm, 6·5cm, x°
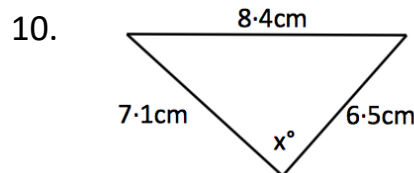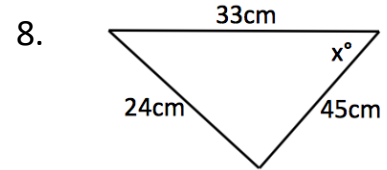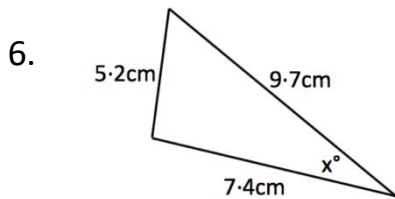
APPLYING QUESTION

Two drones leave from the same position, P.

Drone A flies 350 metres on a bearing of 063°.

Drone B flies 470 metres on a bearing of 134°.

Calculate the distance between the two drones.

The questions in this series of worksheets appear frequently.
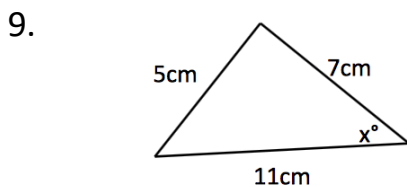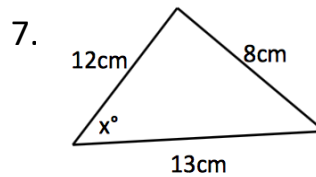
These are the GIFTS you must take to succeed

Pythagoras in Circles   (Calculator)

Calculate x:

1.

O
13cm
A ←——24cm——→ B
x cm

2.

O
20cm
A ←——32cm——→ B
x cm

3.

O
15cm
A ←——24cm——→ B
x cm

4.

O
17cm
A ←——30cm——→ B
x cm

5.

O
41cm
A ←——80cm——→ B
x cm
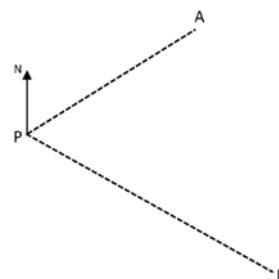
6.

O
25cm
A ←——48cm——→ B
x cm

7.

O
61cm
A ←—— x cm ——→ B
50 cm

8.

O
37cm
A ←—— x cm ——→ B
49 cm

9.

O
29cm
A ←—— x cm ——→ B
49 cm
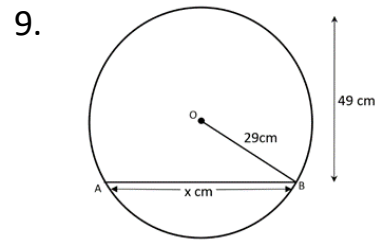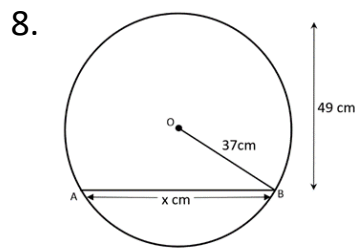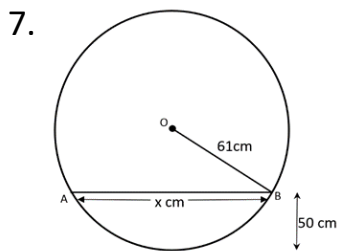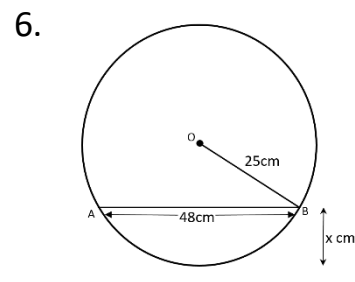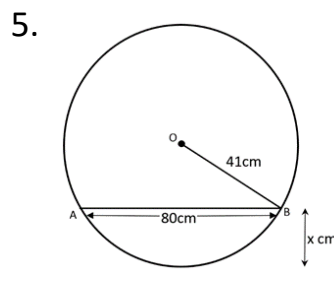
APPLYING QUESTION

A pendant is designed as shown in the diagram.

