Armadale Academy

National 5 Maths

Procedures Booklet

The aim of this booklet is to give you a guide on how to answer common exam questions.

You should use this to help you learn the approach to solving questions which you need to target.

You should be aiming to use this booklet less the closer you get to the exam.

Contents

1. Exam Language

Write down – use the information given and write down the answer.

State - Read the question and write down the answer.

Calculate – work out a numerical answer.

Evaluate – complete a calculation to give a numerical answer.

Find – work out, complete a calculation.

Determine – work out.

Solve – find the numerical value for the letter given.

Simplify – reduce the numbers, reduce the number of terms.

Express – re-write giving your answer in the desired form.

Change – re-write giving your answer in the desired form.

Algebraically – using algebra skills.

Given that – use this information to help in your solution.

Respectively – information was given in the same order.

Hence – use what you have just worked out to help answer this part of the question.

Hence or otherwise – use what you have just worked out or use another valid strategy

BOLD – clear instructions on how to answer the question.

Show that – answer is given, you need to show working on how that answer is obtained.

DO NOT USE – a clear instruction on a skill you should not use to answer this question.

Show all your working – marks are awarded at specific places, not showing working will risk no marks.

Give a reason for your answer – comparison of 2 values, use = ≠ < ≤ > ≥.

Give your answer … - instructions on how to answer the question.

Diagrams – contain useful information as a visual aid.

2. Factorising – Procedure

Often asked as factorise **fully** which suggests that **more** than 1 step is required.

Step 1: Look for a common factor.

Step 2: Identify if there are 2 or 3 terms to be factorised.

Step 3: 2 terms – complete by difference of two squares. (DOTs)

3 terms – complete by trinomials.

Factorising – Example

2. Factorise fully

$$5x^2 - 45.$$

Step 1: **5** is a common factor therefore

$$5x^2 - 45 = 5(x^2 - 9)$$

Step 2: There are **2** terms left to be factorised in the bracket:

$$x^2 - 9$$

Step 3: Use DOTs: $x^2 - 9 = (x - 3)(x + 3)$

$$5x^2 - 45 = 5(x - 3)(x + 3)$$

3. Percentages – Procedure

Often asked as an appreciation or depreciation over a number of years.

Step 1: Identify if the percentage is being added or subtracted.

Step 2: Calculate the multiplier.

Step 3: Identify the number of years required and raise the multiplier to this power.

Step 4: Multiply the original value by your multiplier.

Step 5: Round your answer to the given specification.

Percentages – Example

1. There are 964 pupils on the roll of Aberleven High School.
   It is forecast that the roll will decrease by 15% per year.
   What will be the expected roll after 3 years?
   Give your answer to the nearest ten.

Step 1: decrease therefore subtraction

Step 2: 100% - 15% = 85% = 0.85

Step 3: **3** years so power of 3

$$0.85^3$$

Step 4: $964 \times 0.85^3 = 592.0165$

Step 5: Round to the nearest **ten**

590 pupils.

4. Equation of a Straight Line – Procedure

Depending on what information you have been given, you may find you can start this procedure at a later step.

Step 1: Identify two co-ordinate points which lie on the line and label as (x_1, y_1) and (x_2, y_2).

Step 2: Calculate the gradient of the line using the formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Step 3: Identify the y-intercept or a point on the line.

Step 4: Substitute into the relevant straight line formula

$y = mx + c$ or $y - b = m(x - a)$

and re-arrange if required.

Step 5: Check the variables and change x and y to the relevant letters if required.

Equation of a Straight Line – Example

8. Find the equation of the line joining the points (–2, 5) and (3, 15). Give the equation in its simplest form.

Step 1: $(\mathbf{-2}, \mathbf{5})$ $(\mathbf{3}, \mathbf{15})$

x_1 y_1 x_2 y_2

Step 2:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{15 - 5}{3 - -2}$$

$$= \frac{10}{5}$$

$$= 2$$

Step 3: (3 , 15)

a b

Step 4:

$$y - b = m(x - a)$$

$$y - 15 = 2(x - 3)$$

$$y - 15 = 2x - 6$$

$$y = 2x + 9$$

Step 5: Not required for this question

5. Volume – Procedure

You may be asked to calculate the volume or to use the volume to calculate a dimension.

