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CFE National 5 - Pack 2

Unit : Relationships (REL)

WORKSHEETS

- **Worksheets covering all the unit topics**
- **+ Answers**

INDEX **RELATIONSHIPS (REL)**

2.3 Identifying features of a quadratic function

- **Identify nature, coordinates of turning point and the equation of axis of symmetry of quadratic in the form $y = k(x + p)^2 + q$ where $k = 1$ or -1**
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1.1 DETERMINING the EQUATION of a STRAIGHT LINE

1. For each line, write down the gradient and the coordinates of the point where it crosses the y – axis.

(a) $y = 3x + 1$ **(b)** $y = \frac{1}{2}x - 5$ **(c)** $y = -2x + 3$

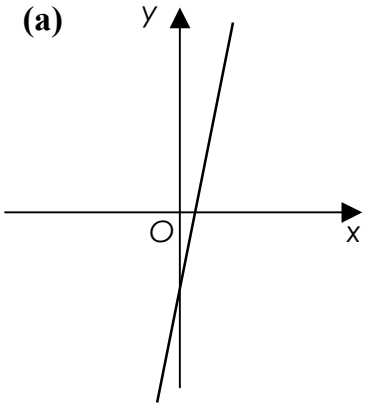
(d) $y = -\frac{1}{4}x - 2$ **(e)** $y = 8x - \frac{1}{2}$ **(f)** $y = -x + 4$

2. Match these equations with the graphs shown below.

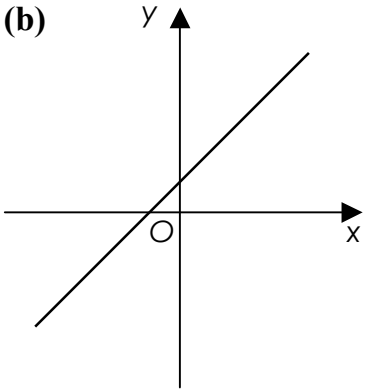
1. $y = x + 1$ **2**. $y = -2x - 3$ **3**. $y = \frac{1}{2}x + 4$

4. $y = -\frac{1}{4}x + 2$ **5**. $y = 6x - 2$ **6**. $y = 3x - 5$

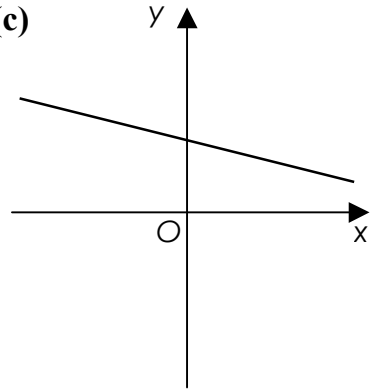
(a)



(b)



(c)



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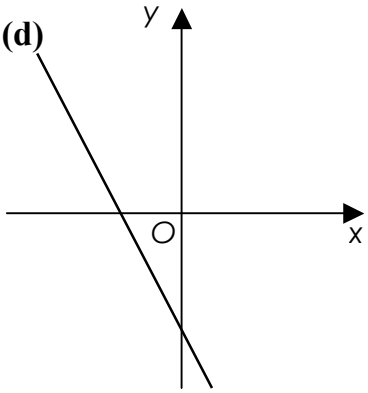

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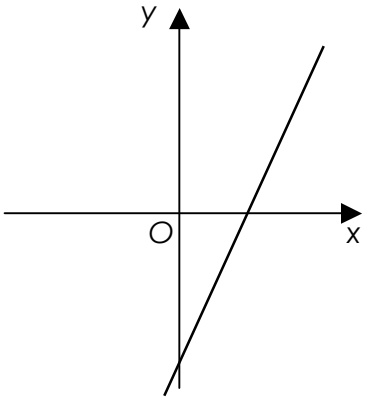
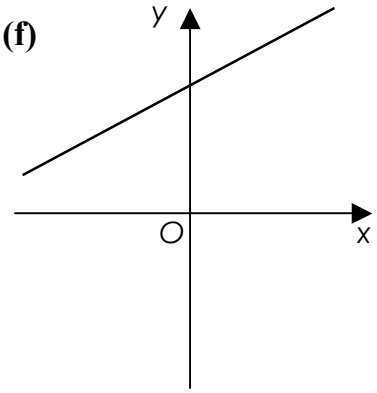


(f)



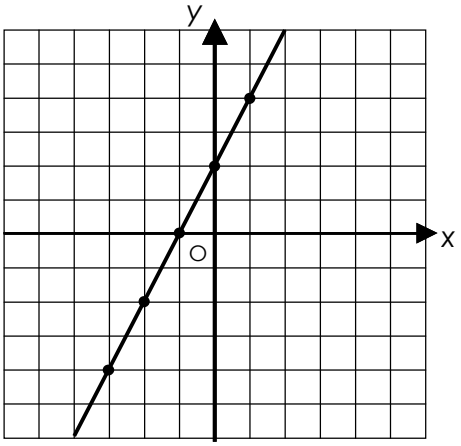
3. Sketch the graphs of lines with equations:

(a) $y = \frac{1}{2}x - 2$ **(b)** $y = -2x - 1$ **(c)** $y = -3x + 2$

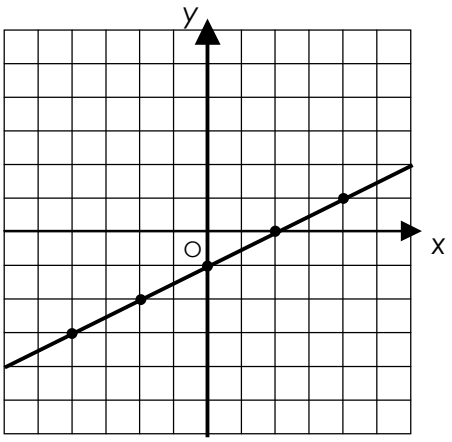
(d) $y = -x + 3$ **(e)** $y = 2x + 3$ **(f)** $y = 4x + 1$

4. Write down the equation of the lines drawn in the diagrams below.

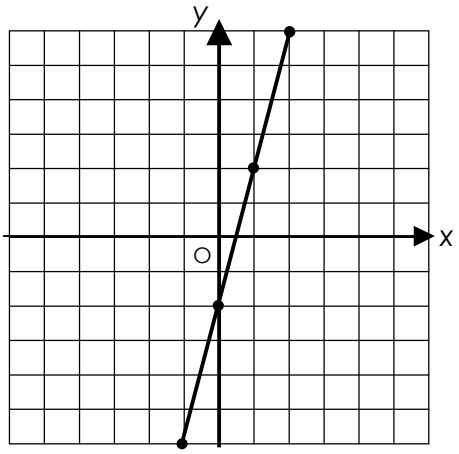
(a)



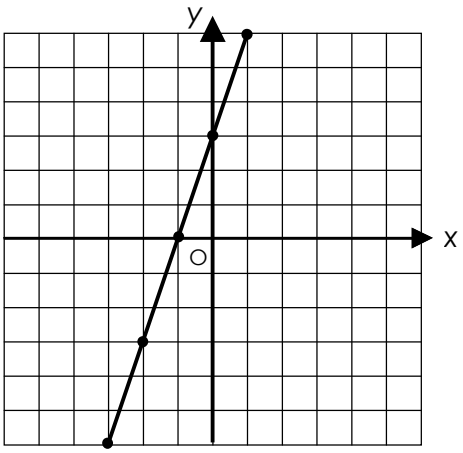
(b)



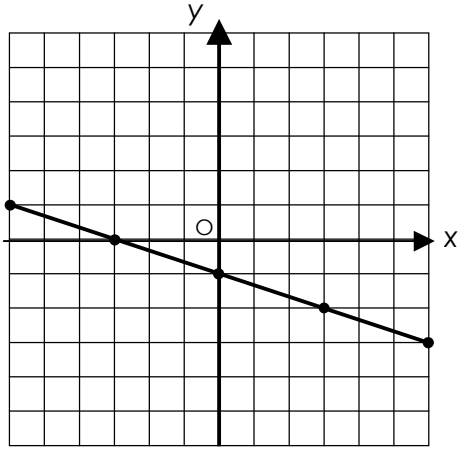
(c)



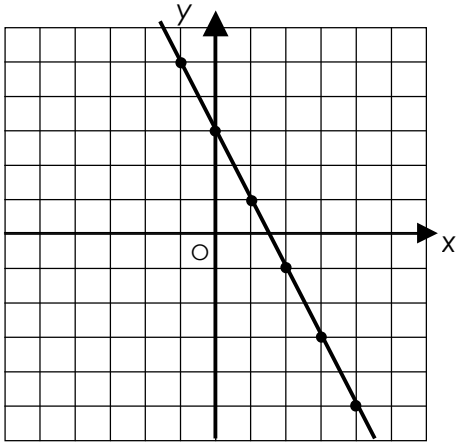
(d)



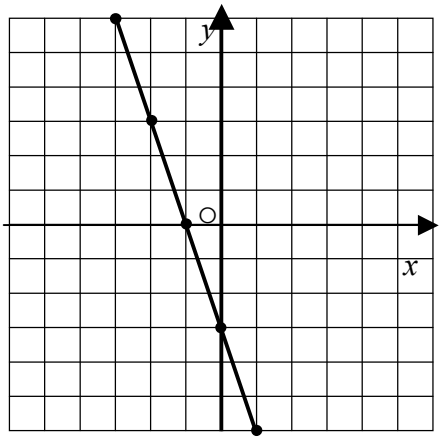
(e)



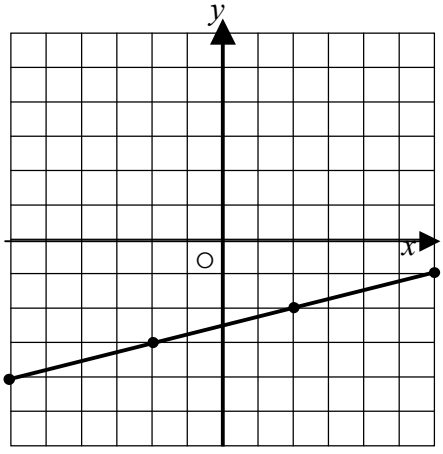
(f)



(g)



(h)



5. Identify the gradient and y – intercept of these lines.

(a) $y = x + 3$ **(b)** $y = -2x - 1$ **(c)** $y = \frac{1}{2}x$

(d) $y = -\frac{1}{2}x + 2$ **(e)** $x + y = 6$ **(f)** $2y = x - 4$

(g) $3y = x + 12$ **(h)** $4x + 5y = 20$ **(i)** $3x - 2y = 12$

6. State the gradient and the y – intercept for each line below.

(a) $y = x - 7$ **(b)** $y = -5x + 3$ **(c)** $5y = 3x - 10$

(d) $y = -4x$ **(e)** $2x + y = 11$ **(f)** $2y = x - 5$

(g) $3y - x = 18$ **(h)** $3x + 7y - 21 = 0$ **(i)** $4x - 5y = 20$

7. Write down the equation of the lines described below:

(a) with gradient 4, passing through the point (0, 5)

(b) with gradient –2, passing through the point (0, 1)

(c) with gradient $\frac{3}{4}$, passing through the point (0, –3)

(d) with gradient 4, passing through the point (3, 1)

(e) with gradient –5, passing through the point (–3, 1)

(f) with gradient $\frac{1}{2}$, passing through the point (–5, –2)

(g) with gradient $\frac{4}{3}$, passing through the point (2, 7)

(h) with gradient $-\frac{3}{4}$, passing through the point (–2, –2)

(i) with gradient $-\frac{3}{2}$, passing through the point (–5, 3)

8. Find the equation of the line joining each pair of points below.

(a) A(4, 3) and B(8, 11) **(b)** C(1, 9) and D(3, 1) **(c)** E(–2, 6) and F(8, 8)

(d) G(5, –9) and H(8, –15) **(e)** I(0, 6) and J(5, 11) **(f)** K(–1, –3) and L(7, –9)

(g) M(–4, 0) and N(–1, 5) **(h)** P(2, 2) and Q(–3, 4) **(i)** R(5, –1) and S(–2, 10)

9. Find the equations of the lines joining the following pairs of points:

(a) (2, 1) and (6, 3) **(b)** (1, 5) and (3, 1) **(c)** (2, 0) and (4, 6)

(d) (–2, –3) and (2, 3) **(e)** (–1, 2) and (5, –1) **(f)** (–4, 2) and (4, –4)

(g) (–6, –2) and (–5, 3) **(h)** (4, –3) and (6, 5) **(i)** (–2, 3) and (0, –2)

10. Establish the equation of the line passing through each pair of points below.

(a) A(2, 1) and B(6, 13) **(b)** C(3, 4) and D(5, –4) **(c)** E(–2, –1) and F(6, 3)

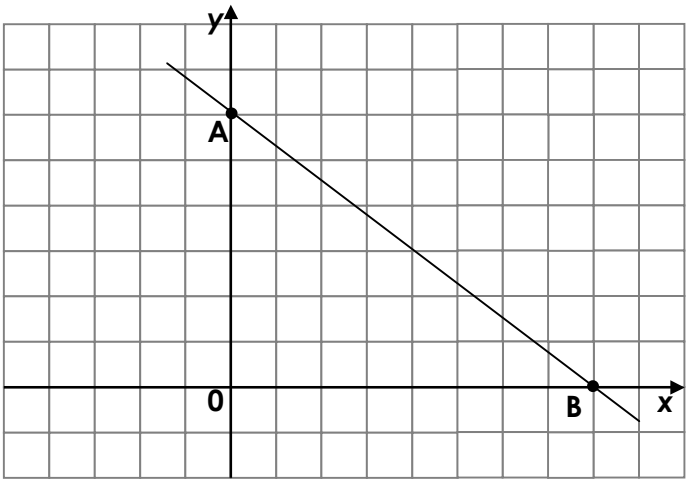
(d) G(4, –13) and H(–2, –1) **(e)** I(2, 8) and J(10, 12) **(f)** K(–3, 2) and L(9, –2)

STRAIGHT LINE EXAM QUESTIONS

1. A straight line has the equation $3x - 2y = -4$.

Find the gradient and y-intercept of the line.

2. The line AB passes through the points (0, 6) and (8, 0) as shown in the diagram.



Find the equation of the line AB.

3. A straight line has equation $2y + 3x = 8$. Which line of these gives its gradient and y – intercept? Show working to explain your answer.

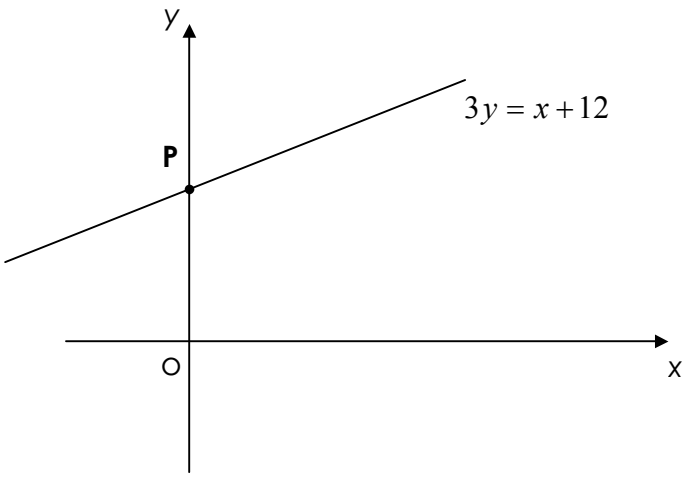
A. 3 and (0, 8) **B.** – 3 and (0. 8)

C. $\frac{3}{2}$ and (0, 4) **D.** $-\frac{3}{2}$ and (0, 4)

4. Find the gradient and y – intercept of the straight line with equation

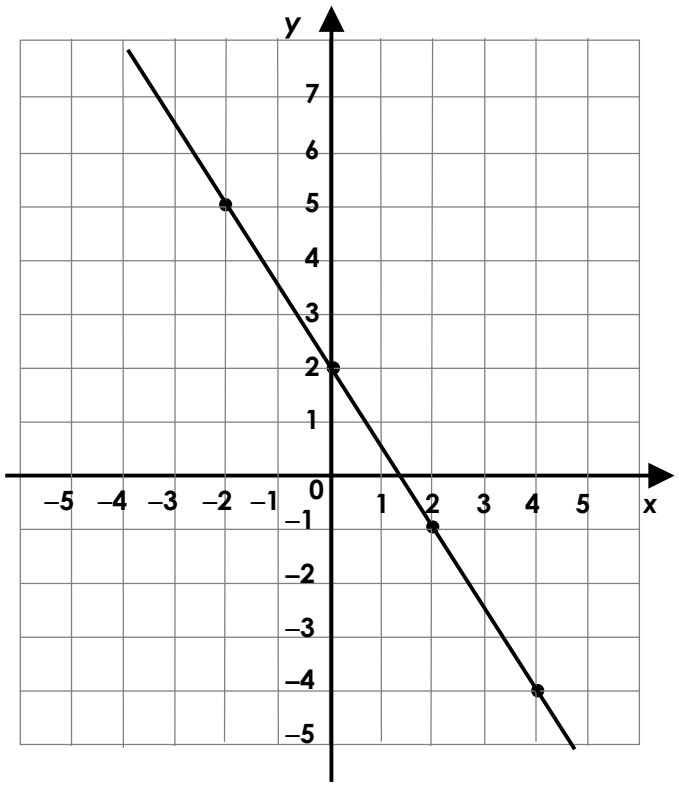
$$3x - 4y = 12 .$$

5. The diagram below shows the line with equation $3y = x + 12$.



Find the coordinates of **P**, the point where the line cuts the y-axis.

6. Find the equation of the line shown in the diagram below.

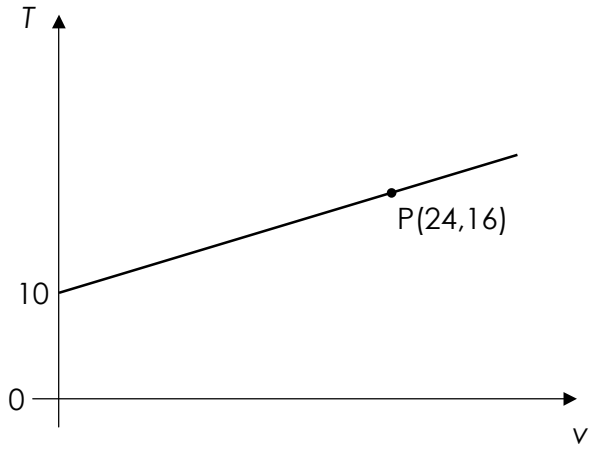


7. A line has equation $2y + 6x = 9$. Find its gradient and y - intercept**.**

8. A line has equation $3y + 4x = 15$. Make a sketch of this line on plain paper showing clearly where it crosses the y - axis.

9. The relationship between variables v and T produces a straight line graph as shown below.

The line passes through the point P(24,16) as shown.



(a) Find the gradient of the line.

(b) Hence, write down the equation of the line in terms of v and T.

10. A straight line has equation $3y - 2x = 6$. Find the gradient and y-intercept of the line.

11. A straight line has equation $3x - 2y = 8$. Find the gradient and y-intercept of the line.

12. Find the equation of the straight line which passes through the point A(3, –2) and is parallel to the line $3y - 2x = 5$

13. **(a)** A straight line has equation $4y - 3x = 6$.

State the gradient and the y-intercept point for this line.

(b) Write down the equation of the line with gradient $-\frac{1}{2}$ which has the same y – intercept point as the line above.

14. **(a)** A straight line has equation $3y - 4x = 12$.

State the gradient and the y-intercept point for this line.

(b) Write down the equation of the line with gradient $-\frac{3}{4}$ which has the same y – intercept point as the line above.

1.1 FUNCTIONAL NOTATION

1. A function is given as $f(x) = 6x - 5$.

Find: **(a)** $f(3)$ **(b)** $f(-1)$ **(c)** $f(\frac{1}{2})$ **(d)** $f(a)$

2. A function is given as $f(x) = x^2 + 4$.

Find: **(a)** $f(2)$ **(b)** $f(4)$ **(c)** $f(-3)$ **(d)** $f(p)$

3. A function is given as $h(a) = 12 - 2a$.

Find: **(a)** $h(4)$ **(b)** $h(6)$ **(c)** $h(-2)$ **(d)** $h(m)$

4. A function is defined as $g(x) = x^2 + 3x$.

Find: **(a)** $g(a)$ **(b)** $g(2p)$ **(c)** $g(m+1)$ **(d)** $g(2-e)$

5. A function is defined as $f(x) = x^2 - 4x$.

Find: **(a)** $f(4)$ **(b)** $f(3a)$ **(c)** $f(a-2)$ **(d)** $f(2p+1)$

6. A function is given as $f(x) = 5x + 3$. For what value of x is :

(a) $f(x) = 23$ **(b)** $f(x) = -2$ **(c)** $f(x) = 5$?

7. A function is given as $h(t) = 20 - 6t$. For what value of t is :

(a) $h(t) = 2$ **(b)** $h(t) = -16$ **(c)** $h(t) = 32$?

8. A function is given as $g(a) = a^2 - 16$. For what value(s) of a is :

(a) $g(a) = 9$ **(b)** $g(a) = -15$ **(c)** $g(a) = 0$?

9. A function is defined as $f(x) = x^2 + 2x$.

(a) Evaluate: **(i)** $f(3)$ **(ii)** $f(-2)$.

(b) Find $f(a+3)$ in its simplest form.

10. A function is defined as $h(a) = 33 - 6a$.

(a) Evaluate: **(i)** $h(4)$ **(ii)** $h(-1)$.

(b) Given that $h(t) = 0$, find the value of t.

(c) Express $h(p-2)$ in its simplest form.

1.2 WORKING with LINEAR EQUATIONS and INEQUATIONS

1. Solve :

(a)	$x + 3 = 5$	**(b)**	$x + 5 = 9$	**(c)**	$x + 9 = 12$	**(d)**	$x + 2 = 7$
(e)	$a + 2 = 4$	**(f)**	$y + 3 = 8$	**(g)**	$p + 7 = 11$	**(h)**	$c + 4 = 5$
(i)	$b - 7 = 9$	**(j)**	$q - 8 = 8$	**(k)**	$d - 5 = 10$	**(l)**	$x - 1 = 6$
(m)	$c - 4 = 6$	**(n)**	$p - 6 = 14$	**(o)**	$a - 2 = 15$	**(p)**	$y - 5 = 14$

2. Solve:

(a)	$2x = 6$	**(b)**	$5x = 20$	**(c)**	$8x = 16$	**(d)**	$3x = 27$
(e)	$4a = 16$	**(f)**	$7y = 28$	**(g)**	$6p = 18$	**(h)**	$5c = 25$
(i)	$9b = 36$	**(j)**	$2q = 18$	**(k)**	$7d = 70$	**(l)**	$4x = 32$
(m)	$8c = 56$	**(n)**	$3p = 15$	**(o)**	$5a = 35$	**(p)**	$6y = 42$

3. Solve:

(a)	$2a = -36$	**(b)**	$-5m = -55$	**(c)**	$-8q = 64$	**(d)**	$-3y = -48$
(e)	$4x = -52$	**(f)**	$-7c = -63$	**(g)**	$-6d = 72$	**(h)**	$-5a = -125$
(i)	$9p = -81$	**(j)**	$-2q = -17$	**(k)**	$-4x = 22$	**(l)**	$-6q = -33$
(m)	$8c = -28$	**(n)**	$-5x = -90$	**(o)**	$-10a = 42$	**(p)**	$-4y = -42$

4. Solve :

(a)	$2x - 3 = 5$	**(b)**	$4x + 5 = 9$	**(c)**	$3x + 3 = -12$	**(d)**	$5x + 2 = 7$
(e)	$2a - 2 = -14$	**(f)**	$5y + 3 = 18$	**(g)**	$2p + 7 = 21$	**(h)**	$3c - 4 = 17$
(i)	$6b + 7 = 49$	**(j)**	$8q - 8 = -8$	**(k)**	$2d - 5 = 35$	**(l)**	$3x + 5 = -25$
(m)	$8c + 4 = 36$	**(n)**	$7p + 6 = 55$	**(o)**	$12a + 2 = 26$	**(p)**	$9y + 5 = 50$

5. Solve :

(a)	$3x - 2 = 7$	**(b)**	$4x - 5 = 11$	**(c)**	$2x - 9 = 3$	**(d)**	$3x - 7 = 5$
(e)	$7a - 2 = 12$	**(f)**	$5y - 3 = 22$	**(g)**	$6p - 7 = 29$	**(h)**	$4c - 3 = 29$
(i)	$8b - 7 = 57$	**(j)**	$10q - 8 = 72$	**(k)**	$3d - 5 = 31$	**(l)**	$9x - 1 = 80$
(m)	$4c - 9 = 15$	**(n)**	$6p - 2 = 40$	**(o)**	$5a - 2 = 73$	**(p)**	$3y - 14 = 40$

6. Multiply out the brackets and solve :

(a)	$2(x + 5) = 12$	**(b)**	$5(y + 7) = 45$	**(c)**	$3(a + 6) = 36$
(d)	$6(x + 4) = 54$	**(e)**	$4(x + 9) = 48$	**(f)**	$3(c + 8) = 30$
(g)	$7(d + 3) = 56$	**(h)**	$5(m + 5) = 55$	**(i)**	$2(y + 14) = 50$
(j)	$8(d - 6) = 24$	**(k)**	$3(s - 8) = 9$	**(l)**	$4(x - 15) = 20$
(m)	$10(w - 2) = 50$	**(n)**	$5(c - 5) = 35$	**(o)**	$3(a - 10) = 33$

7. Solve :

(a)	$6y + 3 = y + 18$	**(b)**	$5a + 7 = a + 15$
(c)	$9c + 5 = c + 21$	**(d)**	$10x + 1 = 4x + 19$
(e)	$5b + 3 = 2b + 9$	**(f)**	$7n + 6 = 3n + 18$
(g)	$3x + 2 = x + 14$	**(h)**	$9c + 58 = 6c + 73$
(i)	$16 + 7y = 2y + 31$	**(j)**	$15a + 4 = 3a + 76$
(k)	$16 + 25x = 5x + 96$	**(l)**	$6n + 3 \cdot 5 = 3n + 5$
(m)	$19b + 8 = 10b + 80$	**(n)**	$14x + 4 = 3x + 125$
(o)	$250 + 3x = 295$	**(p)**	$20y + 4 = 3y + 55$
(q)	$13a + 6 = a + 150$	**(r)**	$50x + 40 = 10x + 200$
(s)	$19y + 3 = 8y + 80$	**(t)**	$5b + 2 = 2b + 50$
(u)	$2 + 14x = 2x + 110$	**(v)**	$20x + 11 = 13x + 60$
(w)	$19x + 10 = 4x + 70$	**(x)**	$205a + 13 = 10a + 403$

8. Solve :

(a)	$6y - 3 = 3y + 15$	**(b)**	$5a - 9 = a + 15$
(c)	$9c - 8 = 4c + 12$	**(d)**	$10x - 1 = 4x + 5$
(e)	$5b - 3 = 2b + 9$	**(f)**	$3n - 10 = n + 2$
(g)	$7x - 14 = 3x + 2$	**(h)**	$6c - 13 = 3c + 59$
(i)	$7y - 16 = 2y + 34$	**(j)**	$15a - 8 = 3a + 76$

(k) $25x - 16 = 5x + 84$ **(l)** $6n - 3 \cdot 5 = 3n + 4$

(m) $b + 13 = 9b - 7$ **(n)** $3x + 12 = 4x - 4$

(o) $x + 25 = 3x - 5$ **(p)** $5y + 4 = 20y - 26$

(q) $a + 6 = 13a - 18$ **(r)** $10x + 40 = 50x - 120$

(s) $8y + 3 = 19y - 74$ **(t)** $2b + 2 = 5b - 16$

(u) $2 + 2x = 10x - 14$ **(v)** $13x + 11 = 20x - 38$

(w) $4x + 10 = 9x - 50$ **(x)** $10a + 13 = 20a - 387$

9. Solve :

(a) $x + 4 > 5$ **(b)** $x + 6 > 9$ **(c)** $x + 8 > 12$ **(d)** $x + 3 > 7$

(e) $a + 1 > 4$ **(f)** $y + 5 > 8$ **(g)** $p + 2 > 11$ **(h)** $c + 4 > 5$

(i) $b + 3 > 9$ **(j)** $q + 8 > 8$ **(k)** $d + 7 > 10$ **(l)** $x + 2 > 6$

(m) $c + 1 > 6$ **(n)** $p + 4 > 13$ **(o)** $a + 3 > 15$ **(p)** $y + 2 > 14$

10. Solve :

(a) $x + 5 < 7$ **(b)** $x + 1 < 8$ **(c)** $x + 3 < 13$ **(d)** $x + 5 < 9$

(e) $a + 3 < 6$ **(f)** $y + 5 < 11$ **(g)** $p + 2 < 10$ **(h)** $c + 1 < 5$

(i) $b + 8 < 13$ **(j)** $q + 3 < 20$ **(k)** $d + 7 < 7$ **(l)** $x + 10 < 15$

(m) $c + 3 < 9$ **(n)** $p + 2 < 16$ **(o)** $a + 4 < 15$ **(p)** $y + 9 < 10$

11. Solve:

(a) $2x > 6$ **(b)** $5x > 20$ **(c)** $8x > 16$ **(d)** $3x > 27$

(e) $4a > 16$ **(f)** $7y > 28$ **(g)** $6p > 18$ **(h)** $5c > 25$

(i) $9b < 36$ **(j)** $2q < 18$ **(k)** $7d < 70$ **(l)** $4x < 32$

(m) $8c < 56$ **(n)** $3p < 15$ **(o)** $5a < 35$ **(p)** $6y < 42$

12. Solve :

(a)	$x-3<4$	**(b)**	$x-5>1$	**(c)**	$x-9>2$	**(d)**	$x-2<7$
(e)	$a-2<4$	**(f)**	$y-3>8$	**(g)**	$p-7<11$	**(h)**	$c-4>5$
(i)	$b-7>9$	**(j)**	$q-8<8$	**(k)**	$d-5>10$	**(l)**	$x-1>6$
(m)	$c-4>6$	**(n)**	$p-6<14$	**(o)**	$a-2<15$	**(p)**	$y-5<14$

13. Solve :

(a)	$2x+1<5$	**(b)**	$4x+1>9$	**(c)**	$3x+3>12$	**(d)**	$5x+2>12$
(e)	$2a+2<8$	**(f)**	$5y+3<13$	**(g)**	$2p+5>21$	**(h)**	$3c+1<16$
(i)	$6b+13>49$	**(j)**	$8q+8<8$	**(k)**	$3d+5<35$	**(l)**	$4x+5>21$
(m)	$8c+12<36$	**(n)**	$7p+6<55$	**(o)**	$12a+2>26$	**(p)**	$9y+23<50$

14. Solve :

(a)	$3x-1>8$	**(b)**	$4x-3>13$	**(c)**	$2x-7<5$	**(d)**	$3x-5>4$
(e)	$7a-1<13$	**(f)**	$5y-2<23$	**(g)**	$6p-5>31$	**(h)**	$4c-7>25$
(i)	$8b-3>61$	**(j)**	$10q-7<73$	**(k)**	$3d-2<34$	**(l)**	$9x-8>73$
(m)	$4c-5<19$	**(n)**	$6p-1<41$	**(o)**	$5a-4<71$	**(p)**	$3y-24<30$

15. Solve each of the following inequations where x can only take values from the set of numbers ….. $\{-2,-1,0,1,2,3,4,5\}$.

(a)	$6x+2\le 3x+5$	**(b)**	$7x\ge 13x+3$
(c)	$3(2x+1)\ge 5x+8$	**(d)**	$2(6+5x)<8x+12$
(e)	$14-2(3-x)\le 8$	**(f)**	$5+3(2-x)\ge 14-6x$
(g)	$2x-(4-x)<x+2$	**(h)**	$3-4(2+x)>6(2-x)-17$

16. Solve each of the following inequations.

(a)	$3a+2\le 17-2a$	**(b)**	$7(2x+3)>8x+27$
(c)	$2(5p-12)\ge 7p-18$	**(d)**	$40+3k<28-k$
(e)	$3-5(2-m)\le 2(m+7)$	**(f)**	$3(2y-4)-1>4(4-y)$
(g)	$2(3-4h)<13-15h$	**(h)**	$2-3(2-x)>2(1-x)-5$

17. Solve each of the following inequations.

(a) $2a + 18 \leq 12 + 4a$ **(b)** $14 - 3x > x + 6$

(c) $3(p - 2) \geq 5p - 10$ **(d)** $16 - 3k < 20 - k$

(e) $7(2 - d) \leq 2(d - 12)$ **(f)** $2(2y - 1) - 8 > 10(1 + y)$

(g) $4(3 - 4h) < 12 + h$ **(h)** $3(2 - y) > 2(1 + 3y) - 7$

18. I think of a whole number, treble it and subtract 3. The answer must be less than or equal to 12.

Form an inequation and solve it to find the possible starting whole numbers.

19. I subtract a whole number from 8 and double the answer. The result must be greater than 10.

Form an inequation and solve it to find the possible starting whole numbers.

20. Fred and Jane are brother and sister. Fred is 3 years older than twice Jane's age.

The sum of their ages is less than 36 years.

Taking Jane's age to be x years form an inequation. What can you say about Jane's age?

1.3 WORKING WITH SIMULTANEOUS EQUATIONS

GRAPHICAL SOLUTION

1. **(a)** Copy and complete the tables below.

Table 1 : y = 9 – *x*

x	0	3	7
y		6	

Table 2 : y = *x* – 1

x	2	5	7
y	1		

(b) Plot the points from table 1. Join them carefully with a straight line.

(c) Plot the points from table 2 on the same graph. Join them with a straight line.

(d) Write down the coordinates of the points where the lines cross.

2. **(a)** Copy and complete the tables below.

Table 1 : y = 8 – *x*

x	0	3	7
y		5	

Table 2 : y = *x* – 2

x	2	5	7
y	0		

(b) Plot the points from table 1. Join them carefully with a straight line.

(c) Plot the points from table 2 on the same graph. Join them with a straight line.

(d) Write down the coordinates of the points where the lines cross.

3. Repeat the questions above for

(a) $y = 7 - x$ and $y = x - 1$ **(b)** $y = 14 - x$ and $y = x - 8$

(c) $y = x - 3$ and $y = 15 - x$ **(d)** $y = x - 7$ and $y = 17 - x$

(e) $y = 12 - x$ and $y = x - 4$ **(f)** $y = 30 - x$ and $y = x - 10$

(g) $y = 18 - x$ and $y = x - 12$ **(h)** $y = 11 - x$ and $y = x - 5$

(i) $x + y = 10$ and $x - y = 4$ **(j)** $x - y = 9$ and $x + y = 17$

4. Solve the following simultaneous equations "**graphically**".

(a) $x + y = 6$, $2x + y = 8$

Draw axes with x and y from 0 to 8

(b) $x + 2y = 8$, $3x + y = 9$

Draw axes with x and y from 0 to 9

(c) $x + 3y = 6$, $x - y = 2$

Draw axes with x from 0 to 8 and y from –2 to 4

(d) $2x + 3y = 12$, $x + y = 5$

Draw axes with x and y from 0 to 7

(e) $3x + 4y = 24$, $3x + 2y = 18$

Draw axes with x and y from 0 to 9

(f) $5x + y = 10$, $x - y = -4$

Draw axes with x from -4 to 4 and y from 0 to 10

5. Find the value of x and y by drawing the graphs of the following pairs of equations.

(a) $3y - x = 9$, $x + y = 11$

(b) $2x - 3y = 6$, $x + 2y = 10$

(c) $x + 2y = 10$, $2x + y = 8$

(d) $x - 2y = -2$, $2x - y = 2$

(e) $x - y = 7$, $3x - 2y = 24$

(f) $3x + 2y = 6$, $x - 2y = 10$

(g) $2y - x = 8$, $3y + x = 17$

(h) $x + y = 2$, $2x - y = 4$

(i) $x - 2y = 3$, $x + y = 0$

(j) $2y - 3x = 0$, $x - y = -2$

(k) $x - y = 2$, $2x + 3y = 4$

(l) $x + y = 0$, $2x + 3y = 6$

(m) $2x + 3y = 4$, $x - 2y = 9$

(n) $3x - 2y = 3$, $x + y = -4$

(o) $5x - y = 6$, $3x + 2y = 1$

ALGEBRAIC SOLUTION

1. Solve each pair of equations below using the method of substitution.

(a) $y = x$ and $3x - y = 10$ **(b)** $y = x$ and $5x - y = 4$

(c) $y = 2x$ and $5x + y = 14$ **(d)** $y = 2x$ and $2x + 3y = 24$

(e) $y = 3x + 1$ and $y = x + 7$ **(f)** $y = 5x - 4$ and $y = 2x + 11$

2. Solve the following pairs of simultaneous equations:

(a) $x + y = 4$, $x - y = 2$ **(b)** $x + y = 9$, $x - y = 5$ **(c)** $x + y = 7$, $x - y = 3$

(d) $x + y = 1$, $x - y = 3$ **(e)** $x + y = 3$, $x - y = 9$ **(f)** $x + y = -1$, $x - y = 9$

(g) $x + y = -5$, $x - y = -1$ **(h)** $x + y = -14$, $x - y = -8$ **(i)** $x + y = -18$, $x - y = 2$

3. Solve the following pairs of simultaneous equations:

(a) $2x + 4y = 24$, $7x - 2y = 4$ **(b)** $4a - 3b = 18$, $2a + 6b = -6$ **(c)** $2e + 7f = 26$, $8e - 5f = 38$

(d) $5x + y = -2$, $3x + 2y = 3$ **(e)** $2x - 3y = 10$, $3x - 6y = 18$ **(f)** $4p + 3q = 1$, $8p + 5q = -1$

(g) $2g + 3h = 1$, $5g - 2h = -26$ **(h)** $-2x + 3y = 6$, $9x - 7y = -1$ **(i)** $2u + 4v = -16$, $11u - 7v = -1$

(j) $2x - 8y = 0$, $5x - 5y = 15$ **(k)** $3p + 2q = -11$, $4p + 3q = -14$ **(l)** $10a - 3b = 46$, $6a - 8b = 40$

4. Solve the following pairs of simultaneous equations:

(a) $x+3y=17$, $3x-2y=-4$ **(b)** $a-3b=6$, $3a+b=8$ **(c)** $2e+f=1$, $5e-2f=-20$

(d) $5x+3y=7$, $4x+y=0$ **(e)** $2x-5y=-14$, $x-2y=-5$ **(f)** $2p+3q=6$, $4p+q=-8$

(g) $2g+h=11$, $7g-8h=96$ **(h)** $3x-2y=25$, $x+5y=-3$ **(i)** $u-4v=10$, $9u-2v=22$

(j) $2x=3y+5$, $x+5y=9$ **(k)** $3p-2q+7=0$, $4p+q=-2$ **(l)** $4a+b-30=0$, $6a+5b-38=0$

5. Solve the following pairs of simultaneous equations:

(a) $2x+y=15$, $x-y=6$ **(b)** $3x+2y=32$, $x-2y=8$ **(c)** $5x+3y=26$, $2x-3y=2$

(d) $3x+y=9$, $x+y=5$ **(e)** $4x+y=11$, $2x+y=5$ **(f)** $7x+2y=36$, $2x+2y=16$

(g) $2x-5y=-21$, $3x+10y=56$ **(h)** $3x+8y=23$, $x-4y=1$ **(i)** $3x+4y=10$, $6x+5y=17$

(j) $5x-2y=16$, $3x+4y=20$ **(k)** $7x+3y=-13$, $3x+y=-5$ **(l)** $3x-5y=8$, $x-7y=8$

6. Solve the following pairs of simultaneous equations:

(a) $5x+2y=9$, $2x-3y=-4$ **(b)** $4x+5y=7$, $7x-3y=24$ **(c)** $5x+2y=14$, $4x-5y=-2$

(d) $3x+y=16$, $2x+3y=13$ **(e)** $8x-3y=19$, $3x-2y=1$ **(f)** $5x+3y=19$, $7x-4y=43$

(g) $2x-5y=21$, $3x+2y=3$ **(h)** $2x-3y=17$, $7x-4y=40$ **(i)** $8x+2y=23$, $5x+6y=31$

(j) $2x+3y=7$, $4x+5y=12$ **(k)** $7x+2y=11$, $6x-5y=-4$ **(l)** $7x-5y=35$, $9x-4y=45$

WORKING with SIMULTANEOUS EQUATIONS in CONTEXT

1. Find two numbers whose sum is 56 and whose difference is 16.

2. Find two numbers whose sum is 22 and where twice the bigger one minus three times the smaller one is 24.

3. Two numbers are such that twice the smaller plus the larger is equal to 18 and the difference between twice the larger and the smaller is 11.

Find the two numbers.

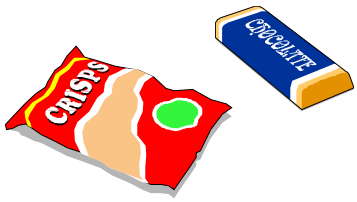
4. Two numbers are such that three times the larger plus twice the smaller is equal to 31 and the sum of twice the smaller plus the larger is 13.

Find the two numbers.

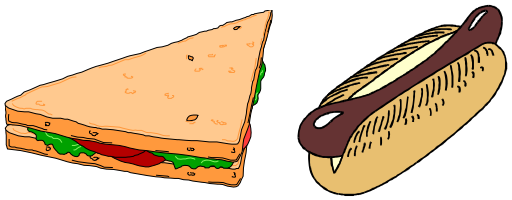
5. Four chocolate bars and six packets of crisps together cost £3.40.

Ten chocolate bars and three packets of crisps cost £4.90.

Form simultaneous equations and solve them to find the cost of each packet of crisps and each bar of chocolate.



6.



Four sandwiches and 3 hot-dogs cost £7.50.

Two sandwiches and 4 hot-dogs cost £6.

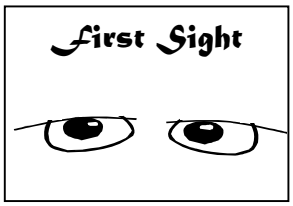
Form simultaneous equations and solve them

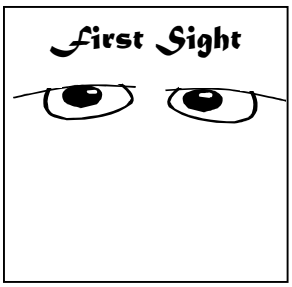
to find the cost of each sandwich and hot-dog.

7. At *Smith's Stationers*, the cost of a ruler and a pencil together is 57p. The ruler costs 23p more than the pencil.

Find the cost of each.

8. Blear's new album **First Sight** is available on CD and as a download.



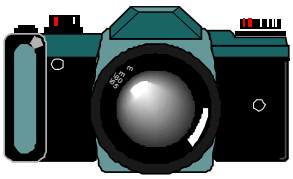


5 downloads and 4 CDs cost £97.

3 downloads and 3CDs cost £66

Calculate the cost of the download and of the CD.

9. A photographer produces 2 sizes of print, Standard and Jumbo.

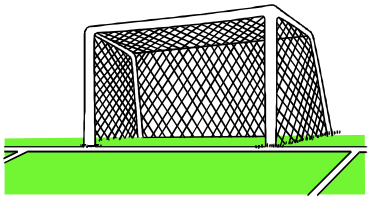


A customer who orders 24 standard and 5 jumbo prints pays £7.79

Another customer pays £8.60 for 20 standard and 8 jumbo prints.