Calculate its total height.

O
0.5cm
A ←——0.8cm——→ B
0.75cm
h cm
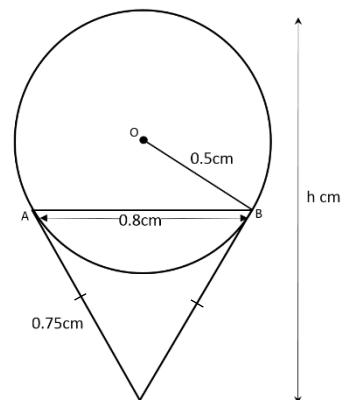
Essential Skills 23

The questions in this series of worksheets appear frequently.

These are the GIFTS you must take to succeed

Percentages: Appreciation   (Calculator)

Calculate:

1.   The interest earned on £3800 at 4% p.a after 3 years.

2.   The number of bacteria after 3 hours if 30 are present initially and are increasing by 42% per
        hour.

3.   The school roll after 5 years if increasing by 8% per year from 630 initially.

4.   The balance after 3 years when £240 is deposited with a 2.8% interest rate.

5.   The value of a work of art, valued at £23000, after 9 years increasing by 12.5% per year.

APPLYING QUESTION

        A car was purchased in 2017 for £18700.

        The value appreciated by 20% in the first year and by 8.2% each subsequent

        year. Calculate, to 3 significant figures, the value of the car in 2021.
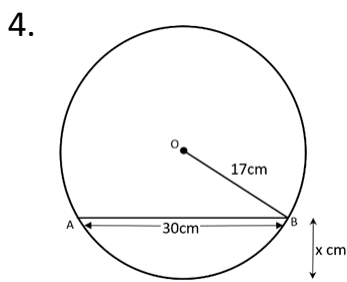
Essential Skills 24

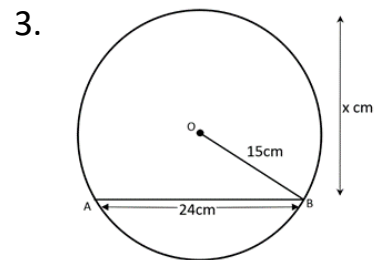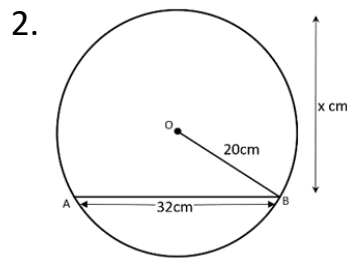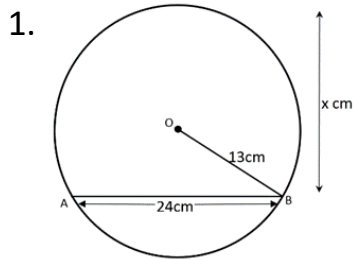The questions in this series of worksheets appear frequently.

These are the GIFTS you must take to succeed

Quadratic Equations (Factorising)   (Non-Calculator)

Solve:

1. $x^2 + 8x = 0$ 2. $2x^2 - x = 0$

3. $x^2 - 25 = 0$ 4. $4x^2 - 1 = 0$

5. $x^2 + 4x + 3 = 0$ 6. $3x^2 - x - 4 = 0$

7. $5x^2 + 8x - 4 = 0$ 8. $3x^2 - 8x - 3 = 0$

9. $10x^2 - 17x + 3 = 0$ 10. $20 + 7x - 6x^2 = 0$

APPLYING QUESTIONS

1. Solve $4x(x + 1) = 15$

2. The dimensions of a picture frame are shown:

    (a) Show that the area of glass at the centre is

$$A = 4x^2 - 42x + 108$$

    (b) If the area of glass is $54cm^2$, find the value of x.