Step 1: Identify the 3D shape and select the correct formula from the formula sheet.

Step 2: Substitute values into the formula.

Step 3: Re-arrange the formula if required for finding a dimension**.**

Step 4: Evaluate the formula.

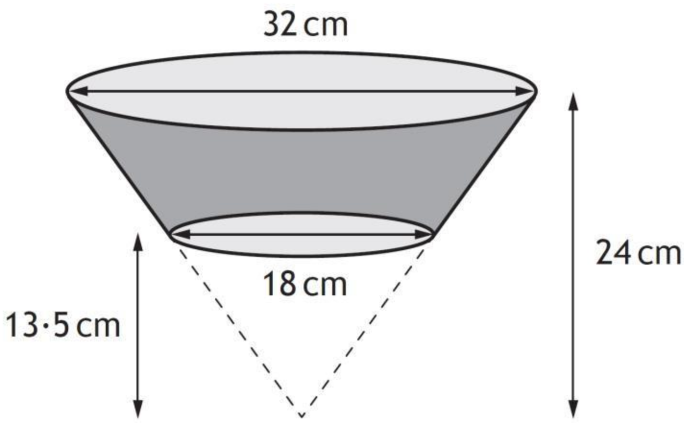
Step 5: Complete the question. If rounding is requested, show unrounded answer first and round appropriately. Include units in your answer.

Volume - Example

7. A carton is in the shape of a large cone with a small cone removed.

The large cone has diameter of 32 cm and height 24 cm.

The small cone has diameter of 18 cm and height 13·5 cm.



Calculate the volume of the carton.

Give your answer correct to 2 significant figures.

Step 1: Cone : $V = \frac{1}{3}\pi r^2 h$

Step 2: $V_{large} = \frac{1}{3} \times \pi \times 16^2 \times 24$ $V_{small} = \frac{1}{3} \times \pi \times 9^2 \times 13.5$

Step 3: Not required

Step 4: $V_{large} = 6433.98$ $V_{small} = 1145.11$

Step 5: Volume of carton = 6433.98 – 1145.11

= 5288.87

= 5300 cm^3

6. Arcs and Sectors - Procedure

You may be asked to find an arc length or sector area or use these to find a missing dimension or angle.

Step 1: Identify if the question is asking for arc length or sector area and write down the relevant formula – **you need to know these**.

$$Sector\ Area = \frac{\theta}{360} \times \pi r^2 \qquad Arc\ Length = \frac{\theta}{360} \times \pi D$$

Step 2: Substitute values into the formula.

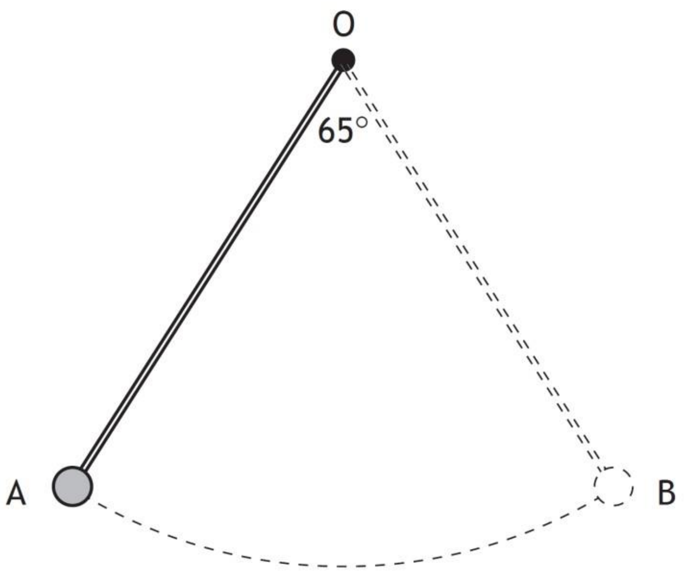
Step 3: Re-arrange if required to find a dimension or angle.

Step 4: Evaluate the formula.

Step 5: Complete the question. If rounding is requested, show unrounded answer first and round appropriately.

Arcs and Sectors – Example

10. The pendulum of a clock swings along an arc of a circle, centre O.



The pendulum swings through an angle of 65°, travelling from A to B.