How much would I have to pay for 1 standard and 1 jumbo print ?

10. There are 2 types of ticket on sale for a football match – Side Stand and Centre Stand.



You are sent to buy tickets for various members of your family and you pay £71.75 for 4 Side and 3 Centre tickets.

Your friend pays £75.25 for 2 Side and 5 Centre tickets.

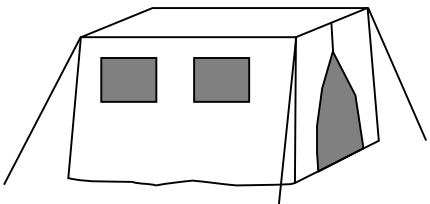
What is the price for each type of ticket?

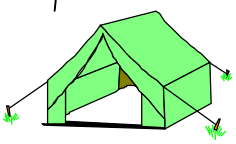
11. Two small glasses and five large glasses together contain 915 ml.

One small glass and three large glasses together hold 530 ml.

How much does each glass hold?

12.





On a camping holiday a group of 30 students take 3 frame tents and 2 ridge tents.

Another group of 25 students take 2 frame tents and 3 ridge tents.

How many people does each type of tent hold ?

13. A magazine pays different rates for ***Star Letters*** and ***Readers' Letters***.

In June the magazine editor paid out £195 for 3 Star Letters and 8 Readers' Letters.

In July £215 was paid out for 2 Star Letters and 11 Readers' Letters.

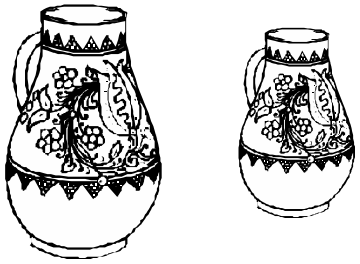
How much does the magazine pay for each type of letter?

14. Brian is a potter and is making 2 different sizes of vase.

Five small vases and four large ones require 17 kg of clay.

Three small vases and two large vases take 9·4 kg of clay.

How much clay is needed for each size of vase?



15. Karen is in charge of ordering the lunches in the office where she works.

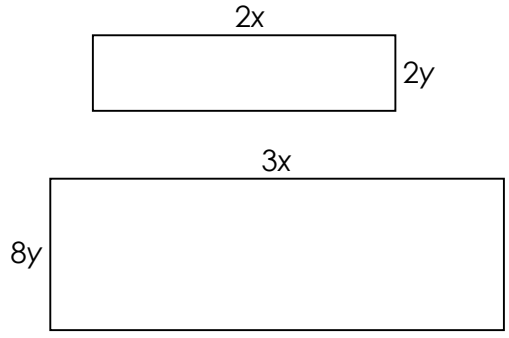
She keeps a note of what she orders and the total costs.

She thinks she has been wrongly charged on one of the days.

By forming and solving pairs of equations, find out if she is correct or not.

	Burger Meals	Chicken Meals	Total Cost(£)
Monday	7	8	29.70
Tuesday	3	12	30.30
Wednesday	8	3	21.35
Thursday	4	7	20.85
Friday	6	6	23.70
Saturday	5	10	30.00

16. Look at the two rectangles opposite.



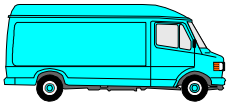
The smaller one has a perimeter of 60cm.

The larger one has a perimeter of twice the smaller.

(a) Form two equations and solve them simultaneously to find the values of x and y.

(b) Hence calculate the area of the smaller rectangle.

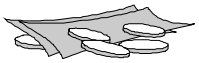
17. A van is carrying eight identical boxes and five identical parcels.



(a) If 3 boxes and 2 parcels weigh a total of 22kg and 4 boxes and 3 parcels weigh 30kg, find the weight of an individual box and a single parcel.

(b) What is the total weight carried by the van?

18. 3 pounds of butter and 4 pints of milk costs £3.84.



5 pounds of butter and 7 pints of milk costs £6.48.

Find the cost of a pound of butter and a single pint of milk.

19. In a certain factory, the basic rate of pay is £4.50 per hour, with overtime at £6.40.

Paul's total wage for a certain week was £215.80.

If he worked a total of 45 hours in all, how many hours did he work at the basis rate?

20. At a concert 500 tickets were sold. Cheap tickets cost £5 whereas more expensive ones cost £9.

If the total receipts were £3 220, how many cheap tickets were sold?

21. John saves money by putting every 50p and every 20p coin he receives in a box. After a while he discovers that he has 54 coins amounting to £17.10.

How many of each coin does he have?

WORKING WITH SIMULTANEOUS EQUATIONS EXAM QUESTIONS

1. A small printing company sends out letters to customers every day.

On Monday they sent out 20 first class letters and 15 second class letters and the charge for postage was £19·50.

On Tuesday they sent out 18 first class letters and 25 second class letters and the charge was £23.30.

How much will it cost on Wednesday to send 10 first class letters and 30 second class?

2. A concert hall sells two types of tickets, stall tickets and balcony tickets. When **all** seats are sold the concert hall holds a total of 640 people.

(a) Let ***s*** be the number of stall tickets and ***b*** the number of balcony tickets.

From the information above write down an equation connecting ***s*** and ***b***.

(b) On a particular night a concert is sold out (all seats are taken) with stall tickets priced at £8.50 and balcony tickets at £12.20. The total takings at the box office for that night was £6143.

From this information write down a second equation connecting ***s*** and ***b***.

(c) Hence find how many stall and balcony seats are in this concert hall.

3. In a fast food restaurant Ian buys 3 burgers and 4 portions of French fries and it costs £5.64.

Sarah buys 2 burgers and 3 portions of French fries and it costs £4.01.

Jack had a voucher to receive one burger and one portion of fries for free.

How much would it cost Jack for 5 burgers and 3 portions of French fries?

4. A hotel owner is buying some new duvets for his hotel.

One week he bought 7 double duvets and 12 single duvets which cost £168.

The next week he bought 4 double duvets and 9 singles for £111.

The hotel owner was given a 14% discount on his next order for 5 double duvets and 5 single duvets.

How much did he pay for this third order?

5. Find the point of intersection of the lines with equations

$$5x - 2y = 16 \text{ and } 3x + 5y = -9$$

6. Clare has baked 60 scones to sell at the school fayre. Some are fruit scones (f) and some are treacle scones (t).

(a) Write down an equation using f and t to illustrate this information.

She sells the fruit scones for 25p and the treacle scones for 20p each.

She sells all the scones for a total of £13.25.

(b) Write down another equation using f and t to illustrate this information.

(c) Hence, find **algebraically** the number of treacle scones Clare sold.

7. At the funfair coloured tokens are awarded as prizes in some of the games. These tokens can be saved up and exchanged for larger items.

3 green tokens and 4 red tokens have a total value of 26 points.

5 green tokens and 2 red tokens have a total value of 20 points.

Dave has 10 green tokens and 10 red tokens.

Does he have enough points to exchange for a large soft toy with a points value of 75?

8. In a week Peter downloads 5 tracks and 4 films and pays £21.23.

In the same week Frank downloads 7 tracks and 3 films and pays £18.49.

Calculate how much Richard would pay if he downloaded 3 tracks and 2 films.

9. Solve, **algebraically**, the equations

$$3x + 2y = 13$$

$$x = y + 1$$

10. Find the point of intersection of the lines with equations:

$$3x - 4y = 18$$

$$2y - 5x = -16$$

11. In the Garden centre there are 2 types of plants on special offer.

This week's specials!

Rose bushes

and

Poppy plants

Carly bought 3 Rose bushes and 2 Poppy plants which cost £15.23

Steph paid £26.71 for 4 Poppy plants and 5 Rose bushes.

How much would Sally pay for a Rose bush and 3 Poppy plants?

12. Peter is buying new furniture for his flat.

Two sofas and one chair will cost him £1145.

For £1310 he can buy one sofa and three chairs.

Find the cost of one sofa and the cost of one chair.

13. Eric orders goods from a mail-order company. 5 books and 2 CDs cost £40·80. 2 books and 3 CDs cost £37·78. Each order includes £2·95 post and packing regardless of the size of the order.

How much would it cost Eric to have 3 books and 1CD and have them delivered?

14. Shereen goes shopping in the summer sales.

The store has an advert in the window.

All tops one price!

SALE

All skirts one price!

Shereen buys 2 tops and 3 skirts and pays £33·90. Her friend Nadia buys 3 tops and 4 skirts and £46·70.

Another friend Kay buys 2 tops and 2 skirts. How much does she pay?

15. Find the point of intersection of the straight lines with these equations.

$$4x + 3y = 7$$

$$y = 2x + 9$$

1.4 CHANGING the SUBJECT of a FORMULA

1. Change the subject of each formula to x.

(a)	$y = x + 3$	**(b)**	$y = x - 5$	**(c)**	$y = x + a$
(d)	$y = x - b$	**(e)**	$y = 3x$	**(f)**	$y = 10x$
(g)	$y = kx$	**(h)**	$y = ax$	**(i)**	$y = 3p + x$
(j)	$y = x - 5t$	**(k)**	$y = 2x + 1$	**(l)**	$y = 3x - 7$
(m)	$y = 7x + 4a$	**(n)**	$y = 3b + 4x$	**(o)**	$y = 8 + 10x$

2. Make a the subject of each formula.

(a)	$b = 4 - a$	**(b)**	$d = 12 - a$	**(c)**	$y = 5x - a$
(d)	$m = 2 - 2a$	**(e)**	$q = 7 - 5a$	**(f)**	$c = 20 - 3a$
(g)	$r = s - 2a$	**(h)**	$t = d - 4a$	**(i)**	$z = 4b - 5a$
(j)	$k = 2h - 7a$	**(k)**	$p = 6q - 11a$	**(l)**	$g = 2x - 9a$

3. Make x the subject of each formula below.

(a)	$y = ax + b$	**(b)**	$y = mx + c$	**(c)**	$t = sx - r$
(d)	$p = qx + 2r$	**(e)**	$m = fx - 3n$	**(f)**	$a = b + cx$
(g)	$k = h - mx$	**(h)**	$d = 3b + cx$	**(i)**	$g = kc - hx$

4. Change the subject of each formula to the letter shown in brackets.

(a)	$P = 4l$	(l)	**(b)**	$V = IR$	(I)	**(c)**	$S = DT$	(T)
(d)	$A = lb$	(b)	**(e)**	$C = \pi d$	(d)	**(f)**	$G = UT$	(U)
(g)	$v = u + at$	(t)	**(h)**	$P = 2l + 2b$	(l)	**(i)**	$H = xy + 5m$	(y)

5. Change the subject of each formula to c.

(a) $b = {}^1/_2\, c$ **(b)** $x = {}^1/_5\, c$ **(c)** $y = {}^1/_4\, c$

(d) $m = {}^1/_6\, c$ **(e)** $k = {}^1/_9\, c$ **(f)** $d = {}^1/_{10}\, c$

(g) $a = {}^1/_2\, c + 2$ **(h)** $h = {}^1/_3\, c - 5$ **(i)** $p = {}^1/_4\, c + q$

(j) $y = {}^1/_{10}\, c - x$ **(k)** $t = {}^1/_8\, c + 2s$ **(l)** $r = {}^1/_5\, c - 3q$

6. Change the subject of each formula to x.

(a) $y = \dfrac{3}{x}$ **(b)** $d = \dfrac{c}{x}$ **(c)** $m = \dfrac{y}{x}$

(d) $s = \dfrac{a+2}{x}$ **(e)** $w = \dfrac{z-1}{x}$ **(f)** $a = \dfrac{b+c}{x}$

(g) $a = \dfrac{x+8}{9}$ **(h)** $k = \dfrac{x-5}{2}$ **(i)** $p = \dfrac{3-x}{4}$

(j) $y = \dfrac{2}{x} + 1$ **(k)** $z = \dfrac{6}{x} - 7$ **(l)** $h = \dfrac{m}{x} + k$

7. Change the subject of each formula to k.

(a) $y = \sqrt{k}$ **(b)** $x = \sqrt{k}$ **(c)** $m = \sqrt{k}$

(d) $a = \sqrt{\dfrac{k}{b}}$ **(e)** $c = \sqrt{\dfrac{k}{d}}$ **(f)** $h = \sqrt{\dfrac{k}{g}}$

(g) $s = \sqrt{\dfrac{t}{k}}$ **(h)** $q = \sqrt{\dfrac{p}{k}}$ **(i)** $w = \sqrt{\dfrac{z}{k}}$

(j) $r = k^2$ **(k)** $ab = k^2$ **(l)** $\dfrac{p}{q} = k^2$

(m) $y = x + k^2$ **(n)** $c = k^2 - d$ **(o)** $x = 3k^2 - 1$

8. Change the subject of each formula to the letter shown in brackets.

(a) $v^2 = u^2 + 2as$ (s) **(b)** $v^2 = u^2 + 2as$ (u)

(c) $V = \pi r^2 h$ (h) **(d)** $V = \pi r^2 h$ (r)

(e) $r = \sqrt{\frac{A}{\pi}}$ (A) **(f)** $L = 3 + \sqrt{6a}$ (a)

(g) $2k = \sqrt{(p+4)}$ (p) **(h)** $x^2 = \frac{4yz}{t}$ (y)

(i) $ar = \frac{1}{2}\sqrt{\frac{x}{b}}$ (b) **(j)** $st = A^2(x - 3y)$ (A)

(k) $R = A^2(x - 3y)$ (x) **(l)** $na = \sqrt{(1 - n^2)}$ (n)

(m) $d = \frac{t(n-1)}{n}$ (n) **(n)** $\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2}$ (R)

(o) $d = \frac{a^2(x+b)}{4}$ (a)

CHANGING the SUBJECT of a FORMULA — EXAM QUESTIONS

1. Change the subject of the formula to c.

$$ab = \frac{1}{2}\sqrt{\frac{x}{c^2}}$$

2. The formula for the velocity that a body must have to escape the gravitational pull of Earth is

$$V = \sqrt{2gR}$$

Change the subject of the formula to g.

3. For the formula given below, change the subject to x

$$A^2 = \sqrt{x} + 5$$

4. The formula for kinetic energy is

$$E = \tfrac{1}{2}mv^2$$

Change the subject of the formula to v.

5. Change the subject of the formula to a:

$$V = 3a^2b$$

6. Change the subject of the formula to k.

$$T = 2\pi\sqrt{\frac{m}{k}}$$

7. A formula to convert temperature from degrees Celsius to degrees Farenheit is

$$F = \frac{9}{5}C + 32$$

Change the subject of the formula to C.

8. The formula for finding the volume of a cone is given by

$$V = \frac{1}{3}\pi r^2 h$$

Change the subject of the formula to r.

9. Change the subject of the formula $m = \dfrac{k^2 + 2}{ab}$ to k.

10. Change the subject of the formula $a = b^2c + 7$ to b.

11. Change the subject of this formula to m $k = \dfrac{mn}{3}$

12. The formula for finding the volume of a sphere is given by

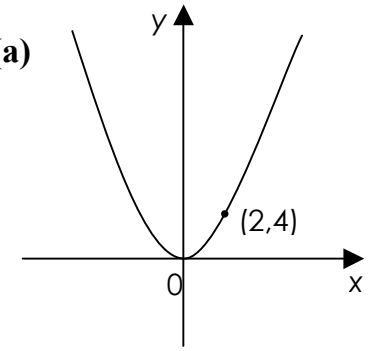
$$V = \frac{4}{3}\pi r^3$$

Change the subject of the formula to r.

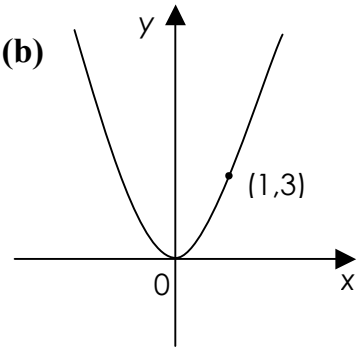
2.1 RECOGNISE and DETERMINE the EQUATIONS of QUADRATICS from their GRAPHS

1. Write down the equation of the graphs shown below, which have the form $y = kx^2$.

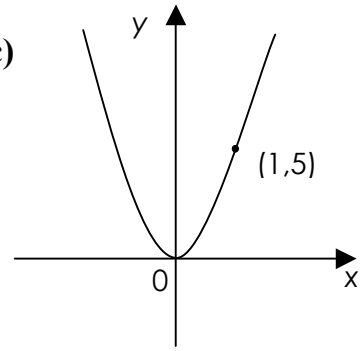
(a)



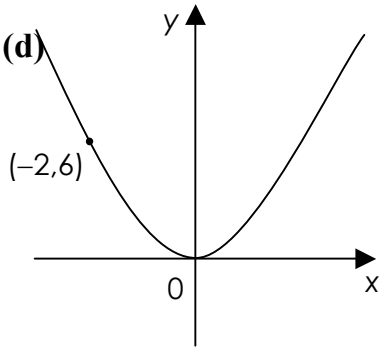
(b)



(c)



(d)



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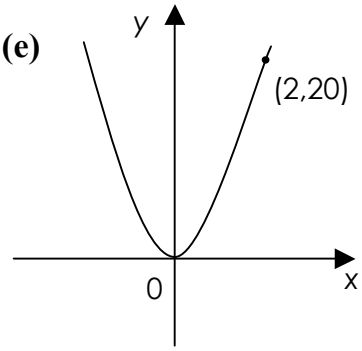
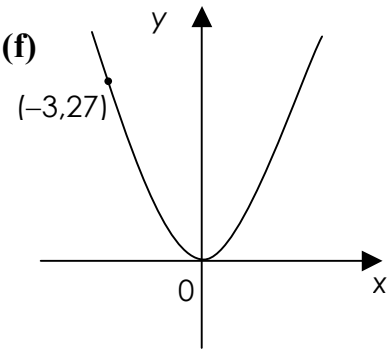


(f)



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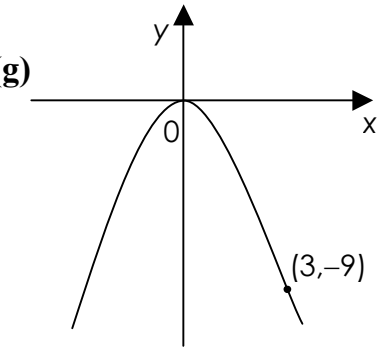
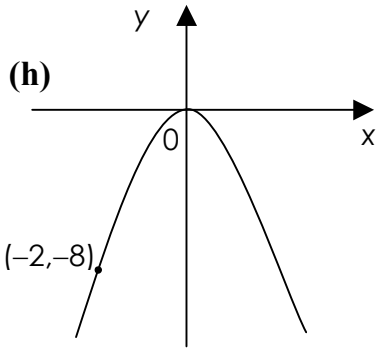
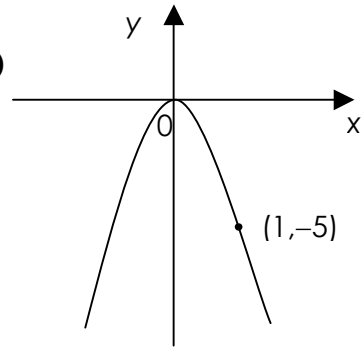


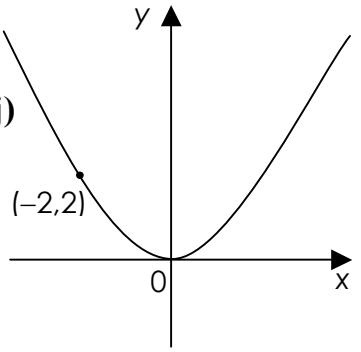
(h)



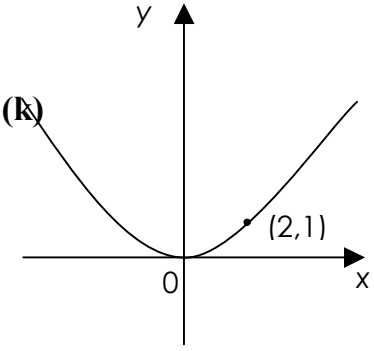
(i)



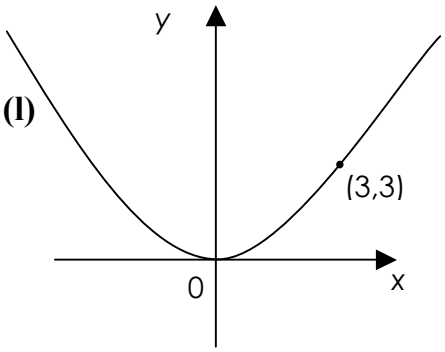
(j)



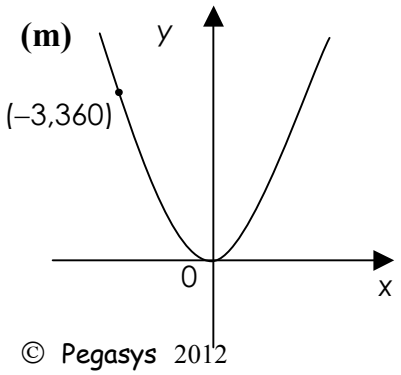
(k)



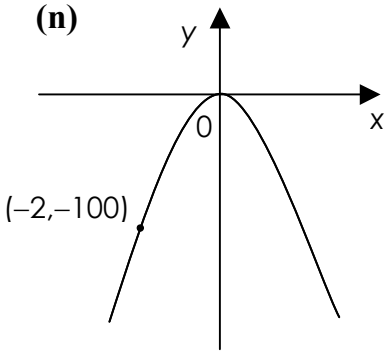
(l)



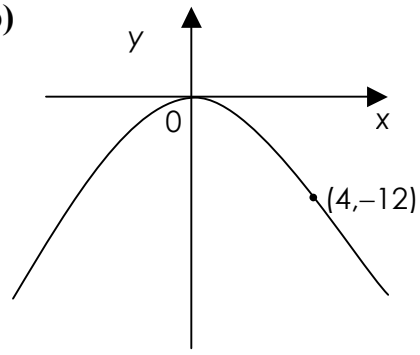
(m)



(n)

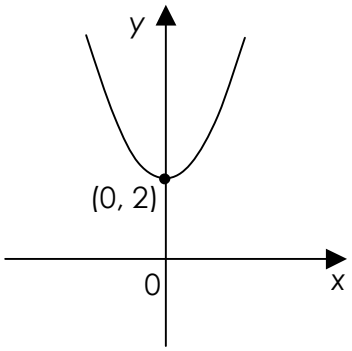


(o)

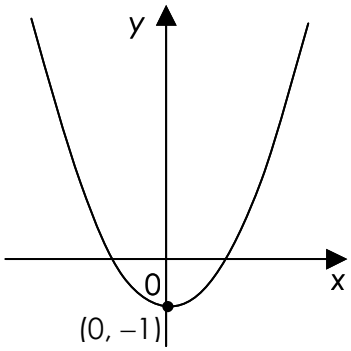


2. Write down the equation of the graphs shown below, which have the form $y = ax^2 + b$.

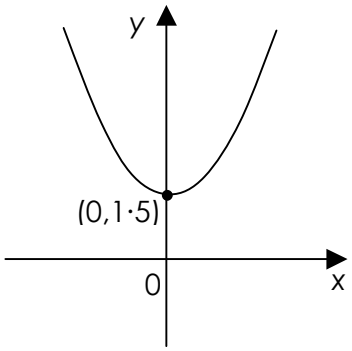
(a)



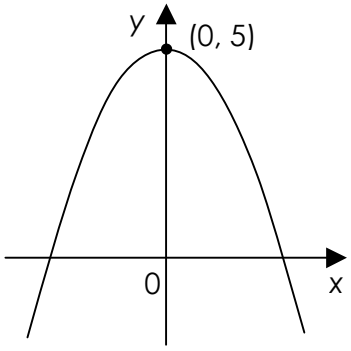
(b)



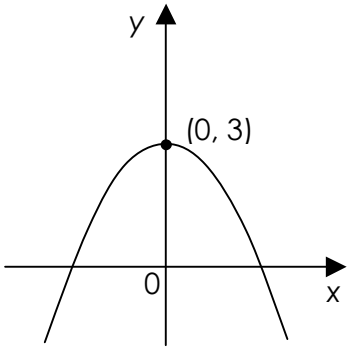
(c)



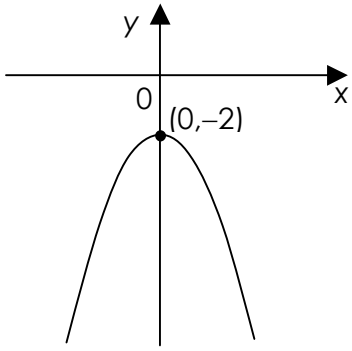
(d)



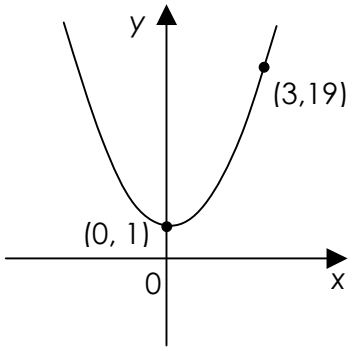
(e)



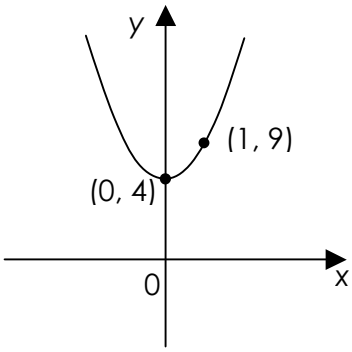
(f)



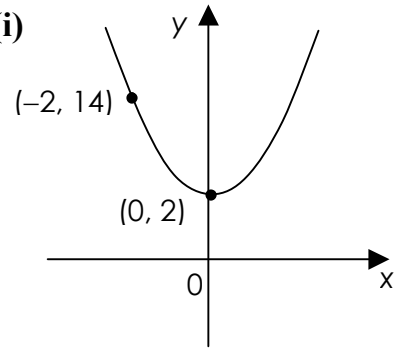
(g)



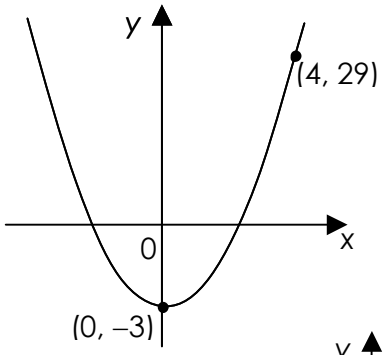
(h)



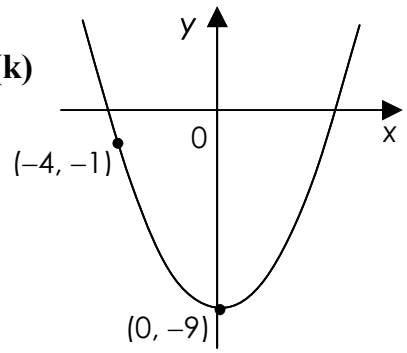
(i)



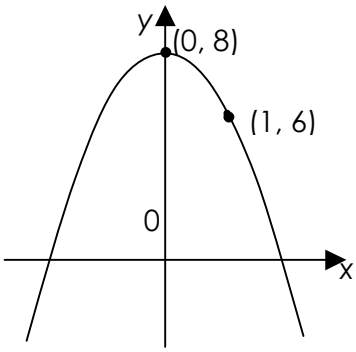
(j)



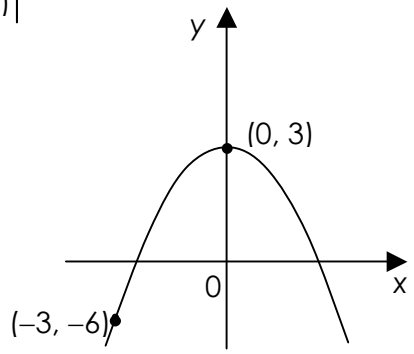
(k)



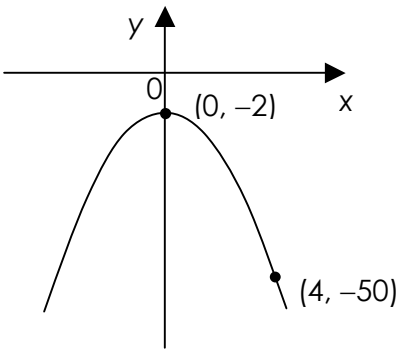
(l)



(m)

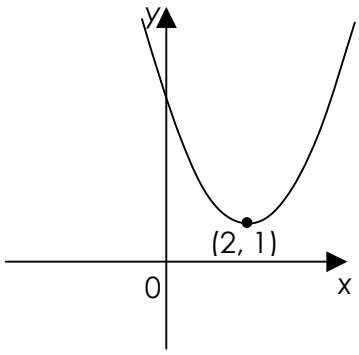


(n)

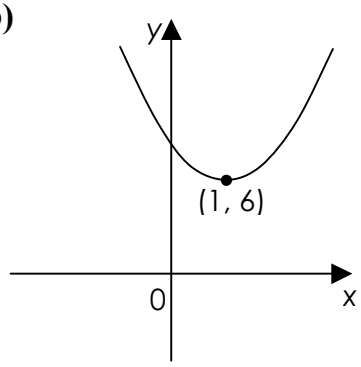


3. Write down the equation of the graphs shown below, which have the form $y = (x + a)^2 + b$.

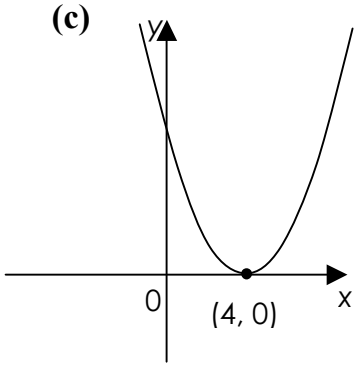
(a)



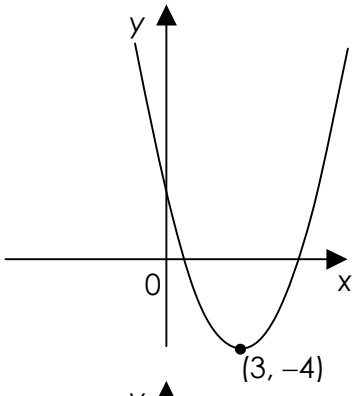
(b)



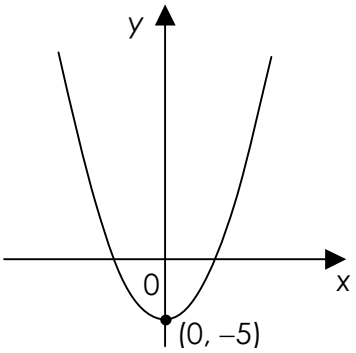
(c)



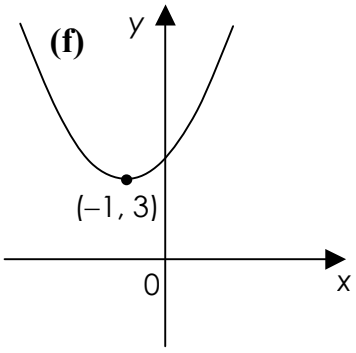
(d)



(e)



(f)



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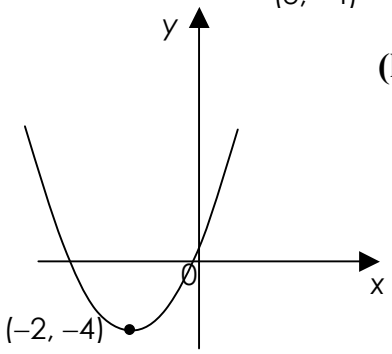
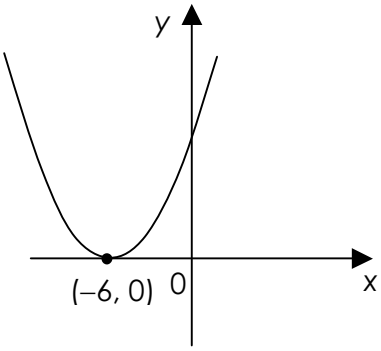
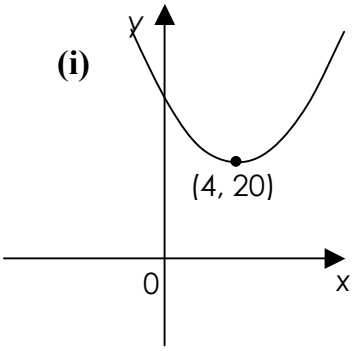


(h)



(i)



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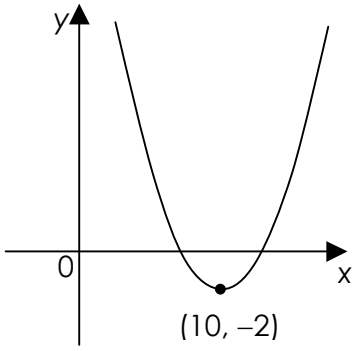
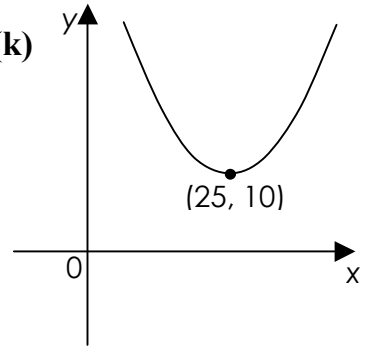
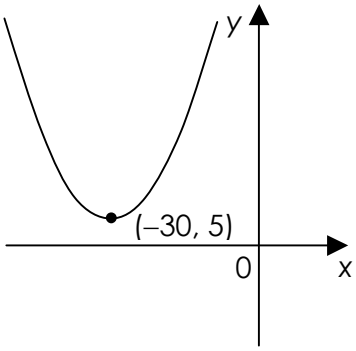


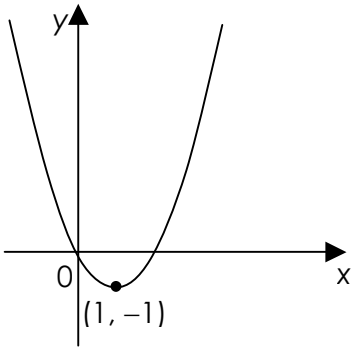
(k)



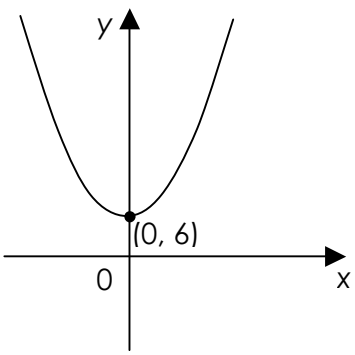
(l)



(m)



(n)



2.2 SKETCHING the GRAPH of a QUADRATIC FUNCTION

1. Sketch the graphs with the following equations

(a) $y = (x - 4)^2 + 1$ **(b)** $y = (x - 2)^2 + 5$ **(c)** $y = (x - 1)^2 + 7$

(d) $y = (x - 2)^2 - 3$ **(e)** $y = (x - 3)^2 - 4$ **(f)** $y = (x - 5)^2 - 2$

(g) $y = (x + 4)^2 + 6$ **(h)** $y = (x + 1)^2 + 5$ **(i)** $y = (x + 8)^2 + 1$

(j) $y = (x + 3)^2 - 1$ **(k)** $y = (x + ½)^2 - ¾$ **(l)** $y = (x + 0{\cdot}5)^2 - 2{\cdot}5$

(m) $y = -(x - 1)^2 + 4$ **(n)** $y = -(x + 6)^2 + 3$ **(o)** $y = -(x + 7)^2 - 2$

(p) $y = (2 - x)^2 + 12$ **(q)** $y = (5 - x)^2 - 1$ **(r)** $y = (4 - x)^2 + 3{\cdot}75$

2. Sketch the graphs with the following equations

(a) $y = (x - 1)(x - 5)$ **(b)** $y = (x - 4)(x - 2)$ **(c)** $y = (x - 3)(x - 7)$

(d) $y = (x - 6)(x - 8)$ **(e)** $y = (x - 5)(x - 2)$ **(f)** $y = (x - 8)(x - 5)$

(g) $y = (x + 2)(x + 3)$ **(h)** $y = (x + 5)(x + 2)$ **(i)** $y = (x + 4)(x + 6)$

(j) $y = (x + 3)(x + 4)$ **(k)** $y = (x + 9)(x + 5)$ **(l)** $y = (x + 3)(x + 8)$

3. Sketch the graphs with the following equations

(a) $y = (x - 1)(x + 5)$ **(b)** $y = (3 + x)(7 - x)$ **(c)** $y = -(3 + x)(5 - x)$

(d) $y = -(x + 8)(x - 4)$ **(e)** $y = (x + 1)(x - 7)$ **(f)** $y = (1 + x)(7 - x)$

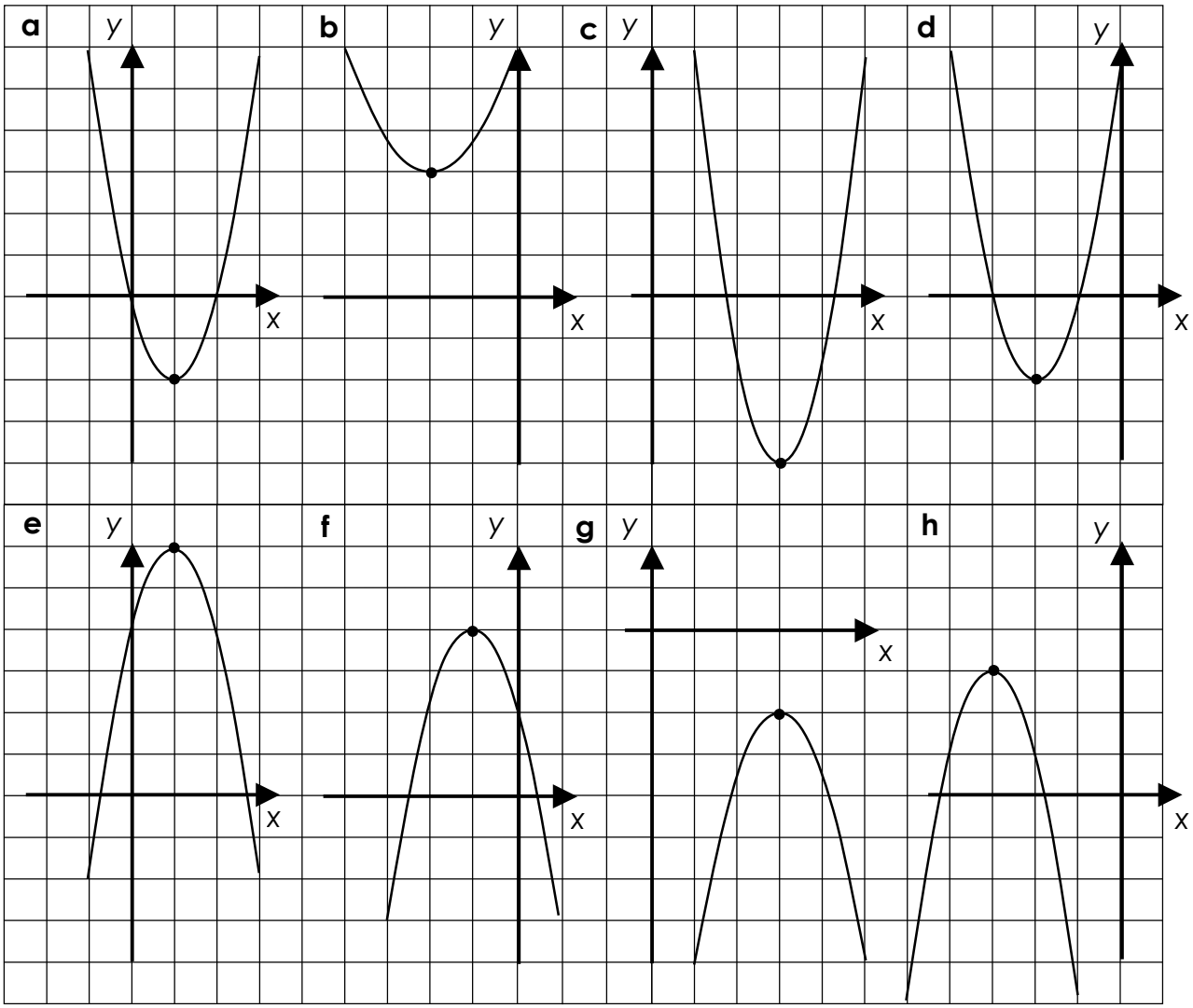
(g) $y = -(x - 3)(x + 9)$ **(h)** $y = (x - 10)(x + 2)$ **(i)** $y = -(x - 9)(x + 7)$

(j) $y = -(x + 4)(x - 6)$ **(k)** $y = (1 + x)(1 - x)$ **(l)** $y = (x + 2)(x - 6)$

(m) $y = (x - 3)(x + 3)$ **(n)** $y = -(x - 7)(x + 1)$ **(o)** $y = -(x + 10)(x - 6)$

2.3 IDENTIFYING FEATURES of a QUADRATIC FUNCTION

1. For each of the graphs below, write down **(i)** the turning point
(ii) its nature
and **(iii)** the equation of the axis of symmetry



2. For each of the equations below, write down **(i)** the turning point
(ii) its nature
and **(iii)** the equation of the axis of symmetry

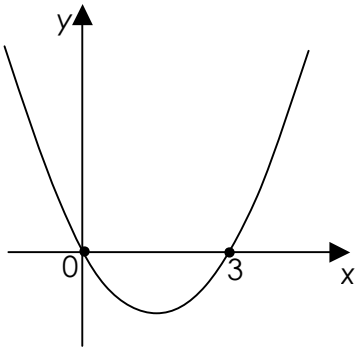
(a)	$y = (x - 4)^2 + 1$	**(b)**	$y = (x - 2)^2 + 5$	**(c)**	$y = (x - 1)^2 + 7$
(d)	$y = (x - 2)^2 - 3$	**(e)**	$y = (x - 3)^2 - 4$	**(f)**	$y = (x - 5)^2 - 2$
(g)	$y = (x + 4)^2 + 6$	**(h)**	$y = (x + 1)^2 + 5$	**(i)**	$y = (x + 8)^2 + 1$
(j)	$y = (x + 3)^2 - 1$	**(k)**	$y = (x + ½)^2 - ¾$	**(l)**	$y = (x + 0{\cdot}5)^2 - 2{\cdot}5$
(m)	$y = -(x - 1)^2 + 4$	**(n)**	$y = -(x + 6)^2 + 3$	**(o)**	$y = -(x + 7)^2 - 2$
(p)	$y = (2 - x)^2 + 12$	**(q)**	$y = (5 - x)^2 - 1$	**(r)**	$y = (4 - x)^2 + 3{\cdot}75$

2.4 WORKING with QUADRATIC EQUATIONS

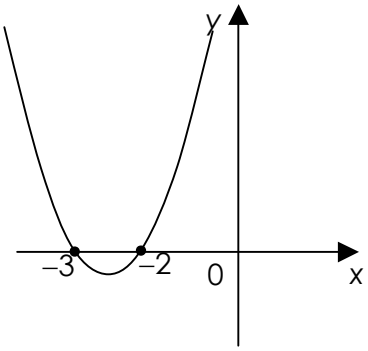
DRAWING GRAPHS

1. Use the sketches below to solve the quadratic equations.

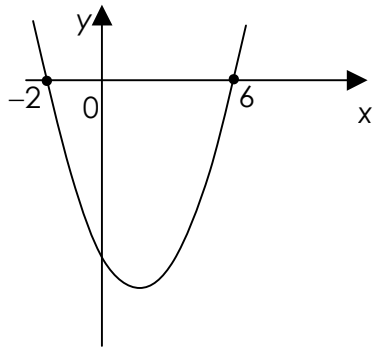
(a) $x^2 - 3x = 0$



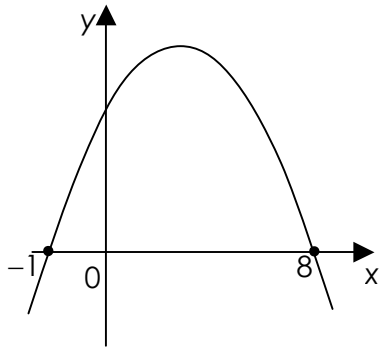
(b) $x^2 + 5x + 6 = 0$



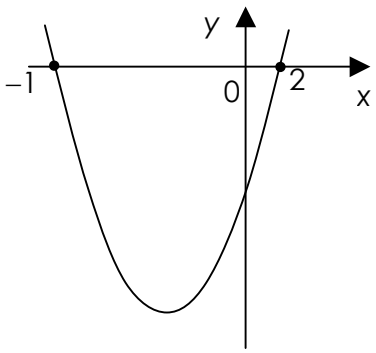
(c) $x^2 - 4x - 12 = 0$



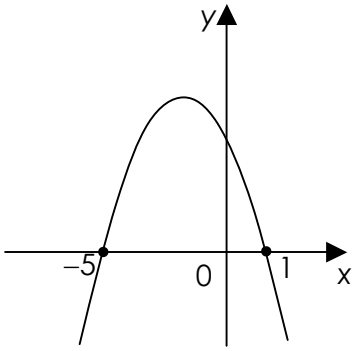
(d) $8 + 7x - x^2 = 0$



(e) $x^2 + 8x - 20 = 0$



(f) $5 - 4x - x^2 = 0$



2. **(i)** Copy and complete the tables below.
(ii) Make a sketch of the graph.
(iii) Write down the roots of the quadratic equation $y = 0$.

