Armadale Academy

Higher Maths

Procedures Booklet

The aim of this booklet is to give you a guide on how to answer common exam questions.

You should use this to help you learn the approach to solving questions which you need to target.

You should be aiming to use this booklet less the closer you get to the exam.

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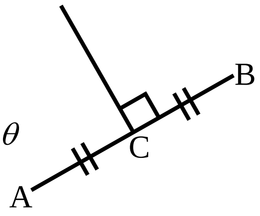
Straight Lines

Perpendicular Bisectors

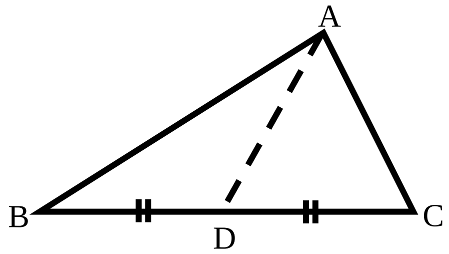
A perpendicular bisector is a line that cuts another line exactly in half and at right angles:



- Find the mid-point C using A&B: $\left(\dfrac{x_1+x_2}{2},\dfrac{y_1+y_2}{2}\right)$
- Find the gradient of AB using: $m=\dfrac{y_2-y_1}{x_2-x_1}$ or sometimes $m=\tan\theta$
- Find perpendicular gradient using: $m_1\times m_2=-1$
- Substitute mid-point C and perpendicular gradient into $y-b=m(x-a)$ then simplify.

Medians

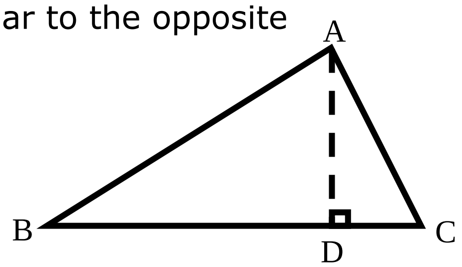
A median of a triangle is a line from a vertex to the mid-point of the opposite side.



- Find the mid-point D using B&C: $\left(\dfrac{x_1+x_2}{2},\dfrac{y_1+y_2}{2}\right)$
- Find the gradient of median AD using: $m=\dfrac{y_2-y_1}{x_2-x_1}$
- Substitute gradient and either A or D into $y-b=m(x-a)$ then simplify.

Altitudes

An altitude of a triangle is a line from a vertex perpendicular to the opposite side.



- Find the gradient of BC using: $m=\dfrac{y_2-y_1}{x_2-x_1}$
- Find the gradient of altitude AD using: $m_1\times m_2=-1$
- Substitute gradient altitude and point A into $y-b=m(x-a)$ then simplify.

Intersections

If finding the point of intersection of lines, we need to use simultaneous equations.

Differentiation

Rate of Change

- Differentiate the given function.
- Substitute given value into the differentiated function.
- Watch for negative and fractional powers
  – using indices rules rearrange to get positive powers and roots before substitution.

Equation of a Tangent to a Curve

- Find the *y*-value of the point of contact: substitute *x* into the original function.
- Find gradient: $m = \frac{dy}{dx}$, so differentiate to find a function for gradient.
- Substitute *x* into $\frac{dy}{dx}$ to give a value for gradient.
- Substitute gradient and point of contact into $y - b = m(x - a)$ then simplify.

Stationary Points and Nature

- State "stationary points occur when $\frac{dy}{dx} = 0$ or $f'(x) = 0$".
- Differentiate and equate to zero.
- Solve for *x* values often using factorising.
- Find *y* values by substituting *x* values into the original function.
- Find nature with a nature table using $\frac{dy}{dx}$ or $f'(x)$.
- Make a statement about stationary points.

Curve Sketching

We need to find where the curve cuts the x & y axes, the co-ordinates of the stationary points and determine their nature.