The length of the arc AB is 28·4 centimetres.

Calculate the length of the pendulum.

Step 1: Arc length given:

Arc length $= \frac{\theta}{360} \times \pi d$

Step 2: $28.4 = \frac{65}{360} \times \pi d$

Step 3: $d = 28.4 \div \frac{65}{360} \div \pi$

$d = 28.4 \times \frac{360}{65} \div \pi$

Step 4: $d = 50.07$

Step 5: Pendulum length $= 50.07 \div 2 = 25\ cm$

7. Similarity - Procedure

Often shown as similar triangles.

Step 1: Identify the similar shapes and calculate the scale factor.

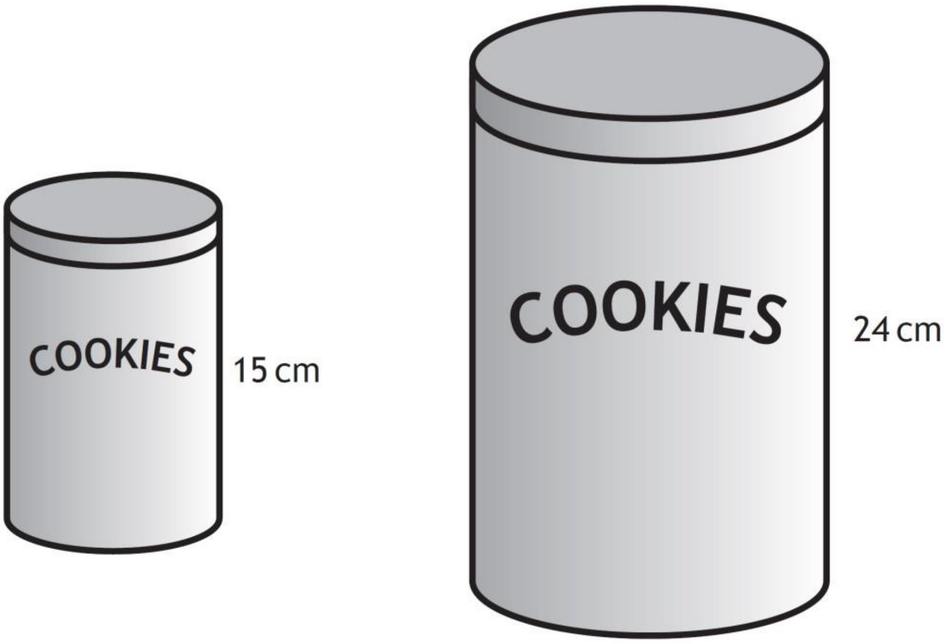
$$scale\ factor = \frac{new}{old}$$

Step 2: Identify if the value to be calculated is length, area or volume. Square or cube the scale factor if required.

Step 3: Multiply the original by the scale factor.

Similarity – Example

5. A supermarket sells cylindrical cookie jars which are mathematically similar.



The smaller jar has a height of 15 centimetres and a volume of 750 cubic centimetres.

The larger jar has a height of 24 centimetres.

Calculate the volume of the larger jar.

Step 1: $scale\ factor = \frac{new}{old} = \frac{24}{15} = 1.6$

Step 2: Volume so cube: 1.6^3

Step 3: Volume $= 750 \times 1.6^3 = 3072\ cm^3$

8. Converse of Pythagoras – Procedure

This is presented as problem solving where you are asked if something is right angled.

Step 1: Calculate c^2

Step 2: Calculate $a^2 + b^2$

Step 3: Compare answers and make the relevant statement including answering the question:

By the converse of Pythagoras, this triangle is/isn't right angled since c^2 =/≠ $a^2 + b^2$. Therefore ……

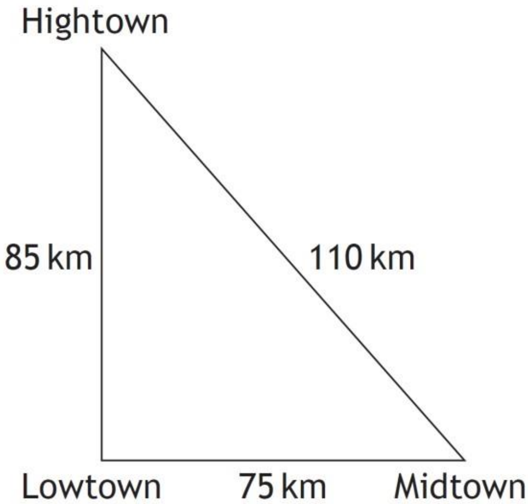
Converse of Pythagoras – Example

6. The diagram below shows the position of three towns.

   Lowtown is due west of Midtown.