(a) $y = x^2 - 2x$

x	-2	-1	0	1	2	3	4
y							

(b) $y = x^2 - 6x + 5$

x	0	1	2	3	4	5	6
y							

(c) $y = x^2 + 4x + 3$

x	-5	-4	-3	-2	-1	0	1
y							

(d) $y = 8 - 2x - x^2$

x	-4	-3	-2	-1	0	1	2
y							

3. For each equation, draw a suitable sketch and find the roots.

(a)	$x^2 - 4x = 0$	**(b)**	$x^2 + 6x = 0$	**(c)**	$x^2 - 5x = 0$
(d)	$x^2 - 8x + 15 = 0$	**(e)**	$x^2 + 6x + 9 = 0$	**(f)**	$x^2 - 4x + 4 = 0$
(g)	$x^2 + 6x + 8 = 0$	**(h)**	$x^2 + 8x + 12 = 0$	**(i)**	$x^2 - 7x + 10 = 0$
(j)	$x^2 - 5x + 4 = 0$	**(k)**	$x^2 + x - 6 = 0$	**(l)**	$x^2 - x - 2 = 0$
(m)	$12 - 4x - x^2 = 0$	**(n)**	$5 + 4x - x^2 = 0$	**(o)**	$2 - x - x^2 = 0$

FACTORISING

1. Solve these quadratic equations, which are already in factorised form.

(a)	$x(x - 5) = 0$	**(b)**	$x(x + 7) = 0$	**(c)**	$x(x - 1) = 0$
(d)	$2b(b - 3) = 0$	**(e)**	$3a(a + 1) = 0$	**(f)**	$5m(m - 2) = 0$
(g)	$(a - 4)(a - 2) = 0$	**(h)**	$(y - 3)(y - 4) = 0$	**(i)**	$(c - 5)(c - 3) = 0$
(j)	$(w + 1)(w + 2) = 0$	**(k)**	$(s + 5)(s + 4) = 0$	**(l)**	$(z + 7)(z + 8) = 0$
(m)	$(x + 3)(x - 1) = 0$	**(n)**	$(t + 2)(t - 12) = 0$	**(o)**	$(y + 1)(y - 9) = 0$
(p)	$(a - 4)(a + 4) = 0$	**(q)**	$(p - 7)(p + 7) = 0$	**(r)**	$(c - 5)(c + 5) = 0$
(s)	$(d - 4)(2d - 1) = 0$	**(t)**	$(2x + 3)(x + 2) = 0$	**(u)**	$(3s + 1)(2s - 5) = 0$

2. Solve these quadratic equations by factorising first.

(a)	$x^2 + 4x = 0$	**(b)**	$c^2 - 2c = 0$	**(c)**	$y^2 + 8y = 0$
(d)	$p^2 - p = 0$	**(e)**	$z^2 + z = 0$	**(f)**	$n^2 + 7n = 0$
(g)	$2t^2 + 4t = 0$	**(h)**	$5x^2 - 20x = 0$	**(i)**	$6b^2 - 18b = 0$
(j)	$4y^2 - 6y = 0$	**(k)**	$6a^2 + 9a = 0$	**(l)**	$14x^2 + 21x = 0$
(m)	$5x - x^2 = 0$	**(n)**	$9b - b^2 = 0$	**(o)**	$2m - m^2 = 0$
(p)	$6w - 4w^2 = 0$	**(q)**	$9c - 12c^2 = 0$	**(r)**	$4y - 10y^2 = 0$

3. Solve these quadratic equations by factorising first.

(a)	$x^2 - 25 = 0$	**(b)**	$b^2 - 1 = 0$	**(c)**	$y^2 - 4 = 0$
(d)	$a^2 - 36 = 0$	**(e)**	$z^2 - 9 = 0$	**(f)**	$k^2 - 64 = 0$
(g)	$x^2 - 16 = 0$	**(h)**	$p^2 - 144 = 0$	**(i)**	$m^2 - 100 = 0$
(j)	$t^2 - 49 = 0$	**(k)**	$a^2 - 81 = 0$	**(l)**	$s^2 - 121 = 0$
(m)	$2a^2 - 18 = 0$	**(n)**	$5c^2 - 80 = 0$	**(o)**	$4y^2 - 64 = 0$

4. Solve these quadratic equations by factorising first.

(a)	$x^2 + 4x + 3 = 0$	**(b)**	$y^2 + 6y + 5 = 0$	**(c)**	$a^2 + 8a + 7 = 0$
(d)	$m^2 + 5m + 6 = 0$	**(e)**	$c^2 + 6c + 8 = 0$	**(f)**	$z^2 + 7z + 12 = 0$
(g)	$15 - 2x - x^2 = 0$	**(h)**	$b^2 - 8b + 16 = 0$	**(i)**	$x^2 - 7x + 10 = 0$
(j)	$w^2 - 12w + 27 = 0$	**(k)**	$18 + 7y - y^2 = 0$	**(l)**	$k^2 - 10k + 24 = 0$
(m)	$8 - 2x - x^2 = 0$	**(n)**	$6 + m - m^2 = 0$	**(o)**	$t^2 - 7t - 30 = 0$
(p)	$a^2 + 5a - 14 = 0$	**(q)**	$c^2 - 2c - 15 = 0$	**(r)**	$12 - 4p - p^2 = 0$

5. Solve these quadratic equations by factorising first.

(a)	$2x^2 + 7x + 5 = 0$	**(b)**	$2p^2 + 11p + 5 = 0$	**(c)**	$3t^2 + 10t + 3 = 0$
(d)	$3k^2 + 7k + 2 = 0$	**(e)**	$3y^2 + 8y + 5 = 0$	**(f)**	$6 - 7a - 5a^2 = 0$
(g)	$3 - 5w - 2w^2 = 0$	**(h)**	$3d^2 - 5d + 2 = 0$	**(i)**	$5x^2 - 16x + 3 = 0$
(j)	$3m^2 - 14m + 8 = 0$	**(k)**	$7 + 5c - 2c^2 = 0$	**(l)**	$1 - 5y - 6y^2 = 0$
(m)	$3x^2 - 2x = 1$	**(n)**	$4q^2 + 5q = 6$	**(o)**	$4t(t - 1) - 3 = 0$
(p)	$3m^2 + 2m = 5$	**(q)**	$36v^2 = -v + 2$	**(r)**	$7s^2 = 4 + 27s$

USING QUADRATIC FORMULA

1. Solve these equations using the quadratic formula.

(a)	$3x^2 + 7x + 2 = 0$	**(b)**	$2a^2 + 5a + 2 = 0$	**(c)**	$3c^2 + 8c + 5 = 0$
(d)	$2p^2 + 11p + 9 = 0$	**(e)**	$2y^2 + 11y + 5 = 0$	**(f)**	$3d^2 + 11d + 6 = 0$
(g)	$2x^2 - 7x + 3 = 0$	**(h)**	$2a^2 - 5a + 3 = 0$	**(i)**	$5p^2 - 17p + 6 = 0$
(j)	$5b^2 - 7b + 2 = 0$	**(k)**	$6x^2 - 7x + 2 = 0$	**(l)**	$4y^2 - 11y + 6 = 0$
(m)	$3x^2 - 2x - 1 = 0$	**(n)**	$2a^2 - a - 3 = 0$	**(o)**	$4p^2 - 4p - 3 = 0$
(p)	$2c^2 + 7c - 4 = 0$	**(q)**	$6y^2 - 11y - 2 = 0$	**(r)**	$3w^2 + 10w - 8 = 0$

2. Solve these equations using the quadratic formula, giving your answers correct to 2 decimal places.

(a)	$x^2 + 5x + 5 = 0$	**(b)**	$b^2 + 9b + 2 = 0$	**(c)**	$p^2 + 4p + 1 = 0$
(d)	$c^2 + 4c + 2 = 0$	**(e)**	$y^2 + 7y + 3 = 0$	**(f)**	$a^2 + 8a + 5 = 0$
(g)	$z^2 - 5z + 1 = 0$	**(h)**	$q^2 - 12q + 4 = 0$	**(i)**	$w^2 - 6w + 2 = 0$
(j)	$d^2 - 10d + 8 = 0$	**(k)**	$x^2 - 3x + 1 = 0$	**(l)**	$m^2 - 7m + 4 = 0$
(m)	$y^2 + 8y - 3 = 0$	**(n)**	$k^2 + 4k - 6 = 0$	**(o)**	$c^2 + 2c - 9 = 0$

3. Solve these equations using the quadratic formula, giving your answers correct to 2 decimal places.

(a)	$3x^2 + 8x + 5 = 0$	**(b)**	$2b^2 + 9b + 3 = 0$	**(c)**	$2p^2 + 5p + 1 = 0$
(d)	$1 + 6c - 4c^2 = 0$	**(e)**	$3y^2 + 7y + 3 = 0$	**(f)**	$5a^2 + 9a + 2 = 0$
(g)	$8z^2 - 7z + 1 = 0$	**(h)**	$3 + 12q - 4q^2 = 0$	**(i)**	$3w^2 - 6w + 2 = 0$
(j)	$5d^2 - 10d + 4 = 0$	**(k)**	$5x^2 - 7x + 1 = 0$	**(l)**	$3 + 8m - 2m^2 = 0$
(m)	$5y^2 + 8y - 2 = 0$	**(n)**	$5 - 2k - 6k^2 = 0$	**(o)**	$10c^2 + 2c - 1 = 0$
(p)	$8 - 9t - 4t^2 = 0$	**(q)**	$3 + 3a - 7a^2 = 0$	**(r)**	$2z^2 + 2z - 9 = 0$

4. Solve these equations using the quadratic formula, giving your answers correct to 3 significant figures.

(a)	$x^2 + 5x + 3 = 0$	**(b)**	$c^2 + 3c + 1 = 0$	**(c)**	$m^2 + 8m + 2 = 0$
(d)	$y^2 + 7y + 7 = 0$	**(e)**	$p^2 + 6p + 2 = 0$	**(f)**	$a^2 + 6a + 3 = 0$
(g)	$b^2 - 5b + 2 = 0$	**(h)**	$z^2 - 9z + 4 = 0$	**(i)**	$q^2 - 7q + 5 = 0$
(j)	$x^2 - 10x + 3 = 0$	**(k)**	$c^2 - 8c + 8 = 0$	**(l)**	$w^2 - 4w + 2 = 0$
(m)	$k^2 + 12k - 20 = 0$	**(n)**	$d^2 + 11d - 15 = 0$	**(o)**	$s^2 + 8s - 17 = 0$
(p)	$a^2 + 3a - 9 = 0$	**(q)**	$y^2 + 2y - 11 = 0$	**(r)**	$c^2 + 3c - 12 = 0$
(s)	$8x^2 + 8x = -1$	**(t)**	$5b^2 + 3b = 9$	**(u)**	$2p^2 - 9p = 3$
(v)	$7m^2 = 6m - 1$	**(w)**	$3x^2 = 8 - 3x$	**(x)**	$4c^2 = 9 + 3c$

MORE QUADRATICS

1. Sketch the graphs of the following quadratic functions marking all relevant points.

   Then …. for each function answer the following questions …

   (i) State the *roots* (or zeros) of the function;

   (ii) write down the equation of the *axis of symmetry*;

   (iii) state the *coordinates* and *nature* of the turning point;

   (iv) give the coordinates of the *y-intercept* point;

(a) $f(x) = x^2 + 2x - 3$ **(b)** $g(x) = x^2 - 2x - 8$ **(c)** $h(x) = x^2 - 4x - 5$

(d) $f(x) = x^2 + 6x$ **(e)** $g(x) = x^2 - 4x$ **(f)** $h(x) = 8x - x^2$

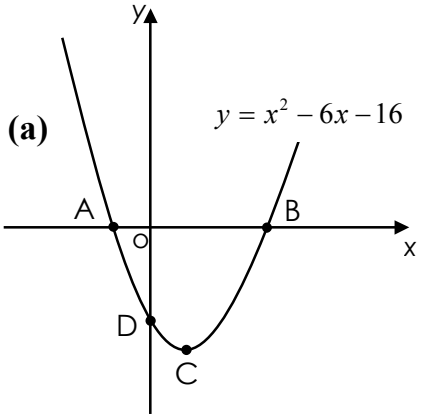
(g) $f(x) = 8 - 2x - x^2$ **(h)** $g(x) = 7 + 6x - x^2$ **(i)** $h(x) = x^2 - 10x + 21$

(j) $f(x) = x^2 - 3x - 4$ **(k)** $g(x) = x^2 + 7x + 6$ **(l)** $h(x) = 5x - x^2$

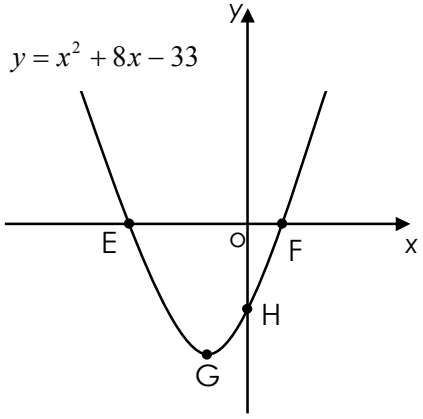
(m) $f(x) = 10 - 3x - x^2$ **(n)** $g(x) = 16 - x^2$ **(o)** $h(x) = x^2 - 9$

2. Find the coordinates of the points marked with letters in the diagrams below.

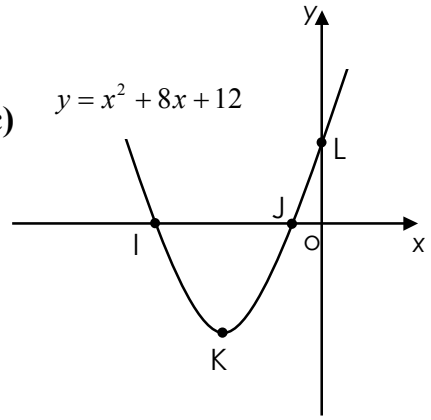
(a)



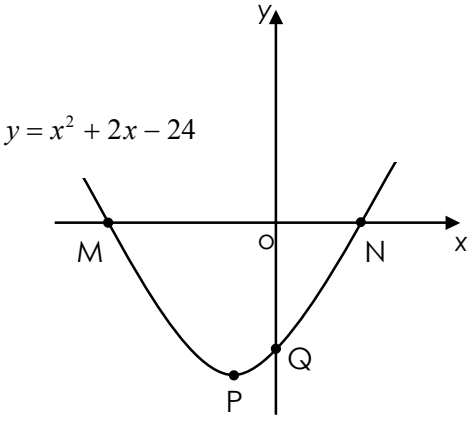
(b)



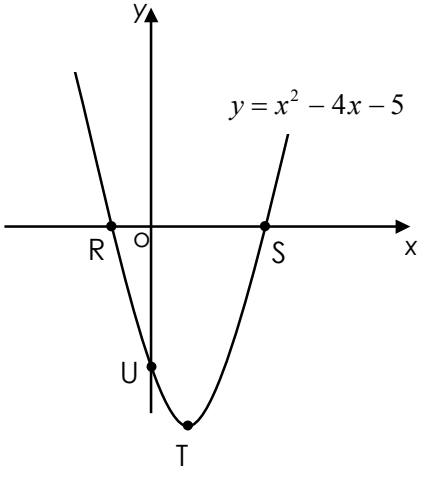
(c)



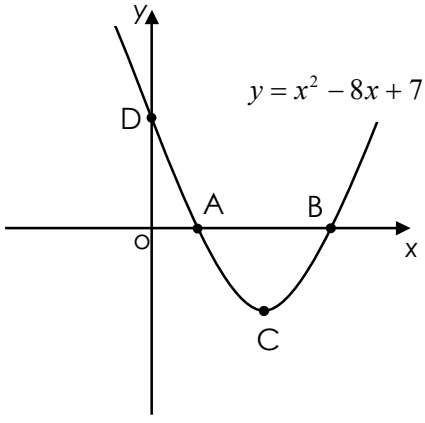
(d)



(e)

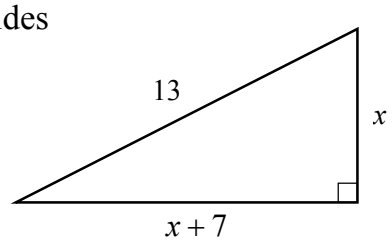


(f)



PROBLEMS INVOLVING QUADRATIC EQUATIONS

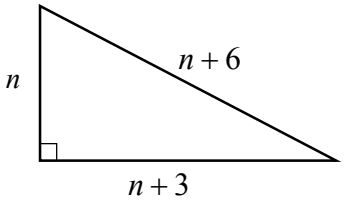
1. The diagram opposite shows a right-angled triangle with sides measuring $13, x+7$ and x centimetres.



(a) Form an equation and solve it to find x.

(b) Hence calculate the perimeter of the triangle.

2.



The sides of a right angled triangle are n, $n + 3$ and $n+6$ centimetres long.

Find n by solving an equation, and hence calculate the area of this triangle.

3. Repeat question 2. for a right angled triangle with short sides measuring n and $2n+4$ and the hypotenuse measuring $3n-4$ millimetres.

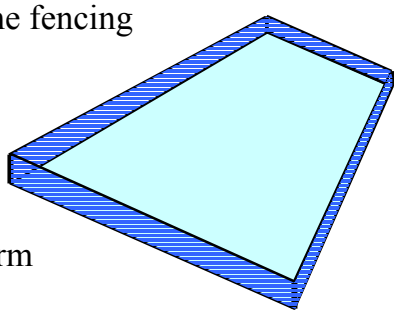
4. A rectangular sheet of glass has an area of 1500 cm^2 and a perimeter of 160 cm.

(a) Taking the length of the glass sheet as x, write down an expression for the width of the sheet in terms of x.

(b) Form an equation in x for the area of the sheet and solve it to find x.

Hence state the dimensions of the glass sheet.

5. A farmer has 260 metres of clear plastic fencing. He uses all the fencing to create a rectangular holding pen.



(a) Taking x as the length of the pen, write down an expression for the width of the pen in terms of x.

(b) Given that the area of the pen is 4000 square metres, form an equation and solve it to find x, the length of the pen.

WORKING with QUADRATICS — EXAM QUESTIONS

1. For the quadratic function $y = (x - ¾)^2 + ½$, write down:

(a) the turning point

(b) its nature

(c) the equation of the axis of symmetry

2. A quadratic function is defined by the formula $f(x) = (x + 3)^2 - 5$.

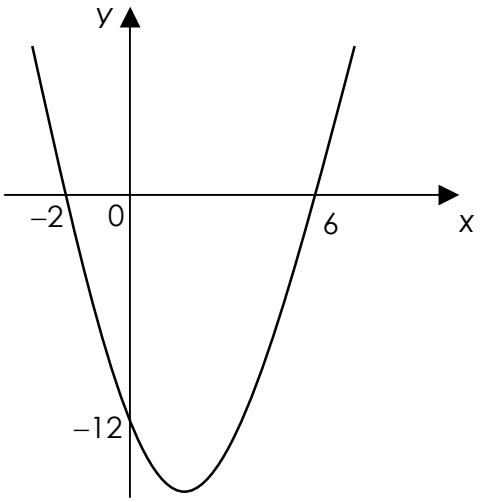
Write down the turning point of the graph of the function.

3. Solve quadratic function

$$10x^2 + 3x - 11 = 0 \quad \text{using an appropriate formula.}$$

Give your answers correct to 1 decimal place.

4. The graph shown is a function of the form $y = (x - b)(x - c)$.



(a) Establish the equation of the function.

(b) State the coordinates and nature of the turning point.

(c) Write the equation of the function in the form

$$y = p(x - q)^2 + r.$$

5. Solve the quadratic equation

$$3x^2 + 10x - 7 = 0 \quad \text{using an appropriate formula.}$$

Give your answers correct to 1 decimal place.

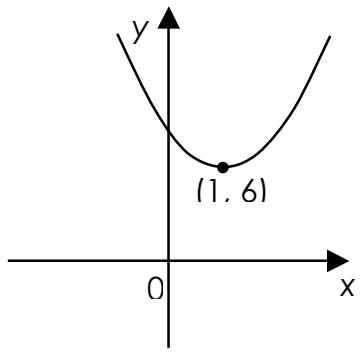
6. For the quadratic function $y = (x + ½)^2 - ¾$, write down the turning point and its nature.

7. Solve the quadratic equation

$$2x^2 - 5x - 10 = 0$$

giving the roots correct to 2 significant figures.

8.



The equation of the parabola is of the form

$$y = (x + p)^2 + q.$$

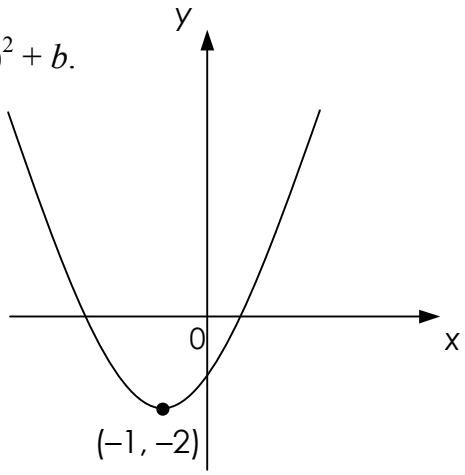
Write down the equation of the parabola.

9. Solve the quadratic equation

$$2x^2 + 11x - 8 = 0$$

Give your answers correct to 1 decimal place.

10. The graph shows a quadratic function of the form $y = (x + a)^2 + b$.



State:

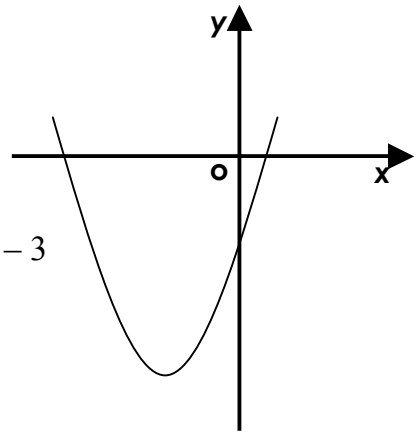
(a) The coordinates of the turning point and its nature.

(b) The equation of the axis of symmetry.

(c) The equation of the function.

11. The diagram shows the graph of $y = 2x^2 + 7x - 3$

Find the x - coordinates of the points where the graph crosses the x - axis giving your answers correct to 1 decimal place.



$y = 2x^2 + 7x - 3$

12. For the quadratic function $y = (¾ - x)^2 + \frac{5}{6}$, write down

(a) the turning point and its nature.

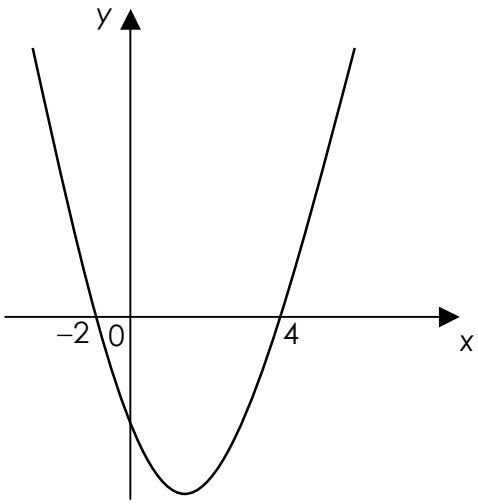
(b) the equation of the axis of symmetry

13. Solve the equation

$$2x^2 - 8x - 19 = 0$$

giving the roots correct to 3 significant figures.

14. The graph shown has the equation of the form $y = (x - b)(x - c)$.



(a) Find the equation of the parabola.

(b) Find the coordinates of where the graph cuts the y-axis.

(c) State the coordinates and nature of the turning point.

(d) State the equation of the axis of symmetry of the parabola.

15. A quadratic graph has equation $y = (x-4)^2 + 7$.

(a) What are the coordinates **and** nature of the turning point of the graph?

(b) Which of the following is the equation of its axis of symmetry?

A $x = -4$

B $x = 4$

` C $x = 7$

D $x = -7$

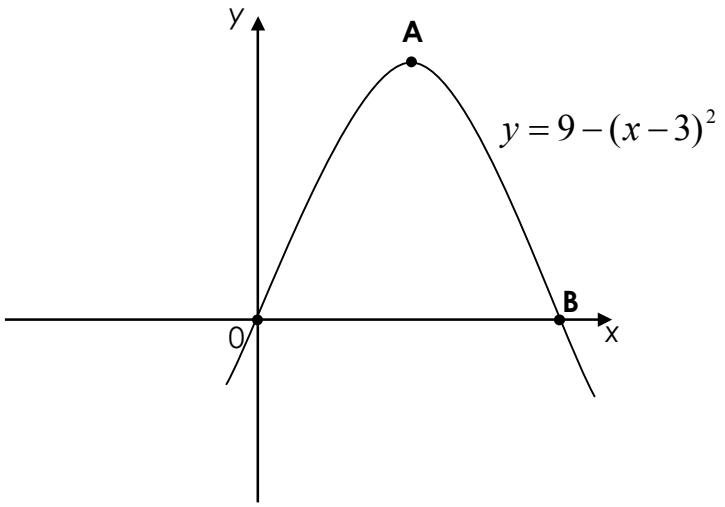
16. Solve the quadratic equation

$$3x^2 - 9x + 2 = 0$$

Give your answers correct to 1 decimal place.

17. There is an arch built over the new Wembley football stadium in London.

It can be represented on suitable axes by the parabola with equation $y = 9 - (x-3)^2$.



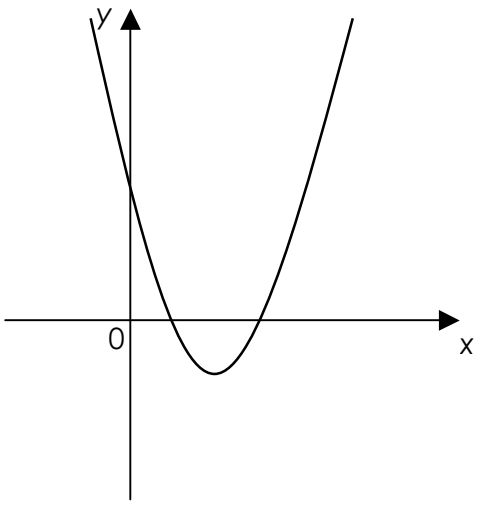
(a) Write down the coordinates of A, the maximum turning point of the parabola.

(b) What is the equation of the axis of symmetry of the parabola?

The parabola cuts the x - axis at the origin and the point B.

(c) Find the coordinates of B.

18. The graph shown is a function of the form $y = (x-3)^2 - 1$.



(a) Write down the turning point of the graph and state its nature.

(b) What is the equation of the axis of symmetry of the graph?

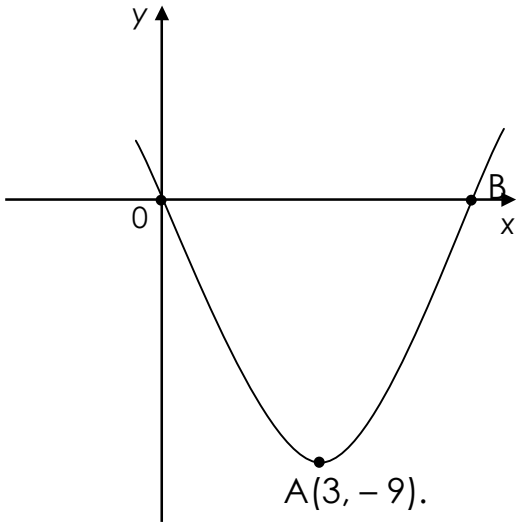
(c) Write the co-ordinates of the points where the graph cuts the x-axis.

19. A parabola has equation $y = 9 - (x+2)^2$

(a) What are the coordinates **and** nature of its turning point?

(b) Write down the equation of the axis of symmetry of the parabola,

20. The diagram shows part of the graph of the parabola which has a minimum turning point at (3, – 9).

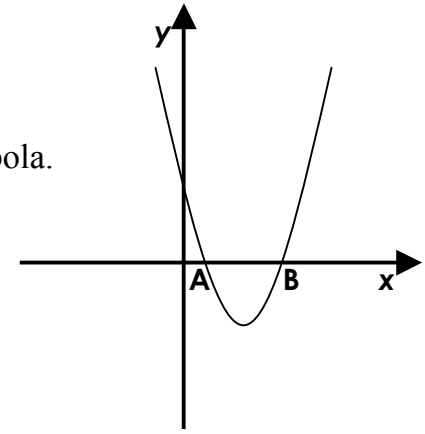


(a) Write down the equation of the parabola in the form $y = (x - a)^2 + b$

The parabola cuts the x - axis at the origin and the point B.

(b) What is the length of OB?

21. The parabola in the diagram has minimum turning point (2, –1) and crosses the x - axis at points A and B.



(a) Write its equation in the form $y = (x - a)^2 + b$.

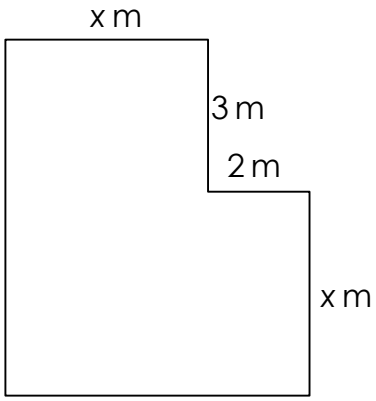
(b) State the equation of the axis of symmetry of the parabola.

(c) Find the coordinates of the point where the parabola cuts the y - axis.

(d) B is the point (3, 0). What are the coordinates of A?

22. **(a)** Show that the area of this L – shaped room is given by the function $A(x) = x^2 + 5x$.

(b) Given that the area is 36 m^2, calculate the value of x.



DISCRIMINANT

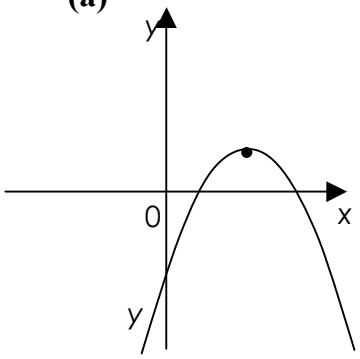
1. Find the discriminant for each of these quadratic equations

(a)	$x^2 + 4x + 3 = 0$	**(b)**	$x^2 + 6x + 9 = 0$	**(c)**	$x^2 + 8x + 7 = 0$
(d)	$3 - 5w - 2w^2 = 0$	**(e)**	$2x^2 + 7x + 5 = 0$	**(f)**	$x^2 - 12x + 36 = 0$
(g)	$x^2 - 7x + 12 = 0$	**(h)**	$2x^2 + 7x + 9 = 0$	**(i)**	$5x^2 - 16x + 3 = 0$
(j)	$6y^2 - 11y - 2 = 0$	**(k)**	$x^2 - 8x + 9 = 0$	**(l)**	$3x^2 + 2x + 7 = 0$
(m)	$2x^2 - 7x + 4 = 0$	**(n)**	$4x^2 - 3x + 4 = 0$	**(o)**	$3x^2 - 2x - 1 = 0$
(p)	$x^2 + 10x + 25 = 0$	**(q)**	$3x^2 - 7x + 5 = 0$	**(r)**	$x^2 - 8x + 16 = 0$

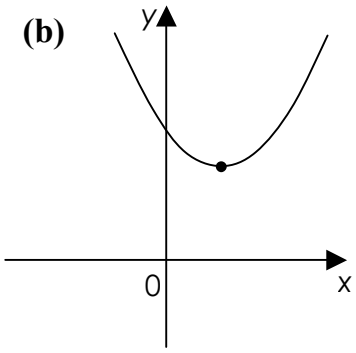
2. Use the discriminants from Q1 to state the nature of the roots of each of the quadratic equations.

3. Here are some graphs of quadratic functions. What can you say about the discriminant for each one?

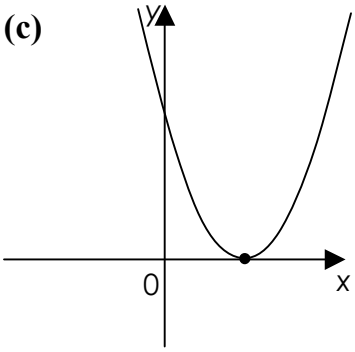
(a)



(b)



(c)



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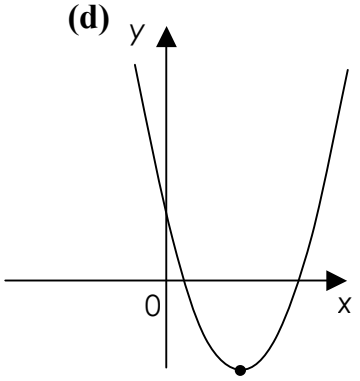
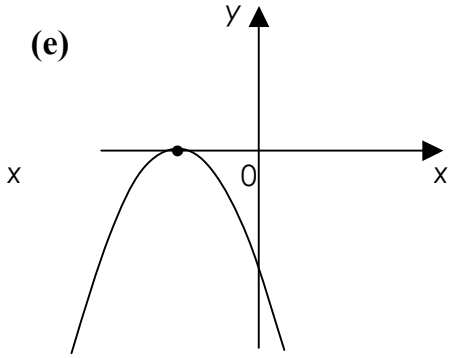


(e)



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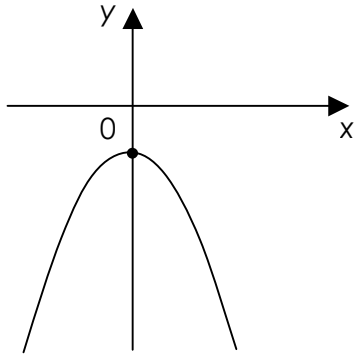
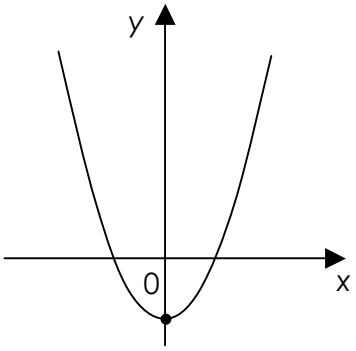


(g)



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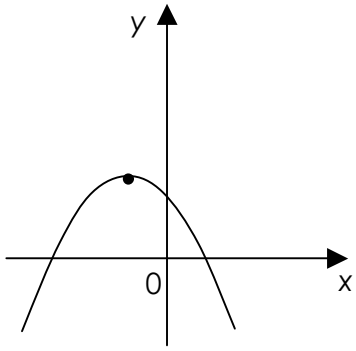
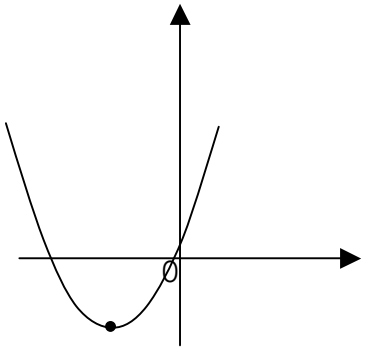
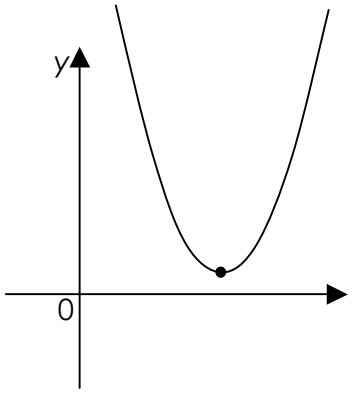


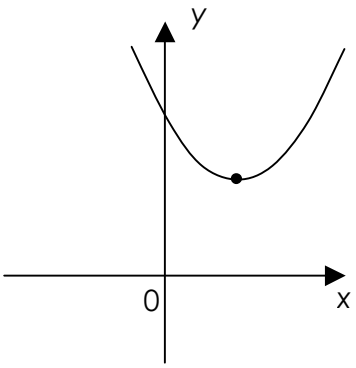
(i)



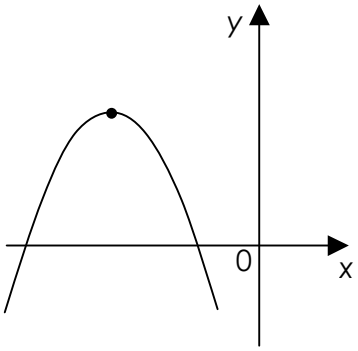
(j)



(k)



(l)



4. Find the value of a so that these quadratic equations have equal roots.

(a) $x^2 - 4x - a = 0$ **(b)** $2x^2 + 10x + a = 0$ **(c)** $ax^2 - 2x + 5 = 0$

(d) $ax^2 + (4a - 3)x + a = 0$ **(e)** $3x^2 + 8x + a = 0$ **(f)** $ax^2 - 7x - 5 = 0$

5. Find the value of k so that these quadratic equations have equal roots.

(a) $kx^2 - 8x + 4 = 0$ **(b)** $kx^2 + 6x + 18 = 0$ **(c)** $x^2 - 2kx + 5 = 0$

(d) $x^2 - 6x + k = 0$ **(e)** $kx^2 + 5x + 10 = 0$ **(f)** $x^2 - 3kx + 36 = 0$

DISCRIMINANT — EXAM QUESTIONS

1. The following words can be used to describe the roots of a quadratic.

I	Real	II	Equal	III	Distinct
IV	Non-real	V	Rational	VI	Irrational

Which of the above words can be used to describe the roots of the equation

$$2x^2 + 3x - 4 = 0?$$

2. **(a)** Find the value of the discriminant for the quadratic equation

$$x^2 - 5x + 3 = 0$$

(b) Use the discriminant to state the nature of the roots in part (a).

3. For what values of p does the equation $x^2 - 2x + p = 0$ have equal roots?

4. The roots of a quadratic equation can be described as:

I	Real	II	Equal	III	Distinct
IV	Non-real	V	Rational	VI	Irrational

Which of the above can be used to describe the roots of the equation $3x^2 - 4x + 5 = 0$?

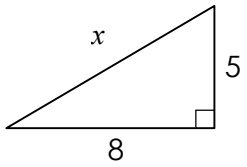
5. **(a)** For the quadratic equation $x^2 - 5x + 3 = 0$, find the value of the discriminant.

(b) Use the words from question 4 to describe the nature of the roots of the equation.

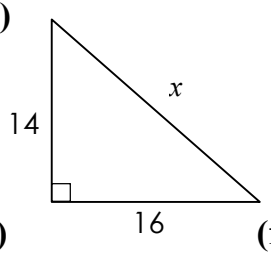
3.1 APPLYING the THEOREM of PYTHAGORAS

1. Calculate the length of the side marked x in each triangle below

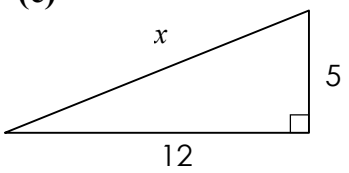
(a)



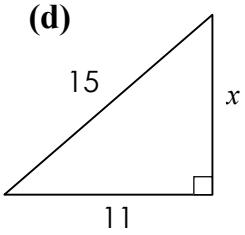
(b)



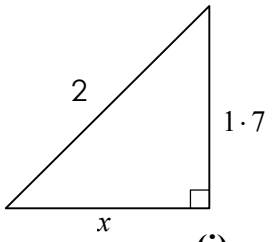
(c)



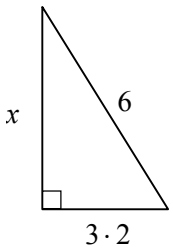
(d)



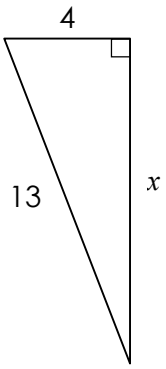
(e)



(f)



(g)



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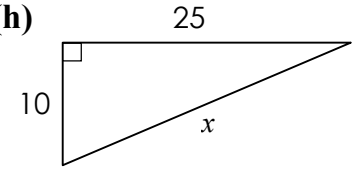
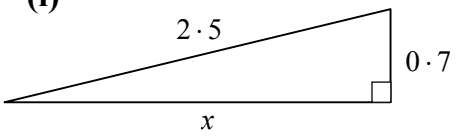
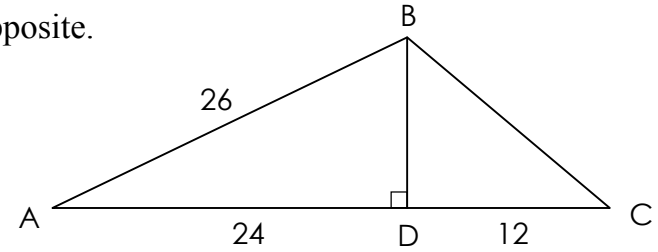


(i)



2. Answer these questions about the framework opposite.

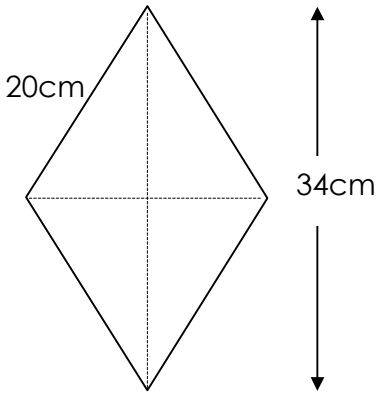


(a) Calculate the length of BD.

(b) Hence calculate the length of BC.

(c) Calculate the area of triangle ABC.

3. A rhombus has sides of 20cm and its longest diagonal measuring 34cm.



(a) Calculate the length of the shorter diagonal.