3.
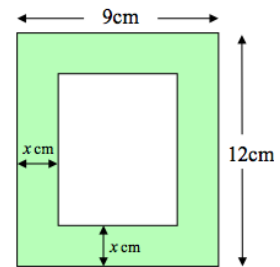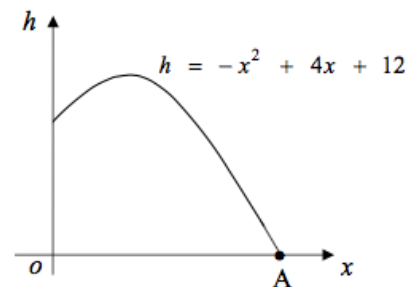
A rock is thrown form a cliff and makes the shape of a parabola.

    (a) How far will it travel before landing in the water?
         (Each unit on the x axis represents 2 metres.)

    (b) What was the maximum height it reached?
         (Each unit on the h axis represents 2 metres.)

The questions in this series of worksheets appear frequently.

These are the GIFTS you must take to succeed

Volume of a Cone and Sphere   (Calculator)

Calculate the volume of each:

1.

3 cm

2.

5 cm

2 cm

3.

13 cm

8 cm

4.

5 cm

5.

14.2 cm

6.

6 cm

4.2 cm

7.

18 cm

12 cm

8.

15 cm

18 cm

9.

31 cm

10.

25 cm

7 cm
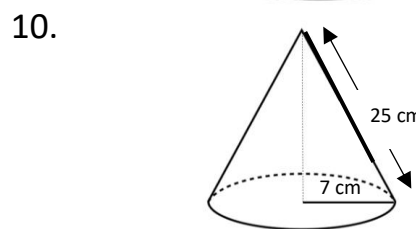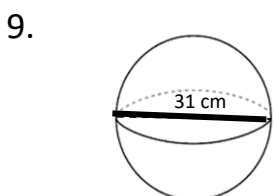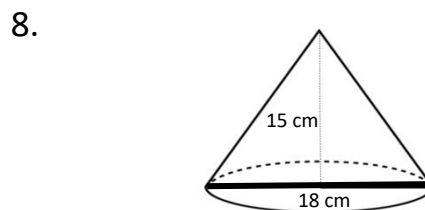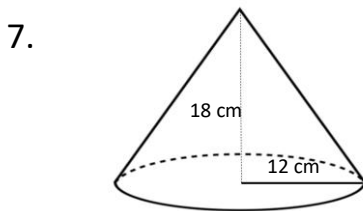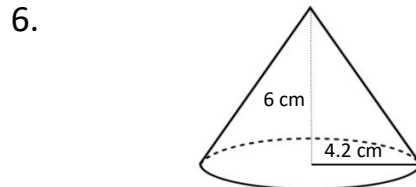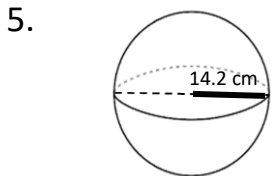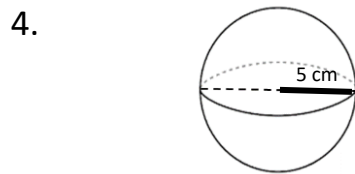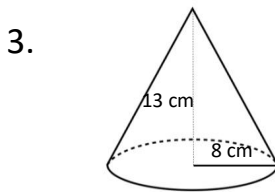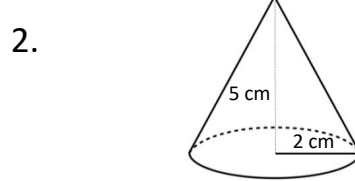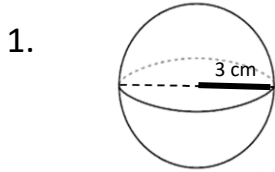
APPLYING QUESTION

The cone and the sphere shown have the same volume.