   The distance from

   - Lowtown to Midtown is 75 kilometres.
   - Midtown to Hightown is 110 kilometres.
   - Hightown to Lowtown is 85 kilometres.



   Is Hightown directly north of Lowtown?

   Justify your answer.

Step 1: $c^2 = 110^2 = 12100$

Step 2: $a^2 + b^2 = 75^2 + 85^2 = 12850$

Step 3: By the converse of Pythagoras, this triangle isn't right angled since $12100 \neq 12850$. Therefore, Hightown is not directly North of Lowtown.

9. Standard Deviation – Procedure

It will be clear when asked to calculate standard deviation and will usually have a part b where you will be asked to compare it with another.

Step 1: Set up your table with the headings $x, x - \bar{x}, (x - \bar{x})^2$

Step 2: List your data down the x column.

Step 3: Calculate the mean.

Step 4: Subtract the mean from each piece of data and write the answers in the 2nd column.

Step 5: Square the numbers in the second column and write the answers in the 3rd column.

Step 6: Total the 3rd column.

Step 7: Substitute values in to the formula:

$$s = \sqrt{\frac{\Sigma(x - \bar{x})^2}{n-1}}$$

Step 8: Evaluate the formula.

Standard Deviation – Example

6. Jack called his internet provider on six occasions to report connection problems.

On each occasion he noted the length of time he had to wait before speaking to an adviser.

The times (in minutes) were as follows:

13 16 10 22 5 12

(a) Calculate the mean and standard deviation of these times.

Step 2: **Step 4:** **Step 5:**

Step 1: →

x	$x - \bar{x}$	$(x - \bar{x})^2$
13	0	0
16	3	9
10	-3	9
22	9	81
5	-8	64
12	-1	1
	Total	164

← **Step 6:**

Step 3: $\bar{x} = \dfrac{13 + 16 + 10 + 22 + 5 + 12}{6} = 13$

Step 7: $s = \sqrt{\dfrac{\Sigma(x - \bar{x})^2}{n-1}}$ **=** $s = \sqrt{\dfrac{164}{5}}$

Step 8: $s = 5.7271 \ldots = \mathbf{5.73}\ (to\ 2.d.p)$

10. Simultaneous Equations – Procedure

Identified when two equations are given where there are the same 2 letters in both equations. Usually a part c to a question where parts a and b are to write the equations from given information. Always find 2 values for these questions even if you think that one is incorrect.

Step 1: Write your equations with the letters in the same order with one equation above the other.

Step 2: Compare the number in front of the 2nd variable and if different multiply one or both equations to make this the same.

Step 3: Compare the signs in both equations to decide whether to add or subtract the equations.

Signs are the **s**ame – **s**ubstract

Signs are **d**ifferent – a**dd**

Step 4: Complete the addition or subtraction and solve the equation for the remaining letter.

Step 5: Substitute this value into an original equation and solve the 2nd letter.

Step 6: Answer the question.

Simultaneous Equations – Example

11. Solve algebraically the system of equations

$$3x + 2y = 17$$
$$2x + 5y = 4.$$

Step 1: Equations are already written in this form.

Step 2: $3x + 2y = 17 \longrightarrow \times 5 \longrightarrow 15x + 10y = 85$

$2x + 5y = 4 \longrightarrow \times 2 \longrightarrow 4x + 10y = 8$

Step 3: Both are +, so same therefore subtract.

Step 4:

$$15x + 10y = 85$$
$$-\quad 4x + 10y = 8$$
$$11x \qquad = 77$$
$$x = 11$$

Step 5:

$$3(11) + 2y = 17$$
$$33 + 2y = 17$$
$$2y = -16$$
$$y = -8$$

Step 6: Not required

11. Solving a Quadratic by Factorising – Procedure

May be asked as solve, find the roots or find where the graph cuts the x-axis. Often appears in a problem solving aspect.