(b) Calculate the area of the rhombus

4. Calculate the distance between each pair of points below.

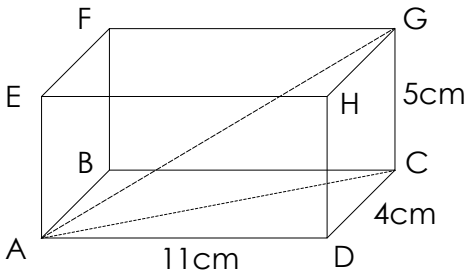
(a) A(2, 5) and B(7, 10)

(b) P(1, 8) and Q(12, 2)

(c) E$(-2,3)$ and F$(2,-4)$

(d) R$(-7,-3)$ and F$(3,-1)$

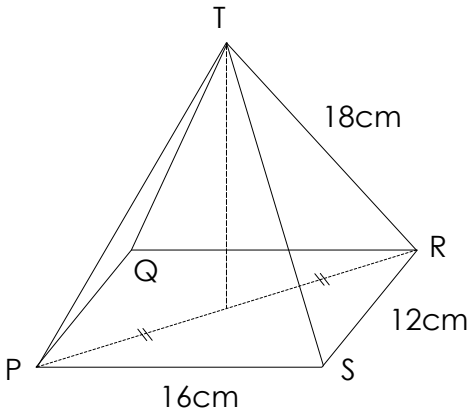
5. Answer the following about the cuboid opposite.



(a) Calculate the length of the face diagonal AC.

(b) Hence calculate the length of the space diagonal AG.

6. The pyramid opposite has a rectangular base.

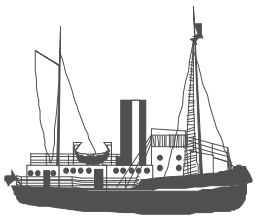


(a) Calculate the length of the base diagonal PR.

(b) Given that edge TR = 18cm, calculate the vertical height of the pyramid.

Start each of the following questions by drawing a diagram.

7. A ship sails 9km due North and then a further 17km due East.



How far is the ship from its starting point?

8. An aircraft flies 400km due West and then a further 150km due South.

How far is the aircraft from its starting point?

9. A ship sailed $8 \cdot 42$ km due East followed by $4 \cdot 7$ km due South.

How far would it have sailed if it had followed a direct course?

10. A ship sails 9km due North and then a further distance x km due West.

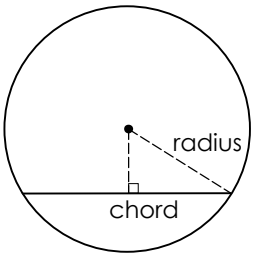
The ship is now 12km from its starting point. Calculate x.

11. How long is the diagonal of a square of side 11mm?

12. A rectangle measures 14cm by 9cm. Calculate the length of its diagonals.

13. A ladder of length 5 metres leans against a vertical wall with the foot of the ladder 2 metres from the base of the wall. How high up the wall does the ladder reach?

14. A ladder is placed against a vertical wall. If the distance between the foot of the ladder and the wall is $1 \cdot 8$ metres, and the ladder reaches 4 metres up the wall, calculate the length of the ladder.



This diagram may help you with questions 15 and 16

15. A circle has a diameter of 20cm.

A chord is drawn which is 6cm from the centre of the circle.

Calculate the length of the chord.

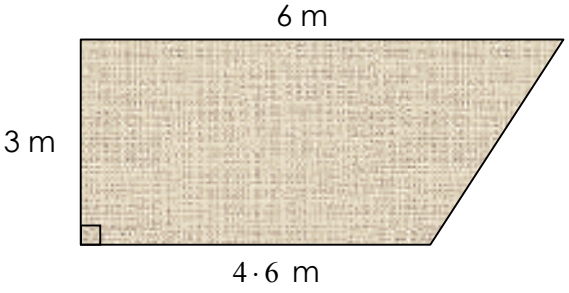
16. A circle has a diameter of 12cm.

A chord is drawn which is 5cm from the centre of the circle.

Calculate the length of the chord.

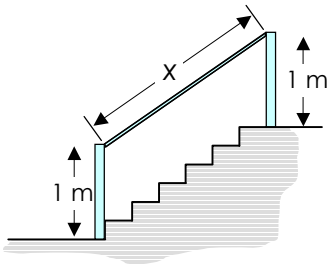
17. The room shown opposite has two parallel sides.

Using the given dimensions calculate the perimeter of the room.



18. Calculate the length of the banister rail shown in the diagram if there are 6 stairs, and if each tread measures 25cm and each riser 20cm.

Give your answer in metres.

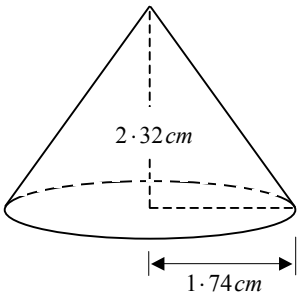


19. A solid cone has a base radius of $1 \cdot 74$ cm and a vertical height of $2 \cdot 32$ cm. Calculate the cone's

(i) slant height;

(ii) total surface area;

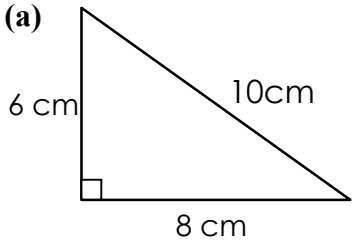
(iii) volume.



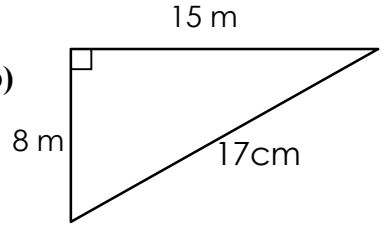
Give all answers correct to 3 significant figures. [Surface Area = $\pi r^2 + \pi rs$; $V = \frac{1}{3}\pi r^2 h$]

20. Use the converse of Pythagoras Theorem to prove that these triangles are right angled.

(a)



(b)



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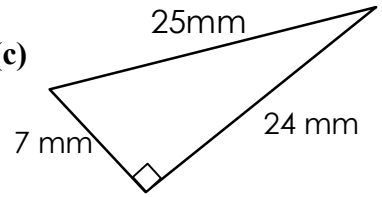
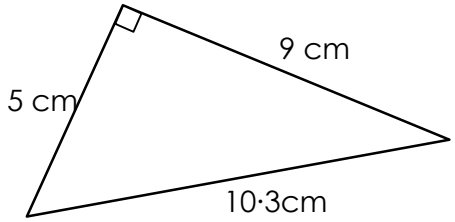
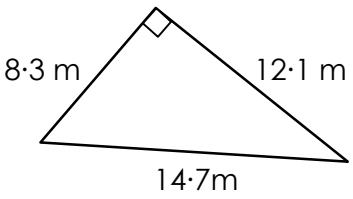


(d)



(e)



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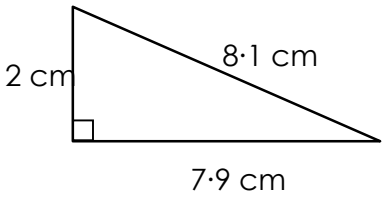
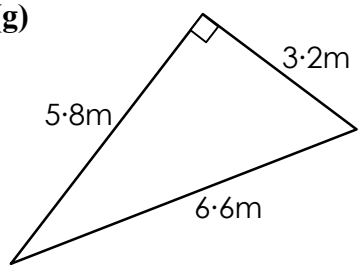
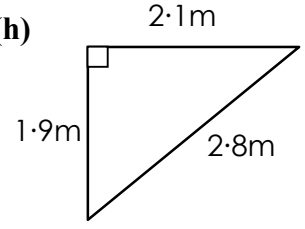


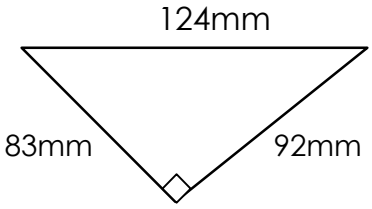
(g)



(h)

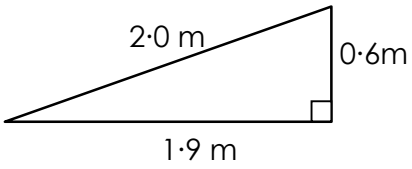


(i)

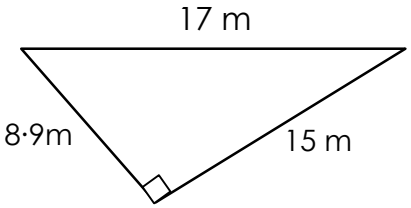


21. Use the converse of Pythagoras Theorem to decide if these triangles are right angled or not.

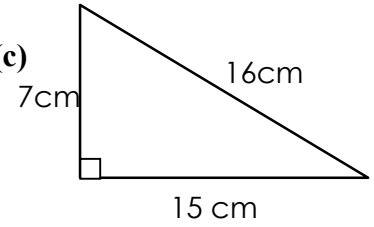
(a)



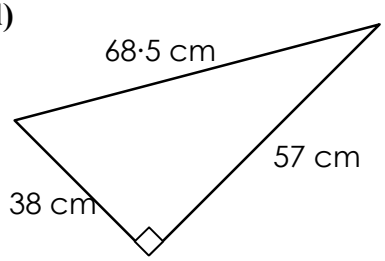
(b)



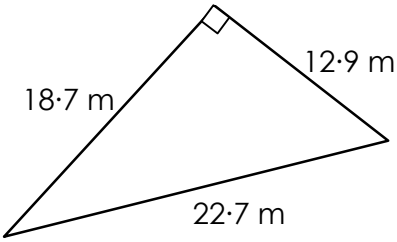
(c)



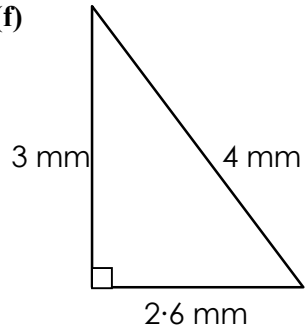
(d)



(e)



(f)



3.1 APPLYING the THEOREM of PYTHAGORAS' EXAM QUESTIONS

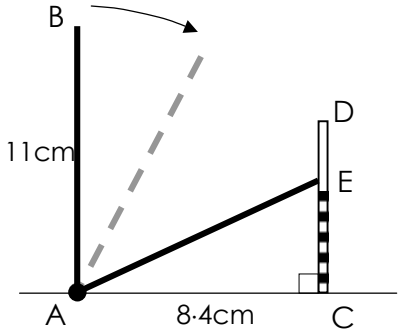
1. In a switch mechanism lever AB rotates around A until it rests against the rod CD.

Point B touches rod CD at E.

AB = 11cm and AC = 8·4cm as shown.

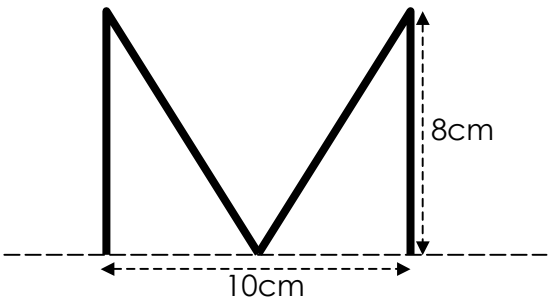
For the switch to work the distance from C to E must be **more than** 7cm.

Will this switch mechanism work?



Your answer must be accompanied by appropriate working and explanation.

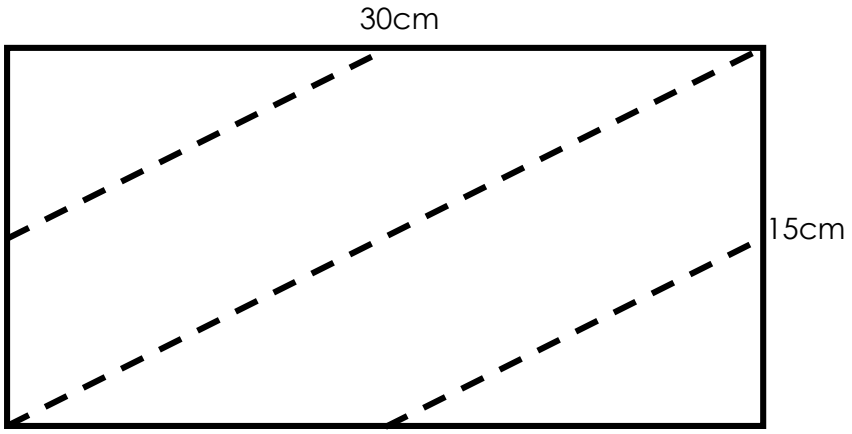
2. The capital letter 'M' can be formed using straight lines as shown below.



Calculate the total length of the lines forming the letter.

3. A wall hanging is decorated with strips of sequences as shown in the diagram. The sequences are represented by broken lines. (– – – –).

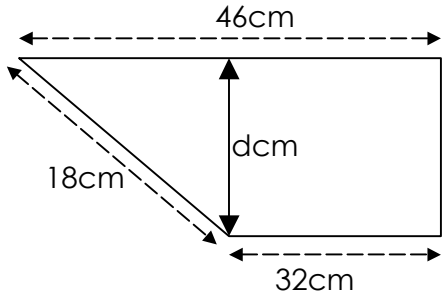
The shorter stripes are half the length of the longest strip.



Calculate the total length of sequences required to make this wall hanging.

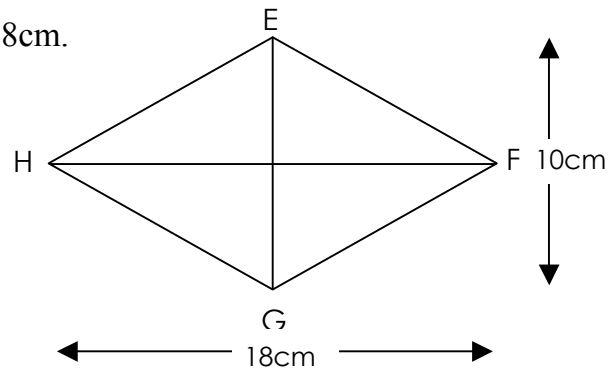
4. The side view of a water trough is as shown in the diagram. The depth of it must by at least 11cm.

Is this container acceptable? Show working and give a reason for your answer.

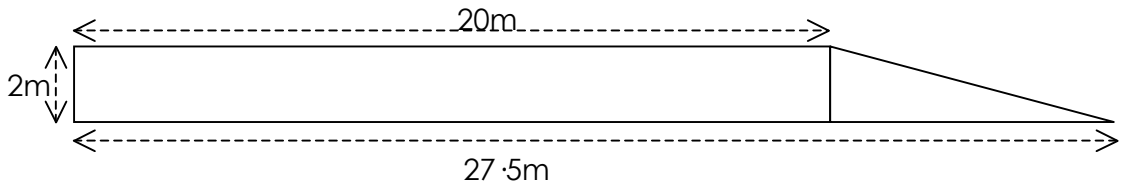


5. EFGH is a rhombus. EG is 10cm and HF is 18cm.

Calculate the perimeter of the rhombus.



6. A special stage is being built for an outdoor concert. It has to be 20 metre wide, 2 metres high and has a ramp on one side.

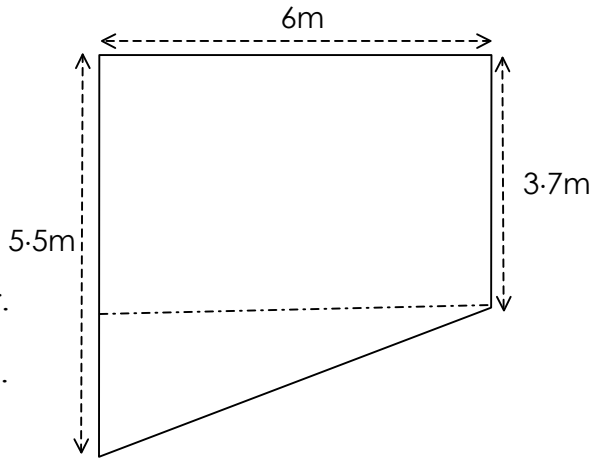


Special non-slip matting has to be laid along the stage and down the ramp.

The cost of the matting is £34 a metre and it is sold in complete metres.

Calculate the cost of the matting.

7. I have just built a new patio area in my garden.



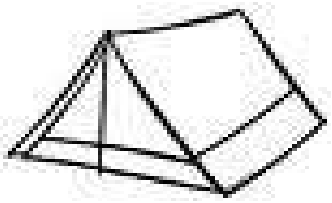
The diagram shows the measurements of it.

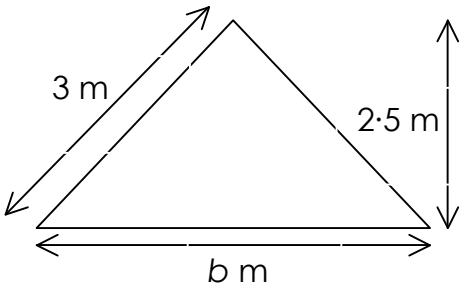
I am going to put a low fence round its perimeter.

Calculate the length of fencing that I will require.

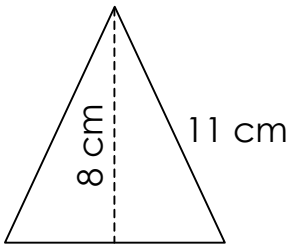
8. The opening of a tent is triangular in shape. The height of the tent is 2·5 metres and the sloping edge of the tent is 3 metres long.

Calculate the length of the base of the tent.





9.

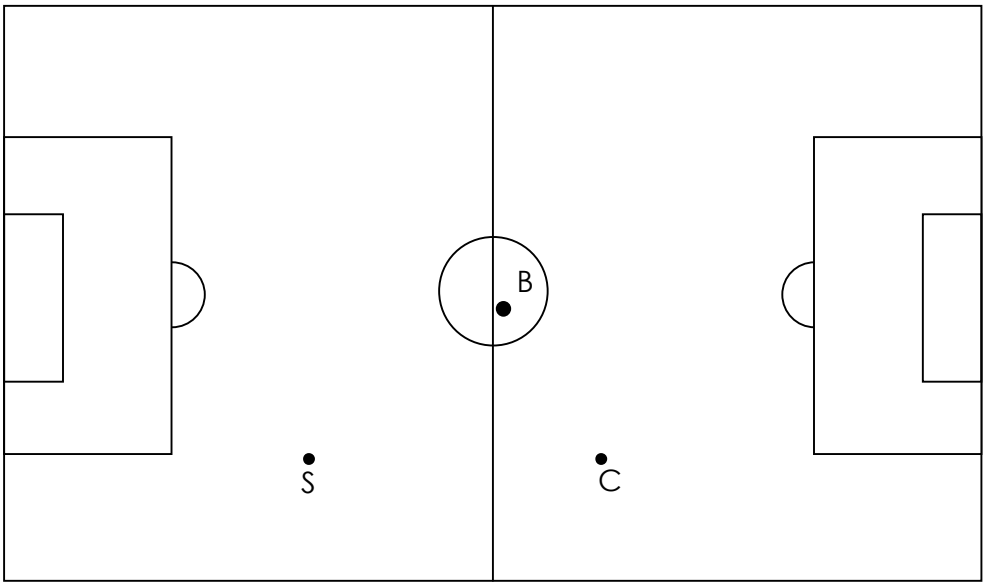


An isosceles triangle has its longest side 11 cm and height 8 cm.

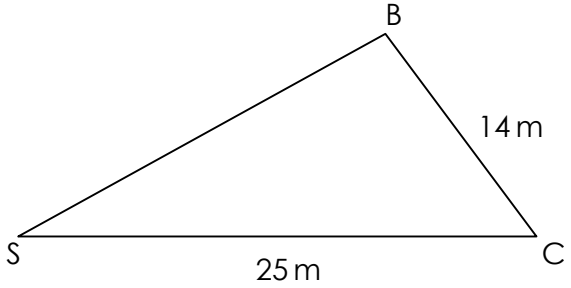
Find the perimeter of the triangle.

10. Two boys, Scott and Callum are playing football.

At one point Callum (C) is 25 m due east of Scott (S). The ball is at position B.



The positions of the 2 boys in relation to the ball are shown in the diagram.

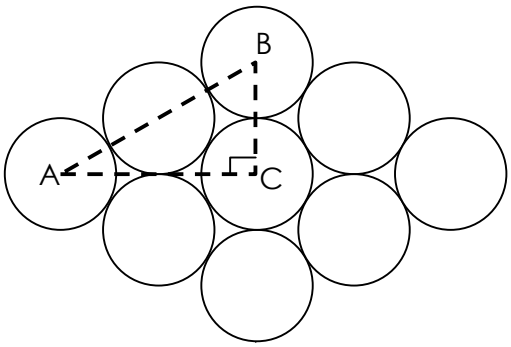


Callum is 14 m away from the ball and angle SBC = 90^{o}.

Calculate how far Scott is away from the ball. **Give your answer correct to 1 decimal place.**

11. The stones in an engagement ring are arranged as shown in the diagram.

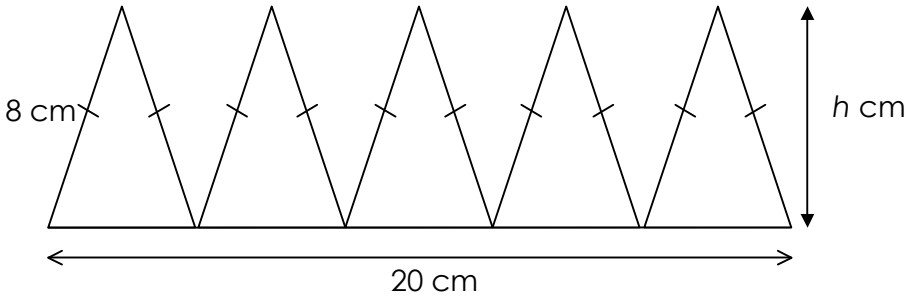
The diameter of each stone is 4 mm and A, B and C are the centres of 3 of the stones.



(a) Write down the lengths of AB and BC.

(b) Calculate the length of AC.

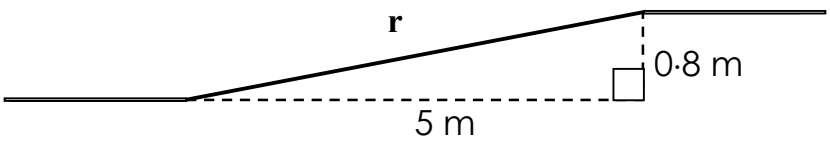
12. The pattern below is formed from 5 congruent isosceles triangles.



The width of the pattern is 20cm and the equal sides of the triangles measure 8cm.

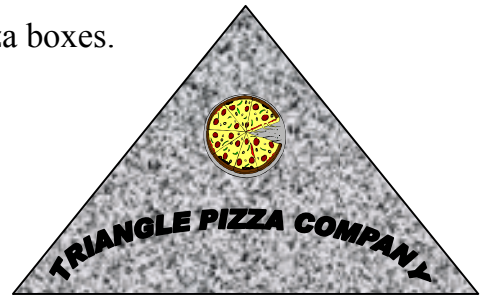
Calculate the height, *h* cm, of the pattern.

13. A ramp is being built to provide wheelchair access to a public building.

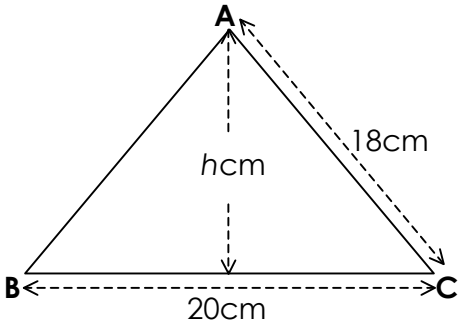


Find the length of the ramp, **r** to the nearest centimetre.

14. The Triangle Pizza Company uses this logo on their pizza boxes.



The logo is in the shape of the isosceles triangle ABC as shown in the diagram below.

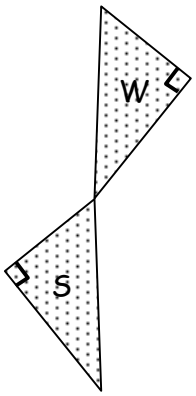


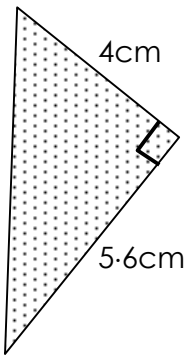
Calculate the height, h cm, of the logo.

Give your answer to the nearest centimetre.

15. The diagram shows the logo of the junior section of a sailing club called the 'Windy Sails' Club.

It consists of two identical right-angled triangles.

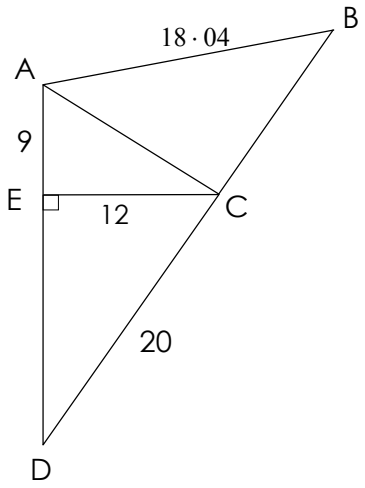




The dimensions of each right-angled triangle are shown in the diagram.
The **outline** of the logo is sewn on to the sailing suits.

Calculate the length of the stitching needed to sew the **complet**e logo.

16. Answer the following questions about the diagram opposite. All lengths are centimetres.



(a) Calculate the length of AC.

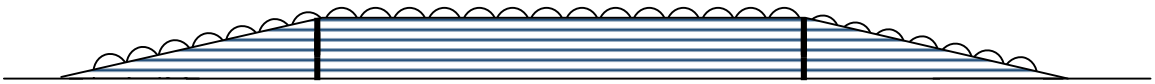
(b) Calculate the length of ED.

(c) Prove that triangle ACD is right-angled at C.

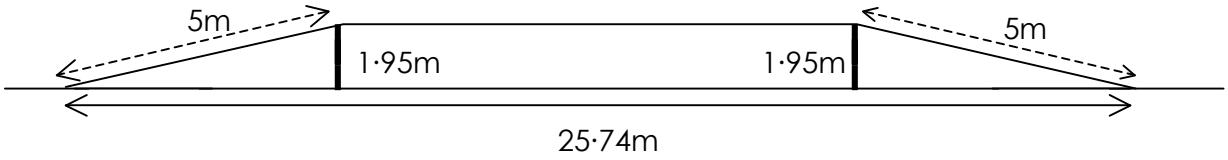
(d) Hence calculate the length of BD and the area of triangle ABD correct to the nearest whole number.

17. At a special school a sensory area was being created with a 'log bridge' as part of it.

The diagram shows the side view of the bridge.



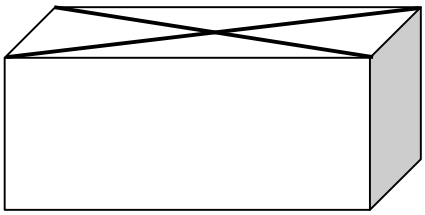
The supports are 1·95 m high and the ramps at each side are 5 metres.



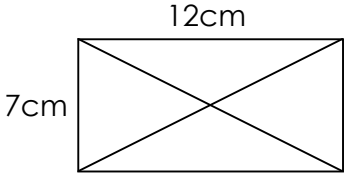
The overall length of the bridge is 25·74metres.

Use the converse of Pythagoras' Theorem to check if the supports are vertical or not.

18. Mrs. Donaldson was wrapping up a parcel for her niece. For decoration, she put a cross of ribbon across the top of it as shown in the diagram.



This diagram shows the top of the parcel.
Each length of ribbon was cut to a length 13·9cm.



Use the converse of Pythagoras Theorem to check if this is long enough. **Give a reason for your answer.**

3.2 APPLYING PROPERTIES of SHAPES

1. Copy and complete the blanks in these statements about QUADRILATERALS.

(a) A RHOMBUS that has 4 RIGHT ANGLES is a _____.

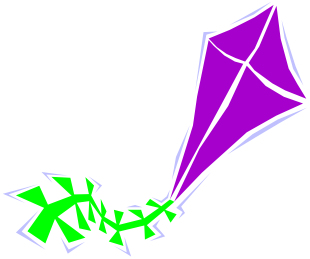
(b) A PARALLELOGRAM has _____ lines of symmetry and _____ of order 2.

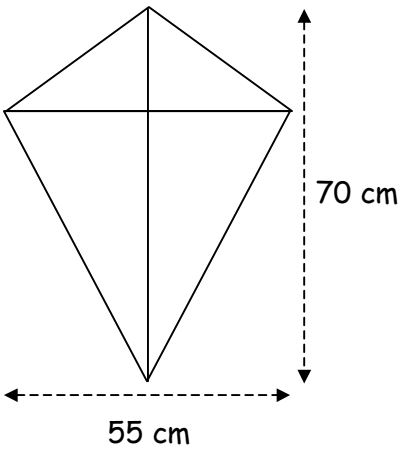
(c) The opposite angles of a PARALELLOGRAM are _____.

(d) The _____ of a SQUARE _____ each other at _____ angles.

(e) A _____ is a QUADRILATERAL that has ONE pair of parallel sides.

2. Pauline wanted to make a kite so she had to work out how much fabric she would need to cover it. She drew a diagram to help her to do this. This is what she drew.

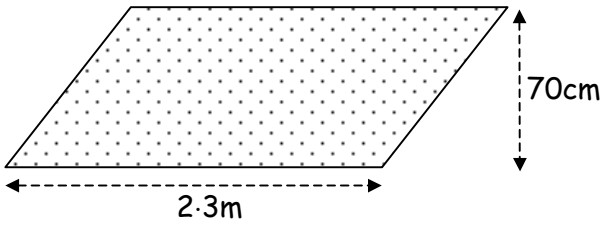




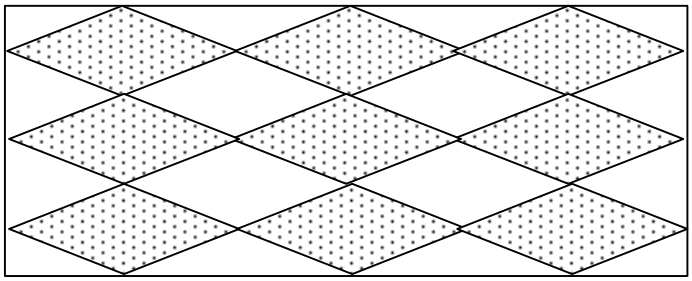
Calculate the area of her kite.

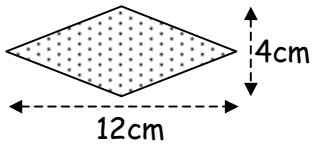
3. **(a)** Calculate the area of this parallelogram giving your answer in cm^2.

(b) How many square metres is this?



4.





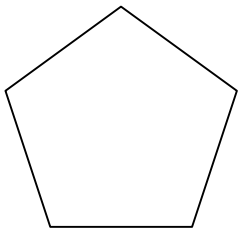
(a) Calculate the area of one rhombus in the above tessellation.

(b) Use the information given about one rhombus to calculate the area of the whole rectangular pattern.

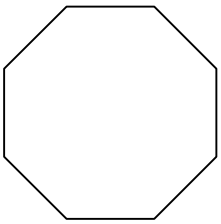
(c) What percentage of the whole rectangle is shaded?

5. Each of these shapes is a REGULAR POLYGON. For each one, write down what kind of POLYGON, the size of the CENTRAL ANGLE, the INTERIOR ANGLE and the EXTERIOR ANGLE.

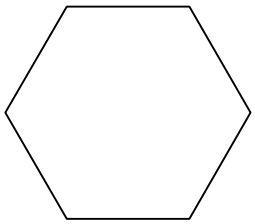
(a)



(b)

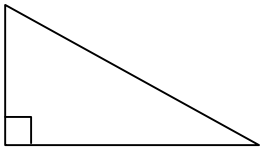


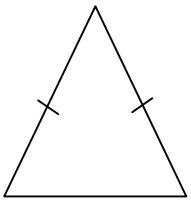
(c)

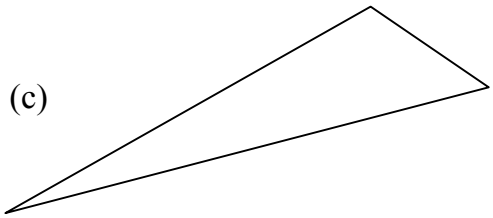


Scalene Right-angled Isosceles Acute-angled Obtuse-angled Equilateral

6. Choose words from the above to describe these triangles as fully as you can.

(a) 

(b) 

(c) 

7. Draw a set of coordinate axes on a coordinate grid and plot the points:

A(2, 4), B(1, -3) and C(-5, 3).

Join the points up to form a triangle.

Write down a FULL description of triangle ABC.

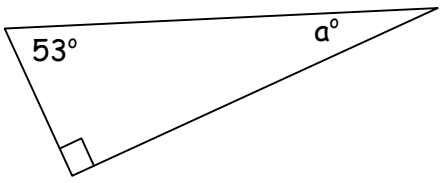
Work out its area.

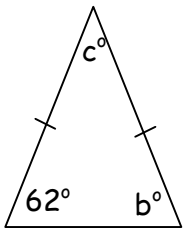
8. Pupils were asked to make up sets of three angles that could be used to draw a triangle.

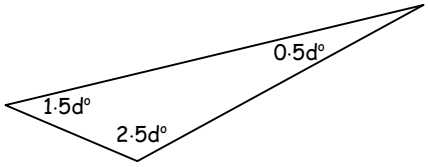
30^o, 60^o, 90^o 47^o, 34^o, 98^o 53^o, 47^o, 90^o 32^o, 36^o, 112^o

Which ones are correct? Why they are correct?

9. Calculate the values of a, b, c and d in these triangles:

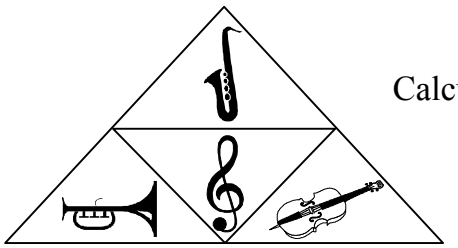
(a) 

(b) 

(c) 

10. Pupils in the art department have been asked to design a logo for the school band. Here is one of the entries.

It consists of 4 isosceles triangles each measuring 12cm on the base and 6cm high.

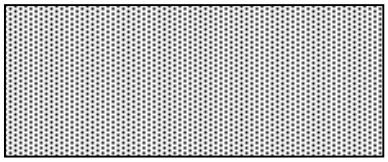


Calculate the area of fabric required to make up 40 of these.

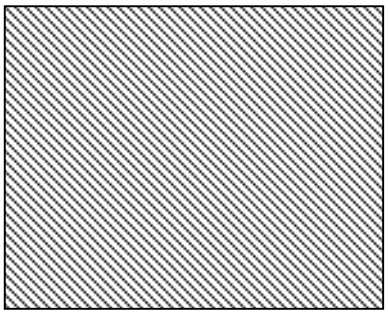
11. The cost of advertising depends on the length of space that the advert takes up. Each column of the paper measures 5cm in width and each cm costs £0·75.

Calculate the cost of these adverts:

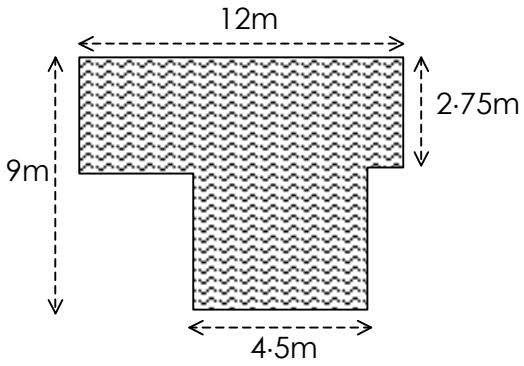
(a)



(b)



12. Calculate the cost of carpeting a room like this if the carpet costs £12·96 per m^2.



13. Say whether the following statements are True or False

(a) If the diameter of a circle is 2·5m, then its radius is 5m.

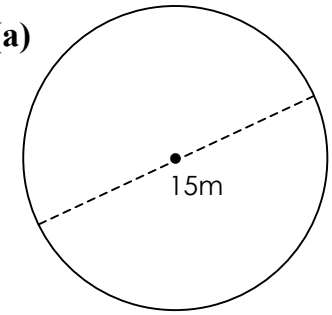
(b) The perimeter of a circle is called its circumference.

(c) To find the circumference of a circle the radius is multiplied by π.

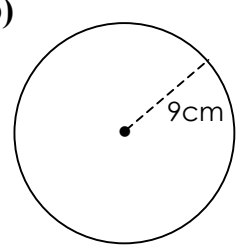
(d) The diameter divides a circle into two semi-circles.

14. Calculate the CIRCUMFERENCES of these circles:

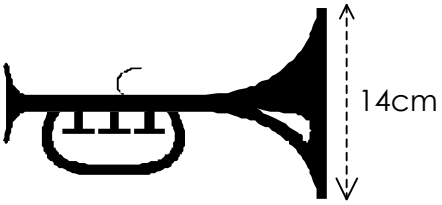
(a)



(b)



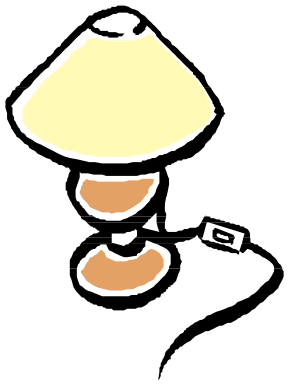
15.



The diameter of the 'bell' on the end of a trumpet measures 14 cm. Calculate its circumference.

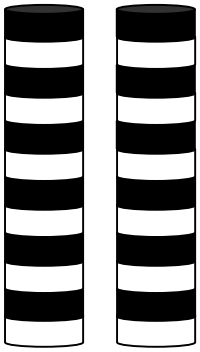
16. Calculate the circumference of the circle drawn with these compasses.

17. The radius of this lampshade is 95mm at the bottom. How much trim would by required to fit round the bottom edge.



If the trim costs £2·75 a metre, how much would it cost to trim the lamp if the trim is only sold in complete metres.

18. A florist is decorating her shop and wants to put pieces of coloured ribbon round white poles to create a striped effect like this:



The pole has a radius of 12cm. Calculate how much ribbon she will need to decorate the two poles.

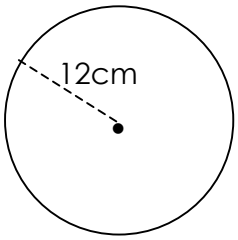
Answer correct to the nearest metre.

19. Linzi's Mum buys a frill of length 38cm to fit round her birthday cake.

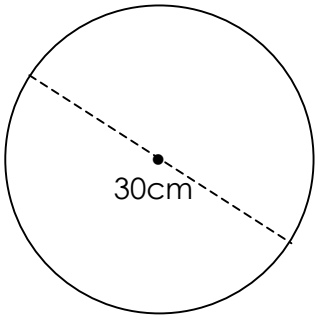
Find out the biggest diameter that the cake can have so that the frill fits.

20. Calculate the area of these circles:

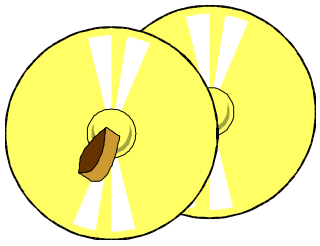
(a)



(b)

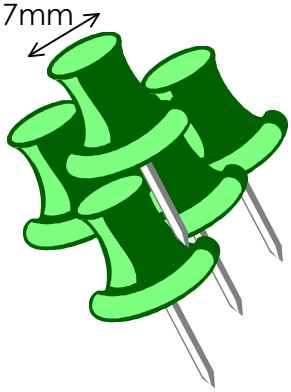


21. The radius of a cymbal is 18cm. Calculate the area of one of them.

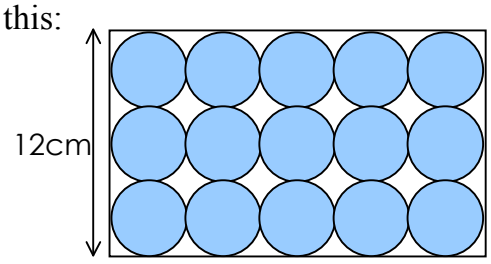


22. The diameter of the top of a pin is 7mm.

Calculate the total area of the tops of 5 of them.



23. Tea-light candles have to be packed into a box like this:



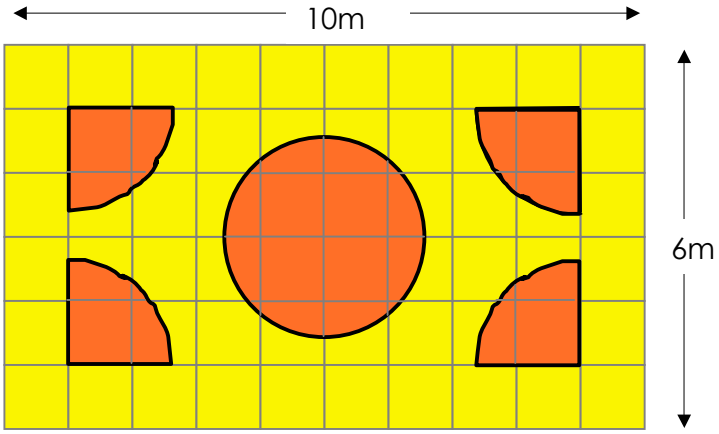
(a) What is the area of 1 tea light?

(b) Calculate the total area taken up by the 15 tea lights on the tray.

(c) What is the area of the top of the tray?

(d) What percentage of space on the tray is **NOT** taken up by the tea lights?

24. Mrs Ahmad has moved into a new house and has to sort out her garden. This is a plan of what she wants to do. It consists of a circular flower-bed with diameter 3m and 4 quarter-circles with radius 1·5m set in a rectangular lawn.



(a) Calculate the total area of the 5 flower beds.

(b) What area is given over to the lawn?

(c) It costs £3·65 for each square metre of lawn and a total of £197·50 for plants.

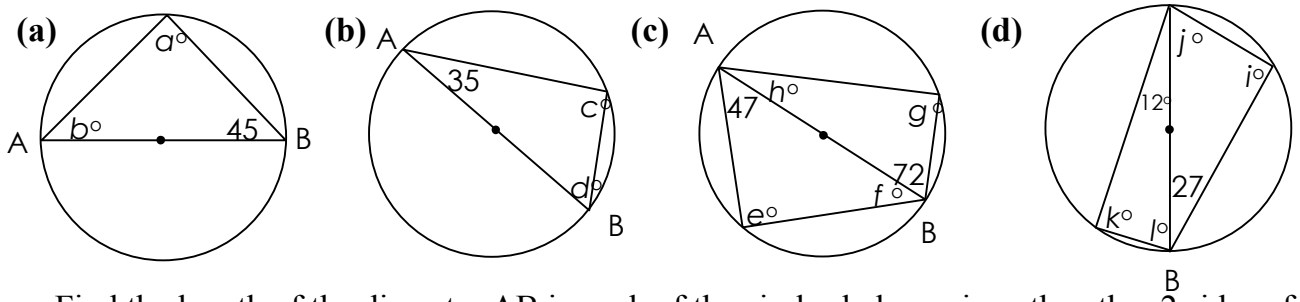
How much would it cost Mrs Ahmad altogether for her new garden?

