Armadale Academy

Higher Maths

Procedures

Exam Questions

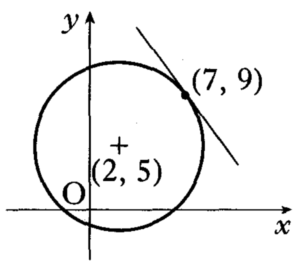
Contents

Straight Line

<table>
<tr>
<td>2008 P1</td>
<td>

5. The diagram shows a circle, centre (2, 5) and a tangent drawn at the point (7, 9).

What is the equation of this tangent?



A $y - 9 = -\frac{5}{4}(x - 7)$

B $y + 9 = -\frac{4}{5}(x + 7)$

C $y - 7 = \frac{4}{5}(x - 9)$

D $y + 9 = \frac{5}{4}(x + 7)$

</td>
<td>2</td>
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<tr>
<td>Ans</td>
<td>A</td>
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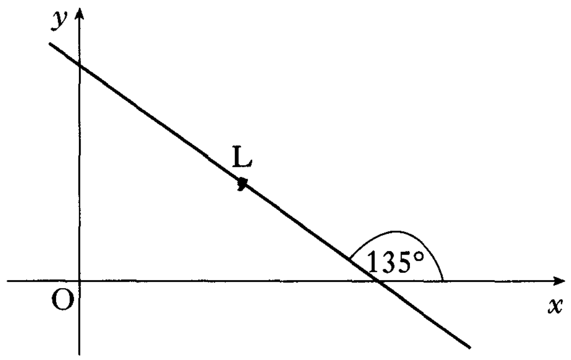
2008 P1

7. The diagram shows a line L; the angle between L and the positive direction of the x-axis is 135°, as shown. 2



What is the gradient of line L?

A $-\frac{1}{2}$

B $-\frac{\sqrt{3}}{2}$

C -1

D $\frac{1}{2}$

Ans C

2008 P2

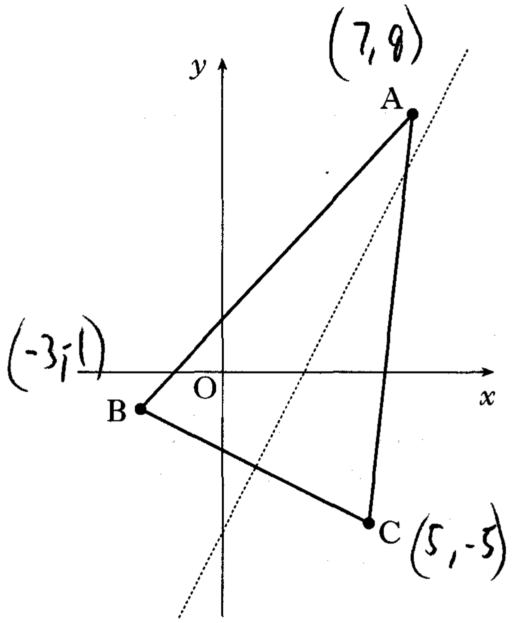
1. The vertices of triangle ABC are A(7, 9), B(–3, –1) and C(5, –5) as shown in the diagram.

The broken line represents the perpendicular bisector of BC.

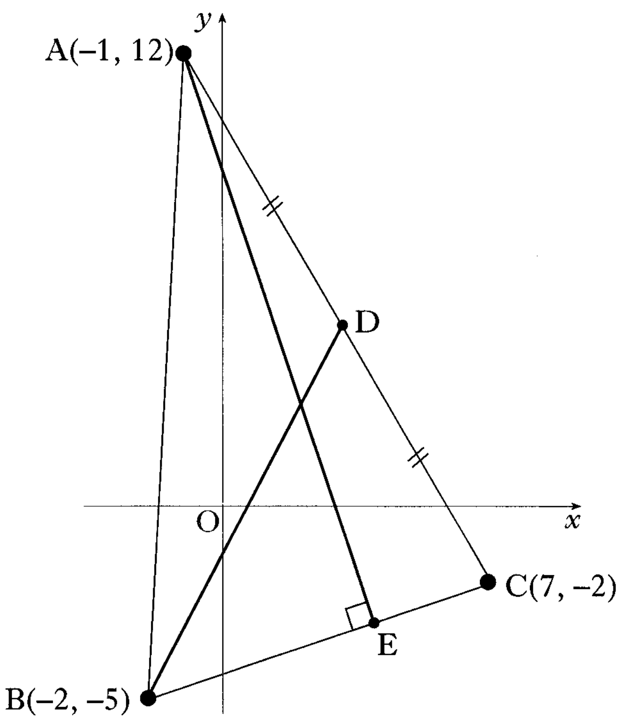
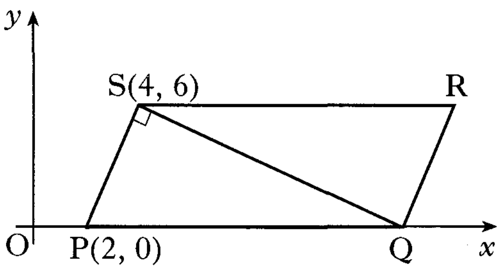
(*a*) Show that the equation of the perpendicular bisector of BC is $y = 2x - 5$. 4

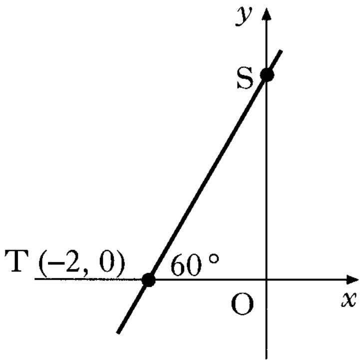
(*b*) Find the equation of the median from C. 3

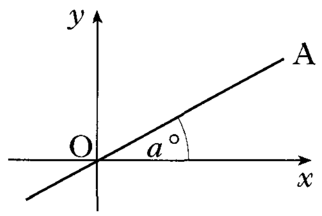
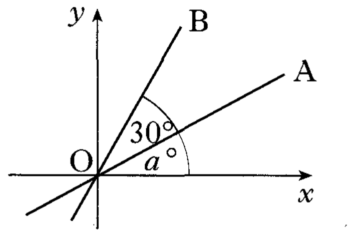
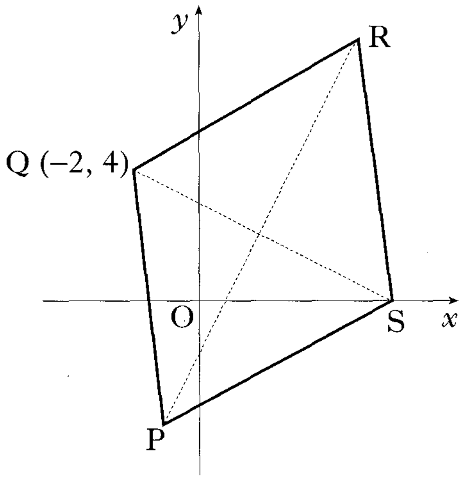
(*c*) Find the coordinates of the point of intersection of the perpendicular bisector of BC and the median from C. 3

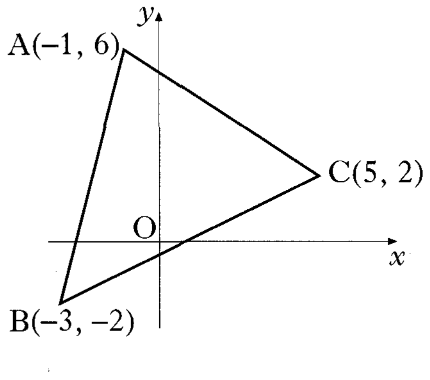
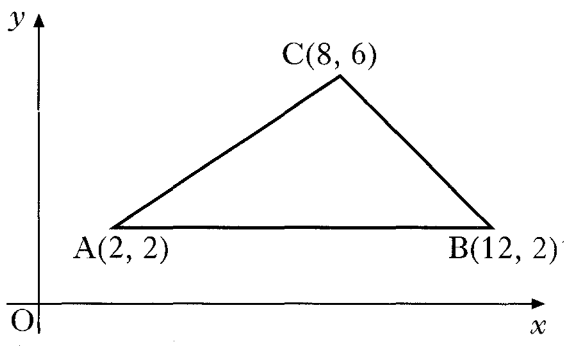