(a)   Calculate the volume of the sphere, rounding to 2 significant figures.
(b)   Hence, calculate the radius of the cone.
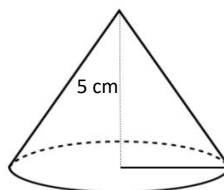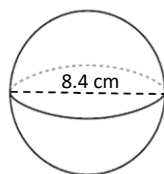
8.4 cm

5 cm

The questions in this series of worksheets appear frequently.

These are the GIFTS you must take to succeed

Area of a Triangle   (Calculator)

Calculate the area of the following:

1.
15 cm
42°
17 cm

2.
4 cm
29°
3.5 cm

3.
9 cm
51°
11 cm

4.
73°
7 cm   9 cm

5.
35 cm
31°
32 cm

6.
2.1 cm
20°
1.8 cm

7.
67°
13.5 cm
8.6 cm

8.
21 cm
76°
12 cm

9.
19 cm
10 cm
46°   70°

10.
54°   8 cm
12 cm
41°   9.8 cm
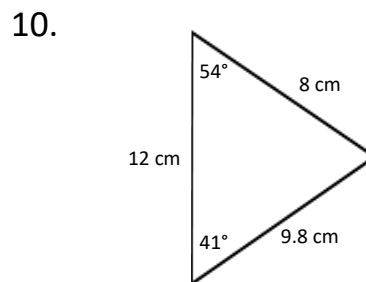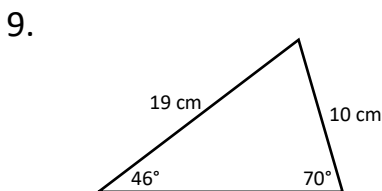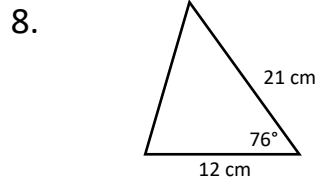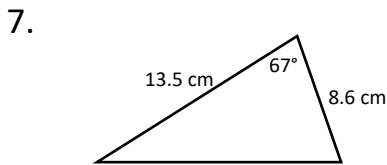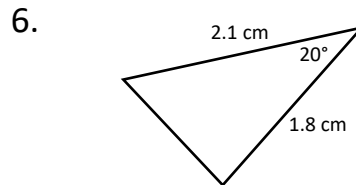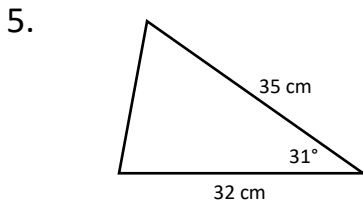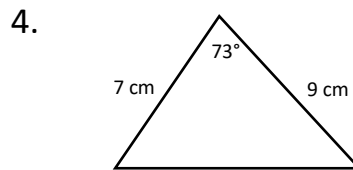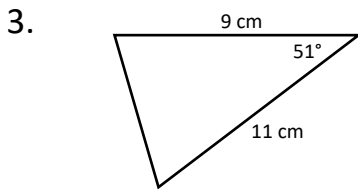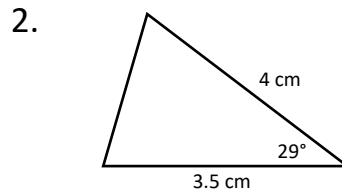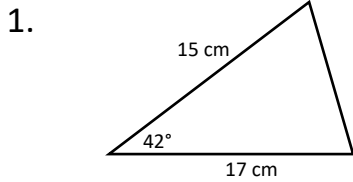
APPLYING QUESTION

Calculate the volume of the regular pentagon shown:

15 cm
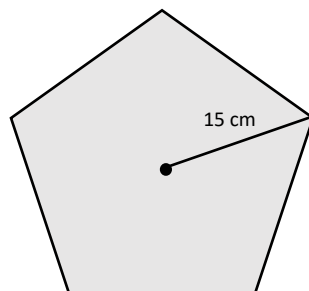
Answers

Essential Skills 1

1	$(x + 4)^2 - 19$
2	$(x - 3)^2 - 10$
3	$(x + 6)^2 - 16$
4	$(x - 9)^2 - 81$
5	$(x - 1)^2 + 6$
6	$(x + 5)^2 - 12$
7	$(x + 2)^2 - 13$
8	$(x - 3)^2 - 3$
9	$(x + 7)^2 - 74$
10	$(x - 2)^2 - 15$
AQ	(a) $(x + 3)^2 - 14$ (b) TP (-3,-14) Y intercept (0,-5) (c) B(2, 11)

Essential Skills 2

1	$4\sqrt{5}$
2	$\sqrt{3}$
3	$2\sqrt{2}$
4	$-4\sqrt{6}$
5	$3\sqrt{10}$
6	0
7	$3\sqrt{11}$
8	$5\sqrt{5}$
9	$-5\sqrt{2}$
10	0
AQ	$2\sqrt{2}$

Essential Skills 3

1	$\dfrac{5x + 11}{(x + 4)(x + 1)}$
2	$\dfrac{7x - 7}{(x - 5)(x + 2)}$
3	$\dfrac{-2x + 1}{(x + 2)(x + 7)}$
4	$\dfrac{2x - 4}{(2x - 1)(x - 1)}$
5	$\dfrac{4x - 4}{(x + 3)(3x + 1)}$
6	$\dfrac{7x + 4}{10}$
7	$\dfrac{7b + 15}{15}$
8	$\dfrac{6p + 2}{(p - 1)(3p + 5)}$
9	$\dfrac{7}{6}$
10	$\dfrac{3 + x}{x^2}$
AQ	(a) $\dfrac{x}{4}$ (b) $\dfrac{5x+6}{12}$

Essential Skills 4

1	$c = \sqrt{\dfrac{A - d}{b}}$
2	$r = \sqrt{\dfrac{V}{\pi h}}$
3	$t = \dfrac{H^2}{f}$
4	$p = \dfrac{d^2}{W}$
5	$v = \dfrac{\sqrt{g}}{ip}$
6	$a = \dfrac{2A}{b\sin C}$
7	$h = \sqrt[3]{\dfrac{w + d}{g}}$
8	$h = \dfrac{tP}{5s}$
9	$a = \dfrac{Df - 3b}{3}$
10	$t = \dfrac{T^3 + 3}{6}$
AQ	(a) $384cm^3$ (b) 4.5cm

Answers

Essential Skills 5	
1	$(a + b)(a - b)$
2	$(x + 3)(x - 3)$
3	$(2a + d)(2a - d)$
4	$(3f + 8)(3f - 8)$
5	$(p + 5)(p - 5)$
6	$(2p + 9)(2p - 9)$
7	$(g + 10h)(g - 10h)$
8	$(3c + 7d)(3c - 7d)$
9	$(x + 11)(x - 11)$
10	$2(2a + 3t)(2a - 3t)$
AQ	(a) $3(j + k)(j - k)$ (b) 14.4

Essential Skills 6	
1	$5x - 19$
2	$21w - 2st + 11$
3	$x^2 + 10x + 24$
4	$x^2 - 15x + 56$
5	$6x^2 + 5x - 4$
6	$5x^2 - 13x + 6$
7	$12x^2 - 5x - 2$
8	$x^2 + 8x + 16$
9	$4x^2 - 4x + 1$
10	$9s^2 - 24st + 16t^2$
AQ	$8y^2 + 10y + 5$