25. The weights at the end of these balloons each have an area of $20cm^2$.

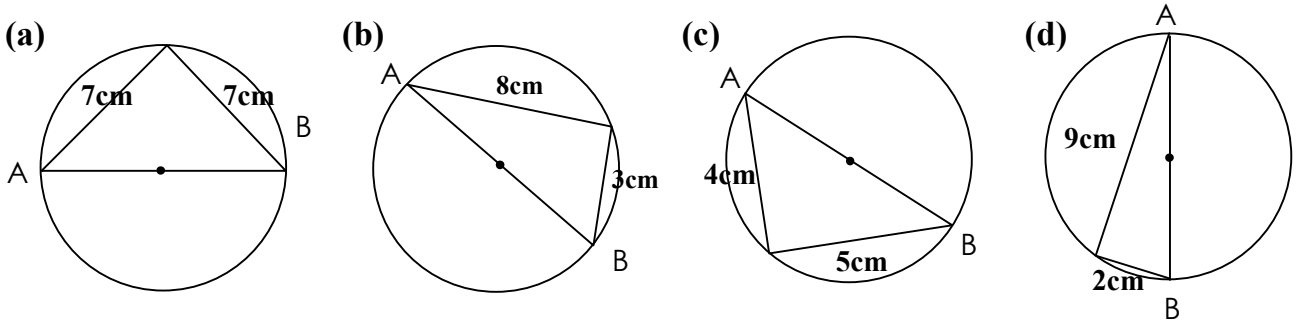
Calculate their radius and then the circumference.

3.2 RELATIONSHIP between the CENTRE, CHORD and PERPENDICULAR BISECTOR

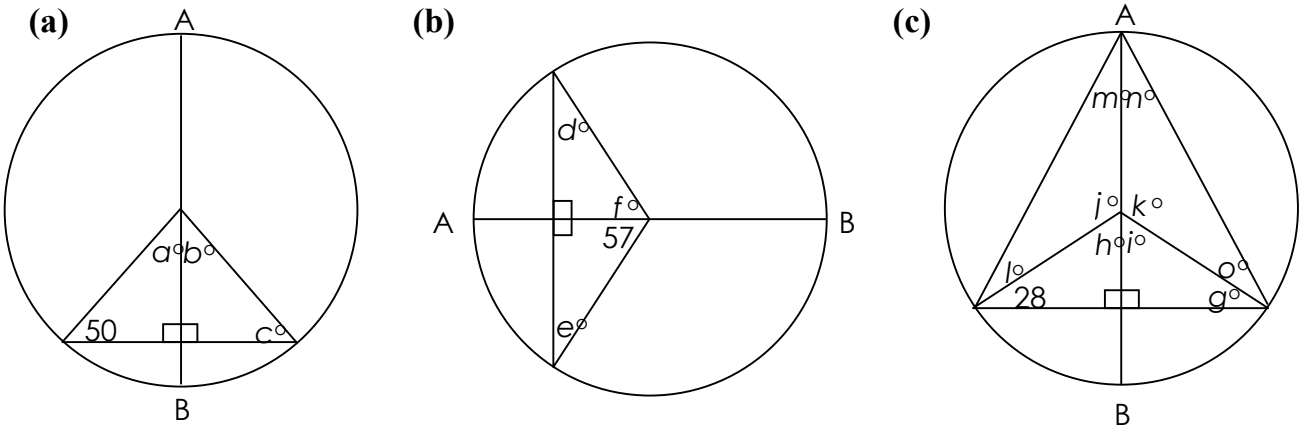
1. In each of the diagrams below AB is a diameter. Find the missing angles in each diagram.



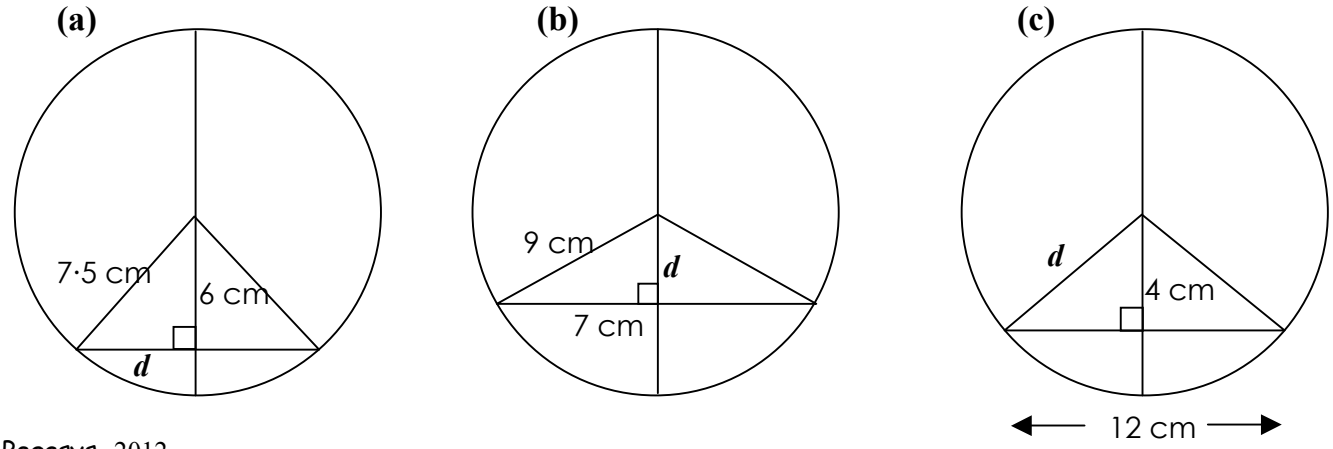
2. Find the length of the diameter AB in each of the circles below, given the other 2 sides of the triangle.



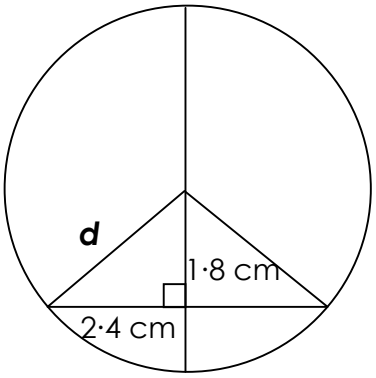
3. Use the symmetry properties of the circle to find the missing angles in the diagrams below. In each diagram AB is a diameter.



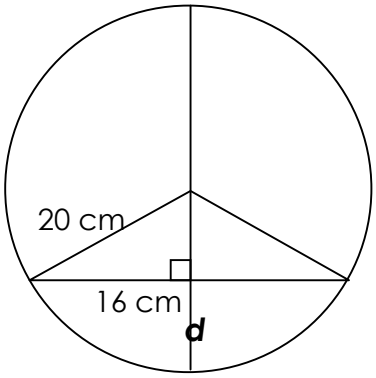
4. Calculate the length of ***d*** in each diagram.



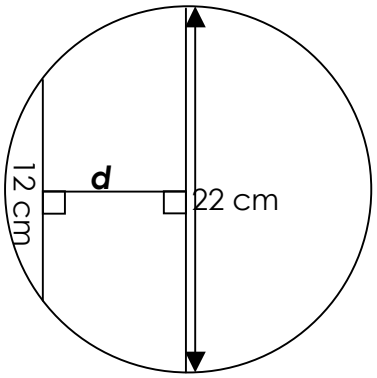
(d)



(e)

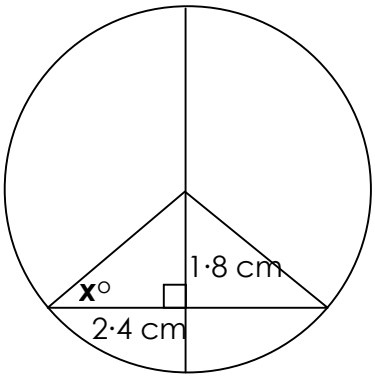


(f)

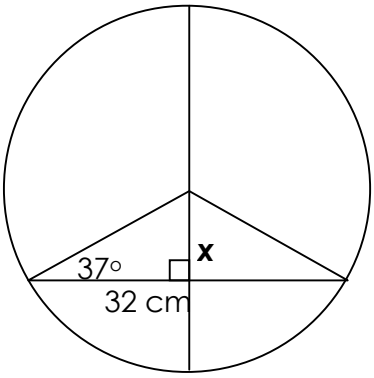


5. Find ***x*** in each of the triangles below.

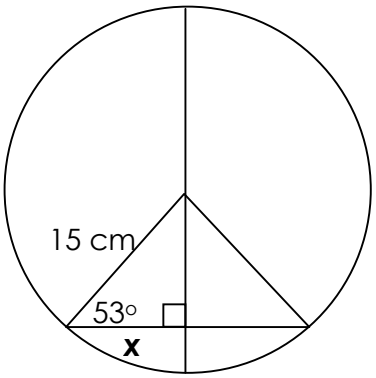
(a)



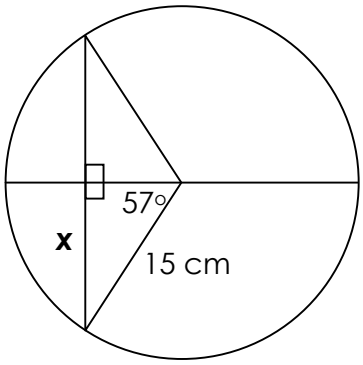
(b)



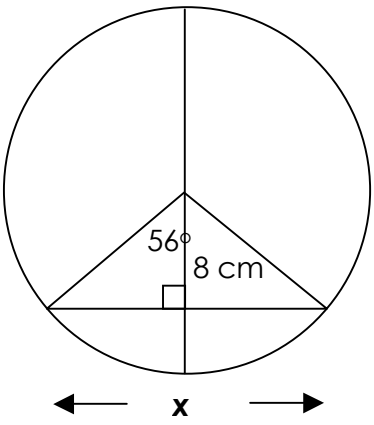
(c)



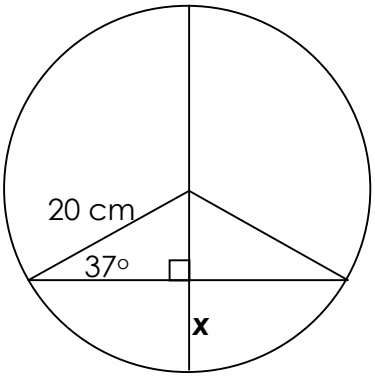
(d)



(e)

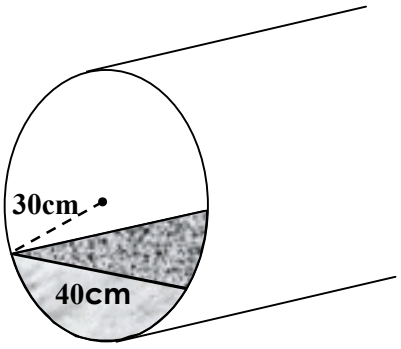


(f)



6. A cylindrical pipe is used to transport water underground.
The radius of the pipe is 30 cm and the width of the water surface is 40 cm.

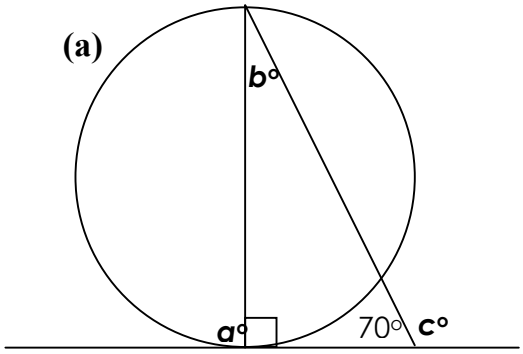
Calculate the height of the pipe above the water.



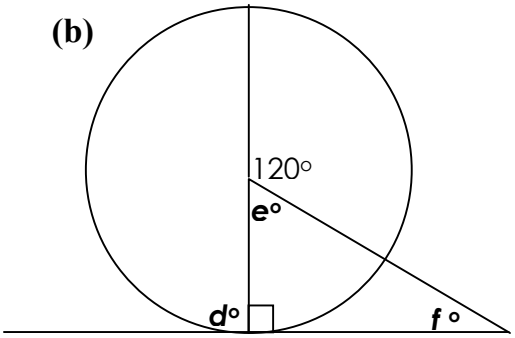
3.2 TANGENTS and ANGLES

1. Calculate the sizes of the angles marked ***a***, ***b***, ...***r***, in the diagrams below.

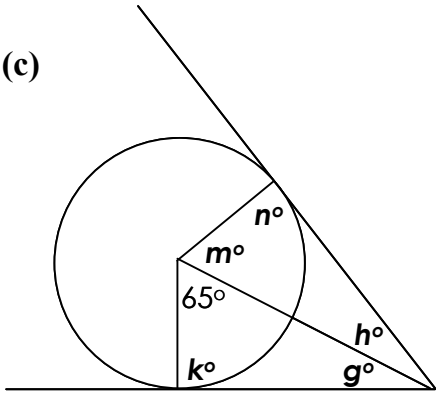
(a)



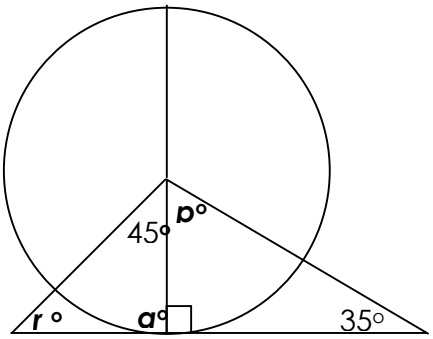
(b)



(c)



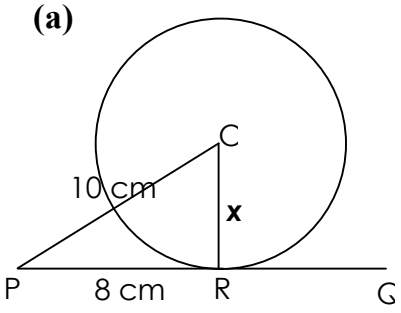
(d)



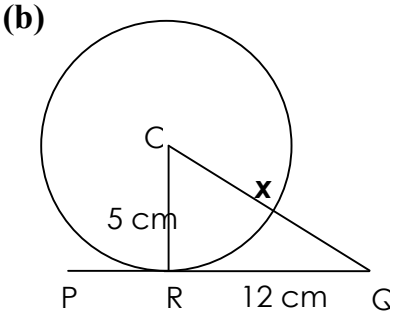
2. In each of the diagrams below, PQ is a tangent which touches the circle at R.

Calculate the lengths of the lines marked ***x***.

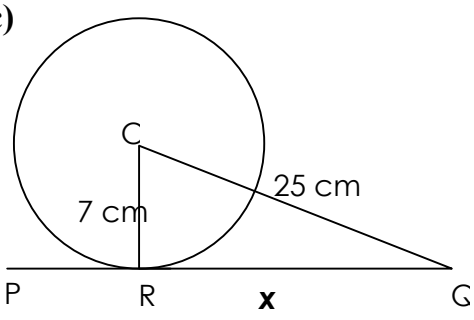
(a)



(b)



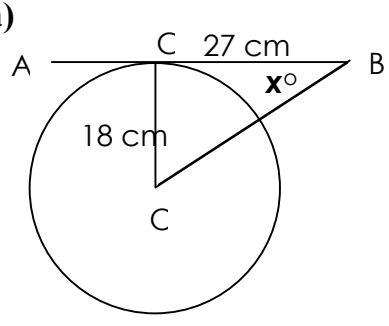
(c)



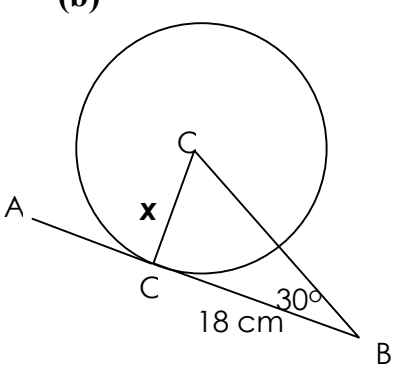
3. In each of the diagrams below, AB is a tangent which touches the circle at C.

Calculate ***x*** for each diagram.

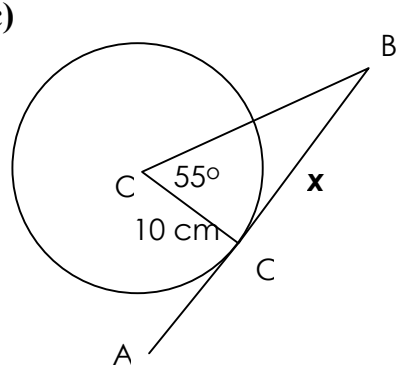
(a)



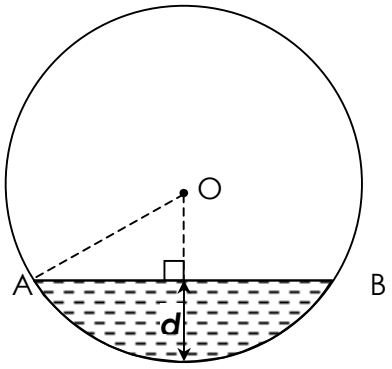
(b)



(c)



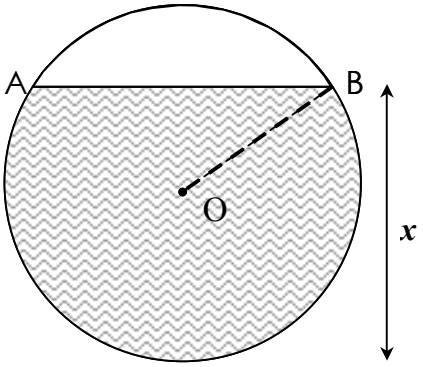
1. The diagram shows a section of a cylindrical drain whose **diameter** is 1 metre. The surface of the water in the drain AB is 70 cm.



(a) Write down the length of OA.

(b) Calculate the depth of water in the pipe, ***d***. (Give your answer to the nearest cm.)

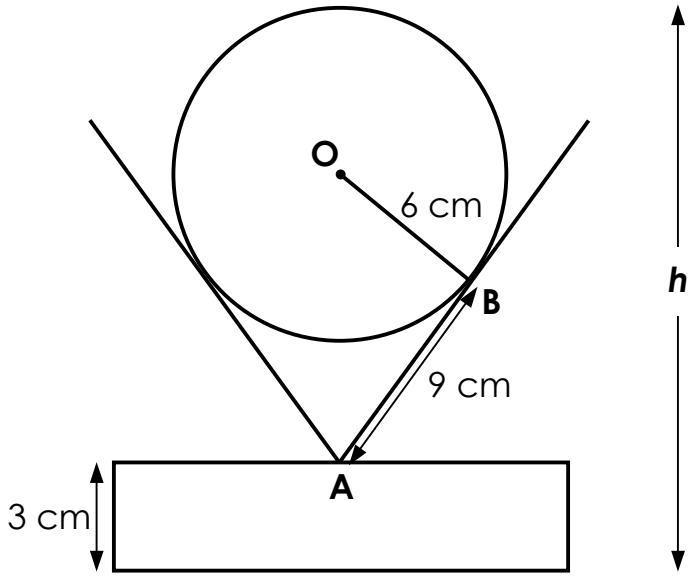
2. The diagram shows a section of a disused mineshaft whose diameter is 2 metres. The surface of the water in the shaft, AB, is 140 cm.



(a) Write down the length of OB.

(b) Calculate the depth of water in the pipe, ***x***. (Give your answer to the nearest cm.)

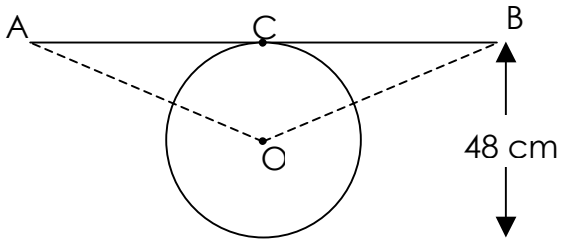
3. A pool trophy is in the shape of a circular disc with two pool cues as tangents to the circle.



The radius of the circle is 6 cm and the length of the tangent to the point of contact (AB) is 9 cm.
The base of the trophy is 3 cm.

Calculate the total height of the trophy, ***h***, to the nearest centimetre.

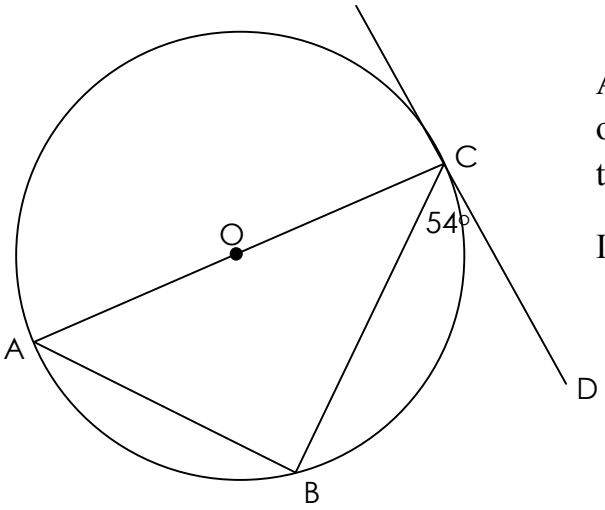
4. A circular bathroom mirror, diameter 48 cm, is suspended from the ceiling by **<u>two</u>** equal wires from the centre of the mirror, O.



The ceiling, AB, is a tangent to the circle at C. AC is 45 cm.

Calculate the total length of wire used to hang the mirror.

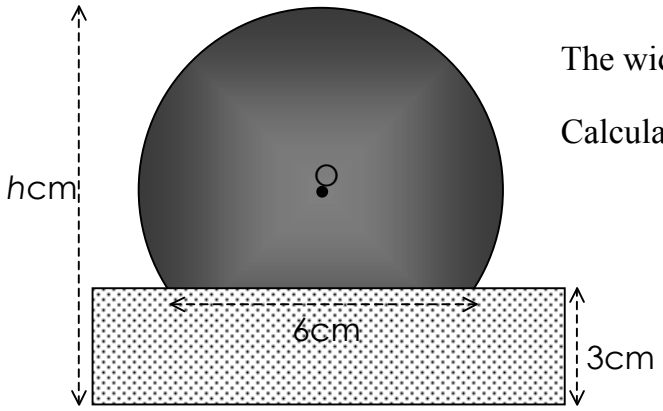
5.



AC is a diameter and O is the centre of the circle shown opposite. CD is a tangent to the circle with C the point of contact.

If $\angle BCD = 54^o$, find the size of $\angle CAB$.

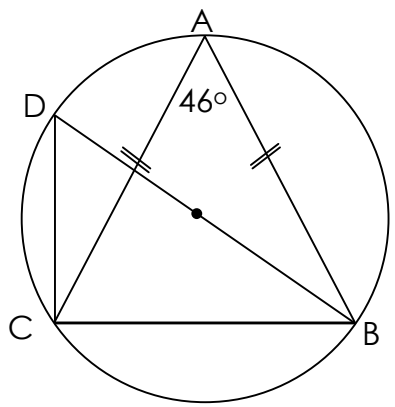
6. A bowling trophy consists of a glass circle set into a rectangular wooden plinth as shown in the diagram. The diameter of the circle, centre O, is 8cm and the height of the plinth is 3 cm.



The width of the glass at the plinth is 6cm.

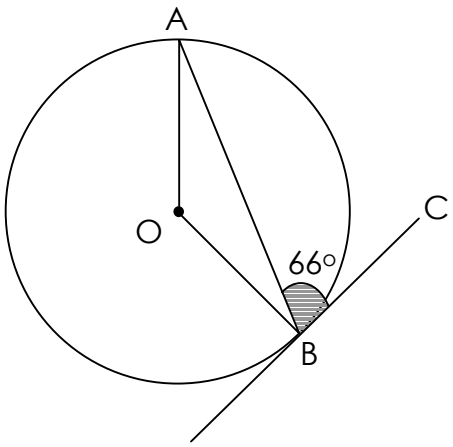
Calculate the height, h cm, of the trophy.

7. In the diagram triangle ABC is isosceles and BD is a diameter of the circle.



Calculate the size of angle ACD.

8.

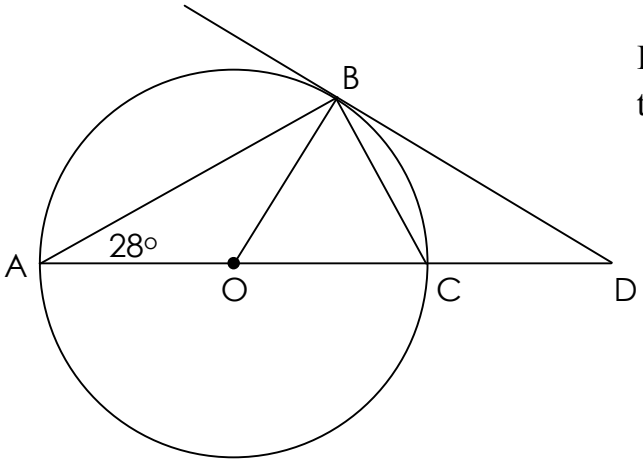


A and B are points on the circumference of a circle centre O. BC is a tangent to the circle.

Angle ABC = 66°.

Calculate the size of angle AOB.

9.



In the diagram shown, BD is a tangent to the circle centre O.

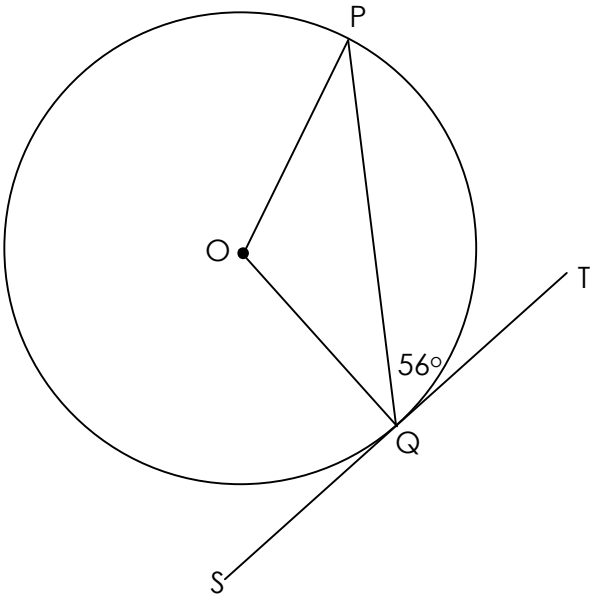
Angle BAC = 28°.

Calculate the size of angle CBD.

10. The diagram shows a circle with centre O. ST is a tangent to the circle with point of contact Q. $\angle PQT = 56^o$.

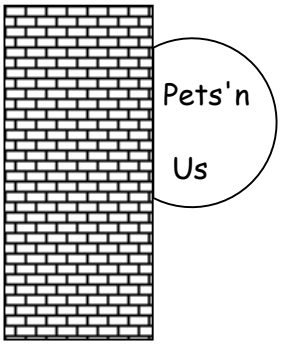
(a) Calculate the size of $\angle POQ$.

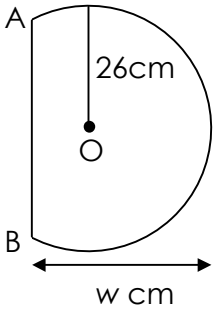
(b) Hence calculate the length of the major arc PQ given that the radius of the circle is 14cm.



11. The sign outside a pet shop is formed from part of a circle.

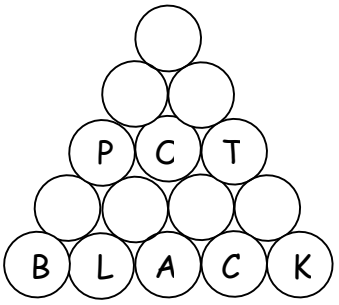
The circle has centre O and radius 26cm.





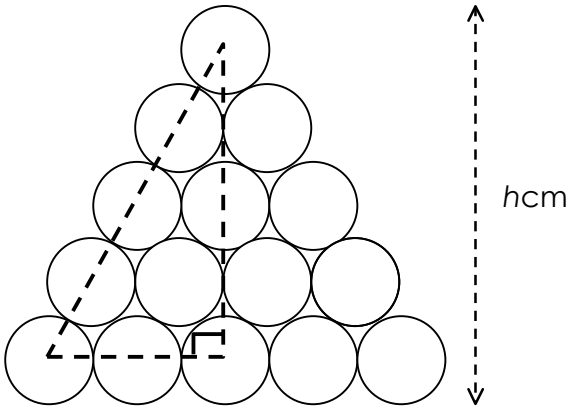
Given that the line AB = 48cm, calculate the width, *w* cm, of the sign.

12. The Pot Black Snooker Club has this sign at its entrance.



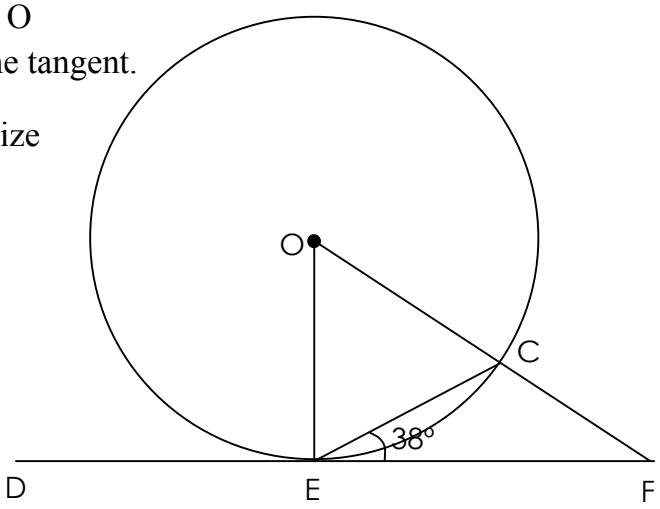
It consists of 10 circles each with radius 8cm.

Calculate the height, *h* cm, of the sign.

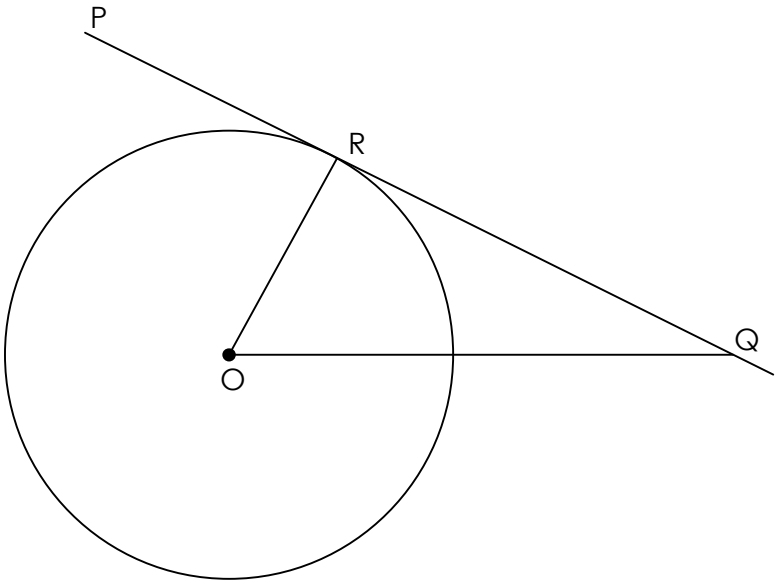


13. The line DF is a tangent to the circle centre O shown below. E is the point of contact of the tangent.

Given that angle CEF is 38°, calculate the size of angle EOC.



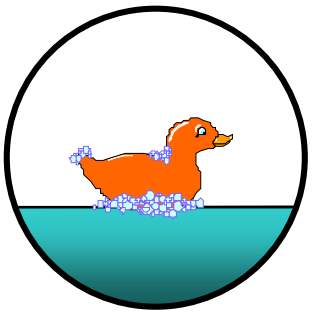
14. The circle in the diagram has centre O and radius 6cm.



R is the point of contacT of the tangent PQ.

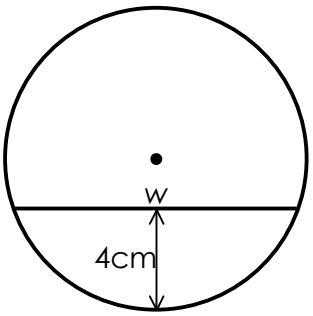
Given that OQ = 10cm calculate the length of RQ.

15. A child's toy is in the shape of a sphere with a duck and some water inside.



As the ball rolls around the water remains at the same level.

The diagram opposite shows the cross section when the sphere has been halved.



Given that the radius of the sphere is 6 cm and that the depth of the water is 4cm, calculate the width of the water surface ($w$cm).

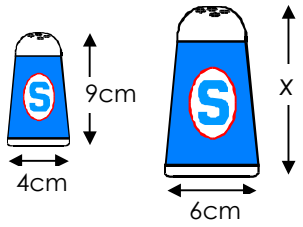
3.3 USING SIMILARITY

LENGTH

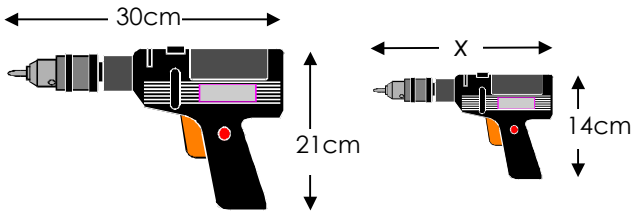
1. Each diagram below shows a pair of similar shapes or objects. For each pair …….

(i) state the scale factor (from left to right) **(ii)** calculate the length marked x .

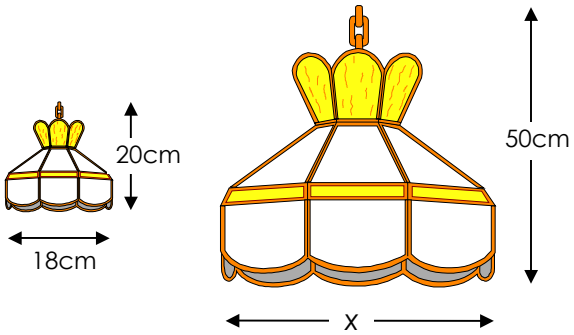
(a)



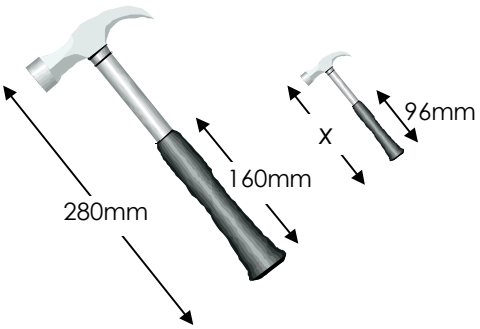
(b)



(c)

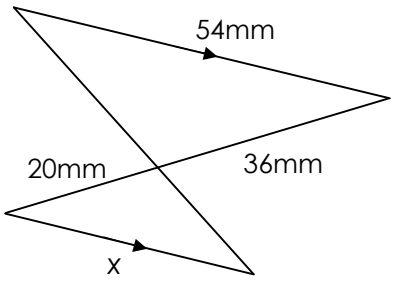


(d)

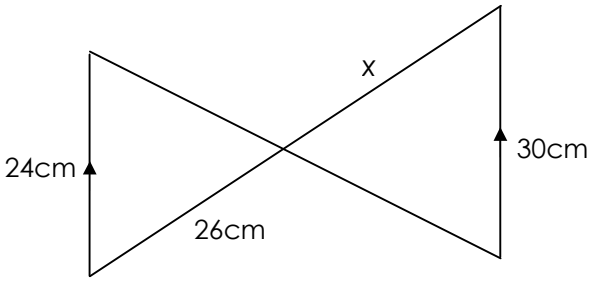


2. Calculate the length of the side marked x in each diagram below.

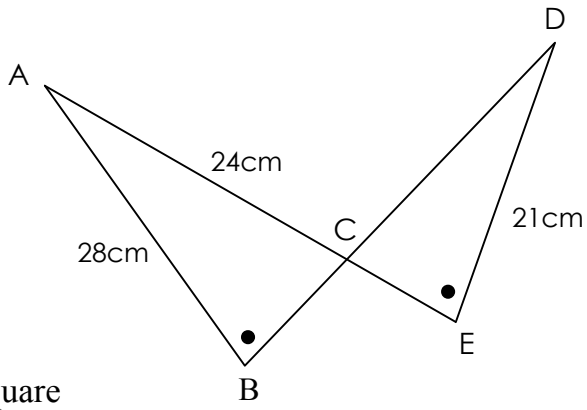
(a)



(b)



3. In the diagram $\angle ABC = \angle CED$, AB = 28cm, AC = 24cm and ED = 21cm.

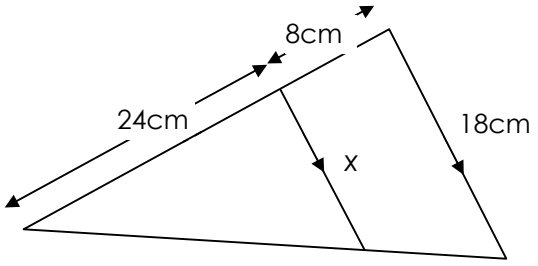


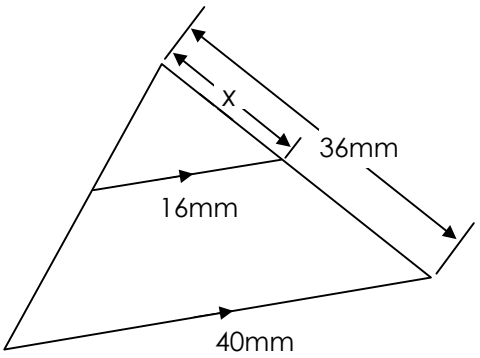
(a) Explain why the triangles ABC and CDE are similar.

(b) Calculate the length of CD.

(c) Given that the area of triangle ABC is 144 square centimetres, calculate the area of triangle CDE.

4. Calculate the length of the side marked x in each diagram below.



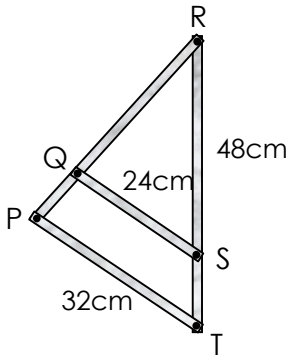


5. The diagram opposite shows an aluminium pipe frame.

The cross members QS and PT are parallel.

RS = 48cm, QS = 24cm and PT = 32cm as shown.

Calculate the length of ST

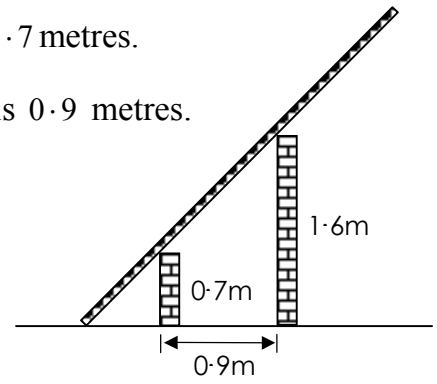


6. In the diagram a ladder is laid against two walls as shown.

The higher wall is $1 \cdot 6$ metres high, and the lower wall is $0 \cdot 7$ metres.

The distance between the two left hand faces of the walls is $0 \cdot 9$ metres.

Calculate the distance between the foot of the ladder and the lower wall.



SIMILARITY and AREA

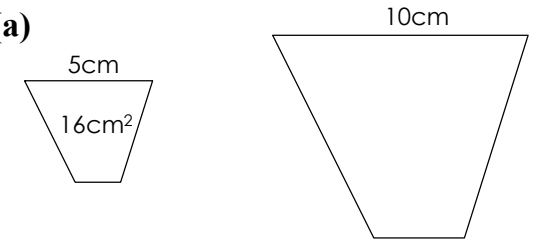
1. For each pair of pictures below

(i) State the enlargement scale factor for the lengths

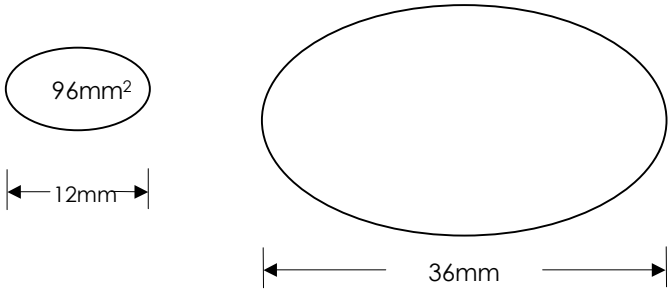
(ii) State the scale factor for the areas.

(iii) Calculate the area of the larger shape.

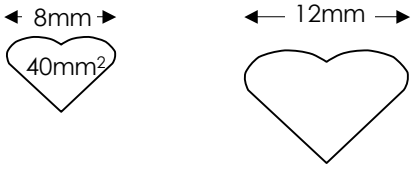
(a)



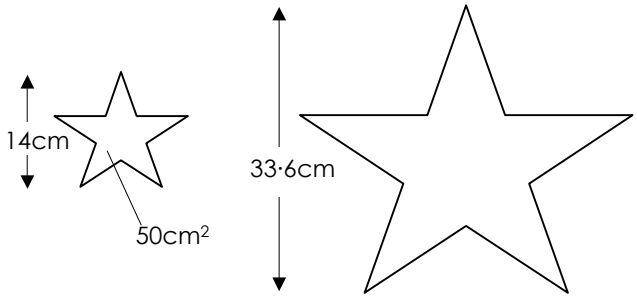
(b)



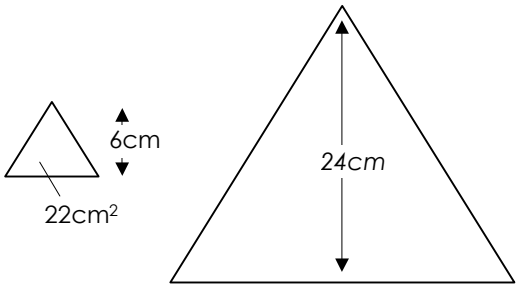
(c)



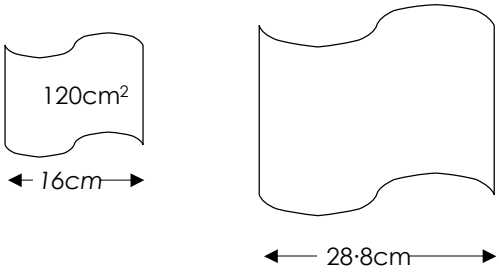
(d)



(e)



(f)



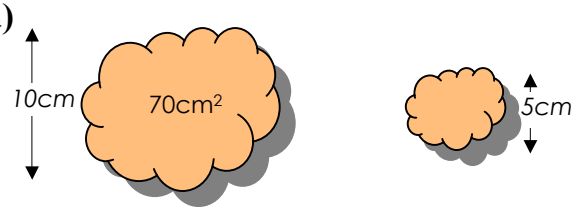
2. For each pair of pictures below

(i) State the reduction scale factor for the lengths.

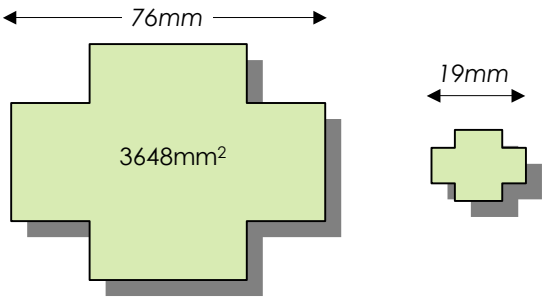
(ii) State the scale factor for the areas.

(iii) Calculate the area of the smaller shape.