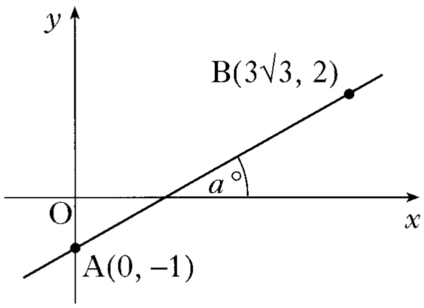
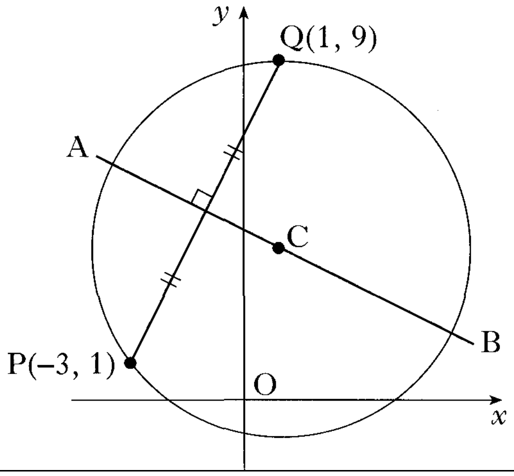
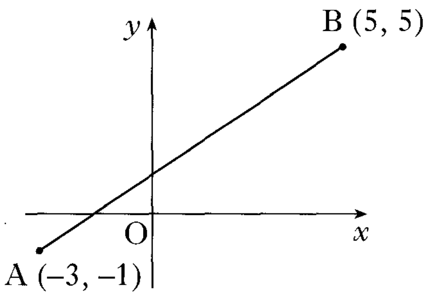
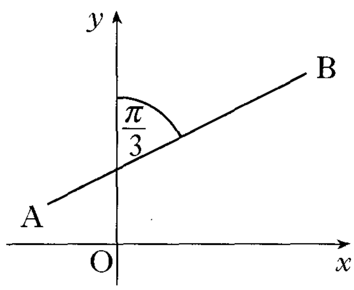
Ans *(a) Proof* *(b) y = -3x+10* *(c) (3,1)*

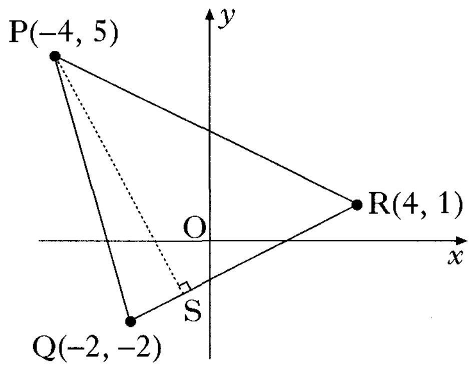
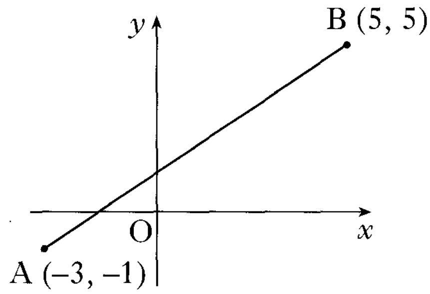
2007 P1	**1.** Find the equation of the line through the point (–1, 4) which is parallel to the line with equation $3x - y + 2 = 0$.	3
Ans	$y = 3x + 7$	
2006 P1	**1.** Triangle ABC has vertices A(–1, 12), B(–2, –5) and C(7, –2). (*a*) Find the equation of the median BD. (*b*) Find the equation of the altitude AE. (*c*) Find the coordinates of the point of intersection of BD and AE.  	 3 3 3
Ans	(*a*) $y - 5 = 2(x - 3)$ or $y + 5 = 2(x - (-2))$ etc (*b*) $y - 12 = -3(x - (-1))$ (*c*) (2, 3)	
2006 P2	**1.** PQRS is a parallelogram. P is the point (2, 0), S is (4, 6) and Q lies on the x-axis, as shown. The diagonal QS is perpendicular to the side PS.  (*a*) Show that the equation of QS is $x + 3y = 22$. (*b*) Hence find the coordinates of Q and R.	 4 2

Ans	(*a*) proof $m_{PS} = 3$ $m_{QS} = -\frac{1}{3}$ $y - 6 = -\frac{1}{3}(x - 4)$ (*b*) Q = (22, 0) R = (24, 6)	
2005 P1	**1.** Find the equation of the line ST, where T is the point (–2, 0) and angle STO is 60°. 	3
Ans	$y - 0 = \sqrt{3}(x - (-2))$	
2004 P1	**1.** The point A has coordinates (7, 4). The straight lines with equations $x + 3y + 1 = 0$ and $2x + 5y = 0$ intersect at B. (*a*) Find the gradient of AB. (*b*) Hence show that AB is perpendicular to only one of these two lines.	3 5
Ans	(*a*) $m_{AB} = 3$ (*b*) $m_{AB} = 3 \Rightarrow m_{perp} = -\frac{1}{3}$ $y = -\frac{1}{3}x - \frac{1}{3}$ $m_{l_1} = -\frac{1}{3}$ $m_{l_2} = -\frac{2}{5}$ so only the 1st line is perpendicular to AB.	

2004 P2	**1.** (*a*) The diagram shows line OA with equation $x - 2y = 0$. The angle between OA and the x-axis is $a°$. Find the value of a.  (*b*) The second diagram shows lines OA and OB. The angle between these two lines is 30°. Calculate the gradient of line OB correct to 1 decimal place. 	3 1
Ans	(*a*) $a = 26{\cdot}6°$ (*b*) 1·5	
2003 P1	**1.** Find the equation of the line which passes through the point (–1, 3) and is perpendicular to the line with equation $4x + y - 1 = 0$.	3
Ans	$x - 4y + 13 = 0$	
2002W P1	**1.** (*a*) Find the equation of the straight line through the points A(–1, 5) and B(3, 1). (*b*) Find the size of the angle which AB makes with the positive direction of the x-axis.	2 2
Ans	(*a*) $y + x = 4$ (*b*) 135°	
2002W P2	**1.** The diagram shows a rhombus PQRS with its diagonals PR and QS. PR has equation $y = 2x - 2$. Q has coordinates (–2, 4). (*a*) (i) Find the equation of the diagonal QS. (ii) Find the coordinates of T, the point of intersection of PR and QS. (*b*) R is the point (5, 8). Write down the coordinates of P. 	6 2
Ans	(*a*) $2y + x = 6$, T(2, 2) (*b*) P(–1, –4)	

2002 P2	**1.** Triangle ABC has vertices A(–1, 6), B(–3, –2) and C(5, 2). Find (*a*) the equation of the line p, the median from C of triangle ABC. (*b*) the equation of the line q, the perpendicular bisector of BC. (*c*) the coordinates of the point of intersection of the lines p and q.  	 3 4 1
Ans	(*a*) $y = 2$ (*b*) $y = -2x + 2$ (*c*) (0, 2)	
2001 P1	**1.** Find the equation of the straight line which is parallel to the line with equation $2x + 3y = 5$ and which passes through the point (2, –1).	3
Ans	$2x + 3y = 1$	
2001 P2	**7.** Triangle ABC has vertices A(2, 2), B(12, 2) and C(8, 6). (*a*) Write down the equation of l_1, the perpendicular bisector of AB. (*b*) Find the equation of l_2, the perpendicular bisector of AC. (*c*) Find the point of intersection of lines l_1 and l_2. (*d*) Hence find the equation of the circle passing through A, B and C.  	 1 4 1 2
Ans	(*a*) $x = 7$ (*b*) $3x + 2y = 23$ (*c*) (7, 1) (*d*) $(x - 7)^2 + (y - 1)^2 = 26$	