Essential Skills 7	
1	$x^3 + 5x^2 + 7x + 3$
2	$3x^3 + 11x^2 + 9x - 2$
3	$2x^3 - 5x^2 + 5x + 4$
4	$x^3 + 3x^2 - 8x - 4$
5	$x^3 - 8x^2 - 25x + 50$
6	$2x^3 - 5x^2 - 6x + 9$
7	$6x^3 + 10x^2 - 7x + 1$
8	$x^3 - 8x^2 + 13x - 6$
9	$3x^3 + 25x^2 + 4x - 32$
10	$2x^3 - 10x^2 - 7x - 4$
AQ	(a)$x^3 - 7x - 6$ (b) $x^3 - 3x + 2$ (c) $8x^3 - 36x^2 - 18x - 27$

Essential Skills 8	
1	$(a + 2)(a + 4)$
2	$(b + 5)(b + 6)$
3	$(c - 2)(c - 6)$
4	$(d - 5)(d - 8)$
5	$(e - 7)(e + 8)$
6	$(f + 6)(f - 9)$
7	$(g + 6)(g + 9)$
8	$(h - 2)(h + 15)$
9	$(j + 5)(j - 11)$
10	$3(k + 9)(k - 7)$
AQ	(a) $(x + 4)(x - 4)$ (b)$\frac{x-7}{x-4}$

Answers

Essential Skills 9	
1	$\dfrac{14}{15}$
2	$1\dfrac{29}{35}$
3	7
4	$3\dfrac{13}{14}$
5	$2\dfrac{7}{24}$
6	$10\dfrac{2}{9}$
7	$2\dfrac{2}{3}$
8	$\dfrac{11}{20}$
9	$1\dfrac{13}{18}$
10	$1\dfrac{13}{16}$
AQ	$\dfrac{23}{29}$

Essential Skills 10	
1	$\bar{x} = 18, s = 3 \cdot 35$
2	$\bar{x} = 8, s = 2 \cdot 58$
3	$\bar{x} = 3 \cdot 6, s = 1 \cdot 56$
4	$\bar{x} = 108, s = 3 \cdot 4$
5	$\bar{x} = 55, s = 5 \cdot 6$
6	$\bar{x} = 2, s = 1 \cdot 15$
7	$\bar{x} = 11, s = 2 \cdot 28$
8	$\bar{x} = 37, s = 2 \cdot 92$
9	$\bar{x} = 1301, s = 4 \cdot 43$
10	$\bar{x} = 38 \cdot 6, s = 5 \cdot 13$
AQ	$\bar{x} = 119, s = 1 \cdot 58$ (b) Competition (c) Mean up by 4, Standard deviation the same.

Essential Skills 11	
1	$x = -0 \cdot 4 \, \& - 5 \cdot 6$
2	$x = 0 \cdot 2 \, \& - 1 \cdot 5$
3	$x = 1 \cdot 0 \, \& - 0 \cdot 8$
4	$x = 1 \cdot 6 \, \& 1 \cdot 3$
5	$x = 0 \cdot 4 \, \& - 4 \cdot 4$
6	$x = -4 \cdot 8 \, \& \, 0 \cdot 8$
7	$x = 0 \cdot 7 \, \& 0 \cdot 2$
8	$x = 1 \, \& - 2 \cdot 5$
9	$x = 1 \cdot 5 \, \& \, 0 \cdot 3$
10	$x = -0 \cdot 7 \, \& \, 2 \cdot 2$
AQ	$A(-0 \cdot 581, 0) \, \& \, B(2 \cdot 58, 0)$

Essential Skills 12	
1	30° & 150°
2	30° & 330°
3	31° & 211°
4	9.6° & 170.4
5	48.2° & 311.8°
6	77.5° & 257.5°
7	203.6° & 336.4°
8	120° & 240°
9	135° & 315°
10	23.6° & 156.4°
AQ	$A(210°, -1) \, \& \, B(330°, -1)$