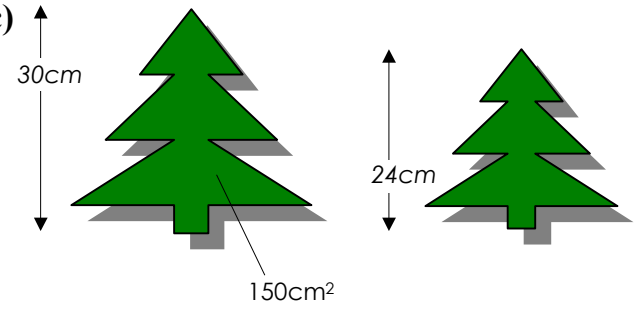
(a)



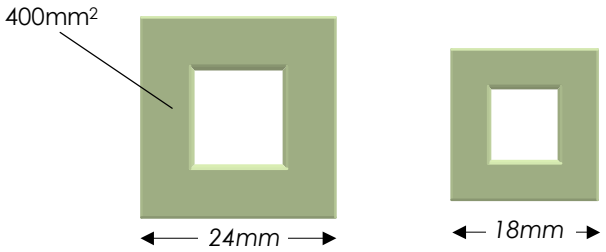
(b)



(c)

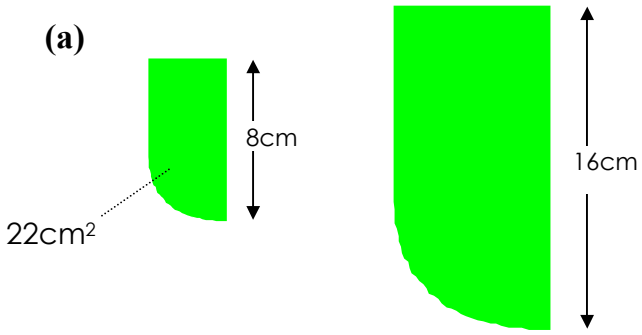


(d)

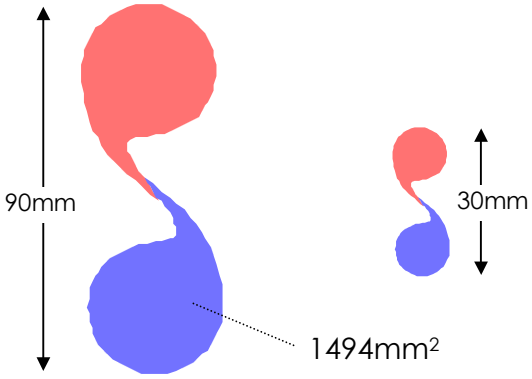


3. Each pair of shapes below is mathematically similar.
Calculate the **area** of each right-hand shape.

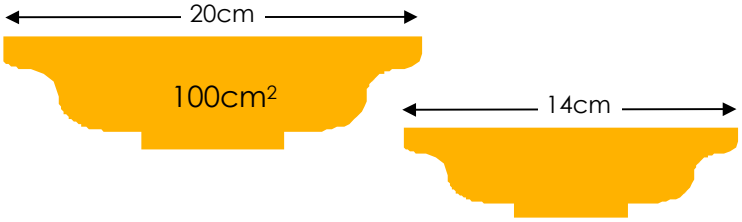
(a)



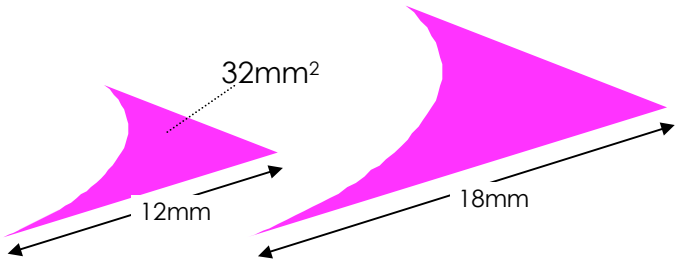
(b)



(c)



(d)



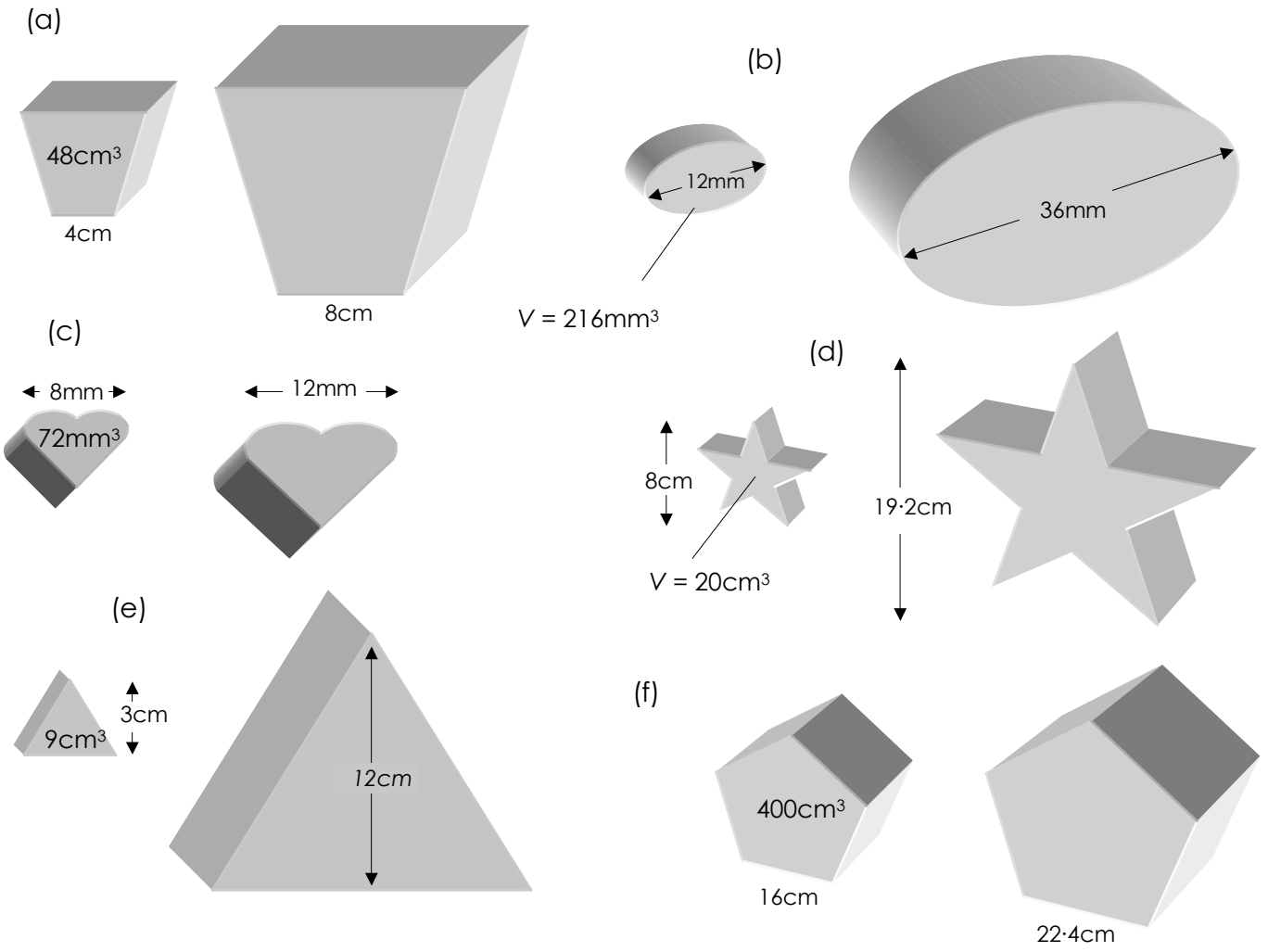
SIMILARITY and VOLUME

1. For each pair of similar pictures below

(i) State the enlargement scale factor for the length.

(ii) State the scale factor for the volumes.

(iii) Calculate the volume of the larger solid.



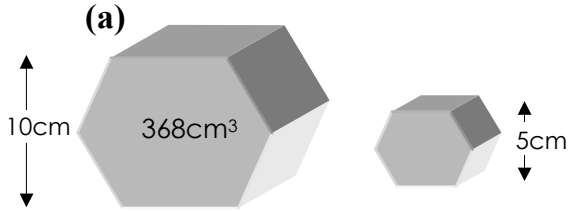
2. For each pair of similar pictures below

(i) State the reduction scale factor for the lengths.

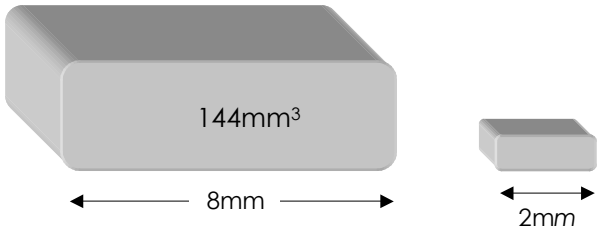
(ii) State the scale factor for the volumes.

(iii) Calculate the volume of the smaller solid.

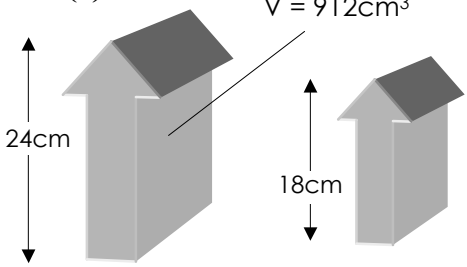
(a)



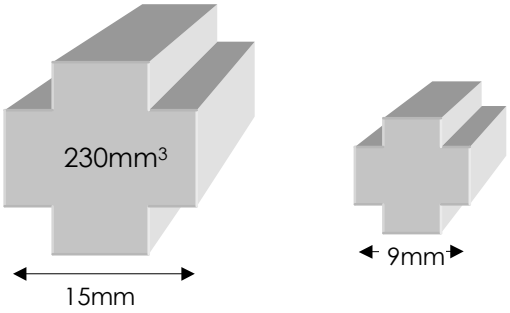
(b)



(c)



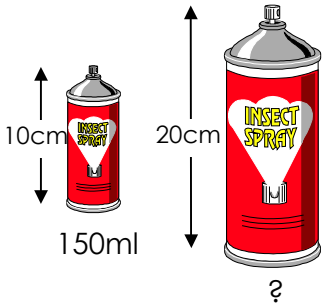
(d)



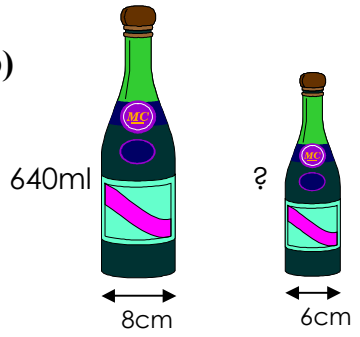
3. Each pair of containers below is mathematically similar.

Calculate the **volume** of each container with a question mark.

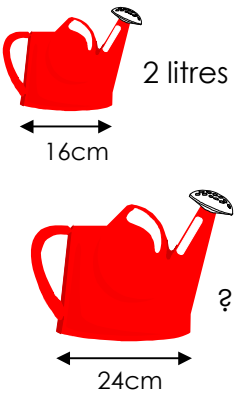
(a)



(b)

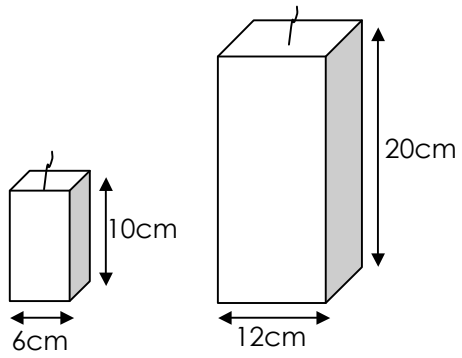


(c)



1. The diagram below shows two candles. Each candle is in the shape of a cuboid with a square base.

The length of time each candle will burn is proportional to its volume.



The small candle burns for 36 hours.

Nadia reckons that because the large candle's measurements are double the small candle's measurements then the large candle should burn for 72 hours.

Is she correct? [You should show all working and give a reason for your answer]

2. An international perfume manufacturer prices their bottles of perfume **by volume**.

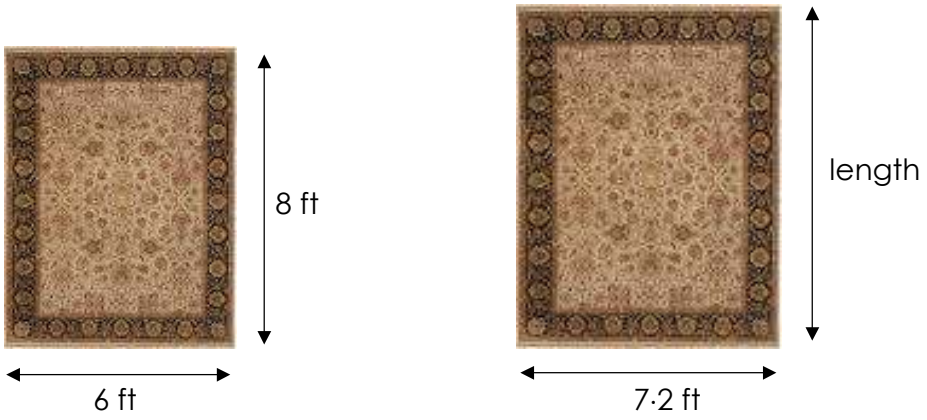
The two bottles below, although containing different volumes, are mathematically similar in shape. Their heights and prices are shown.

The larger of the two bottles is for sale in France.

Assuming the smaller bottle to be priced correctly, determine whether or not the larger bottle has the correct price tag given that the exchange rate is £1 = 1·10 euros.

3. John is looking to buy a new rug for his main room.

The two rugs below are **mathematically similar** in shape.



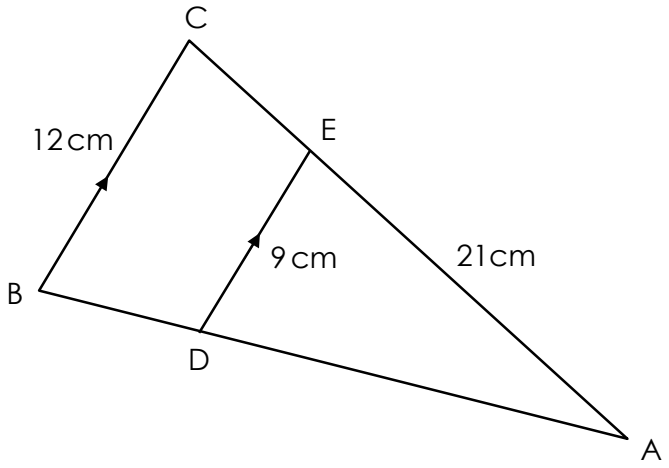
He is hoping that the length of the large rug will be enough to make the **area** of the large rug **at least 72 square feet**.

Does the large rug have the required area?

You must show appropriate working with your answer.

4. In the diagram below triangles ABC and ADE are **mathematically similar**.

BC = 12 cm, DE = 9 cm and AE = 21 cm.



Find the length of CE.

4.1 WORKING with TRIGONOMETRIC FUNCTIONS – BASIC GRAPHS

1. **(a)** With the help of a calculator, copy and complete the table below.

x^o	0	30	60	90	120	150	180	210	240	270	300	330	360
$\sin x^o$													

(b) Plot the points from your table.

(c) Join the points with a smooth curve.

(d) Write down the equation of the curve.

2. **(a)** With the help of a calculator, copy and complete the table below.

x^o	0	30	60	90	120	150	180	210	240	270	300	330	360
$\cos x^o$													

(b) Plot the points from your table.

(c) Join the points with a smooth curve.

(d) Write down the equation of the curve.

3. **.(a)** With the help of a calculator, copy and complete the table below.

x^o	0	30	60	90	120	150	180	210	240	270	300	330	360
$\tan x^o$													

(b) Plot the points from your table.(Be careful with the scale on the y-axis)

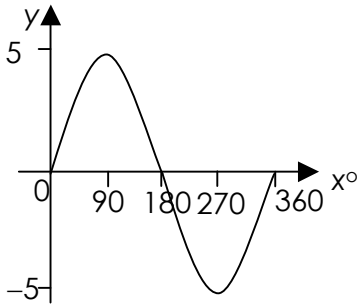
(c) Join the points with a smooth curve.

(d) Write down the equation of the curve.

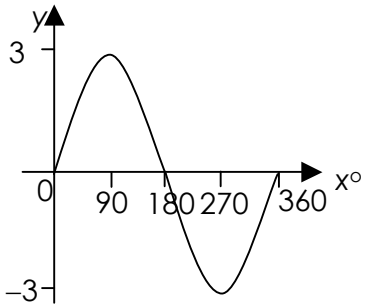
4.1 WORKING with the GRAPHS of TRIGONOMETRIC FUNCTIONS (1)

1. The graphs represent the functions $a \sin x^o$ and $a \cos x^o$. Write down the equation for each.

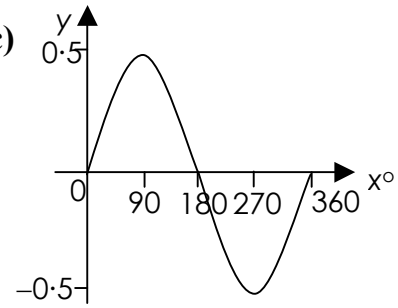
(a)



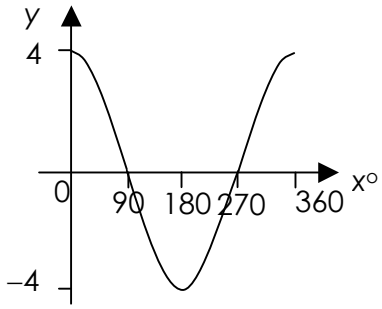
(b)



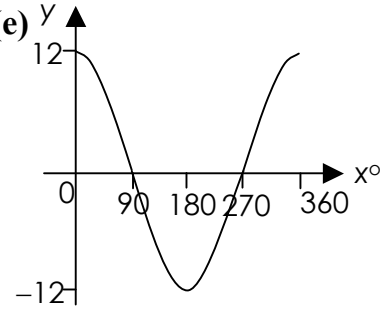
(c)



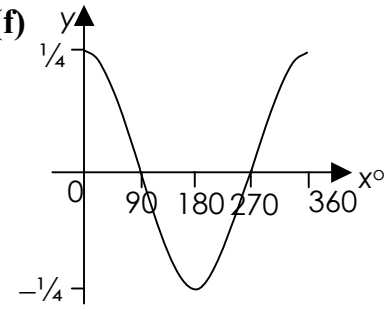
(d)



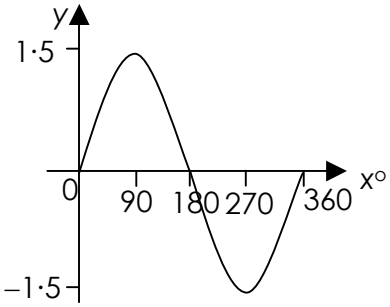
(e)



(f)



(g)



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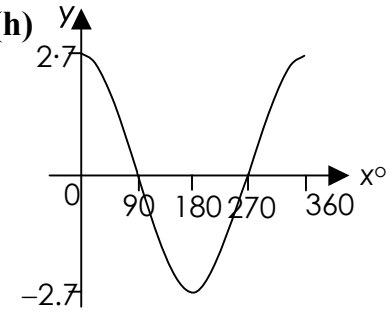

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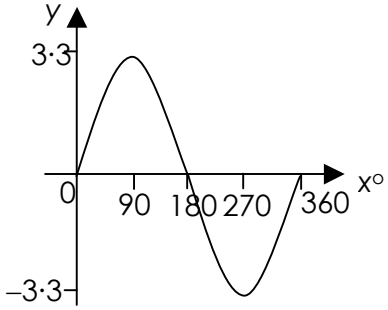

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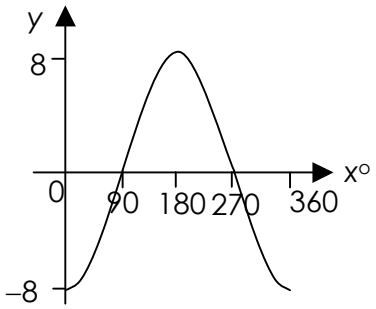

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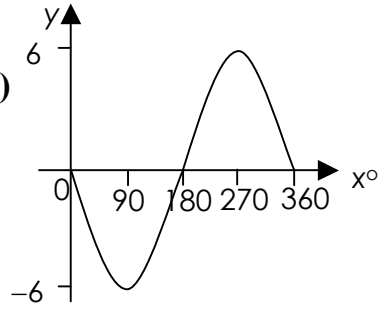
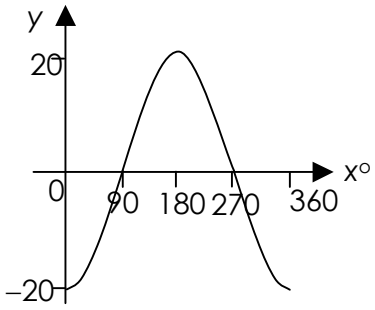


(l)



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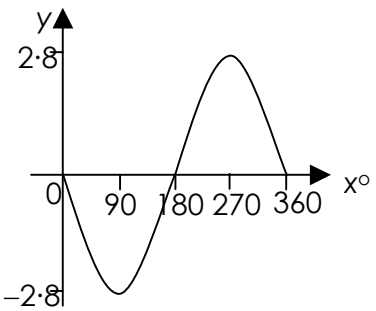

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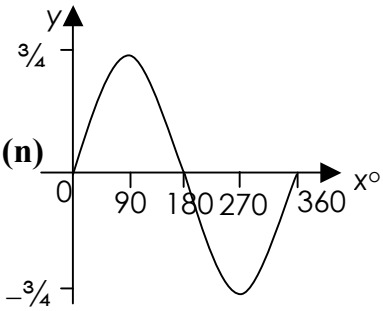
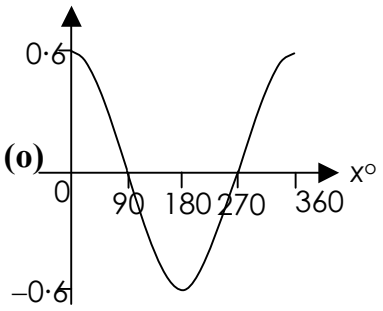
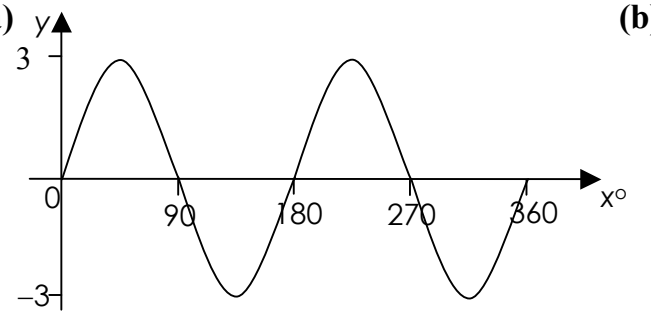


(o)

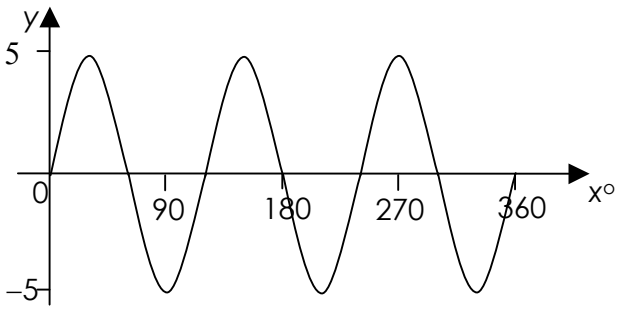


2. The graphs represent trigonometric functions. Write down the equation for each.

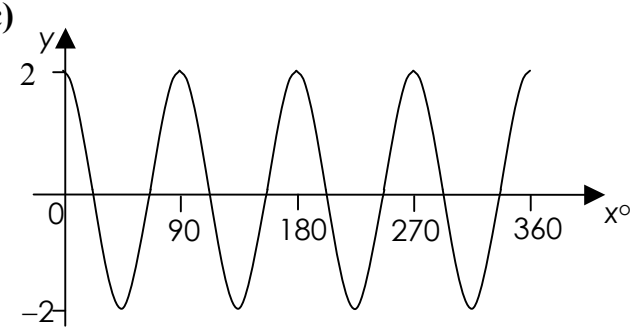
(a)



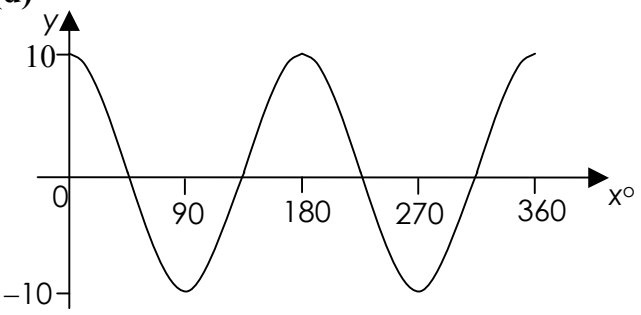
(b)



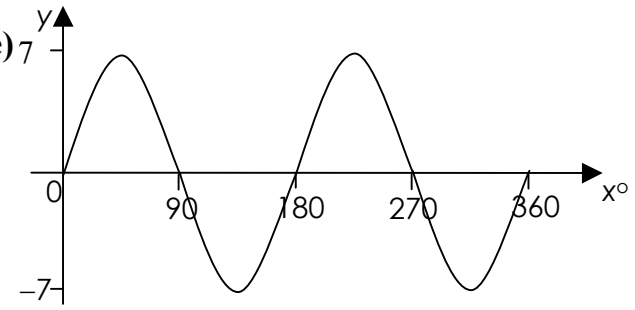
(c)



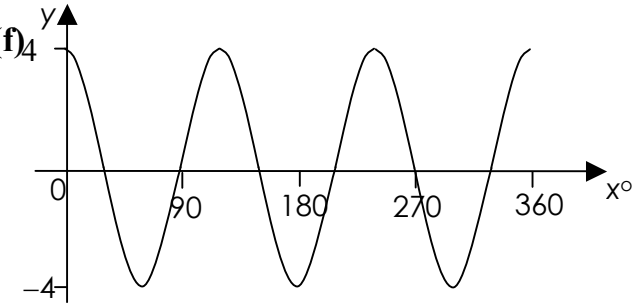
(d)



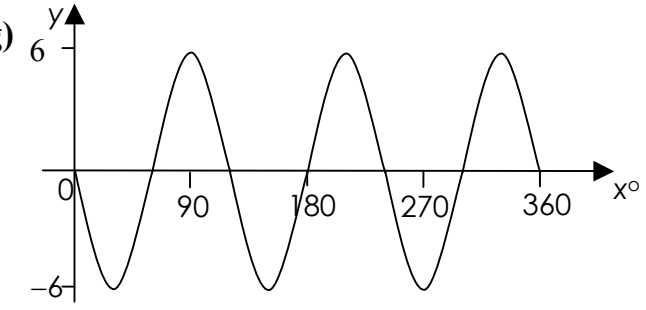
(e)



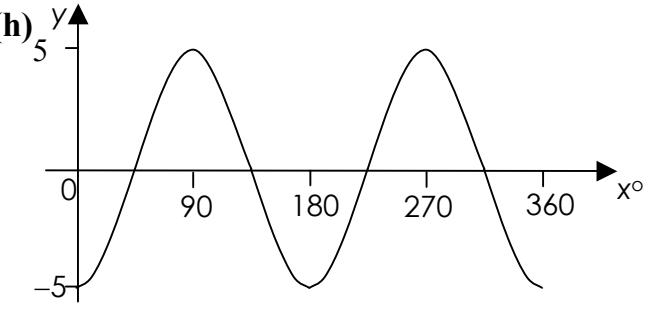
(f)



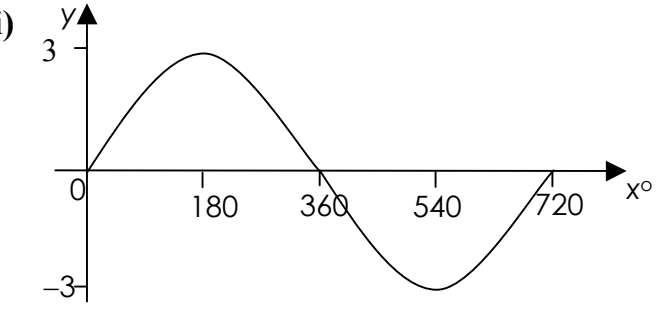
(g)



(h)



(i)



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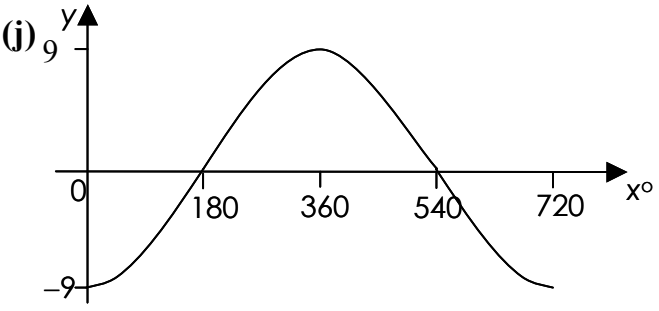
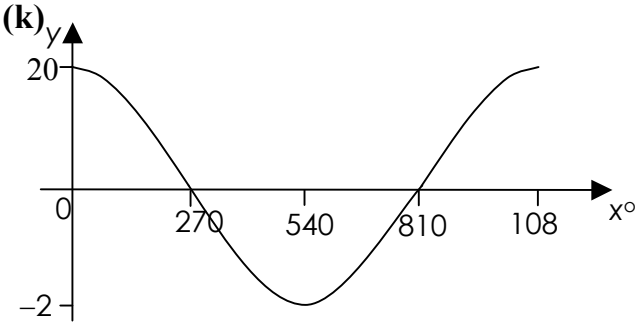


(k)



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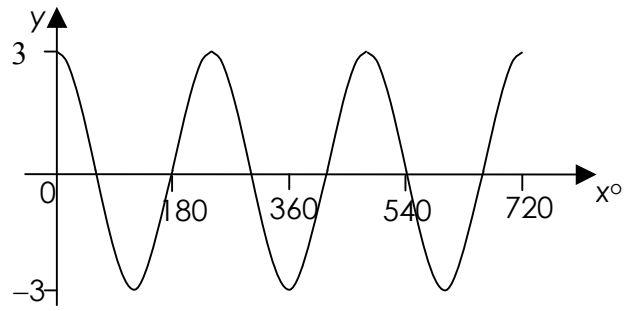
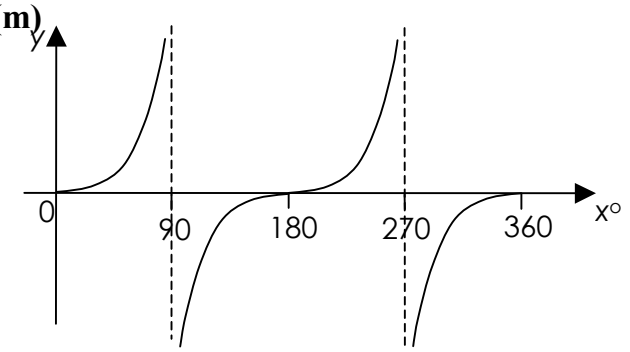


(m)



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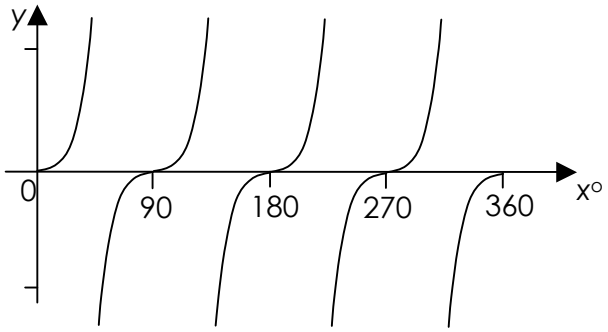
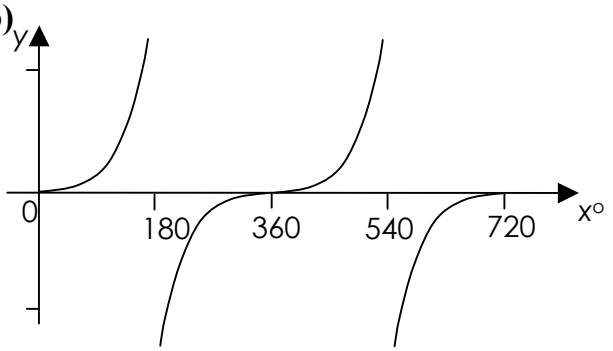
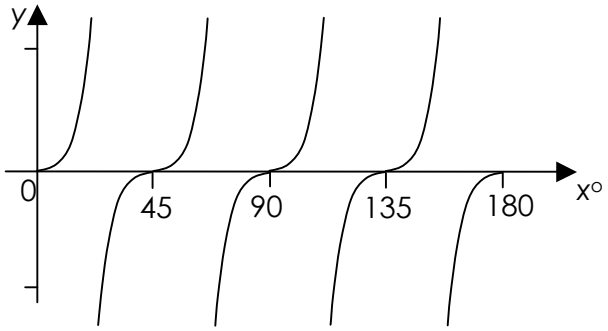


(o)



(p)

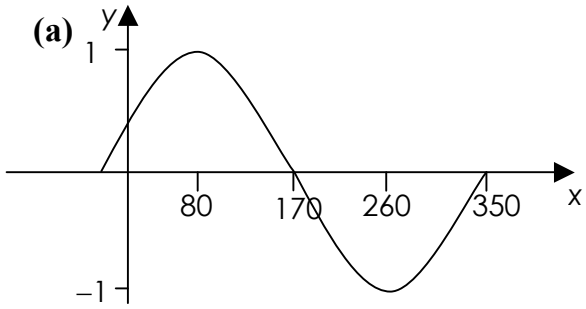


3. Make sketches of the following functions, $0 \leq x < 360$, clearly marking any important points.

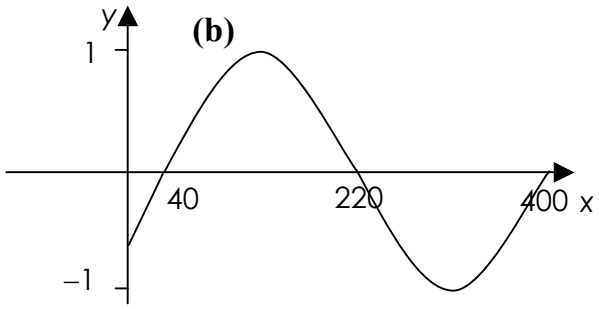
(a)	$y = \cos x^o$	**(b)**	$y = \sin x^o$	**(c)**	$y = \tan x^o$
(d)	$y = 3 \sin x^o$	**(e)**	$y = 2 \cos x^o$	**(f)**	$y = \sin 2x^o$
(g)	$y = \cos 3x^o$	**(h)**	$y = 2 \sin 3x^o$	**(i)**	$y = 3 \cos 2x^o$
(j)	$y = 4 \cos 3x^o$	**(k)**	$y = 3 \sin \frac{1}{2}x^o$	**(l)**	$y = 5 \cos \frac{3}{2}x^o$
(m)	$y = \tan 2x^o$	**(n)**	$y = -2 \sin 3x^o$	**(o)**	$y = -8 \cos 4x^o$

4. The graphs represent the functions $\sin(x \pm a)^o$ and $\cos(x \pm a)^o$. Write down the equation for each.

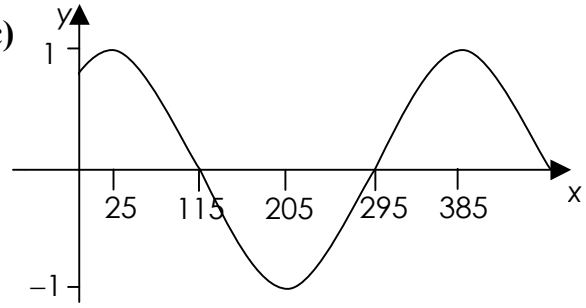
(a)



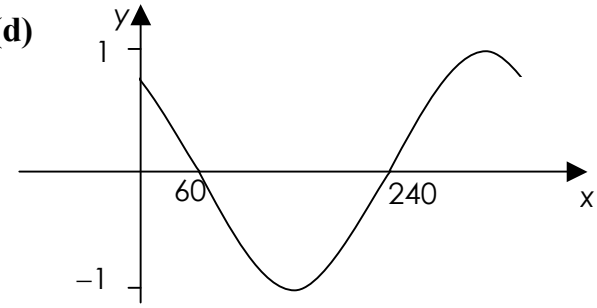
(b)



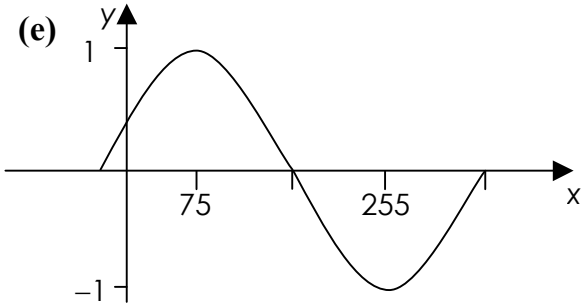
(c)



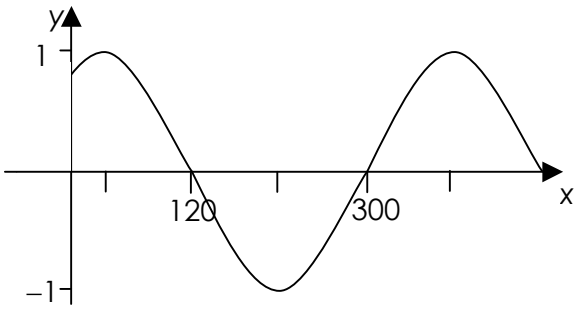
(d)



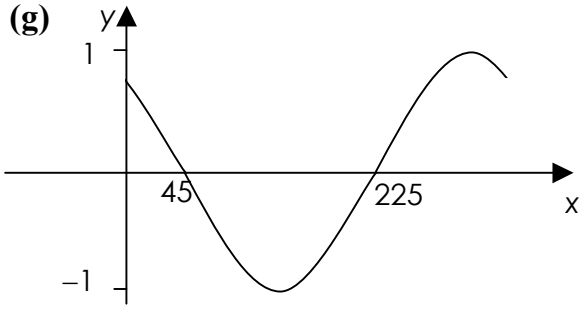
(e)



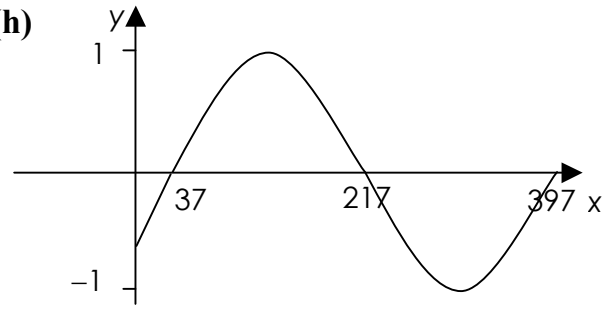
(f)



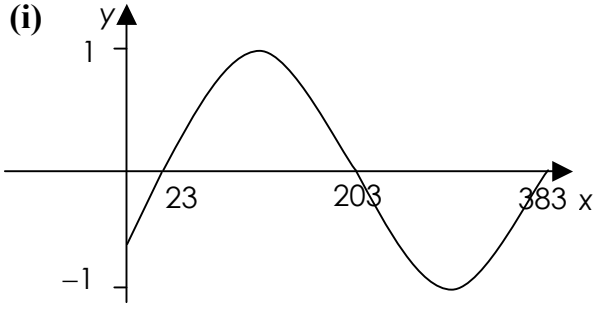
(g)



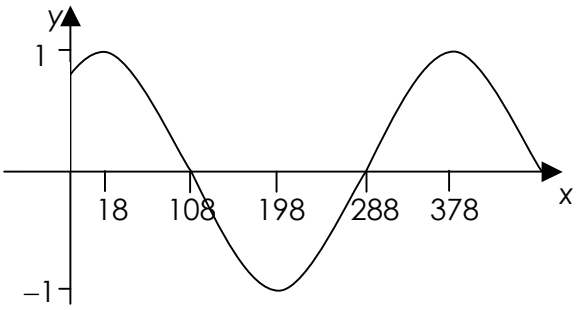
(h)



(i)

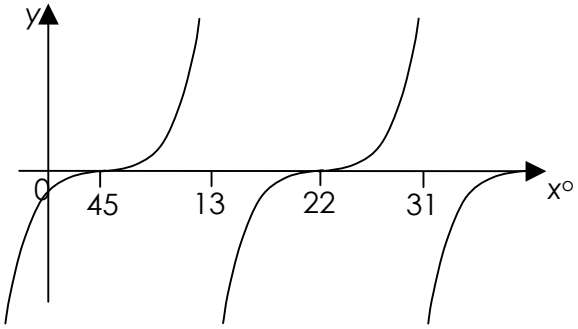


(j)

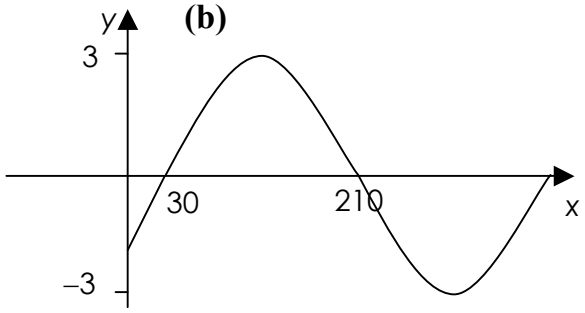


5. Write down the equation for each graph shown below.

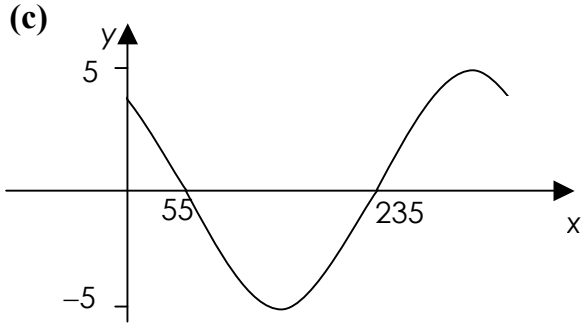
(a)



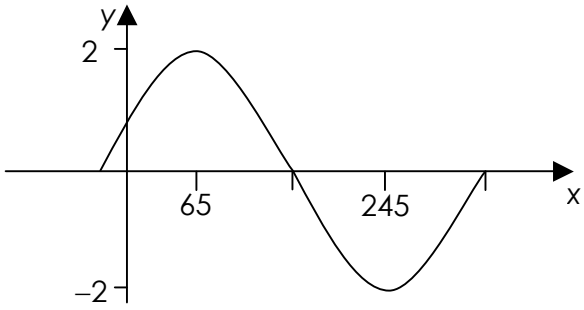
(b)



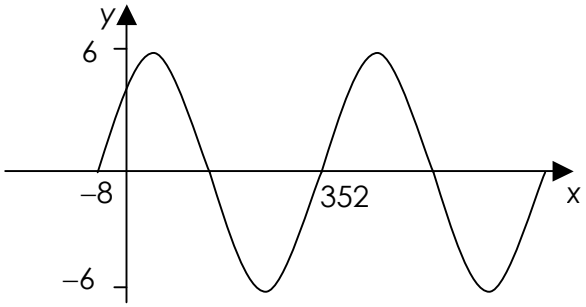
(c)