2000 P1	**3.** Find the size of the angle $a°$ that the line joining the points A(0, –1) and B($3\sqrt{3}$, 2) makes with the positive direction of the x-axis. 	3
Ans	30 degrees	
2000 P2	**2.** (a) Find the equation of AB, the perpendicular bisector of the line joining the points P(–3, 1) and Q(1, 9). 	4
Ans	(a) $x + 2y = 9$	
Specimen 2 P1	**2.** A and B are the points (–3, –1) and (5, 5). Find the equation of the perpendicular bisector of AB. 	4
Ans	$m_{AB} = \frac{3}{4} \Rightarrow m_{perp} = -\frac{4}{3}$ midpoint $= (1,2)$ $y - 2 = -\frac{4}{3}(x - 1)$	
Specimen 2 P1	**4.** The line AB makes an angle of $\frac{\pi}{3}$ radians with the y-axis, as shown in the diagram. Find the exact value of the gradient of AB. 	2
Ans	angle between line and x-axis $= \frac{\pi}{2} - \frac{\pi}{3}$ gradient $= \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$	

Specimen 1 P1	**1.** P(−4, 5), Q(−2, −2) and R(4, 1) are the vertices of triangle PQR as shown in the diagram. Find the equation of PS, the altitude from P. 	3
Ans	$y = -2x - 3$	
Specimen 1 P2	**2.** A and B are the points (−3, −1) and (5, 5). Find the equation of the perpendicular bisector of AB. 	3
Ans	$y = 2x - 5$	

Differentiation

2008 P1

21. A function f is defined on the set of real numbers by $f(x) = x^3 - 3x + 2$.

(*a*) Find the coordinates of the stationary points on the curve $y = f(x)$ and determine their nature. 6

(*b*) (i) Show that $(x - 1)$ is a factor of $x^3 - 3x + 2$.

(ii) Hence or otherwise factorise $x^3 - 3x + 2$ fully. 5

(*c*) State the coordinates of the points where the curve with equation $y = f(x)$ meets both the axes and hence sketch the curve. 4

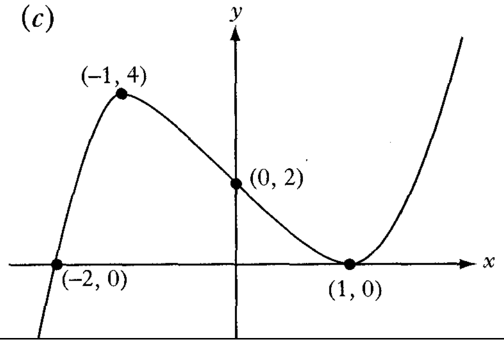
Ans

(*a*) $(-1, 4)$ maximum
$(1, 0)$ minimum

(*b*) (i) $x = 1,\ f(x) = 0$ so $(x - 1)$ is a factor

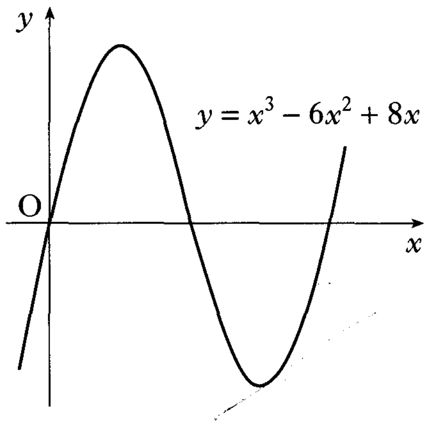
(ii) $(x - 1)(x - 1)(x + 2)$

(*c*)



2008 P1

22. The diagram shows a sketch of the curve with equation $y = x^3 - 6x^2 + 8x$.



(*a*) Find the coordinates of the points on the curve where the gradient of the tangent is -1. 5

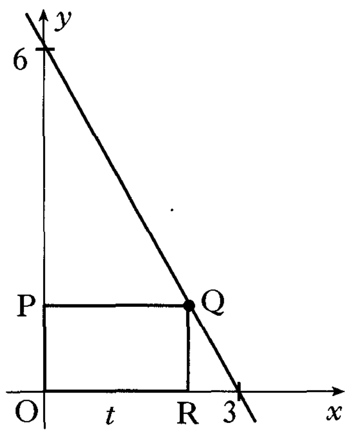
(*b*) The line $y = 4 - x$ is a tangent to this curve at a point A. Find the coordinates of A. 2

Ans (a) (1,3), (3,-3) (b) (1,3)

2008 P2

6. In the diagram, Q lies on the line joining (0, 6) and (3, 0).

OPQR is a rectangle, where P and R lie on the axes and OR = t.



(*a*) Show that QR = $6 - 2t$. 3

(*b*) Find the coordinates of Q for which the rectangle has a maximum area. 6

Ans (a) proof (b) (1.5,3)

2007 P1

9. A function f is defined by the formula $f(x) = 3x - x^3$.

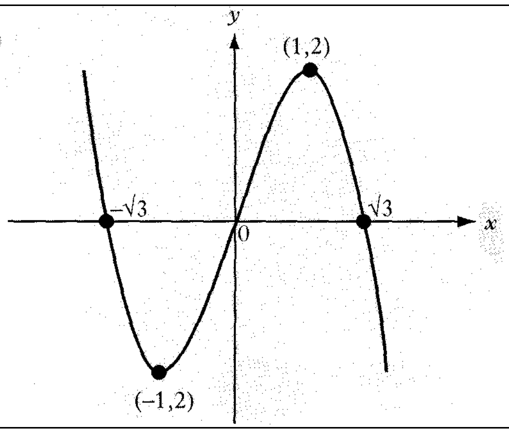
(*a*) Find the exact values where the graph of $y = f(x)$ meets the x- and y-axes. 2

(*b*) Find the coordinates of the stationary points of the function and determine their nature. 7

(*c*) Sketch the graph of $y = f(x)$. 1

Ans

(*c*)



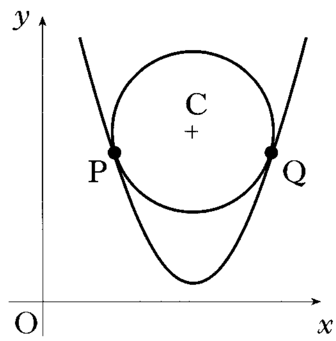
(*a*) $(-\sqrt{3}, 0)$, $(0,0)$, $(\sqrt{3},0)$

(*b*) $(1,2)$: maximum

$(-1,-2)$: minimum

2007 P2

5. A circle centre C is situated so that it touches the parabola with equation $y = \frac{1}{2}x^2 - 8x + 34$ at P and Q.



(*a*) The gradient of the tangent to the parabola at Q is 4. Find the coordinates of Q. 5

(*b*) Find the coordinates of P. 2

(*c*) Find the coordinates of C, the centre of the circle. 2

Ans

(*a*) Q = (12,10)

(*b*) P = (4,10)

(*c*) C = (8,11)

2008 P1

15. What is the derivative of $(x^3 + 4)^2$? 2

A $(3x^2 + 4)^2$

B $\frac{1}{3}(x^3 + 4)^3$

C $6x^2(x^3 + 4)$

D $2(3x^2 + 4)^{-1}$

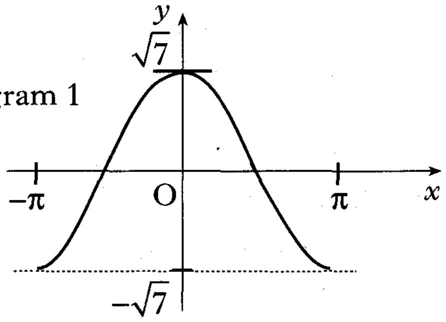
Ans	C

2008 P2

3. (*a*) (i) Diagram 1 shows part of the graph of $y = f(x)$, where $f(x) = p\cos x$.