Step 1: Set the trinomial equal to zero. $y = 0$ for roots.

Step 2: Factorise the trinomial.

Step 3: Set each bracket equal to 0.

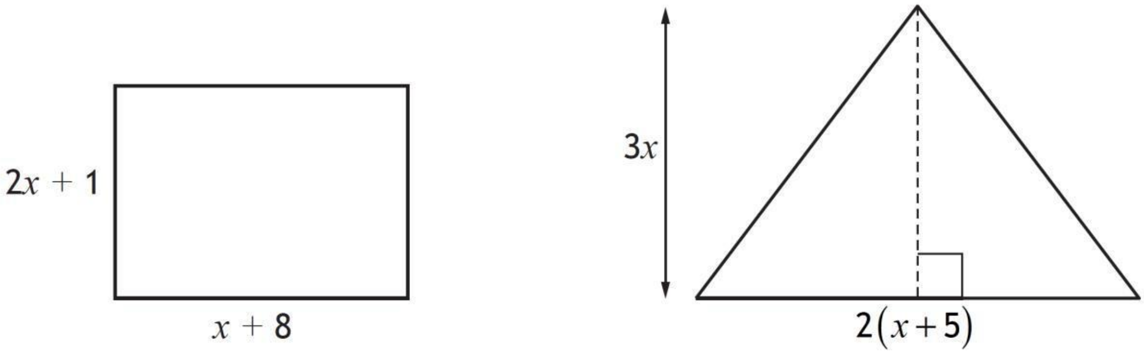
Step 4: Solve both equations for x.

Step 5: Answer question if applicable.

Solving a Quadratic by Factorising – Example

12. The diagrams below show a rectangle and a triangle.

All measurements are in centimetres.



(a) Find an expression for the area of the **rectangle**.

(b) Given that the area of the rectangle is equal to the area of the triangle, show that $x^2 - 2x - 8 = 0$.

(c) Hence find, **algebraically**, the length and breadth of the rectangle.

Part (c) is solving the quadratic.

Step 1: Already equal to zero.

Step 2: $x^2 - 2x - 8 = 0$

$(x - 4)(x + 2) = 0$

Step 3: either $(x - 4) = 0$ or $(x + 2) = 0$

Step 4: $x = 4$ or $x = -2$

Step 5: x must be 4 as a length cannot be negative therefore

Length = 4 + 8 = 12 and breadth = 2(4) + 1 = 9

12. Quadratic Formula – Procedure

You use the quadratic formula to solve a trinomial when you cannot factorise. You are usually asked to solve to a given number of decimal places or significant figures.

Step 1: Identify a, b and c.

Step 2: Copy the quadratic formula from the formula sheet and substitute in values.

Step 3: Evaluate the discriminant.

Step 4: Split the formula up into a + and – to form 2 solutions.

Step 5: Complete both calculations and write down unrounded answers.

Step 6: Round both answers as required.

Quadratic Formula – Example

4. Solve the equation $2x^2 + 5x - 4 = 0$.
Give your answers correct to one decimal place.

Step 1: a = 2 b = 5 c = -5

Step 2: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{5^2 - 4 \times 2 \times -5}}{2 \times 2}$

Step 3: $x = \frac{-5 \pm \sqrt{65}}{4}$

Step 4: $x = \frac{-5 + \sqrt{65}}{4}$ and $x = \frac{-5 - \sqrt{65}}{4}$

Step 5: $x = 0.765564$ and $x = -3.265564$

Step 6: $x = 0.8$ and $x = -3.3$

13. Discriminant – Procedure

Usually asked as determine the nature of the roots.

Step 1: Identify a, b and c.

Step 2: Substitute into the discriminant. $b^2 - 4ac$

Step 3: Evaluate the discriminant.

Step 4: Compare to zero and make relevant statement:

Since $b^2 - 4ac < 0$ there are no real roots

Since $b^2 - 4ac = 0$ the roots are real and equal

Since $b^2 - 4ac > 0$ the roots are real and distinct

Discriminant – Example

6. Determine the nature of the roots of the function $f(x) = 7x^2 + 5x - 1$.

Step 1: a = 7 b = 5 c = -1

Step 2: $b^2 - 4ac = 5^2 - 4 \times 7 \times -1$

Step 3: $= 53$

Step 4: $53 > 0$,

since $b^2 - 4ac > 0$ the roots are real and distinct.