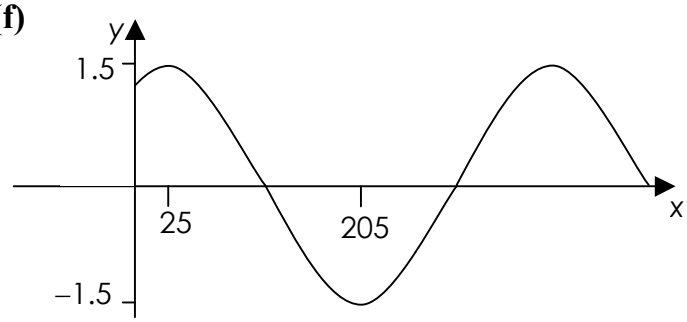
(d)



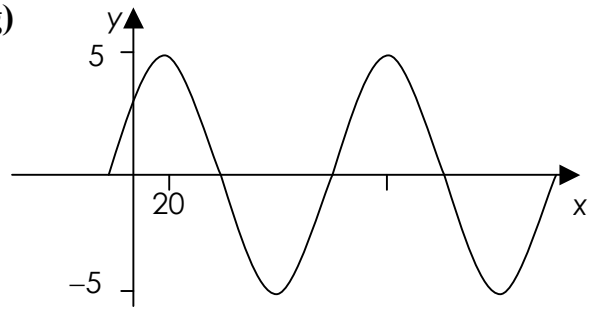
(e)



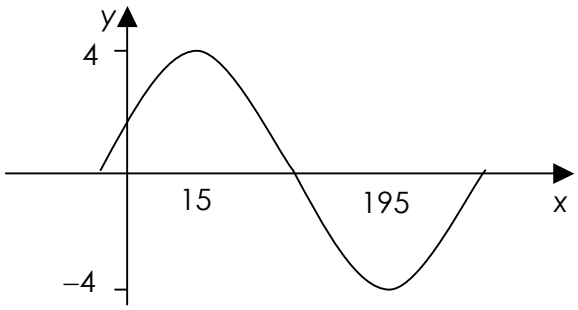
(f)



(g)



(h)



6. Make a neat sketch of these trig. functions showing the important values, $0 \le x \le 360$.

(a) $y = \sin (x - 50)^o$ **(b)** $y = \sin (x + 30)^o$

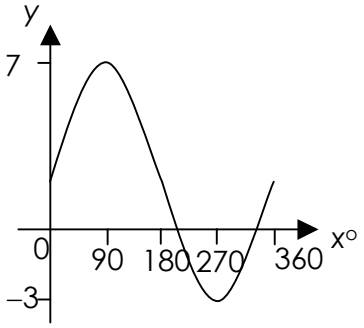
(c) $y = \cos (x - 20)^o$ **(d)** $y = \cos (x + 60)^o$

(e) $y = \tan (x - 30)^o$ **(f)** $y = \sin (x - 45)^o$

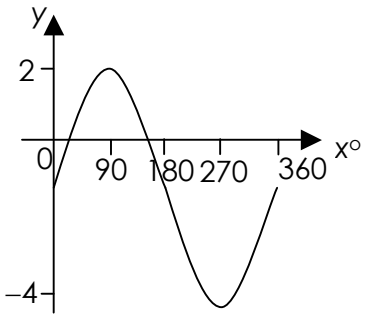
4.1 WORKING with the GRAPHS of TRIGONOMETRIC FUNCTIONS (2)

1. For each graph below, determine the amplitude and the equation of the graph.

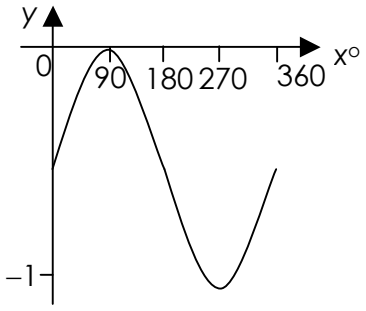
(a)



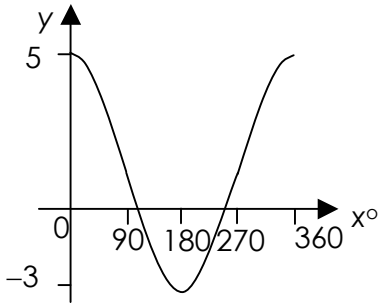
(b)



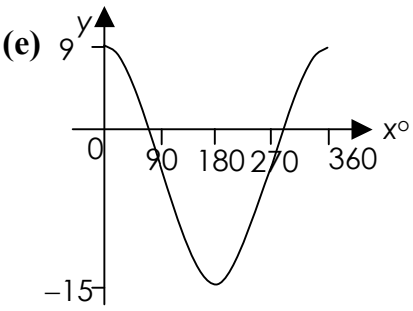
(c)



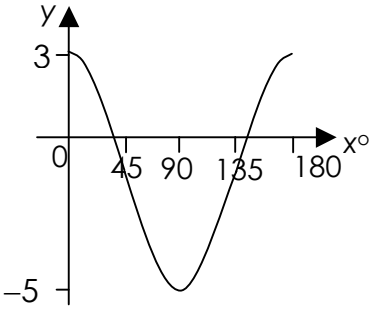
(d)



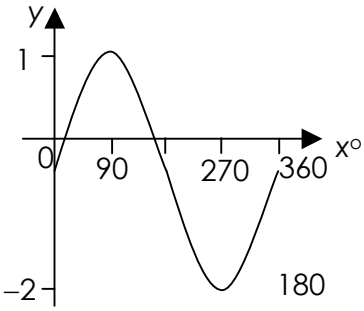
(e)



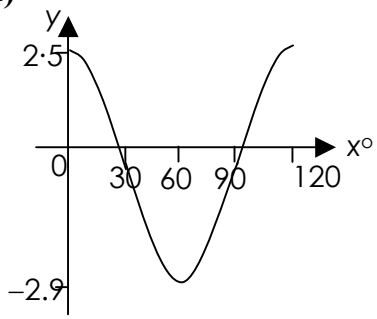
(f)



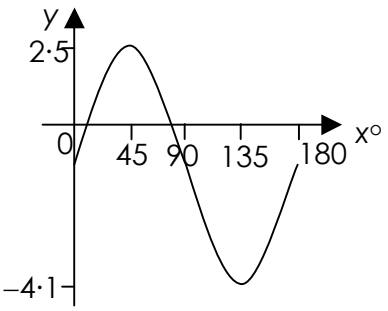
(g)



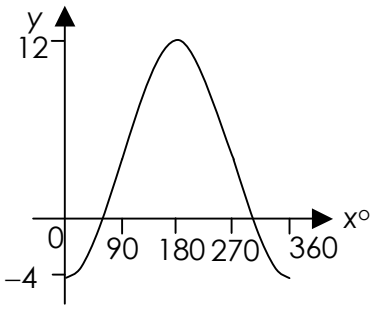
(h)



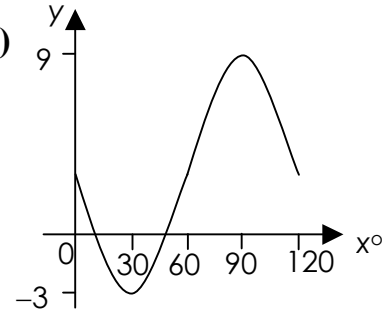
(i)



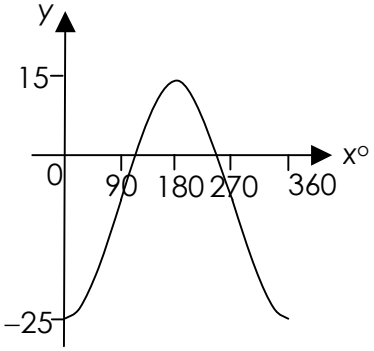
(j)



(k)



(l)



2. Make sketches of the following functions, $0 \leq x < 360$, clearly marking any important points.

(a) $y = 3\cos x^{\circ} + 1$ **(b)** $y = 2\sin 3x^{\circ} - 2$ **(c)** $y = 1{\cdot}5\sin 2x^{\circ} + 3$

(d) $y = 5\sin 2x^{\circ} + 1$ **(e)** $y = -3\cos 2x^{\circ} - 1$ (f) $y = -\sin x^{\circ} - 3$

4.2 WORKING with TRIGONOMETRIC RELATIONSHIPS in DEGREES

SINE, COSINE and TANGENT of angles 0 – 360°

1. Copy and complete this table to show the values where sin, cos and tan are positive (+) or negative (–).

	$0 < x < 90$	$90 < x < 180$	$180 < x < 270$	$270 < x < 360$
$\sin x^o$	+			–
$\cos x^o$		–		
$\tan x^o$	+			

2. Write down the sign (+ or –) for the following

(a) $\cos 22^o$ **(b)** $\tan 123^o$ **(c)** $\sin 315^o$ **(d)** $\sin 15^o$

(e) $\tan 196^o$ **(f)** $\cos 295^o$ **(g)** $\tan 66^o$ **(h)** $\sin 132^o$

(i) $\cos 170^o$ **(j)** $\sin 218^o$ **(k)** $\cos 200^o$ **(l)** $\tan 310^o$

3. For the angles in Question 2, write down the equivalent acute angle.

4.2 WORKING with TRIGONOMETRIC RELATIONSHIPS in DEGREES

EXACT VALUES

1. Write down the exact values of

(a) $\sin 30^o$ **(b)** $\sin 150^o$ **(c)** $\sin 210^o$ **(d)** $\sin 330^o$

(e) $\cos 30^o$ **(f)** $\cos 150^o$ **(g)** $\cos 210^o$ **(h)** $\cos 330^o$

(i) $\tan 30^o$ **(j)** $\tan 150^o$ **(k)** $\tan 210^o$ **(l)** $\tan 330^o$

2. Write down the exact values of

(a) $\sin 60^o$ **(b)** $\sin 120^o$ **(c)** $\sin 240^o$ **(d)** $\sin 300^o$

(e) $\cos 60^o$ **(f)** $\cos 120^o$ **(g)** $\cos 240^o$ **(h)** $\cos 300^o$

(i) $\tan 60^o$ **(j)** $\tan 120^o$ **(k)** $\tan 240^o$ **(l)** $\tan 300^o$

3. Write down the exact values of

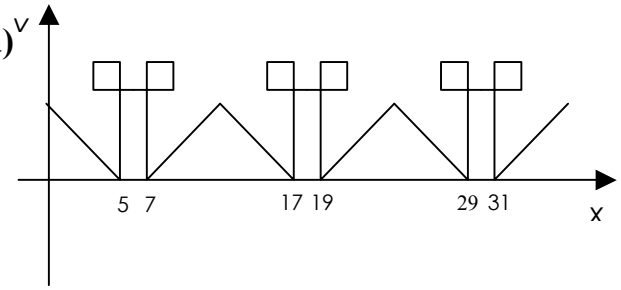
(a)	$\sin 45^o$	**(b)**	$\sin 135^o$	**(c)**	$\sin 225^o$	**(d)**	$\sin 315^o$
(e)	$\cos 45^o$	**(f)**	$\cos 135^o$	**(g)**	$\cos 225^o$	**(d)**	$\cos 315^o$
(i)	$\tan 45^o$	**(j)**	$\tan 135^o$	**(k)**	$\tan 225^o$	**(l)**	$\tan 315^o$

4.2 WORKING with TRIGONOMETRIC RELATIONSHIPS in DEGREES

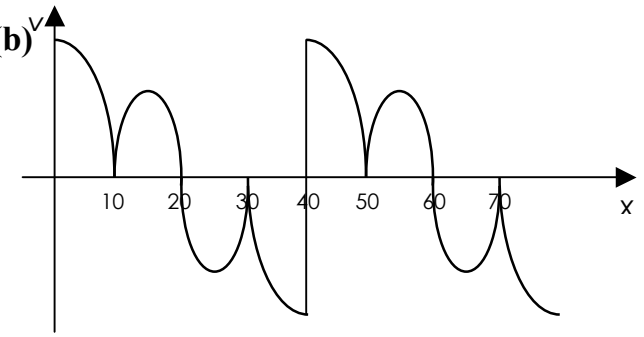
PERIOD

1. Write down the period of the following graphs

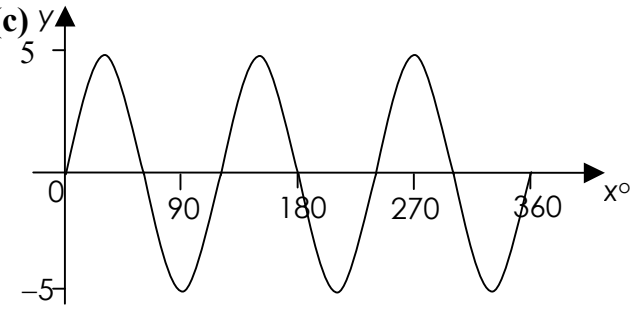
(a)



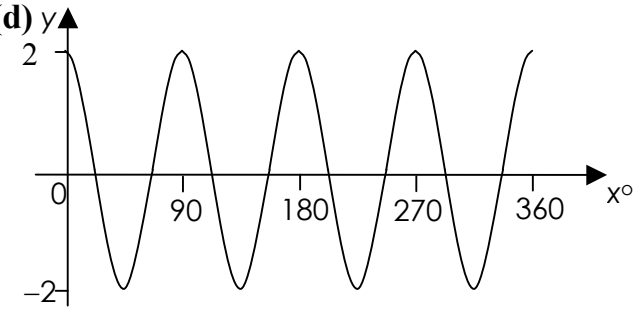
(b)



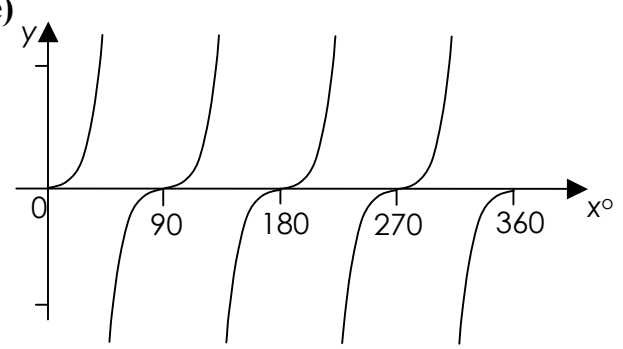
(c)



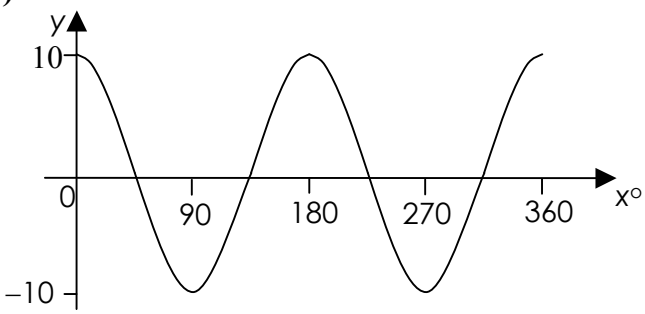
(d)



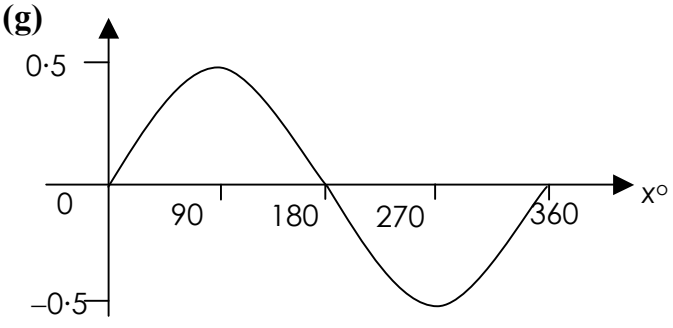
(e)



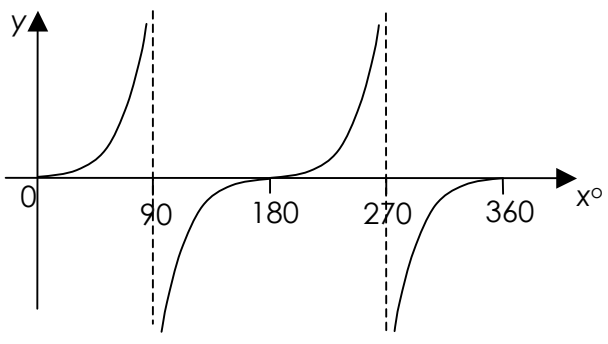
(f)



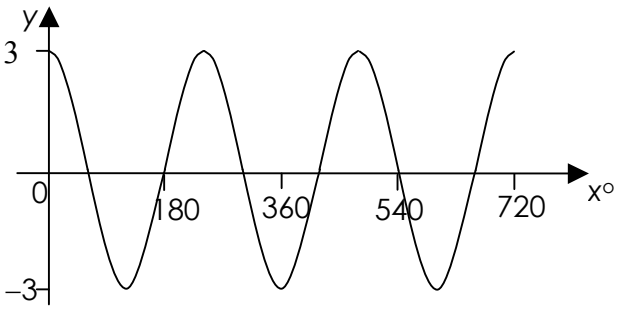
(g)



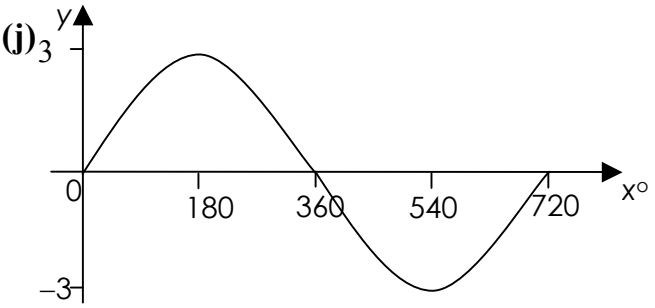
(h)



(i)



(j)



2. Write down the period of each of the following trigonometrical functions.

(a)	$y = \sin 2x^{\circ}$	**(b)**	$y = \tan 2x^{\circ}$	**(c)**	$y = \cos 2x^{\circ}$
(d)	$y = \tan 3x^{\circ}$	**(e)**	$y = \cos 4x^{\circ}$	**(f)**	$y = \sin 3x^{\circ}$
(g)	$y = \cos 1{\cdot}5x^{\circ}$	**(h)**	$y = \sin 4{\cdot}5x^{\circ}$	**(i)**	$y = \tan 0{\cdot}5x^{\circ}$
(j)	$y = \sin 8x^{\circ}$	**(k)**	$y = \tan 6x^{\circ}$	**(l)**	$y = \cos 12x^{\circ}$
(m)	$y = \tan 18x^{\circ}$	**(n)**	$y = \cos 9x^{\circ}$	**(o)**	$y = \sin 30x^{\circ}$
(p)	$y = \cos 15x^{\circ}$	**(q)**	$y = \sin 10x^{\circ}$	**(r)**	$y = \tan 4x^{\circ}$

3. Write down the period of each of the following trigonometrical functions.

(a)	$y = \sin \frac{1}{2}x^{\circ}$	**(b)**	$y = \tan {}^{1}/_{3}x^{\circ}$	**(c)**	$y = \cos \frac{1}{4}x^{\circ}$
(d)	$y = \tan {}^{1}/_{5}x^{\circ}$	**(e)**	$y = \cos {}^{1}/_{6}x^{\circ}$	**(f)**	$y = \sin {}^{2}/_{3}x^{\circ}$
(g)	$y = 3\cos 2x^{\circ}$	**(h)**	$y = 4\sin 3x^{\circ}$	**(i)**	$y = 2\tan 2x^{\circ}$
(j)	$y = 5\sin 2x^{\circ}$	**(k)**	$y = 4\tan x^{\circ}$	**(l)**	$y = 2\cos 4x^{\circ}$

4.2 WORKING with TRIGONOMETRIC RELATIONSHIPS in DEGREES

SOLVING BASIC EQUATIONS

1. Solve the following equations where $0 \le x \le 360$

(a)	$\sin x^o = 0{\cdot}5$	**(b)**	$\cos x^o = 0{\cdot}866$	**(c)**	$\tan x^o = 1$
(d)	$\cos x^o = -0{\cdot}5$	**(e)**	$\tan x^o = -0{\cdot}577$	**(f)**	$\sin x^o = -0{\cdot}866$
(g)	$\tan x^o = 1{\cdot}732$	**(h)**	$\sin x^o = 0{\cdot}707$	**(i)**	$\cos x^o = 0{\cdot}707$
(j)	$\sin x^o = -0{\cdot}707$	**(k)**	$\cos x^o = -0{\cdot}866$	**(l)**	$\tan x^o = -1{\cdot}732$

2. Solve the following equations where $0 \le x \le 360$

(a)	$\sin x^o = 0{\cdot}313$	**(b)**	$\cos x^o = 0{\cdot}425$	**(c)**	$\tan x^o = 5{\cdot}145$
(d)	$\cos x^o = -0{\cdot}087$	**(e)**	$\tan x^o = -0{\cdot}869$	**(f)**	$\sin x^o = -0{\cdot}191$
(g)	$\tan x^o = 11{\cdot}43$	**(h)**	$\sin x^o = 0{\cdot}695$	**(i)**	$\cos x^o = 0{\cdot}755$
(j)	$\sin x^o = -0{\cdot}358$	**(k)**	$\cos x^o = -0{\cdot}682$	**(l)**	$\tan x^o = -0{\cdot}268$

3. Solve the following equations where $0 \le x \le 360$

(a)	$2 \sin x^o = 1$	**(b)**	$3 \cos x^o = 2$	**(c)**	$3 \tan x^o = 5$
(d)	$2 \cos x^o = -1$	**(e)**	$2 \tan x^o = -8$	**(f)**	$4 \sin x^o = -3$
(g)	$5 \tan x^o = 23{\cdot}5$	**(h)**	$5 \sin x^o = 2$	**(i)**	$6 \cos x^o = 1$
(j)	$8 \sin x^o = -3$	**(k)**	$11 \cos x^o = -9$	**(l)**	$10 \tan x^o = -9$

4. Solve the following equations where $0 \le x \le 360$

(a)	$\sin x^o - 1 = 0$	**(b)**	$\cos x^o + 1 = 0$	**(c)**	$\tan x^o - 1 = 0$
(d)	$2 \sin x^o + 1 = 0$	**(e)**	$2 \cos x^o - 1 = 0$	**(f)**	$2 \tan x^o - 1 = 0$
(g)	$4 \cos x^o - 3 = 0$	**(h)**	$3 \sin x^o - 2 = 0$	**(i)**	$5 \cos x^o + 2 = 0$
(j)	$3 \tan x^o - 2 = 0$	**(k)**	$3 \cos x^o + 1 = 0$	**(l)**	$7 \sin x^o + 3 = 0$

5. Solve the following equations where $0 \le x \le 360$

(a)	$4 \cos x^o + 3 = 2$	**(b)**	$10 \sin x^o - 4 = 3$	**(c)**	$2 \tan x^o - 3 = 17$
(d)	$7 + 10 \cos x^o = 12$	**(e)**	$2 \tan x^o + 3 = 5$	**(f)**	$17 - 5 \cos x^o = 20$
(g)	$5 \sin x^o + 3 = 5$	**(h)**	$21 + 2 \cos x^o = 20$	**(i)**	$2 \sin x^o - 1{\cdot}6 = 0$
(j)	$3 \cos x^o + \sqrt{2} = 0$	**(k)**	$7 \sin x^o - 1 = 4$	**(l)**	$2 \sin x^o + \sqrt{3} = 2\sqrt{2}$

4.2 WORKING with TRIGONOMETRIC RELATIONSHIPS in DEGREES

IDENTITIES INVOLVING $\cos^2 x + \sin^2 x = 1$ and $\tan x = \sin x / \cos x$

1. Simplify

(a) $3\cos^2 x + 3\sin^2 x$ **(b)** $1 - \cos^2 x$ **(c)** $\cos A \tan A$

(d) $5 - 5\sin^2 B^o$ **(e)** $\dfrac{4\sin a^o}{4\cos a^o}$ **(f)** $\dfrac{4\tan x^o}{2\cos x^o}$

(g) $\dfrac{(1-\sin^2 x)}{2\cos x}$ **(h)** $\dfrac{8-8\cos^2 x}{2\sin x}$ **(i)** $\dfrac{3\sin x\cos x}{6\tan x}$

(j) $4\sin^2 A + 3\cos^2 A - 3$ **(k)** $4\cos^2 B - 2\sin^2 B + 2$

(l) $(\cos x + \sin x)^2 - 2\sin x\cos x$ **(m)** $\tan^2 a(1 - \sin^2 a)$

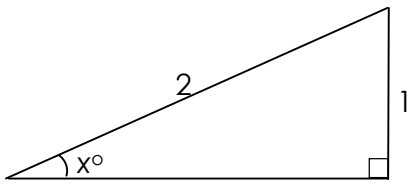
2. Prove that

(a) $2\cos^2 A + 3\sin^2 A = 3 - \cos^2 A$ **(b)** $\dfrac{1}{\tan x} + \tan x = \dfrac{1}{\sin x \cos x}$

(c) $(2\cos B + 3\sin B)^2 + (3\cos B - 2\sin B)^2 = 13$

(d) $(1 + \sin x)(1 - \sin x) = \cos^2 x$ **(e)** $\sin\theta.\tan\theta = \dfrac{1-\cos^2\theta}{\cos\theta}$

3. Consider the diagram opposite :



(a) Write down the **exact** values of .....

$\sin x^\circ$, $\cos x^\circ$ **and** $\tan x^\circ$.

(b) Prove that $\sin^2 x^\circ + \cos^2 x^\circ = 1$.

(c) Show that $\dfrac{\sin x^\circ}{\cos x^\circ} = \tan x^\circ$.

4. **(a)** If $\cos a^\circ = \dfrac{1}{\sqrt{5}}$, find the **exact** values of $\sin a^\circ$ **and** $\tan a^\circ$ where $0 < a < 90$.

(b) Prove that $\cos^2 a^\circ = 1 - \sin^2 a^\circ$.

(c) Show that $\dfrac{\sin^2 a^\circ}{\cos^2 a^\circ} = \tan^2 a^\circ$.

(d) Show that $2(3\sin a^\circ + 4\cos a^\circ) = 4\sqrt{5}$.

5. Prove that

(a) $3\cos^2 a + 3\sin^2 a = 3$

(b) $(\cos x + \sin x)^2 = 1 + 2\sin x \cos x$

(c) $(\cos x + \sin x)(\cos x - \sin x) = 2\cos^2 x - 1$

(d) $\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x} = \dfrac{1}{\cos x \sin x}$

(e) $\tan^2 p - \tan^2 p \sin^2 p = \sin^2 p$

(f) $\cos^4 x - \sin^4 x = 2\cos^2 x - 1$

(g) $3\sin^2 \theta + 2\cos^2 \theta = 2 + \sin^2 \theta$

6. Show that $(2\cos x + 5\sin x)^2 + (5\cos x - 2\sin x)^2 = 29$

7. Given that $p = \cos\theta + \sin\theta$ and $q = \cos\theta - \sin\theta$, prove that :

(a) $pq = 2\cos^2 \theta - 1$ **(b)** $\sin^2 \theta = \frac{1}{2}(1 - pq)$

EXAM QUESTIONS

1. Solve the following equation

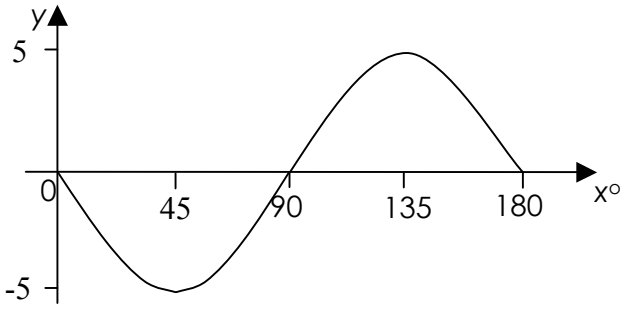
$3 - 5\sin x° = -1$ where $0 \le x \le 360$

2. Sketch the graph of $f(x) = \cos (x - 30)^o$ for $0 \le x \le 360$

3. Simplify

$$\frac{1 - \cos^2 x}{\cos^2 x}$$

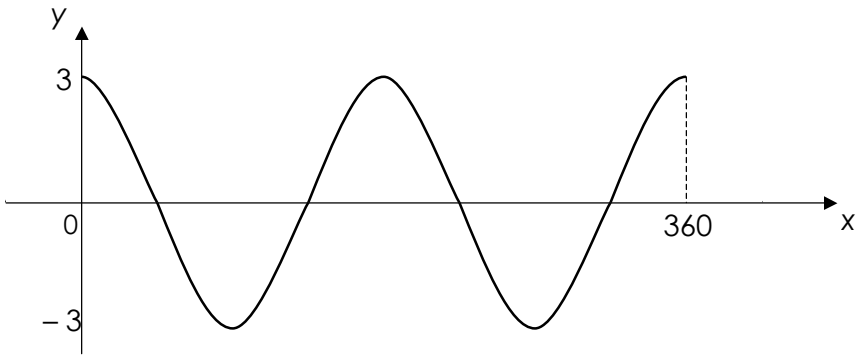
4. The diagram shows the graph of $y = a \sin bx^\circ$.



Write down the values of a and b.

5. Solve $3 \sin x^\circ + 2 = 0, \ 0 \le x \le 360$.

6. The diagram below shows the graph of $y = a\cos bx^\circ$ for $0 \le x \le 360$.



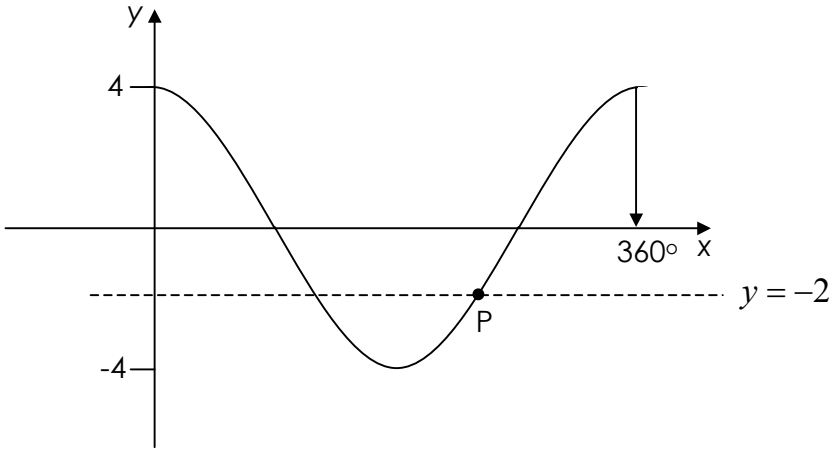
Write down the values of a and b.

7. Solve the equation $3\tan x^\circ - 4 = 0$ $0 \le x \le 360$

8. Solve the equation

$4 \sin x^\circ - 3 = 0, \qquad 0 \le x \le 360.$

9. The graph in the diagram has equation of the form $y = a\cos x^{\circ}$.

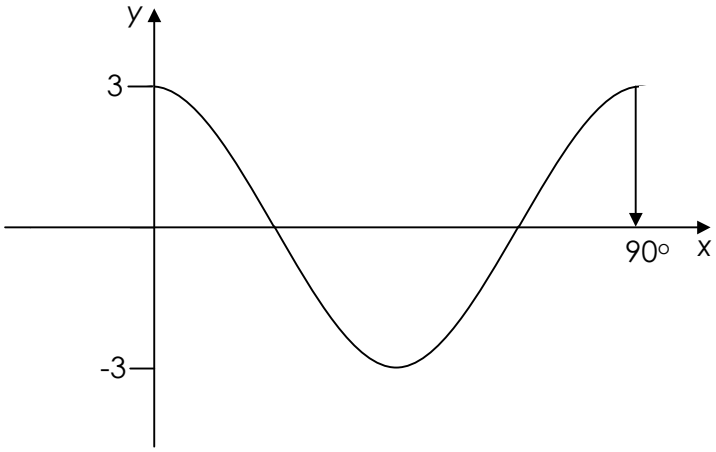


(a) Write down the value of *a*.

The broken line in the diagram has equation $y = -2$.

(b) Establish the coordinates of the point P.

10. The graph in the diagram has equation of the form $y = a\cos bx$.



Write down the values of *a* and *b*.

11. Sketch the graph of $f(x) = 3\sin 2x^{\circ}$ for $0 \le x \le 360$

12. Sketch the graph of $y = 4\sin 2x°$ for the interval $0 \le x \le 360$

13. Simplify $\dfrac{1-\cos^2 x}{\sin^2 x}$

14. Solve the trig equation $3\sin x = -2$ for $180 < x < 270$.

15. Solve the equation $7\cos x° + 5 = 0$ for $0 \le x \le 360$

16. Give a value for x where $0 \le x \le 360$ which satisfies these conditions

$\sin x^o$ is negative
and $\tan x^o$ is positive
and $\cos x^o$ is negative

17. Sketch the graph

$y = 2\cos x + 1$ for the interval $0 \le x \le 360$

18. Solve the equation

$2 \sin x^o + 3 = 2{\cdot}5, \qquad 0 \le x \le 360.$

19. Simplify

$(\sin x + \cos x)^2$

20. Show clearly that $\sin x \tan x = \dfrac{1 - \cos^2 x}{\cos x}$

21. Solve the trig equation $5\cos x° = -2$ for $0 < x < 360$.

22. Which **two** of the following are equal to $\cos 60^o$?

A. $\sin 210^o$

B. $\cos 240^o$

C. $\sin 150^o$

D. $\cos 300^o$

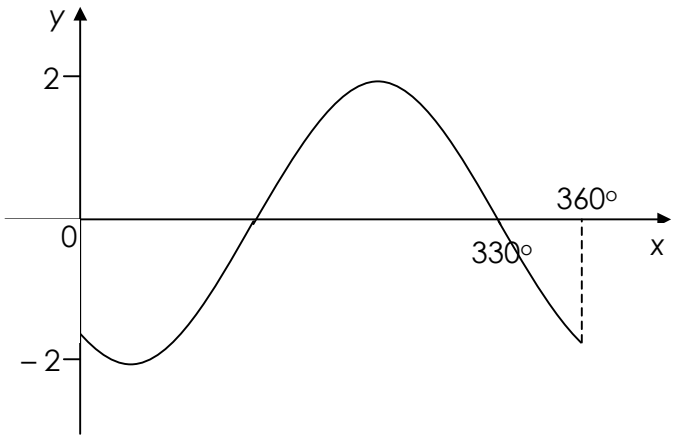
23. Solve the equation $1 + 4\cos x° = 0$ for $0 \le x \le 360$

24. Solve **algebraically** the equation

$4\tan x^\circ + 3 = 0$, for $0 \le x < 360$.

25. The graph below has equation of the form $y = a\sin(x + b)$.

Find the values of *a* and *b*.



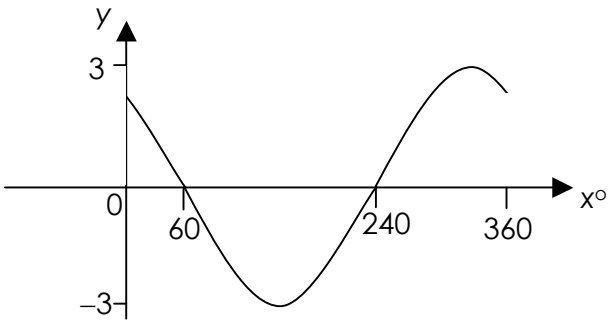
26. Simplify $\dfrac{1}{\tan x}(1 - \cos^2 x)$

27. Solve, **algebraically**, the equation

$$3\cos x^{\circ} + 4 = 3{\cdot}1, \quad \text{for} \quad 0 \le x \le 360.$$

28. Show that $\dfrac{4 - 4\sin^2 x}{8\cos^2 x} = \dfrac{1}{2}$.

29. Write down the equation of the graph shown below.



National 5 Relationships

ANSWERS

1.1 DETERMINING the EQUATION of a STRAIGHT LINE

1. **(a)** 3; (0, 1) **(b)** ½; (0, – 5) **(c)** –2; (0, 3)
(d) –¼; (0, – 2) **(e)** 8; (0, – ½) **(f)** –1; (0, 4)

2. **1** and **(b)** **2** and **(d)** **3** and **(f)** **4** and **(c)** **5** and **(a)** **6** and **(e)**

3. Lines sketched**.**

4. **(a)** $y = 2x + 2$ **(b)** $y = \frac{1}{2}x - 1$ **(c)** $y = 4x - 2$ **(d)** $y = 3x + 3$

(e) $y = -\frac{1}{3}x - 1$ **(f)** $y = -\frac{1}{2}x + 3$ **(g)** $y = -3x - 3$ **(h)** $y = \frac{1}{4}x - 2 \cdot 5$

5. **(a)** $1;\ (0, 3)$ **(b)** $-2;\ (0, -1)$ **(c)** $\frac{1}{2};\ (0, 0)$

(d) $-\frac{1}{2};\ (0, 2)$ **(e)** $-1;\ (0, 6)$ **(f)** $\frac{1}{2};(0, -2)$

(g) $\frac{1}{3};(0, 4)$ **(h)** $-\frac{4}{5};(0, 4)$ **(i)** $\frac{3}{2};(0, -6)$

6. **(a)** $1;\ (0, -7)$ **(b)** $-5;\ (0, 3)$ **(c)** $\frac{3}{5};(0, -2)$

(d) $-4;\ (0, 0)$ **(e)** $-2;(0, 11)$ **(f)** $\frac{1}{2};(0, -\frac{5}{2})$

(g) $\frac{1}{3};(0, 6)$ **(h)** $-\frac{3}{7};(0, 3)$ **(i)** $\frac{4}{5};(0, -4)$

7. **(a)** $y = 4x + 5$ **(b)** $y = -2x + 1$ **(c)** $y = \frac{3}{4}x - 3$

(d) $y = 4x - 11$ **(e)** $y = -5x - 14$ **(f)** $2y - x = 1$

(g) $3y - 4x = 13$ **(h)** $3x + 4y = -14$ **(i)** $2x + 3y = -9$

8. **(a)** $y = 2x - 5$ **(b)** $y + 4x = 13$ **(c)** $5y = x + 32$

(d) $y = 2x - 1$ **(e)** $y = x + 6$ **(f)** $4x + 3y = -15$

(g) $3y - 5x = 20$ **(h)** $2x + 5y = 14$ **(i)** $7y + 11x = 48$

9. **(a)** $2y - x = 0$ **(b)** $y + 2x = 7$ **(c)** $y = 3x - 6$

(d) $2x - 3y = 0$ **(e)** $2y + x = 3$ **(f)** $4y + 3x = -4$

(g) $y = 5x + 28$ **(h)** $y = 4x - 19$ **(i)** $2y + 5x = -4$

10. **(a)** $y = 3x - 5$ **(b)** $y + 4x = 16$ **(c)** $2y - x = 0$

(d) $y + 2x = -5$ **(e)** $2y - x = 14$ **(f)** $3y + x = 3$

STRAIGHT LINE EXAM QUESTIONS

1. $\frac{3}{2}$; $(0, 2)$ **2.** $y = -\frac{3}{4}x + 6$ **3.** D **4.** $\frac{3}{4}$; $(0, 3)$

5. P(0, 4) **6.** $y = -\frac{3}{2}x + 2$ **7.** -3; $(0, 4 \cdot 5)$

8. Line crossing at (0, 5) with gradient $-\frac{4}{3}$ **9.** **(a)** $\frac{1}{4}$ **(b)** $T = \frac{1}{4}v + 10$

10. $\frac{2}{3}$; $(0, 2)$ **11.** $\frac{3}{2}$; $(0, -4)$ **12.** $3y - 2x = -12$

13. **(a)** $\frac{3}{4}$; $(0, 1 \cdot 5)$ **(b)** $2y + x = 3$ **14.** **(a)** $\frac{4}{3}$; $(0, 4)$ **(b)** $4y + 3x = 16$

1.1 FUNCTIONAL NOTATION

1. **(a)** 13 **(b)** -11 **(c)** -2 **(d)** $6a - 5$

2. **(a)** 8 **(b)** 20 **(c)** 13 **(d)** $p^2 + 4$

3. **(a)** 4 **(b)** 0 **(c)** 16 **(d)** $12 - 2m$

4. **(a)** $a^2 + 3a$ **(b)** $4p^2 + 6p$ **(c)** $m^2 + 5m + 4$ **(d)** $e^2 - 7e + 10$

5. **(a)** 0 **(b)** $9a^2 - 12a$ **(c)** $a^2 - 8a + 12$ **(d)** $4p^2 - 4p - 3$

6. **(a)** 4 **(b)** -1 **(c)** $\frac{2}{5}$

7. **(a)** 3 **(b)** 6 **(c)** -2

8. **(a)** ± 5 **(b)** ± 1 **(c)** ± 4

9. **(a)** **(i)** 15 **(ii)** 0 **(b)** $a^2 + 8a + 15$

10. **(a)** **(i)** 9 **(ii)** 39 **(b)** $5 \cdot 5$ **(c)** $45 - 6p$

1.2 WORKING with LINEAR EQUATIONS and INEQUATIONS

1.	**(a)**	2	**(b)**	4	**(c)**	3	**(d)**	5	**(e)**	2	**(f)**	5
	(g)	4	**(h)**	1	**(i)**	16	**(j)**	16	**(k)**	15	**(l)**	7
	(m)	10	**(n)**	20	**(o)**	17	**(p)**	19				
2.	**(a)**	3	**(b)**	4	**(c)**	2	**(d)**	9	**(e)**	4	**(f)**	4
	(g)	2	**(h)**	5	**(i)**	4	**(j)**	9	**(k)**	10	**(l)**	8
	(m)	7	**(n)**	5	**(o)**	7	**(p)**	7				
3.	**(a)**	−18	**(b)**	11	**(c)**	−8	**(d)**	16	**(e)**	−13	**(f)**	9
	(g)	−12	**(h)**	25	**(i)**	−9	**(j)**	8·5	**(k)**	−5·5	**(l)**	5·5
	(m)	−3·5	**(n)**	18	**(o)**	−4·2	**(p)**	10·5				
4.	**(a)**	4	**(b)**	1	**(c)**	−5	**(d)**	1	**(e)**	−6	**(f)**	3
	(g)	7	**(h)**	7	**(i)**	7	**(j)**	0	**(k)**	20	**(l)**	−10
	(m)	4	**(n)**	7	**(o)**	2	**(p)**	5				
5.	**(a)**	3	**(b)**	4	**(c)**	6	**(d)**	4	**(e)**	2	**(f)**	5
	(g)	6	**(h)**	8	**(i)**	8	**(j)**	8	**(k)**	12	**(l)**	9
	(m)	6	**(n)**	7	**(o)**	15	**(p)**	18				
6.	**(a)**	1	**(b)**	2	**(c)**	6	**(d)**	5	**(e)**	3	**(f)**	2
	(g)	5	**(h)**	6	**(i)**	11	**(j)**	9	**(k)**	11	**(l)**	20
	(m)	7	**(n)**	12	**(o)**	21						
7.	**(a)**	3	**(b)**	2	**(c)**	2	**(d)**	3	**(e)**	2	**(f)**	3
	(g)	6	**(h)**	5	**(i)**	3	**(j)**	6	**(k)**	4	**(l)**	0·5
	(m)	8	**(n)**	11	**(o)**	15	**(p)**	3	**(q)**	12	**(r)**	4
	(s)	7	**(t)**	16	**(u)**	9	**(v)**	7	**(w)**	4	**(x)**	2
8.	**(a)**	6	**(b)**	6	**(c)**	4	**(d)**	1	**(e)**	4	**(f)**	6
	(g)	4	**(h)**	24	**(i)**	8	**(j)**	7	**(k)**	5	**(l)**	2·5
	(m)	2·5	**(n)**	16	**(o)**	15	**(p)**	2	**(q)**	2	**(r)**	4
	(s)	7	**(t)**	6	**(u)**	2	**(v)**	7	**(w)**	12	**(x)**	40