Write down the value of p.

Diagram 1



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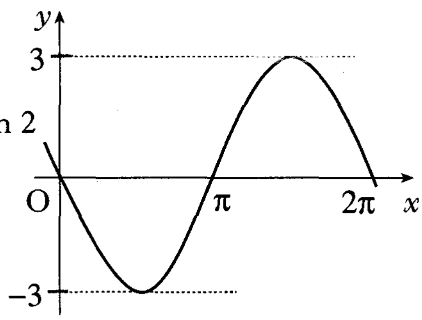
(ii) Diagram 2 shows part of the graph of $y = g(x)$, where $g(x) = q\sin x$.

Write down the value of q.

Diagram 2



(*b*) Write $f(x) + g(x)$ in the form $k\cos(x + a)$ where $k > 0$ and $0 < a < \frac{\pi}{2}$. 4

(*c*) Hence find $f'(x) + g'(x)$ as a single trigonometric expression. 2

Ans (a) $p = \sqrt{7}$, $q=-3$ (b) $4Cos(x+0.848)$ (c) $-4Sin(x+0.848)$

2007 P1

10. Given that $y = \sqrt{3x^2 + 2}$, find $\frac{dy}{dx}$. 3

Ans $\frac{1}{2}(3x^2 + 2)^{-\frac{1}{2}} \times 6x \; (= 3x(3x^2 + 2)^{-\frac{1}{2}})$

2006 P1

5. A function f is defined by $f(x) = (2x - 1)^5$.

Find the coordinates of the stationary point on the graph with equation $y = f(x)$ and determine its nature. 7

Ans point of inflexion at $(\frac{1}{2}, 0)$

2006 P2

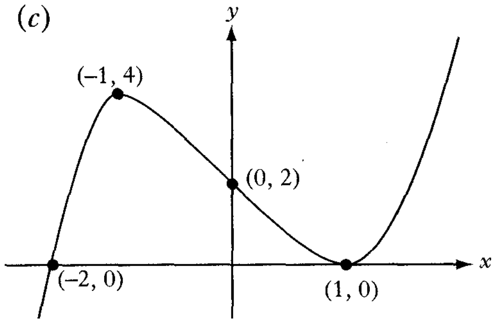
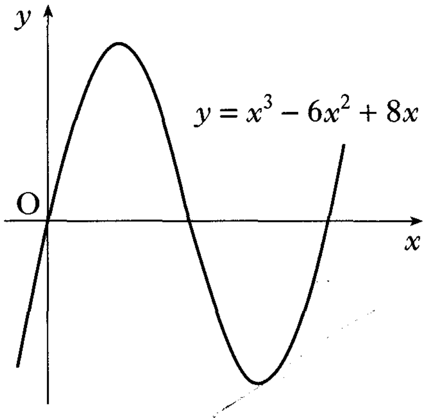
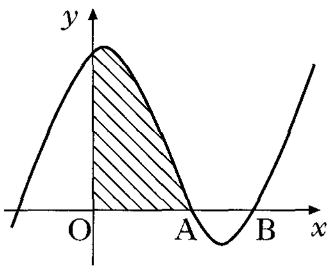
9. If $y = \frac{1}{x^3} - \cos 2x$, $x \neq 0$, find $\frac{dy}{dx}$. 4

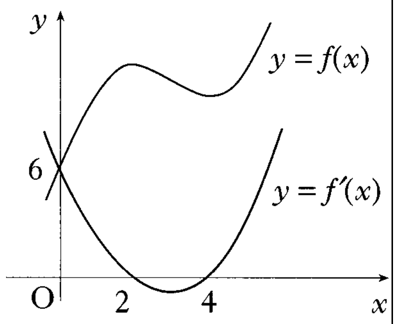
Ans $-3x^{-4} + 2\sin 2x$

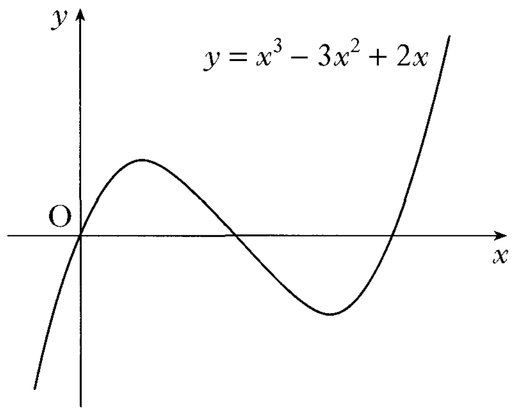
2005 P1	**5.** Differentiate $(1 + 2\sin x)^4$ with respect to x.	2
Ans	$4(1 + 2\sin(x))^3$ $\ldots\times 2\cos(x)$	
2004 P1	**6.** Given that $y = 3\sin(x) + \cos(2x)$, find $\frac{dy}{dx}$.	3
Ans	$3\cos(x) - 2\sin(2x)$	
2003 P2	**6.** If $f(x) = \cos(2x) - 3\sin(4x)$, find the exact value of $f'\left(\frac{\pi}{6}\right)$.	4
Ans	$6 - \sqrt{3}$	
2002W P1	**10.** A function f is defined by $f(x) = 2x + 3 + \frac{18}{x-4}$, $x \neq 4$. Find the values of x for which the function is increasing.	5
Ans	$f'(x) = 2 - \frac{18}{(x-4)^2}$ $f'(x) = 0 \Rightarrow x = 1, 7$ $f'(x) > 0 \Rightarrow x < 1, x > 7$	
2002 P1	**10.** (*a*) Find the derivative of the function $f(x) = (8 - x^3)^{\frac{1}{2}}$, $x < 2$. (*b*) Hence write down $\int \frac{x^2}{(8-x^3)^{\frac{1}{2}}}\,dx$.	2 1
Ans	(*a*) $f'(x) = -\frac{3}{2}x^2(8 - x^3)^{-\frac{1}{2}}$ (*b*) $-\frac{2}{3}(8 - x^3)^{\frac{1}{2}} + c$	
2002 P2	**6:** Find the equation of the tangent to the curve $y = 2\sin\left(x - \frac{\pi}{6}\right)$ at the point where $x = \frac{\pi}{3}$.	4
Ans	$y = \sqrt{3}x + 1 - \frac{\pi}{\sqrt{3}}$	

2000 P2	**8.** Given that $f(x) = (5x - 4)^{\frac{1}{2}}$, evaluate $f'(4)$.	3
Ans	$\frac{5}{8}$	
Specimen 1 P1	**9.** Find $\frac{dy}{dx}$ given that $y = \sqrt{1 + \cos x}$.	3
Ans	$-\frac{1}{2}\sin x(1 + \cos x)^{-\frac{1}{2}}$	

Polynomials

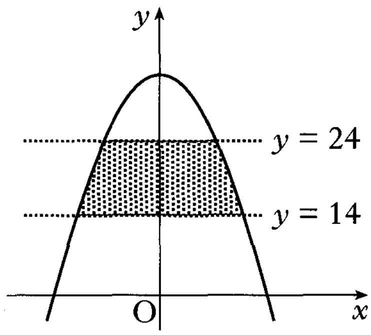
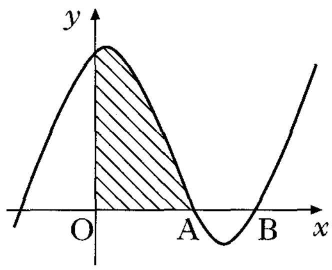
2008 P1	**21.** A function f is defined on the set of real numbers by $f(x) = x^3 - 3x + 2$. (a) Find the coordinates of the stationary points on the curve $y = f(x)$ and determine their nature. (b) (i) Show that $(x - 1)$ is a factor of $x^3 - 3x + 2$. (ii) Hence or otherwise factorise $x^3 - 3x + 2$ fully. (c) State the coordinates of the points where the curve with equation $y = f(x)$ meets both the axes and hence sketch the curve.	6 5 4
Ans	(a) $(-1, 4)$ maximum $(1, 0)$ minimum (b) (i) $x = 1,\ f(x) = 0$ so $(x - 1)$ is a factor (ii) $(x - 1)(x - 1)(x + 2)$ (c) 	
2008 P1	**22.** The diagram shows a sketch of the curve with equation $y = x^3 - 6x^2 + 8x$.  (a) Find the coordinates of the points on the curve where the gradient of the tangent is -1. (b) The line $y = 4 - x$ is a tangent to this curve at a point A. Find the coordinates of A.	5 2
Ans	(a) (1,3), (3,-3) (b) (1,3)	
2007 P1	**8.** The diagram shows a sketch of the graph of $y = x^3 - 4x^2 + x + 6$.  (a) Show that the graph cuts the x-axis at $(3, 0)$. (b) Hence or otherwise find the coordinates of A.	1 3