14. Sine and Cosine Rules – Procedure

Usually given as problem solving and may require the use of SOH CAH TOA to answer the question.

Step 1: Identify which rule you are going to use.

If in doubt, try sine rule, and if it does not work use cosine.

Step 2: Re-label your triangle A, B and C. Be careful with where you place A if using the cosine rule.

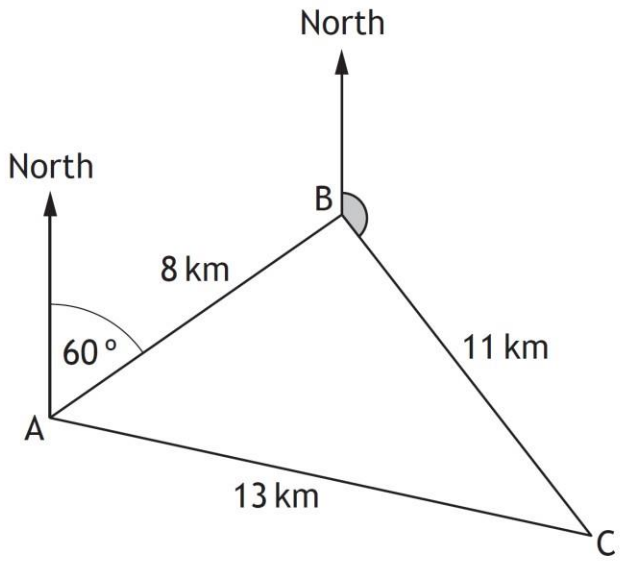
Step 3: Substitute values into the formula.

Step 4: Re-arrange if required.

Step 5: Evaluate.

Sine and Cosine Rules – Example

10. In a race, boats sail round three buoys represented by A, B, and C in the diagram below.



B is 8 kilometres from A on a bearing of 060°.

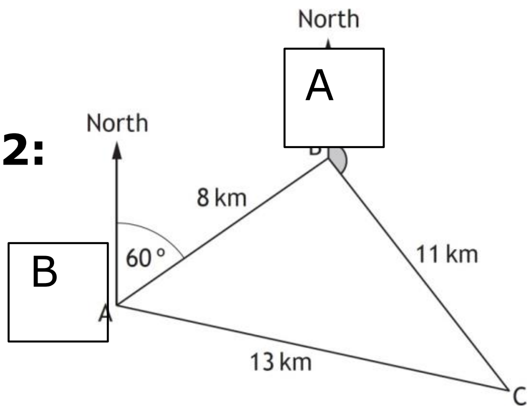
C is 11 kilometres from B.

A is 13 kilometres from C.

(a) Calculate the size of angle ABC.

Step 1: Asked to find an angle, and no other angles given, therefore sine rule will not work. Must use cosine rule.

Step 2:



Step 3: $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$$\cos A = \frac{11^2 + 8^2 - 13^2}{2 \times 11 \times 8}$$

Step 4: Not required

Step 5: $\cos A = 0.040404$

$A = \cos^{-1}(0.040404)$

$A = 87.7°$ angle ABC = 87.7°

15. Trig Equations - Procedure

Often given in a complex form, where re-arranging is required. Can be asked in a problem solving context.

Step 1: Re-arrange the equation so that you have the trig function by itself.

Step 2: Calculate the related angle.

This step may require some re-arranging or ignoring a negative.

Step 3: Use the CAST diagram to identify which quadrants require to be calculated.

Step 3: Calculate the two values of x.

Trig Equations – Example

12. Solve the equation $11\cos x° - 2 = 3$, for $0 \leq x \leq 360$.

Step 1: $11\cos x = 5$

$\cos x = \frac{5}{11}$

Step 2: $x = \cos^{-1}\left(\frac{5}{11}\right)$

$x = 62.96°$

Step 2:

S	A✓
T	C✓

cos is positive.

Step 3: Quadrant 1: x = 62.96°

Quadrant 4: x = 360 – 62.96 = 297.04°