Answers

Essential Skills 13	
1	$(a)\ 2\ \ (b)\ -7$
2	$(a)\ 15\ \ (b)\ 3$
3	$(a)\ 57\ \ (b)\ 1$
4	$(a)\ 75\ \ (b)\ 48$
5	$(a)\ 47\ \ (b)\ 11$
6	$(a)\ 4\ \ (b)\ 14$
7	$(a)\ 1\ \ (b)\ -4$
8	$(a)\ -1\ \ (b)\ 64$
9	$(a)\ 29\ (b)\ 13$
10	$(a)\ -1\ \ (b)\ 2$
AQ	(a) 39 (b) $44-5p$ (c) t= -7 (d) x = 3

Essential Skills 14	
1	$y = 3x - 1$
2	$y = 2x + 7$
3	$y = x + 5$
4	$y = 2x - 2$
5	$y = -x + 3$
6	$y = -2x - 10$
7	$4y = -5x + 20$
8	$2y = -5x - 6$
9	$2y - 3x + 7 = 0$
10	$3y + 4x = 7$
AQ	

Essential Skills 15	
1	$x = 2, y = 3$
2	$x = 1, y = 5$
3	$x = 2, y = 2$
4	$x = 1, y = -1$
5	$x = 3, y = 5$
6	$x = 4, y = -2$
7	$x = -5, y = -2$
8	$x = 0, y = 4$
9	$x = -1, y = 4$
10	$x = 2, y = 7$
AQ	(1) (7, 6) (2) (a) $x + y = 12000$ (b) $28 \cdot 5x + 41y = 472500$ (c) 1560 standing, 10440 seated

Essential Skills 16	
1	$16 \cdot 8cm$
2	$12 \cdot 99cm,$
3	$11 \cdot 8m,$
4	$12 \cdot 2cm,$
5	$16 \cdot 9cm,$
6	$1 \cdot 0m,$
7	$17 \cdot 0cm,$
8	$68 \cdot 8cm,$
9	$3 \cdot 3cm,$
10	$101 \cdot 3mm,$
AQ	(1) $33 \cdot 5cm$

Answers

Essential Skills 17	
1	$\dfrac{x+3}{3}$
2	$\dfrac{x-3}{2}$
3	$\dfrac{x+2}{x}$
4	$\dfrac{x-7}{x+5}$
5	$\dfrac{2x+1}{x+3}$
6	$\dfrac{1}{x+5}$
7	$\dfrac{2(x+5)}{4x+1}$
8	$\dfrac{2x+1}{x-1}$
9	$\dfrac{2x+1}{3x-1}$
10	$\dfrac{x-7}{2x-5}$
AQ	(a) $(x+6)(x-9)$ (b) $\dfrac{x-9}{3x-1}$

Essential Skills 18	
1	£200
2	150g
3	£12500
4	€7500
5	400ml
6	£80
7	$99
8	£35000
9	£80
10	€6·80
AQ	£1280

Essential Skills 19	
1	x^8
2	x
3	$3x^{11}$
4	$10x^2$
5	$2x^4$
6	$\dfrac{6}{7}$
7	$\dfrac{2x^{10}}{3}$
8	$\dfrac{3}{x^3}$
9	$3x^3y^2$
10	$\dfrac{2y}{x^2}$
AQ	(a) $\dfrac{2x^2}{3y}$ (b)3

Essential Skills 20	
1	19·75cm
2	58·2cm
3	44·5cm
4	63·9cm
5	11·4cm
6	36·8°
7	11°
8	62·8°
9	42·0°
10	120·1°
AQ	26·5 metres