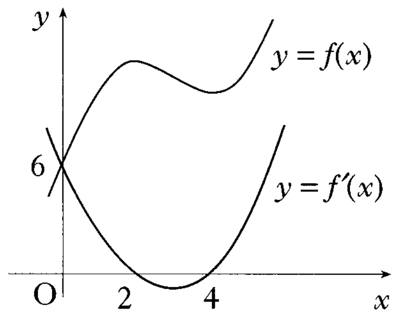
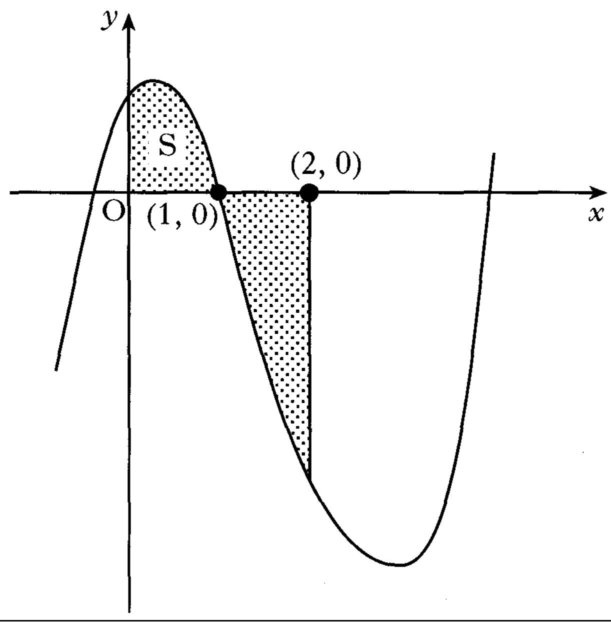
Ans	(a) To cut the x-axis, $y = 0$. So $0 = x^3 - 4x^2 + x + 6$ $= (x - 3)(x^2 - x - 2)$ $= (x - 3)(x - 2)(x + 1)$ So graph cuts x-axis at $x = -1, 3, 2$. (b) (2,0)	
2007 P2	**10.** The diagram shows the graphs of a cubic function $y = f(x)$ and its derived function $y = f'(x)$. Both graphs pass through the point (0, 6). The graph of $y = f'(x)$ also passes through the points (2, 0) and (4, 0).  (a) Given that $f'(x)$ is of the form $k(x - a)(x - b)$: (i) write down the values of a and b; (ii) find the value of k.	3
Ans	(a) (i) $a = 2$, $b = 4$ (ii) $k = \frac{3}{4}$	
2005 P1	**8.** A function f is defined by the formula $f(x) = 2x^3 - 7x^2 + 9$ where x is a real number. (a) Show that $(x - 3)$ is a factor of $f(x)$, and hence factorise $f(x)$ fully. (b) Find the coordinates of the points where the curve with equation $y = f(x)$ crosses the x- and y-axes. (c) Find the greatest and least values of f in the interval $-2 \leq x \leq 2$.	5 2 5
Ans	(a)$(x - 3)(2x - 3)(x + 1)$ (b)$(-1,0)$, $(\frac{3}{2}, 0)$, $(3, 0)$ (c)greatest value = 9 least value = -35	
2005 P2	**11.** (a) Show that $x = -1$ is a solution of the cubic equation $x^3 + px^2 + px + 1 = 0$. (b) Hence find the range of values of p for which all the roots of the cubic equation are real.	1 7
Ans	(a)$f(-1) = -1 + p - p + 1 = 0$ (b)$p \leq -1$, $p \geq 3$	

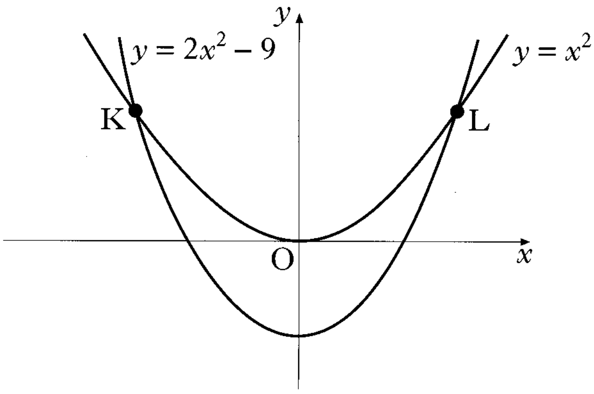
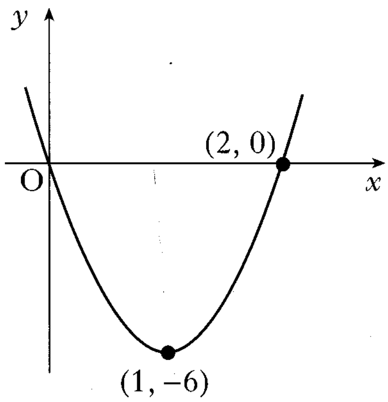
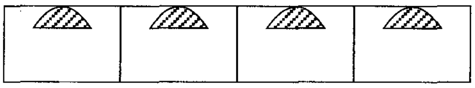
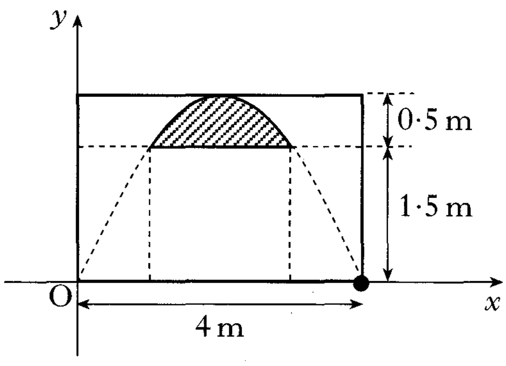
2004 P1	**2.** $f(x) = x^3 - x^2 - 5x - 3$. (*a*) (i) Show that $(x + 1)$ is a factor of $f(x)$. (ii) Hence or otherwise factorise $f(x)$ fully. (*b*) One of the turning points of the graph of $y = f(x)$ lies on the x-axis. Write down the coordinates of this turning point.	5 1
Ans	$(x + 1)(x + 1)(x - 3)$ $(-1, 0)$	
2003 P2	**1.** $f(x) = 6x^3 - 5x^2 - 17x + 6$. (*a*) Show that $(x - 2)$ is a factor of $f(x)$. (*b*) Express $f(x)$ in its fully factorised form.	4
Ans	(*b*) $(x - 2)(2x + 3)(3x - 1)$	
2002W P1	**5.** Given that $(x - 2)$ and $(x + 3)$ are factors of $f(x)$ where $f(x) = 3x^3 + 2x^2 + cx + d$, find the values of c and d.	5
Ans	$c = -19$, $d = 6$	
2001 P2	**1.** (*a*) Given that $x + 2$ is a factor of $2x^3 + x^2 + kx + 2$, find the value of k. (*b*) Hence solve the equation $2x^3 + x^2 + kx + 2 = 0$ when k takes this value.	3 2
Ans	(*a*) $k = -5$ (*b*) $x = -2, \frac{1}{2}, 1$	
2000 P2	**1.** The diagram shows a sketch of the graph of $y = x^3 - 3x^2 + 2x$. (*a*) Find the equation of the tangent to this curve at the point where $x = 1$. (*b*) The tangent at the point (2, 0) has equation $y = 2x - 4$. Find the coordinates of the point where this tangent meets the curve again.  	5 5
Ans	(*a*) $x + y = 1$ (*b*) $(-1, -6)$	