- Cuts the x-axis when $y = 0$: Equate function to zero and solve for x.
- Cuts the y-axis when $x = 0$: Make x's zero and calculate for y.
- State "stationary points occur when $\frac{dy}{dx} = 0$ or $f'(x) = 0$".
- Differentiate and equate to zero.
- Solve for x values often using factorising.
- Find y values by substituting x values into the original function.
- Find nature with a nature table using $\frac{dy}{dx}$ or $f'(x)$.
- Make a statement about stationary points.
- Make a neat sketch of the curve fully annotating all points.

Increasing & Decreasing Functions

- Differentiate the given function.
- Substitute given value into the differentiated function.
- If the answer is positive then the function is increasing.
- If the answer is negative then the function is decreasing.

Maximum & Minimum Values Within a Closed Interval

The values of a function are the y-coordinates. We need to find the y-values of the stationary points and the y-values of the given end points of the interval.

- State "stationary points occur when $\frac{dy}{dx} = 0$ or $f'(x) = 0$".
- Differentiate and equate to zero.
- Solve for x values often using factorising.
- Find y values by substituting x values into the original function.
- Substitute x-values of end points into the original function.
- We now pick our highest and lowest y-values.
- CAUTION: sometimes some of our x-values will be outside the given range, we should disregard these stating 'outside of range'.

Optimisation

Often we will be asked to derive a formula for a given situation. This part often proves challenging for students. However, the formula is usually given so make sure you attempt the optimisation part of the question following this procedure:

- State "max/min at $\frac{dy}{dx} = 0$".
- Differentiate and equate to zero.
- Solve for *x* values often using factorising.
- Check for max/min using a nature table.
- Use your max/min value to fully answer the question in the given context.

Polynomials

Finding the function from a graph

- Using the roots from the given graph set up the equation:
    - $y = k(x - a)(x - b)(x - c)$
    - Remember; if the graph passes through the origin one of the brackets will be x on its own, if the graph has a turning point on the x–axis this will give a repeated root and two of the brackets will be the same.
- Using the y – intercept or another given point, substitute (x, y) into your equation to find the value of k.
- If required multiply out the brackets and multiply through by k to get your final function for the graph.

Integration

Area Between 2 Curves (let *f(x)* be the top function and *g(x)* the bottom function)

- Find the points of intersection (if not given):
  - set *f(x) = g(x)* re-arrange and solve. These values will be your limits for integration.
- Find the function to be integrated:
  - top – bottom; *f(x)* – *g(x)* then collect like terms, this is now the function to be integrated.
- Integrate this function between the limits worked out earlier.
- Substitute in limits and calculate the final answer.

Differential Equations

These questions come in the form: $\frac{dy}{dx} = 3x^2 + 5x$ find an equation for *y*.

They may be set in a context.

- Integrate the given equation and include '+ C'.
- Substitute in the given values for *x* and *y*, then solve for C.
- Write out your final equation for *y*.

Circles

Equations of Tangents

- Use the centre and the point of contact with $m = \frac{y_2 - y_1}{x_2 - x_1}$ find the gradient of the radius m_r.
- Find gradient of tangent using: $m_r \times m_t = -1$
- Substitute the point of contact and the gradient of the tangent into $y - b = m(x - a)$ then simplify.

Intersections of Lines and Circles

Finding Point(s) of Contact.

- Re-arrange the line equation into either, *x* = or *y* = .
- Substitute the re-arranged line equation into the circle equation.
- Multiply out collect like terms until you have a quadratic that equals zero.
- Factorise and solve for *x*–value(s). (these may be *y*–value(s) depending on your re-arranged line equation).
- Substitute *x*–value(s) back into the line equation to find *y*–value(s).

Tangency

- Re-arrange the line equation into either, *x* = or *y* = .
- Substitute the re-arranged line equation into the circle equation.
- Multiply out collect like terms until you have a quadratic that equals zero.
- At this point we can use the discriminant, $b^2 - 4ac$.
- The three possibilities are, if;
    - $b^2 - 4ac < 0$ no solutions so no points of contact
    - $b^2 - 4ac = 0$ one solution so tangent
    - $b^2 - 4ac > 0$ two solutions so two points of contact
- You could also factorise and two identical brackets would indicate one solution so this would be a tangent.