Answers

<table>
<tr><th colspan="2">Essential Skills 21</th></tr>
<tr><td>1</td><td>3·1m</td></tr>
<tr><td>2</td><td>7·4cm</td></tr>
<tr><td>3</td><td>88·8km</td></tr>
<tr><td>4</td><td>28·0mm</td></tr>
<tr><td>5</td><td>12·5m</td></tr>
<tr><td>6</td><td>32·0°</td></tr>
<tr><td>7</td><td>37·1°</td></tr>
<tr><td>8</td><td>31·2°</td></tr>
<tr><td>9</td><td>19·7°</td></tr>
<tr><td>10</td><td>76·1°</td></tr>
<tr><td>AQ</td><td>486·1m</td></tr>
</table>

<table>
<tr><th colspan="2">Essential Skills 22</th></tr>
<tr><td>1</td><td>18cm</td></tr>
<tr><td>2</td><td>32cm</td></tr>
<tr><td>3</td><td>24cm</td></tr>
<tr><td>4</td><td>9cm</td></tr>
<tr><td>5</td><td>32cm</td></tr>
<tr><td>6</td><td>18cm</td></tr>
<tr><td>7</td><td>120cm</td></tr>
<tr><td>8</td><td>70cm</td></tr>
<tr><td>9</td><td>42cm</td></tr>
<tr><td>10</td><td></td></tr>
<tr><td>AQ</td><td>1.4cm</td></tr>
</table>

<table>
<tr><th colspan="2">Essential Skills 23</th></tr>
<tr><td>1</td><td>£474·48 interest</td></tr>
<tr><td>2</td><td>85 bacteria</td></tr>
<tr><td>3</td><td>925 pupils</td></tr>
<tr><td>4</td><td>£260·73</td></tr>
<tr><td>5</td><td>£66389·67</td></tr>
<tr><td>6</td><td></td></tr>
<tr><td>7</td><td></td></tr>
<tr><td>8</td><td></td></tr>
<tr><td>9</td><td></td></tr>
<tr><td>10</td><td></td></tr>
<tr><td>AQ</td><td>£28400</td></tr>
</table>

<table>
<tr><th colspan="2">Essential Skills 24</th></tr>
<tr><td>1</td><td>$x = 0, x = -8$</td></tr>
<tr><td>2</td><td>$x = 0, x = \frac{1}{2}$</td></tr>
<tr><td>3</td><td>$x = -5, x = 5$</td></tr>
<tr><td>4</td><td>$x = -\frac{1}{2}, x = \frac{1}{2}$</td></tr>
<tr><td>5</td><td>$x = -3, x = -1$</td></tr>
<tr><td>6</td><td>$x = \frac{4}{3}, x = -1$</td></tr>
<tr><td>7</td><td>$x = \frac{2}{5}, x = -2$</td></tr>
<tr><td>8</td><td>$x = -\frac{1}{3}, x = 3$</td></tr>
<tr><td>9</td><td>$x = \frac{1}{5}, x = \frac{3}{2}$</td></tr>
<tr><td>10</td><td>$x = -\frac{4}{3}, x = \frac{5}{2}$</td></tr>
<tr><td>AQ</td><td>(1) $x = -\frac{5}{2}, x = \frac{3}{2}$
(2) (a) $l = 12 - 2x, b = 9 - 2x$, proof (b) $x = \frac{3}{2}$
(3) (a) 12m (b) 32m</td></tr>
</table>

Answers

Essential Skills 25	
1	113.1 cm³
2	20.9 cm³
3	871.3 cm³
4	523.6 cm³
5	11993.7 cm³
6	110.8 cm³
7	2714.3 cm³
8	1272.3 cm³
9	15598.5 cm³
10	1231.5 cm³
AQ	(a) 310 cm³ (b) 7.7cm

Essential Skills 26	
1	85.3 cm²
2	3.4 cm²
3	38.5 cm²
4	30.1 cm²
5	288.4 cm²
6	0.6 cm²
7	53.4 cm²
8	122.3 cm²
9	85.4 cm²
10	38.8 cm²
AQ	535.0 cm²

Adapted from G Rennie