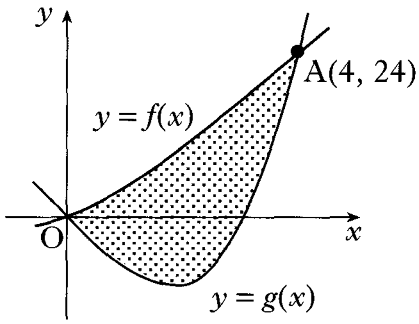
Specimen 2 P1	**1.** Show that $x = 2$ is a root of the equation $y = 2x^3 + x^2 - 13x + 6 = 0$ and hence, or otherwise, find the other roots.	4
Ans	$\begin{array}{c\|cccc} 2 & 2 & 1 & -13 & 6 \\ & & 4 & 10 & -6 \\ \hline & 2 & 5 & -3 & 0 \end{array}$ remainder = 0 $\Rightarrow x = 2$ is a root $2x^2 + 5x - 3 = 0 \Rightarrow x = \frac{1}{2}, -3$	
Specimen 1 P1	**3.** (*a*) Show that $(x - 1)$ is a factor of $f(x) = x^3 - 6x^2 + 9x - 4$ and find the other factors. (*b*) Write down the coordinates of the points at which the graph of $y = f(x)$ meets the axes.	3 1
Ans	(*a*) $f(1) = 0$, $(x - 4)$, $(x - 1)$ (*b*) $(1,0)$, $(4,0)$, $(0, -4)$	

Integration

2008 P1	**14.** Find $\int 4\sin(2x+3)\,dx$. A $-4\cos(2x+3)+c$ B $-2\cos(2x+3)+c$ C $4\cos(2x+3)+c$ D $8\cos(2x+3)+c$	2
Ans	B	
2008 P2	**7.** The parabola shown in the diagram has equation $y = 32 - 2x^2$. The shaded area lies between the lines $y = 14$ and $y = 24$. Calculate the shaded area. 	8
Ans	$50\frac{2}{3}$	
2007 P1	**8.** The diagram shows a sketch of the graph of $y = x^3 - 4x^2 + x + 6$. (*a*) Show that the graph cuts the x-axis at (3, 0). (*b*) Hence or otherwise find the coordinates of A. (*c*) Find the shaded area. 	1 3 5
Ans	(*a*) To cut the x-axis, $y = 0$. So $0 = x^3 - 4x^2 + x + 6$ $= (x-3)(x^2 - x - 2)$ $= (x-3)(x-2)(x+1)$ So graph cuts x-axis at $x = -1, 3, 2$. (*b*) (2,0) (*c*) $\frac{22}{3}$	

2007 P2	**10.** The diagram shows the graphs of a cubic function $y = f(x)$ and its derived function $y = f'(x)$. Both graphs pass through the point (0, 6). The graph of $y = f'(x)$ also passes through the points (2, 0) and (4, 0).  (*a*) Given that $f'(x)$ is of the form $k(x - a)(x - b)$: (i) write down the values of a and b; (ii) find the value of k. (*b*) Find the equation of the graph of the cubic function $y = f(x)$.	3 4
Ans	(*a*) (i) $a = 2$, $b = 4$ (ii) $k = \frac{3}{4}$ (*b*) $y = \frac{1}{4}x^3 - \frac{9}{4}x^2 + 6x + 6$	
2006 P1	**6.** The graph shown has equation $y = x^3 - 6x^2 + 4x + 1$. The total shaded area is bounded by the curve, the x-axis, the y-axis and the line $x = 2$. (*a*) Calculate the shaded area labelled S. (*b*) Hence find the total shaded area. 	4 3
Ans	(*a*) $\frac{5}{4}$ or equivalent (*b*) $\frac{9}{2}$ or equivalent	
2006 P2	**5.** The curve $y = f(x)$ is such that $\frac{dy}{dx} = 4x - 6x^2$. The curve passes through the point (–1, 9). Express y in terms of x.	4
Ans	$y = 2x^2 - 2x^3 + 5$	

2005 P2	**1.** Find $\int \frac{4x^3 - 1}{x^2}\,dx, \quad x \neq 0.$	4
Ans	$2x^2 + x^{-1} + c$	
2005 P2	**5.** The curves with equations $y = x^2$ and $y = 2x^2 - 9$ intersect at K and L as shown. Calculate the area enclosed between the curves.  	8
Ans	36	
2004 P1	**11.** The diagram shows a parabola passing through the points (0, 0), (1, –6) and (2, 0). (*a*) The equation of the parabola is of the form $y = ax(x - b)$. Find the values of a and b. (*b*) This parabola is the graph of $y = f'(x)$. Given that $f(1) = 4$, find the formula for $f(x)$.  	3 5
Ans	(*a*) $a = 6$, $b = 2$ (*b*) $f(x) = 2x^3 - 6x^2 + 8$	
2004 P2	**11.** An architectural feature of a building is a wall with arched windows. The curved edge of each window is parabolic. The second diagram shows one such window. The shaded part represents the glass. The top edge of the window is part of the parabola with equation $y = 2x - \frac{1}{2}x^2$. Find the area in square metres of the glass in one window.   	8
Ans	$\frac{2}{3}$	

2003 P2	**3.** The incomplete graphs of $f(x) = x^2 + 2x$ and $g(x) = x^3 - x^2 - 6x$ are shown in the diagram. The graphs intersect at A(4, 24) and the origin. Find the shaded area enclosed between the curves. 		5
Ans	$A = \int [(x^2 + 2x) - (x^3 - x^2 - 6x)]\, dx$ $42\frac{2}{3}$		
2002W P1	**7.** Find $\int \left(\sqrt[3]{x} - \frac{1}{\sqrt{x}}\right) dx$.		4
Ans	$\frac{3}{4}x^{\frac{4}{3}} - 2x^{\frac{1}{2}} + c$		

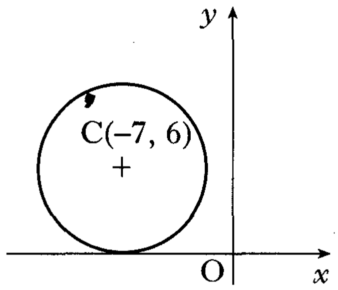
2004 P1	**7.** Find $\int_0^2 \sqrt{4x+1}\; dx$.	5
Ans	$\frac{13}{3}$	
2003 P1	**8.** Find $\int_0^1 \frac{dx}{(3x+1)^{\frac{1}{2}}}$.	4
Ans	$\frac{2}{3}$	
2002W P2	**8.** Find $\int_0^1 \left(\cos(3x) - \sin\left(\frac{1}{3}x + 1\right)\right) dx$ correct to 3 decimal places.	3
Ans	$a = -0{\cdot}868$	
2001 P2	**10.** A curve for which $\frac{dy}{dx} = 3\sin(2x)$ passes through the point $\left(\frac{5}{12}\pi, \sqrt{3}\right)$. Find y in terms of x.	4
Ans	$y = -\frac{3}{2}\cos(2x) + \frac{1}{4}\sqrt{3}$	

2000 P1	**8.** The graph of $y = f(x)$ passes through the point $\left(\frac{\pi}{9}, 1\right)$. If $f'(x) = \sin(3x)$ express y in terms of x.	4
Ans	$y = -\frac{1}{3}\cos(3x) + \frac{7}{6}$	
2000 P2	**10.** Find $\int \frac{1}{(7-3x)^2}\,dx$.	2
Ans	$\frac{1}{3(7-3x)} + c$	

Circle

<table>
<tr>
<td>2008 P1</td>
<td>

2. The x-axis is a tangent to a circle with centre (–7, 6) as shown in the diagram.



What is the equation of the circle?

A $(x + 7)^2 + (y - 6)^2 = 1$

B $(x + 7)^2 + (y - 6)^2 = 49$

C $(x - 7)^2 + (y + 6)^2 = 36$

D $(x + 7)^2 + (y - 6)^2 = 36$

</td>
<td>2</td>
</tr>
<tr>
<td>Ans</td>
<td colspan="2">D</td>
</tr>
<tr>
<td>2008 P1</td>
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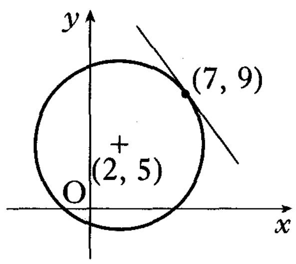
5. The diagram shows a circle, centre (2, 5) and a tangent drawn at the point (7, 9).

What is the equation of this tangent?



A $y - 9 = -\frac{5}{4}(x - 7)$

B $y + 9 = -\frac{4}{5}(x + 7)$

C $y - 7 = \frac{4}{5}(x - 9)$

D $y + 9 = \frac{5}{4}(x + 7)$

</td>
<td>2</td>
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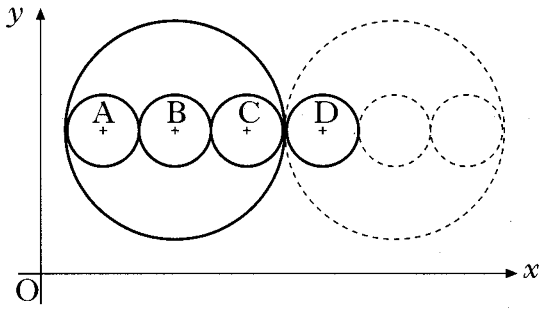
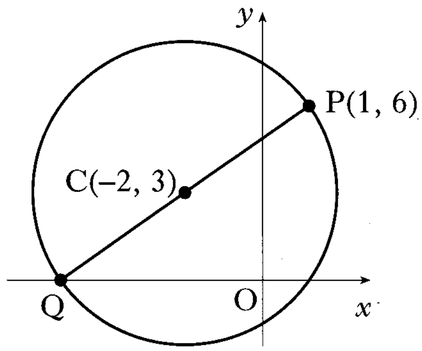
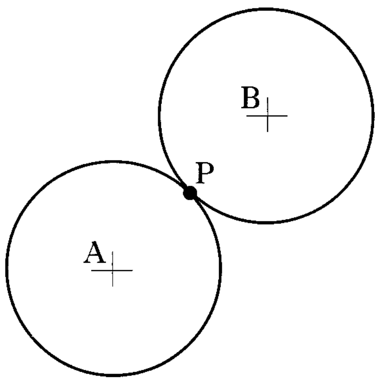
<tr>
<td>Ans</td>
<td colspan="2">A</td>
</tr>
</table>

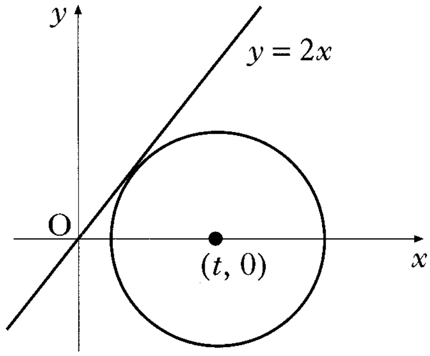
2008 P2	**4.** (*a*) Write down the centre and calculate the radius of the circle with equation $x^2 + y^2 + 8x + 4y - 38 = 0$. (*b*) A second circle has equation $(x-4)^2 + (y-6)^2 = 26$. Find the distance between the centres of these two circles and hence show that the circles intersect. (*c*) The line with equation $y = 4 - x$ is a common chord passing through the points of intersection of the two circles. Find the coordinates of the points of intersection of the two circles.	2 4 5
Ans	(a) (-4,-2), √58 (b) d = √128 (c) (3,1), (-1,5)	
2007 P1	**5.** The large circle has equation $x^2 + y^2 - 14x - 16y + 77 = 0$. Three congruent circles with centres A, B and C are drawn inside the large circle with the centres lying on a line parallel to the x-axis. This pattern is continued, as shown in the diagram. Find the equation of the circle with centre D.  	5
Ans	$(x-15)^2 + (y-8)^2 = 2^2$	
2007 P2	**3.** Show that the line with equation $y = 6 - 2x$ is a tangent to the circle with equation $x^2 + y^2 + 6x - 4y - 7 = 0$ and find the coordinates of the point of contact of the tangent and the circle.	6
Ans	$x^2 + (6-2x)^2 + 6x - 4(6-2x) - 7 = 0$ $x^2 + 36 - 24x + 4x^2 + 6x - 24 + 8x - 7 = 0$ $5x^2 - 10x + 5 = 0$ $5(x^2 - 1) = 0$ Only one root so line is tangential to circle. Point of contact is (1,4).	

2006 P1	**2.** A circle has centre C(–2, 3) and passes through P(1, 6). (*a*) Find the equation of the circle. (*b*) PQ is a diameter of the circle. Find the equation of the tangent to this circle at Q.  	2 4
Ans	(*a*) $r^2 = 18$ (*b*) $y - 0 = -(x - (-5))$	
2006 P2	**4.** The circles with equations $(x-3)^2 + (y-4)^2 = 25$ and $x^2 + y^2 - kx - 8y - 2k = 0$ have the same centre. Determine the radius of the larger circle.	5
Ans	$\sqrt{37}$, 5 and "2nd" circle	
2005 P1	**2.** Two congruent circles, with centres A and B, touch at P. Relative to suitable axes, their equations are $x^2 + y^2 + 6x + 4y - 12 = 0$ and $x^2 + y^2 - 6x - 12y + 20 = 0$. (*a*) Find the coordinates of P. (*b*) Find the length of AB.  	3 2
Ans	(*a*) P = (0,2) (*b*) AB = 10	

2005 P1

11. (*a*) A circle has centre $(t, 0)$, $t > 0$, and radius 2 units.
Write down the equation of the circle. **1**

(*b*) Find the exact value of t such that the line $y = 2x$ is a tangent to the circle. **5**



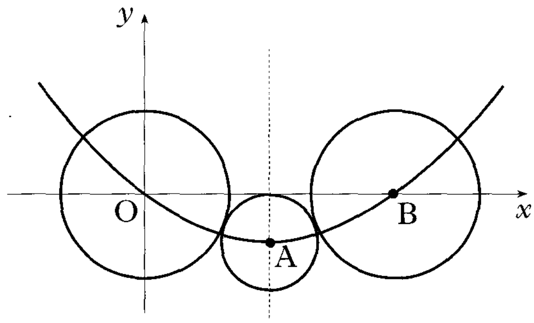
Ans

(*a*) $(x - t)^2 + (y - 0)^2 = 2^2$

(*b*) $t = \sqrt{5}$

2003 P1

11.
- O, A and B are the centres of the three circles shown in the diagram below.
- The two outer circles are congruent and each touches the smallest circle.
- Circle centre A has equation $(x - 12)^2 + (y + 5)^2 = 25$.
- The three centres lie on a parabola whose axis of symmetry is shown by the broken line through A.



(*a*) (i) State the coordinates of A and find the length of the line OA. **2**

(ii) Hence find the equation of the circle with centre B. **3**

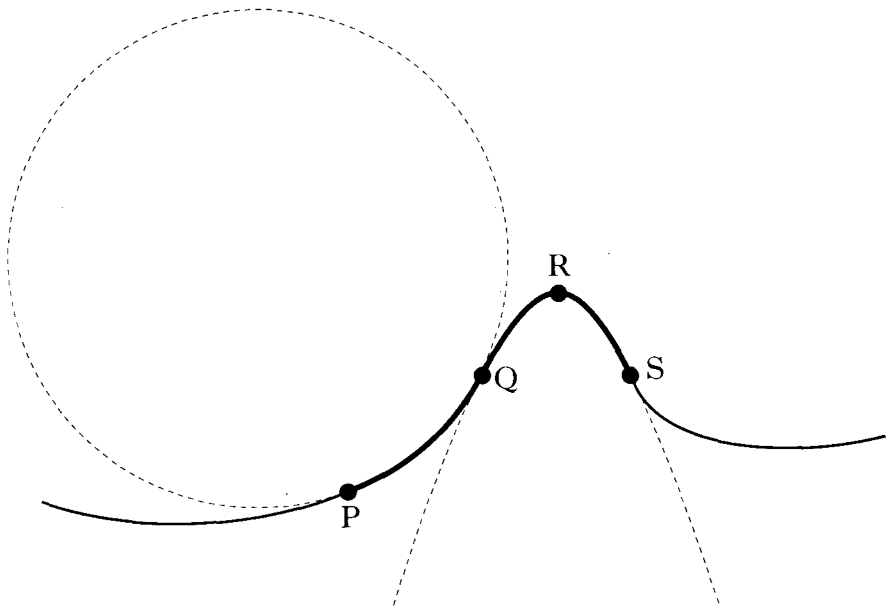
(*b*) The equation of the parabola can be written in the form $y = px(x + q)$.
Find the values of p and q. **2**

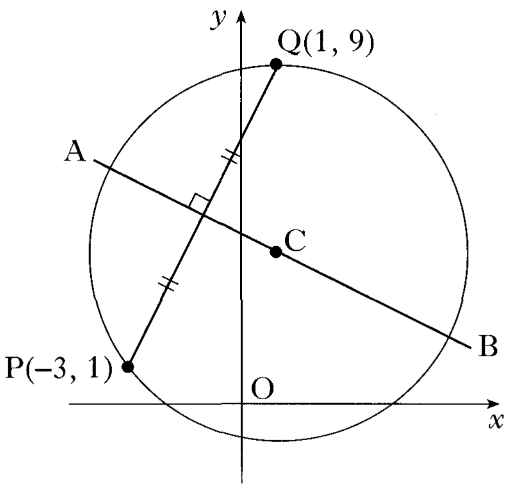
Ans

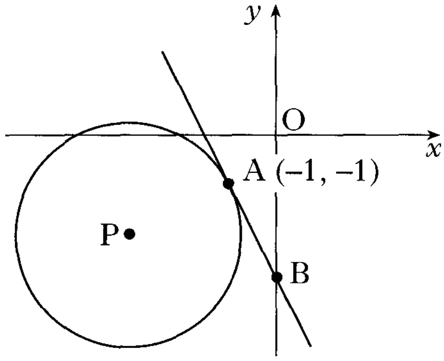
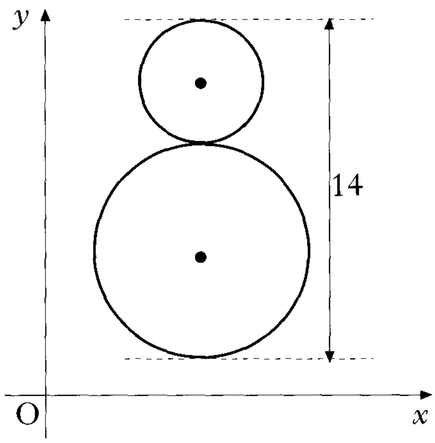
(*a*) (i) A(12, –5), OA = 13

(ii) $(x - 24)^2 + y^2 = 64$

(*b*) $p = \frac{5}{144}$, $q = -24$

2002W P1	**6.** The side view of part of a roller coaster ride is shown by the path PQRS. The curve PQ is an arc of the circle with equation $x^2 + y^2 + 4x - 10y + 9 = 0$. The curve QRS is part of the parabola with equation $y = -x^2 + 6x - 5$. The point Q has coordinates (2, 3).  (*a*) Find the equation of the tangent to the circle at Q. (*b*) Show that this tangent to the circle at Q is also the tangent to the parabola at Q.	4 2
Ans	(*a*) $y - 2x = -1$ (*b*) proof For parabola • $\frac{dy}{dx} = -2x + 6$ • $m = 2$ (same as gradient of tangent to circle)	
2002W P2	**10.** The line $y + 2x = k$, $k > 0$, is a tangent to the circle $x^2 + y^2 - 2x - 4 = 0$. (*a*) Find the value of k. (*b*) Deduce the coordinates of the point of contact.	7 2
Ans	(*a*) $x^2 + (k - 2x)^2 - 2x - 4 = 0$ and apply discriminant $k=7$ (*b*) (3, 1)	

2002 P1	**1.** The point P(2, 3) lies on the circle $(x+1)^2 + (y-1)^2 = 13$. Find the equation of the tangent at P.	4
Ans	$2y + 3x = 12$	
2001 P1	**11.** Circle P has equation $x^2 + y^2 - 8x - 10y + 9 = 0$. Circle Q has centre (–2, –1) and radius $2\sqrt{2}$. (*a*) (i) Show that the radius of circle P is $4\sqrt{2}$. (ii) Hence show that circles P and Q touch. (*b*) Find the equation of the tangent to circle Q at the point (–4, 1). (*c*) The tangent in (*b*) intersects circle P in two points. Find the *x*-coordinates of the points of intersection, expressing your answers in the form $a \pm b\sqrt{3}$.	4 3 3
Ans	(*a*) $r_P = 4\sqrt{2}$ $r_P + r_Q = 6\sqrt{2}$ $C_P = (4,5)$ $C_PC_Q = \sqrt{6^2 + 6^2} = 6\sqrt{2}$ and "so touch" (*b*) $y = x + 5$ (*c*) $x = 2 \pm 2\sqrt{3}$	
2000 P1	**6.** For what range of values of k does the equation $x^2 + y^2 + 4kx - 2ky - k - 2 = 0$ represent a circle?	5
Ans	for all k	
2000 P2	**2.** (*a*) Find the equation of AB, the perpendicular bisector of the line joining the points P(–3, 1) and Q(1, 9). (*b*) C is the centre of a circle passing through P and Q. Given that QC is parallel to the y-axis, determine the equation of the circle. (*c*) The tangents at P and Q intersect at T. Write down (i) the equation of the tangent at Q (ii) the coordinates of T. 	4 3 2

Ans	(*a*) $x + 2y = 9$ (*b*) $(x-1)^2 + (y-4)^2 = 25$ (*c*) (i) $y = 9$ (ii) T = (–9, 9)	
Specimen 2 P1	**5.** (*a*) The diagram shows a circle, centre P, with equation $x^2 + y^2 + 6x + 4y + 8 = 0$. Find the equation of the tangent at the point A (–1, –1) on the circle. (*b*) The tangent crosses the y-axis at B. Find the equation of the circle with AB as diameter. 	4 3
Ans	(*a*) P = (–3,–2) $m_{\text{PA}} = \frac{1}{2} \Rightarrow m_{tgt} = -2$ $y - 1 = -2(x - (-1))$ (*b*) B = (0,3) centre C = $\text{mid}_{\text{AB}} = (-\frac{1}{2}, -2)$ $\text{radius}^2 = \text{CB}^2 = \frac{5}{4}$ $(x + \frac{1}{2})^2 + (y+2)^2 = \frac{5}{4}$	
Specimen 1 P1	**6.** A bakery firm makes ginger-bread men each 14 cm high with a circular "head" and "body". The equation of the "body" is $x^2 + y^2 - 10x - 12y + 45 = 0$ and the line of centres is parallel to the y-axis. Find the equation of the "head". 	5
Ans	$(x-5)^2 + (y-13)^2 = 9$	