9. **(a)** $x > 1$ **(b)** $x > 3$ **(c)** $x > 4$ **(d)** $x > 4$ **(e)** $a > 3$ **(f)** $y > 3$

(g) $p > 9$ **(h)** $c > 1$ **(i)** $b > 6$ **(j)** $q > 0$ **(k)** $d > 3$ **(l)** $x > 4$

(m) $c > 5$ **(n)** $p > 9$ **(o)** $a > 12$ **(p)** $y > 12$

10. **(a)** $x < 2$ **(b)** $x < 7$ **(c)** $x < 10$ **(d)** $x < 4$ **(e)** $a < 3$ **(f)** $y < 6$

(g) $p < 8$ **(h)** $c < 4$ **(i)** $b < 5$ **(j)** $q < 17$ **(k)** $d < 0$ **(l)** $x < 5$

(m) $c < 6$ **(n)** $p < 14$ **(o)** $a < 11$ **(p)** $y < 1$

11. **(a)** $x > 3$ **(b)** $x > 4$ **(c)** $x > 4$ **(d)** $x > 9$ **(e)** $a > 4$ **(f)** $y > 4$

(g) $p > 3$ **(h)** $c > 5$ **(i)** $b < 4$ **(j)** $q < 9$ **(k)** $d < 10$ **(l)** $x < 8$

(m) $c < 7$ **(n)** $p < 5$ **(o)** $a < 7$ **(p)** $y < 7$

12. **(a)** $x < 7$ **(b)** $x > 6$ **(c)** $x > 11$ **(d)** $x < 9$ **(e)** $a < 6$ **(f)** $y > 11$

(g) $p < 18$ **(h)** $c > 9$ **(i)** $b > 16$ **(j)** $q < 16$ **(k)** $d > 15$ **(l)** $x > 7$

(m) $c > 10$ **(n)** $p < 20$ **(o)** $a < 17$ **(p)** $y < 19$

13. **(a)** $x < 2$ **(b)** $x > 2$ **(c)** $x > 3$ **(d)** $x > 2$ **(e)** $a < 3$ **(f)** $y < 2$

(g) $p > 8$ **(h)** $c < 5$ **(i)** $b > 6$ **(j)** $q < 0$ **(k)** $d < 10$ **(l)** $x > 4$

(m) $c < 3$ **(n)** $p < 7$ **(o)** $a > 2$ **(p)** $y < 3$

14. **(a)** $x > 3$ **(b)** $x > 4$ **(c)** $x < 6$ **(d)** $x > 3$ **(e)** $a < 2$ **(f)** $y < 5$

(g) $p > 6$ **(h)** $c > 8$ **(i)** $b > 8$ **(j)** $q < 8$ **(k)** $d < 12$ **(l)** $x > 9$

(m) $c < 6$ **(n)** $p < 7$ **(o)** $a < 15$ **(p)** $y < 18$

15. **(a)** $\{-2, -1, 0, 1\}$ **(b)** $\{-2, -1\}$ **(c)** $\{5\}$ **(d)** $\{-2, -1\}$

(e) $\{-2, -1, 0\}$ **(f)** $\{1,2,3,4,5\}$ **(g)** $\{-2, -1, 0, 1, 2\}$ **(h)** $\{1,2,3,4,5\}$

16. **(a)** $a \leq 3$ **(b)** $x > 1$ **(c)** $p \geq 2$ **(d)** $k < -3$

(e) $m \leq 7$ **(f)** $y > 2 \cdot 9$ **(g)** $h < 1$ **(h)** $x > \frac{1}{5}$

17. **(a)** $a \geq 3$ **(b)** $x < 2$ **(c)** $p \leq 2$ **(d)** $k > -2$

(e) $d \geq \frac{38}{9}$ **(f)** $y < -\frac{10}{3}$ **(g)** $h > 0$ **(h)** $y < \frac{11}{9}$

18. $\{0, 1, 2, 3, 4, 5\}$ **19.** $\{0, 1, 2\}$ **20.** Jane must be younger than 11

1.3 WORKING WITH SIMULTANEOUS EQUATIONS

GRAPHICAL SOLUTION

1. **(a)** Tables completed Table 1: 9, 6, 2 Table 2: 1, 4, 6

(b) and **(c)** Graphs drawn

(d) (5, 4)

2. **(a)** Tables completed Table 1: 0, 5, 1 Table 2: 0, 3, 5

(b) and **(c)** Graphs drawn

(d) (5, 3)

3. **(a)** (4, 3) **(b)** (11, 3) **(c)** (9, 6) **(d)** (12, 5)

(e) (8, 4) **(f)** (20, 10) **(g)** (15, 3) **(h)** (8, 3)

(i) (7, 3) **(j)** (13, 4)

4. **(a)** (2, 4) **(b)** (2, 3) **(c)** (3, 1) **(d)** (3, 2)

(e) (4, 3) **(f)** (1, 5)

5. **(a)** (6, 5) **(b)** (6, 2) **(c)** (2, 4) **(d)** (2, 2)

(e) (10, 3) **(f)** (4, –3) **(g)** (2, 5) **(h)** (2, 0)

(i) (1, –1) **(j)** (4, 6) **(k)** (2, 0) **(l)** (–6, 6)

(m) (5, –2) **(n)** (–1, –3) **(o)** (1, –1)

ALGEBRAIC SOLUTION

1. **(a)** $x = 5$ and $y = 5$ **(b)** $x = 1$ and $y = 1$ **(c)** $x = 2$ and $y = 4$

(d) $x = 3$ and $y = 6$ **(e)** $x = 3$ and $y = 10$ **(f)** $x = 5$ and $y = 21$

2. **(a)** $x = 3$ and $y = 1$ **(b)** $x = 7$ and $y = 2$ **(c)** $x = 5$ and $y = 2$

(d) $x = 2$ and $y = -1$ **(e)** $x = 6$ and $y = -3$ **(f)** $x = 4$ and $y = -5$

(g) $x = -3$ and $y = -2$ **(h)** $x = -11$ and $y = -3$ **(i)** $x = -8$ and $y = -10$

3. **(a)** $x = 2$ and $y = 5$ **(b)** $a = 3$ and $b = 2$ **(c)** $e = 6$ and $f = 2$

(d) $x = -1$ and $y = 3$ **(e)** $x = 2$ and $y = -2$ **(f)** $p = -2$ and $q = 3$

(g) $g = -4$ and $h = 3$ **(h)** $x = 3$ and $y = 4$ **(i)** $u = -2$ and $v = -3$

(j) $x = 4$ and $y = 1$ **(k)** $p = -5$ and $q = 2$ **(l)** $a = 4$ and $b = -2$

4. **(a)** $x = 2$ and $y = 5$ **(b)** $a = 3$ and $b = -1$ **(c)** $e = -2$ and $f = 5$

(d) $x = -1$ and $y = 4$ **(e)** $x = 3$ and $y = 4$ **(f)** $p = -3$ and $q = 4$

(g) $g = 8$ and $h = -5$ **(h)** $x = 7$ and $y = -2$ **(i)** $u = 2$ and $v = -2$

(j) $x = 4$ and $y = 1$ **(k)** $p = -1$ and $q = 2$ **(l)** $a = 8$ and $b = -2$

5. **(a)** $x = 7$ and $y = 1$ **(b)** $x = 10$ and $y = 1$ **(c)** $x = 4$ and $y = 2$

(d) $x = 2$ and $y = 3$ **(e)** $x = 3$ and $y = -1$ **(f)** $x = 4$ and $y = 4$

(g) $x = 2$ and $y = 5$ **(h)** $x = 5$ and $y = 1$ **(i)** $x = 2$ and $y = 1$

(j) $x = 4$ and $y = 2$ **(k)** $x = -1$ and $y = -2$ **(l)** $x = 1$ and $y = -1$

6. **(a)** $x = 1$ and $y = 2$ **(b)** $x = 3$ and $y = -1$ **(c)** $x = 2$ and $y = 2$

(d) $x = 5$ and $y = 1$ **(e)** $x = 5$ and $y = 7$ **(f)** $x = 5$ and $y = -2$

(g) $x = 3$ and $y = -3$ **(h)** $x = 4$ and $y = -3$ **(i)** $x = 2$ and $y = 3{\cdot}5$

(j) $x = 0{\cdot}5$ and $y = 2$ **(k)** $x = 1$and $y = 2$ **(l)** $x = 5$ and $y = 0$

WORKING with SIMULTANEOUS EQUATIONS in CONTEXT

1. 36 and 20 **2.** 18 and 4 **3.** 8 and 5 **4.** 9 and 2

5. Chocolate costs 40p and crisps cost 30p

6. Sandwich costs £1.20 and hotdog costs 90p.

7. Ruler costs 49p and pencil costs 26p.

8. Download is £9 and CD is £13

9. Standard print is 21p and Jumbo costs 55p.

10. Centre costs £11.25 and Side costs £9.50

11. Large glass holds 145ml and small holds 95ml.

12. Frame tent holds 8 and ridge tent holds 3.

13. Reader's letter pays £15 and Star letter pays £25.

14. Small takes 1·8kg and the large takes 2kg.

15. Thursday should have been £21.95.

16. **(a)** $4x + 4y = 60$; $6x + 16y = 120$; $x = 12$ and $y = 3$ **(b)** 144cm²

17. **(a)** Box weighs 6kg and parcel weighs 2kg. **(b)** 58kg

18. Milk is 24p and butter is 96p.

19. 38 hours basic and 7 hours overtime

20. 320 cheaper tickets were sold

21. 33 × 20p coins and 21 × 50p coins

WORKING WITH SIMULTANEOUS EQUATIONS — EXAM QUESTIONS

1. £21

2. **(a)** $s + b = 640$ **(b)** $8{\cdot}5s + 12{\cdot}2b = 6143$

(c) 450 stalls tickets and 190 balcony tickets

3. £5.02 **4.** £81.70 **5.** (2, –3)

6. **(a)** $f + t = 60$ **(b)** $25f + 20t = 1325$

(c) Clare sold 35 treacle scones

7. 75 points are needed – tokens give only 70 points so not enough.

8. £11.01. **9.** $x = 3$ and $y = 2$ **10.** (2, –3)

11. £9.72 **12.** Sofa costs £425 and chair costs £295.

13. £23.87 **14.** £25.60 **15.** (–2, 5)

1.4 CHANGING the SUBJECT of a FORMULA

1. **(a)** $x = y - 3$ **(b)** $x = y + 5$ **(c)** $x = y - a$ **(d)** $x = y + b$

(e) $x = \frac{y}{3}$ **(f)** $x = \frac{y}{10}$ **(g)** $x = \frac{y}{k}$ **(h)** $x = \frac{y}{a}$

(i) $x = y - 3p$ **(j)** $x = y + 5t$ **(k)** $x = \frac{y-1}{2}$ **(l)** $x = \frac{y+7}{3}$

(m) $x = \frac{y-4a}{7}$ **(n)** $x = \frac{y-38}{4}$ **(o)** $x = \frac{y-8}{10}$

2. **(a)** $a = 4 - b$ **(b)** $a = 12 - d$ **(c)** $a = 5x - y$ **(d)** $a = \frac{2-m}{2}$

(e) $a = \frac{7-q}{5}$ **(f)** $a = \frac{20-c}{3}$ **(g)** $a = \frac{s-r}{2}$ **(h)** $a = \frac{d-t}{4}$

(i) $a = \frac{4b-z}{5}$ **(j)** $a = \frac{2h-k}{7}$ **(k)** $a = \frac{6p-q}{11}$ **(l)** $a = \frac{2x-g}{9}$

3. **(a)** $x = \frac{y-b}{a}$ **(b)** $x = \frac{y-c}{m}$ **(c)** $x = \frac{t+r}{s}$ **(d)** $x = \frac{p-2r}{q}$

(e) $x = \frac{m+3n}{f}$ **(f)** $x = \frac{a-b}{c}$ **(g)** $x = \frac{h-k}{m}$ **(h)** $x = \frac{d-3b}{c}$

(i) $x = \frac{kc-g}{h}$

4. **(a)** $l = \frac{P}{4}$ **(b)** $I = \frac{V}{R}$ **(c)** $T = \frac{S}{D}$ **(d)** $b = \frac{A}{l}$

(e) $d = \frac{C}{\pi}$ **(f)** $U = \frac{G}{T}$ **(g)** $t = \frac{v-u}{a}$ **(h)** $l = \frac{P-2b}{2}$

(i) $y = \frac{H-5m}{x}$

5. **(a)** $c = 2b$ **(b)** $c = 5x$ **(c)** $c = 4y$ **(d)** $c = 6m$

(e) $c = 9k$ **(f)** $c = 10d$ **(g)** $c = 2a - 4$ **(h)** $c = 3h + 15$

(i) $c = 4p - 4q$ **(j)** $c = 10y + 10x$ **(k)** $c = 8t - 16s$ **(l)** $c = 5r + 15q$

6. **(a)** $x = \frac{3}{y}$ **(b)** $x = \frac{c}{d}$ **(c)** $x = \frac{y}{m}$ **(d)** $x = \frac{a+2}{s}$

(e) $x = \frac{z-1}{w}$ **(f)** $x = \frac{b+c}{a}$ **(g)** $x = 9a - 8$ **(h)** $x = 2k + 5$

(i) $x = 3 - 4p$ **(j)** $x = \frac{2}{y-1}$ **(k)** $x = \frac{6}{z-7}$ **(l)** $x = \frac{m}{h-k}$

7. **(a)** $k = y^2$ **(b)** $k = x^2$ **(c)** $k = m^2$ **(d)** $k = a^2b$

(e) $k = c^2d$ **(f)** $k = gh^2$ **(g)** $k = \frac{t}{s^2}$ **(h)** $k = \frac{p}{q^2}$

(i) $k = \frac{z}{w^2}$ **(j)** $k = \sqrt{r}$ **(k)** $k = \sqrt{ab}$ **(l)** $k = \sqrt{\frac{p}{q}}$

(m) $k = \sqrt{y-x}$ **(n)** $k = \sqrt{c+d}$ **(o)** $k = \sqrt{\frac{x+1}{3}}$

8. **(a)** $s = \frac{v^2-u^2}{2a}$ **(b)** $u = \sqrt{v^2-2as}$ **(c)** $h = \frac{V}{\pi r^2}$ **(d)** $r = \sqrt{\frac{V}{\pi h}}$

(e) $A = \pi r^2$ **(f)** $a = \frac{(L-3)^2}{6}$ **(g)** $p = 4k^2 - 4$ **(h)** $y = \frac{tx^2}{4z}$

(i) $b = \frac{x}{4a^2r^2}$ **(j)** $A = \sqrt{\frac{st}{x-3y}}$ **(k)** $x = \frac{R+3A^2y}{A^2}$ **(l)** $n = \sqrt{\frac{1}{a^2+1}}$

(m) $n = \dfrac{t}{t-d}$ **(n)** $R = \dfrac{r_1 r_2}{r_2 + r_1}$ **(o)** $a = \sqrt{\dfrac{4d}{x+b}}$

CHANGING the SUBJECT of a FORMULA — EXAM QUESTIONS

1. $\sqrt{\dfrac{x}{4a^2b^2}}$ or $\dfrac{\sqrt{x}}{2ab}$ **2.** $g = \dfrac{V^2}{2R}$ **3.** $x = (A^2 - 5)^2$

4. $v = \sqrt{\dfrac{2E}{m}}$ **5.** $a = \sqrt{\dfrac{v}{3b}}$ **6.** $k = \dfrac{4\pi^2 m}{T^2}$

7. $C = \dfrac{5}{9}(F - 32)$ **8.** $r = \sqrt{\dfrac{3V}{\pi h}}$ **9.** $k = \sqrt{mab - 2}$

10. $b = \sqrt{\dfrac{a-7}{c}}$ **11.** $m = \dfrac{3k}{n}$ **12.** $r = \sqrt[3]{\dfrac{3V}{4\pi}}$

2.1 RECOGNISE and DETERMINE the EQUATIONS of QUADRATICS from their GRAPHS

1. **(a)** $y = x^2$ **(b)** $y = 3x^2$ **(c)** $y = 5x^2$ **(d)** $y = 1 \cdot 5x^2$

(e) $y = 5x^2$ **(f)** $y = 3x^2$ **(g)** $y = -x^2$ **(h)** $y = -2x^2$

(i) $y = -5x^2$ **(j)** $y = \dfrac{1}{2}x^2$ **(k)** $y = \dfrac{1}{4}x^2$ **(l)** $y = \dfrac{1}{3}x^2$

(m) $y = 40x^2$ **(n)** $y = -25x^2$ **(o)** $y = -\dfrac{3}{4}x^2$

2. **(a)** $y = x^2 + 2$ **(b)** $y = x^2 - 1$ **(c)** $y = x^2 + 1 \cdot 5$ **(d)** $y = -x^2 + 5$

(e) $y = -x^2 + 3$ **(f)** $y = -x^2 - 2$ **(g)** $y = 2x^2 + 1$ **(h)** $y = 5x^2 + 4$

(i) $y = 3x^2 + 2$ **(j)** $y = 2x^2 - 3$ **(k)** $y = \dfrac{1}{2}x^2 - 9$ **(l)** $y = -2x^2 + 8$

(m) $y = -x^2 + 3$ **(n)** $y = -3x^2 - 2$

3. **(a)** $y = (x-2)^2 + 1$ **(b)** $y = (x-1)^2 + 6$ **(c)** $y = (x-4)^2$

(d) $y = (x-3)^2 - 4$ **(e)** $y = x^2 - 5$ **(f)** $y = (x+1)^2 + 3$

(g) $y = (x+2)^2 - 4$ **(h)** $y = (x+6)^2$ **(i)** $y = (x-4)^2 + 20$

(j) $y = (x-10)^2 - 2$ **(k)** $y = (x-25)^2 + 10$ **(l)** $y = (x+30)^2 + 5$

(m) $y = (x-1)^2 - 1$ **(n)** $y = x^2 + 6$

2.2 SKETCHING a QUADRATIC FUNCTION

1. Graphs should show the following:

(a) Turning point (4, 1); minimum; y – intercept (0, 17)

(b) Turning point (2, 5); minimum; y – intercept (0, 9)

(c) Turning point (1, 7); minimum; y – intercept (0, 8)

(d) Turning point (2, –3); minimum; y – intercept (0, 1)

(e) Turning point (3, –4); minimum; y – intercept (0, 5)

(f) Turning point (5, –2); minimum; y – intercept (0, 23)

(g) Turning point (–4, 6); minimum; y – intercept (0, 22)

(h) Turning point (–1, 5); minimum; y – intercept (0, 6)

(i) Turning point (–8, 1); minimum; y – intercept (0, 65)

(j) Turning point (–3, –1); minimum; y – intercept (0, 8)

(k) Turning point (–½, –¾); minimum; y – intercept (0, –½)

(l) Turning point (–0·5, –2·5); minimum; y – intercept (0, –2·25)

(m) Turning point (1, 4); maximum; y – intercept (0, 3)

(n) Turning point (–6, 3); maximum; y – intercept (0, –33)

(o) Turning point (–7, –2); maximum; y – intercept (0, –51)

(p) Turning point (2, 12); minimum; y – intercept (0, 16)

(q) Turning point (5, –1); minimum; y – intercept (0, 24)

(p) Turning point (4, 3·75); minimum; y – intercept (0, 19·75)

2. Graphs should all be minimum T.P. and show the following points:

(a) (1, 0), (5, 0) and (3, –4)

(b) (2, 0), (4, 0) and (3, –1)

(c) (3, 0), (7, 0) and (5, –4)

(d) (6, 0), (8, 0) and (7, –4)

(e) (2, 0), (5, 0) and (3·5, –2·25)

(f) (5, 0), (8, 0) and (6·5, –2·25)

(g) (−2, 0), (−3, 0) and (−2·5, –0·25)

(h) (−2, 0), (−5, 0) and (−3·5, –2·25)

(i) (−4, 0), (−6, 0) and (−5, –1)

(j) (−3, 0), (−4, 0) and (−3·5, –0·25)

(k) (−9, 0), (−5, 0) and (−7, –4)

(l) (−8, 0), (−3, 0) and (−5·5, –6·25)

3. Graphs should show the following:

(a) (1, 0), (–5, 0), (–2, –9)**,** minimum **(b)** (7, 0), (–3, 0), (2, 25)**,** maximum

(c) (5, 0), (–3, 0), (1, 16)**,** minimum **(d)** (4, 0), (–8, 0), (–2, 36)**,** maximum

(e) (7, 0), (–1, 0), (3, –16)**,** minimum **(f)** (7, 0), (–1, 0), (3, 16)**,** maximum

(g) (3, 0), (–9, 0), (–3, 36)**,** maximum **(h)** (10, 0), (–2, 0), (4, –36)**,** minimum

(i) (–7, 0), (9, 0), (1, 64)**,** maximum **(j)** (6, 0), (–4, 0), (1, 25)**,** maximum

(k) (1, 0), (–1, 0), (0, 1)**,** maximum **(l)** (6, 0), (–2, 0), (2, –16)**,** minimum

(m) (3, 0), (–3, 0), (0, –9)**,** minimum **(n)** (7, 0), (–1, 0), (3, 16)**,** maximum

(o) (6, 0), (–10, 0), (–2, 64)**,** maximum

2.3 IDENTIFYING FEATURES of a QUADRATIC FUNCTION

1. **(a)** (1, –2); minimum; $x = 1$ **(b)** (–2, 3); minimum; $x = –2$

(c) (3, –4); minimum; $x = 3$ **(d)** (–2, –2); minimum; $x = –2$

(e) (1, 6); maximum; $x = 1$ **(f)** (–1, 4); maximum; $x = –1$

(g) (3, –2); maximum; $x = 3$ **(h)** (–3, 3); maximum; $x = –3$

2. **(a)** (4, 1); minimum; $x = 4$ **(b)** (2, 5); minimum; $x = 2$

(c) (1, 7); minimum; $x = 1$ **(d)** (2, –3); minimum; $x = 2$

(e) (3, –4); minimum; $x = 3$ **(f)** (5, –2); minimum; $x = 5$

(g) (–4, 6); minimum; $x = –4$ **(h)** (–1, 5); minimum; $x = –1$

(i) (–8, 1); minimum; $x = –8$ **(j)** (–3, –1); minimum; $x = –3$

(k) $(-\frac{1}{2}, -\frac{3}{4})$; minimum; $x = -\frac{1}{2}$ **(l)** (–0·5, – 2·5); minimum; $x = –0·5$

(m) (1, 4); maximum; $x = 1$ **(n)** (–6, 3); maximum; $x = –6$

(o) (–7, –2); maximum; $x = –7$ **(p)** (2, 12); minimum; $x = 2$

(q) (5, –1); minimum; $x = 5$ **(r)** (4, 3·75); minimum; $x = 4$

2.4 WORKING with QUADRATIC EQUATIONS

DRAWING GRAPHS

1. **(a)** $x = 0$ or 3 **(b)** $x = –3$ or $–2$ **(c)** $x = –2$ or 6 **(d)** $x = –1$ or 8

(e) $x = –10$ or 2 **(f)** $x = –5$ or 1

2. **(a)** 8, 3, 0, –1, 0, 3, 8; roots 0 and 2 **(b)** 5, 0, –3, –4, –3, 0 5; roots 1 and 5

(c) 8, 3, 0, –1, 0 3, 8; roots –3 and –1 **(d)** 0, 5, 8, 9, 8, 5, 0; roots –4 and 2

3. Graphs drawn with roots:

(a)	0 and 4	**(b)**	0 and –6	**(c)**	0 and 5	**(d)**	5 and 3
(e)	–3 (twice)	**(f)**	2 (twice)	**(g)**	–2 and –4	**(h)**	–6 and –2
(i)	2 and 5	**(j)**	1 and 4	**(k)**	–3 and 2	**(l)**	2 and –1
(m)	–6 and 2	**(n)**	–1 and`	**(o)**	–2 and 1		

FACTORISING

1. **(a)**	0 and 5	**(b)**	0 and –7	**(c)**	0 and 1	**(d)**	0 and 3
(e)	0 and –1	**(f)**	0 and 2	**(g)**	2 and 4	**(h)**	3 and 4
(i)	3 and 5	**(j)**	–2 and –1	**(k)**	–5 and –4	**(l)**	–7 and –8
(m)	–3 and 1	**(n)**	–2 and 12	**(o)**	–1 and 9	**(p)**	–4 and 4
(q)	–7 and 7	**(r)**	–5 and 5	**(s)**	4 and ½	**(t)**	$-\frac{3}{2}$ and –2
(u)	$-\frac{1}{3}$ and $\frac{5}{2}$						

2. **(a)**	0 and –4	**(b)**	0 and 2	**(c)**	0 and –8	**(d)**	0 and 1
(e)	0 and –1	**(f)**	0 and –7	**(g)**	0 and –2	**(h)**	0 and 4
(i)	0 and 3	**(j)**	0 and $\frac{3}{2}$	**(k)**	0 or $-\frac{3}{2}$	**(l)**	0 or $-\frac{3}{2}$
(m)	0 and 5	**(n)**	0 and 9	**(o)**	0 and 2		
(p)	0 and $\frac{3}{2}$	**(q)**	0 and $\frac{3}{4}$	**(r)**	0 and $\frac{2}{5}$		
3. **(a)**	–5 and 5	**(b)**	–1 and 1	**(c)**	–2 and 2	**(d)**	–6 and 6
(e)	–3 and 3	**(f)**	–8 and 8	**(g)**	–4 and 4	**(h)**	–12 and 12
(i)	–10 and 10	**(j)**	–7 and 7	**(k)**	–9 and 9	**(l)**	–11 and 11
(m)	–3 and 3	**(n)**	–4 and 4	**(o)**	–4 and 4		
4. **(a)**	–3 and –1	**(b)**	–5 and –1	**(c)**	–7 and –1	**(d)**	–3 and –2
(e)	–4 and –2	**(f)**	–3 and –4	**(g)**	–5 and 3	**(h)**	4 (twice)
(i)	5 and 2	**(j)**	3 and 9	**(k)**	9 and –2	**(l)**	4 and 6
(m)	–4 and 2	**(n)**	–2 and 3	**(o)**	–3 and 10	**(p)**	–7 and 2
(q)	–3 and 5	**(r)**	–6 and 2				

5. **(a)** $-\frac{5}{2}$ and -1 **(b)** $-\frac{1}{2}$ and -5 **(c)** $-\frac{1}{3}$ and -3 **(d)** $-\frac{1}{3}$ and -2

(e) $-\frac{5}{3}$ and -1 **(f)** $\frac{3}{5}$ and -2 **(g)** $\frac{1}{2}$ and -3 **(h)** $\frac{2}{3}$ and 1

(i) $\frac{1}{5}$ and 3 **(j)** $\frac{2}{3}$ and 4 **(k)** $\frac{7}{2}$ and -1 **(l)** $\frac{1}{6}$ and -1

(m) $-\frac{1}{3}$ and 1 **(n)** $\frac{3}{4}$ and -2 **(o)** $-\frac{1}{2}$ and $\frac{3}{2}$ **(p)** $-\frac{5}{3}$ and 1

(q) $\frac{2}{9}$ and $-\frac{1}{4}$ **(r)** $-\frac{1}{7}$ and 4

USING QUADRATIC FORMULA

1. **(a)** $-\frac{1}{3}$ and -2 **(b)** $-\frac{1}{2}$ and -2 **(c)** $-\frac{5}{3}$ and -1 **(d)** $-\frac{9}{2}$ and -1

(e) $-\frac{1}{2}$ and -5 **(f)** $-\frac{2}{3}$ and -3 **(g)** $\frac{1}{2}$ and 3 **(h)** $\frac{3}{2}$ and 1

(i) $\frac{2}{5}$ and 3 **(j)** $\frac{2}{5}$ and 1 **(k)** $\frac{2}{3}$ and $\frac{1}{2}$ **(l)** $\frac{3}{4}$ and 2

(m) $-\frac{1}{3}$ and 1 **(n)** $\frac{3}{2}$ and -1 **(o)** $-\frac{1}{2}$ and $\frac{3}{2}$ **(p)** $\frac{1}{2}$ and -4

(q) $-\frac{1}{6}$ and 2 **(r)** $\frac{2}{3}$ or -4

2. **(a)** –1·38 and –3·62 **(b)** –0·23 and –8·77 **(c)** –0·27 and –3·73

(d) –0·59 and –3·41 **(e)** –0·46 and –6·54 **(f)** –0·68 and –7·32

(g) 4·79 and 0·21 **(h)** 11·66 and 0·34 **(i)** 5·65 and 0·35

(j) 9·12 and 0·88 **(k)** 2·62 and 0·38 **(l)** 6·37 and 0·63

(m) 0·36 and –8·36 **(n)** 1·16 and –5·16 **(o)** 2·16 and –4·16

3. **(a)** –1 and –1·67 **(b)** –0·36 and –4·14 **(c)** –0·22 and –2·28

(d) –0·15 and 1·65 **(e)** –0·57 and –1·77 **(f)** –0·26 and –1·54

(g) 0·70 and 0·18 **(h)** –3·23 and 0·23 **(i)** 1·58 and 0·42

(j) –1·45 and –0·55 **(k)** 1·24 and 0·16 **(l)** –0·35 and 4·35

(m) 0·22 and –1·82 **(n)** –1·09 and 0·76 **(o)** 0·23 and –0·43

(p) –2·93 and 0·68 **(q)** –0·47 and 0·90 **(r)** 1·68 and –2·68

4. **(a)** –0·697 and –4·30 **(b)** –0·382 and –2·62 **(c)** –0·258 and –7·74

(d) –1·21 and –5·79 **(e)** –0·354 and –5·65 **(f)** –0·551 and –5·45

(g) 0·438 and 4·56 **(h)** 0·469 and 8·53 **(i)** 0·807 and 6·19

(j) 0·310 and 9·69 **(k)** 1·17 and 6·83 **(l)** 0·586 and 3·41

(m) –13·5 and 1·48 **(n)** –12·2 and 1·23 **(o)** –9·75 and 1·75

(p) –4·85 and 1·85 **(q)** –4·46 and 2·46 **(r)** –5·28 and 2·28

(s) –0·146 and –0·854 **(t)** –1·68 and 1·08 **(u)** –0·312 and 4·81

(v) 0·631 and 0·227 **(w)** 1·21 and –2·21 **(x)** 1·92 and –1·17

MORE QUADRATICS

1. **(a)** –3 and 1; $x = -1$; (–1, –4) Minimum; (0, –3)

(b) –2 and 4 ; $x = 1$; (1, –9) Minimum; (0, –8)

(c) –1 and 5; $x = 2$; (1, –9) Minimum; (0, –5)

(d) –6 and 0; $x = -3$; (–3, –9) Minimum; (0, 0)

(e) 0 and 4; $x = 2$; (2, –4) Minimum; (0, 0)

(f) 0 and 8; $x = 4$; (4, 16) Maximum; (0, 0)

(g) –4 and 2; $x = -1$; (–1, 9) Maximum; (0, 8)

(h) –1 and 7; $x = 3$; (3, 16) Maximum; (0, 7)

(i) 3 and 7; $x = 5$; (5, –4) Minimum; (0, 21)

(j) –1 and 4; $x = 1{\cdot}5$; (1·5, –6·25) Minimum; (0, –4)

(k) –6 and –1; $x = -3{\cdot}5$; (–3·5, –6·25) Minimum; (0, 6)

(l) 0 and 5; $x = 2{\cdot}5$; (2·5, 6·25) Maximum; (0, 0)

(m) 2 and –5; $x = -1{\cdot}5$; (–1·5, 12·25) Maximum; (0, 10)

(n) –4 and 4; $x = 0$; (0, 16) Maximum; (0, 16)

(o) –3 and 3; $x = 0$; (0, –9) Minimum; (0, –9)

2. **(a)** A(–2, 0); B(8, 0); C(3, –25); D(0, –16)

(b) E(–11, 0); F(3, 0); G(–4, –49); H(0, –33)

(c) I(–6, 0); J(–2, 0); K(–4, –4); L(0, 12)

(d) M(–6, 0); N(4, 0); O(–1, –25); P(0, –24)

(e) R(–1, 0); S(5, 0); T(2, –9); U(0, –5)

(f) A(1, 0); B(7, 0); C(4, –9); D(0, 7)

PROBLEMS INVOLVING QUADRATIC EQUATIONS

1. **(a)** $x^2 + (x+7)^2 = 13^2$; $x = 5$ **(b)** 30 cm **2.** $n = 9$; 54 cm²

3. $n = 10$; 65 mm² **4.** **(a)** $80 - x$ **(b)** $x(80 - x) = 1500$; 30×50

5. **(a)** $130 - x$ **(b)** $x(130 - x) = 4000$; 80m

WORKING with QUADRATICS

EXAM QUESTIONS

1. **(a)** (¾ , ½) **(b)** minimum **(c)** $x = ¾$ **2.** $(-3, -5)$

3. 0·9 and – 1·2 **4**. **(a)** $y = (x + 2)(x – 6)$ **(b)** (2, – 16); minimum

(c) $y = (x – 2)^2 – 16$.

5. 0·6 and –3·9 **6.** (–½,–¾) ; minimum

7. 3·8 and –1·3 **8**. $y = (x – 1)^2 + 6$. **9.** 0·65 and –6·2

10. **(a)** (–1, –2) minimum **(b)** $x = –1$ **(c)** $y = (x + 1)^2 – 2$

11. 0·4 and –3·9 **12.** **(a)** (¾, ⅚) minimum **(b)** $x = ¾$

13. 5·67 and –1·67

14. **(a)** $y = (x + 2)(x – 4)$ **(b)** (0, –8) **(c)** (1, –9); min **(d)** $x = 1$

15. **(a)** (4, 7); minimum **(b)** B **16.** 2·8 and 0·2

17. **(a)** (3, 9) **(b)** $x = 3$ **(c)** (6, 0)

18. **(a)** (3, –1); minimum **(b)** $x = 3$ **(c)** (4, 0) and (2, 0)

19. **(a)** (–2, 9) maximum **(b)** $x = –2$

20. **(a)** $y = (x-3)^2 - 9$ **(b)** 6 units

21. **(a)** $y = (x-2)^2 - 1.$ **(b)** $x = 2$ **(c)** (0, 3) **(d)** A(1, 0)

22. **(a)** proof **(b)** $x = 4$

DISCRIMINANT

1. **(a)** 4 **(b)** 0 **(c)** 36 **(d)** 49 **(e)** 9 **(f)** 0

(g) 1 **(h)** –23 **(i)** 196 **(j)** 169 **(k)** 28 **(l)** –80

(m) 17 **(n)** –55 **(o)** 16 **(p)** 0 **(q)** –11 **(r)** 0

2. **(a)** real, rational, distinct **(b)** equal **(c)** real, rational, distinct

(d) real, rational, distinct **(e)** real, rational, distinct **(f)** equal

(g) real, rational, distinct **(h)** non real **(i)** real, rational, distinct

(j) real, rational, distinct **(k)** real, irrational, distinct **(l)** non real

(m) real, irrational, distinct **(n)** non real **(o)** Real, rational, distinct

(p) equal **(q)** non real **(r)** equal

3. **(a)** $b^2 - 4ac > 0$ **(b)** $b^2 - 4ac < 0$ **(c)** $b^2 - 4ac = 0$ **(d)** $b^2 - 4ac > 0$

(e) $b^2 - 4ac = 0$ **(f)** $b^2 - 4ac < 0$ **(g)** $b^2 - 4ac > 0$ **(h)** $b^2 - 4ac > 0$

(i) $b^2 - 4ac > 0$ **(j)** $b^2 - 4ac < 0$ **(k)** $b^2 - 4ac < 0$ **(l)** $b^2 - 4ac > 0$

4. **(a)** –4 **(b)** 12·5 **(c)** 0·2

(d) 0·5 or 1·5 **(e)** $5\frac{1}{3}$ **(f)** $-2\frac{9}{20}$

5. **(a)** 4 **(b)** $\frac{1}{2}$ **(c)** $\pm\sqrt{5}$

(d) 9 **(e)** $\frac{5}{8}$ **(f)** ± 4

EXAM QUESTIONS

1. 41, real, irrational and distinct **2.** **(a)** 13 **(b)** real, irrational and distinct

3. 1 **4.** Non real **5.** 13, real, irrational and distinct

3.1 APPLYING the THEOREM of PYTHAGORAS

1. **(a)** 9·43 **(b)** 21·3 **(c)** 13 **(d)** 10·2 **(e)** 1·05 **(f)** 5·07

(g) 12·4 **(h)** 26·9 **(i)** 2·4

2. **(a)** 10 **(b)** 15·6 **(c)** 180cm²

3. **(a)** 21 **(b)** 357cm²

4. **(a)** 7·07 **(b)** 12·5 **(c)** 8·06 **(d)** 10·2

5. **(a)** 11.7cm **(b)** 12·7cm

6. **(a)** 20cm **(b)** 15·0cm

7. 19·2km **8.** 427·2km **9.** 9·46km

10. 7·9km **11.** 15·6mm **12.** 16·6cm

13. 4·6m **14.** 4·4m **15.** 16cm

16. 6·6cm **17.** 16·9m **18.** 1·92m

19. **(i)** 2·9cm **(ii)** 25·4cm² **(iii)** 7·35cm³

20. proofs

21. **(a)** no **(b)** no **(c)** no

(d) yes **(e)** no **(f)** yes

EXAM QUESTIONS – PYTHAGORAS

1. Mechanism will work since 7·1 > 7 **2.** 34·8cm

3. 67cm **4.** Suitable since 11·3 > 11 **5.** 41·2cm

6. £952 **7.** 21·5m **8.** 3·3m **9.** 37cm

10. 20·7m **11.** **(a)** AB = 8mm; BC = 4mm **(b)** 6·9mm

12. 7·7cm **13.** 5·06m or 506 cm **14.** 15cm **15.** 33cm

16. **(a)** 15cm **(b)** 16cm **(c)** proof **(d)** 30cm; 225cm²

17. supports are vertical **18.** 13·9 > 13·89 so just long enough

3.2 APPLYING PROPERTIES OF SHAPES

1. **(a)** square **(b)** 0; rotational symmetry **(c)** equal

(d) diagonals; bisect; right **(e)** trapezium

2. 1925cm² **3.** **(a)** 16 100cm² **(b)** 1·61m³

4. **(a)** 24cm² **(b)** 432cm² **(c)** 50%

5. **(a)** pentagon; 72^o; 108^o, 72^o **(b)** octagon; 45^o; 135^o; 45^o

(c) hexagon; 60^o; 120^o; 60^o

6. **(a)** right angled scalene **(b)** acute angled isosceles

(c) obtuse angled scalene

7. Acute angled isosceles; 23·8 units²

8. 30^o, 60^o, 90^o and 32^o, 36^o, 112^o angles of a triangle add up to 180^o

9. **(a)** 37^o **(b)** 62^o and 56^o **(c)** 40^o

10. 5760cm² **11.** **(a)** £1.50 **(b)** £3

12. £792.18 **13.** **(a)** false **(b)** true **(c)** false **(d)** true

14. **(a)** 47·1m **(b)** 56·52cm **15.** 43·96cm

16. 33·3cm **17.** 596·6mm; £2.75 **18.** 9m **19.** 12·1cm

20. **(a)** 452·16cm² **(b)** 706·5cm² **21.** 1017·36cm² **22.** 192mm²

23. **(a)** 12·56cm² **(b)** 188·4cm² **(c)** 240cm² **(d)** 21·5%

24. **(a)** 14·13m² **(b)** 45·87m² **(c)** £364.93

25. 2·5cm; 15·7cm

3.2 RELATIONSHIP between the CENTRE, CHORD and PERPENDICULAR BISECTOR

1. **(a)** 90^o **(b)** 45^o **(c)** 90^o **(d)** 55^o **(e)** 90^o
(f) 43^o **(g)** 90^o **(h)** 18^o **(i)** 90^o **(j)** 63^o
(k) 90^o **(l)** 78^o

2. **(a)** 9·9 cm **(b)** 8·5 cm **(c)** 6·4 cm **(d)** 9·2 cm

3. **(a)** 40^o **(b)** 40^o **(c)** 50^o **(d)** 33^o **(e)** 33^o
(f) 57^o **(g)** 28^o **(h)** 62^o **(i)** 62^o **(j)** 118^o
(k) 118^o **(l)** 31^o **(m)** 31^o **(n)** 31^o **(o)** 31^o

4. **(a)** 4·5 cm **(b)** 5·7 cm **(c)** 7·2 cm **(d)** 3 cm **(e)** 8 cm
(f) 9·2 cm

5. **(a)** $36{\cdot}9^o$ **(b)** 24·1 cm **(c)** 9·0 cm **(d)** 12·6 cm
(e) 23·7 cm **(f)** 8 cm **6**. 37·6 cm

TANGENTS and ANGLES

1. **(a)** 90^o **(b)** 20^o **(c)** 110^o **(d)** 90^o **(e)** 60^o
(f) 30^o **(g)** 35^o **(h)** 35^o **(k)** 90^o **(m)** 65^o
(n) 90^o **(p)** 55^o **(q)** 90^o **(r)** 45^o

2. **(a)** 6 cm **(b)** 13 cm **(c)** 24 cm

3. **(a)** $33{\cdot}7^o$ **(b)** 10·4 cm **(c)** 14·3 cm

EXAM QUESTIONS

1. **(a)** 50cm **(b)** 14cm **2.** **(a)** 100cm **(b)** 171cm

3. 20cm **4.** 102cm **5.** 54^o **6.** 9·6cm **7.** 23^o **8.** 132^o

9. 28^o **10.** **(a)** 112^o **(b)** 60·6cm **11.** 36cm **12.** 71·4cm

13. 76^o **14.** 8cm **15.** 11·3cm

3.3 USING SIMILARITY

LENGTH

1. **(a)** **(i)** s.f. = $\frac{3}{2}$ or 1·5 **(ii)** 13·5cm **(b)** **(i)** s.f. = $\frac{2}{3}$ of 0·66... **(ii)** 20cm

(c) **(i)** s.f. = $\frac{5}{2}$ of 2·5 **(ii)** 45cm **(d)** **(i)** s.f. = $\frac{3}{5}$ or 0·6 **(ii)** 168mm

2. **(a)** $x = 30\text{mm}$ **(b)** $x = 32 \cdot 5\,\text{cm}$

3. **(a)** Because they are equiangular **(b)** CD = 18cm **(c)** 81 cm^2

4. **(a)** $x = 13 \cdot 5\,\text{cm}$ **(b)** $x = 14 \cdot 4\,\text{m}$ **5.** ST = 16cm **6.** distance = $0 \cdot 7\,\text{m}$

SIMILARITY and AREA

1. **(a)** s.f.(L) = 2; s.f. (A) = 4; $A = 64\text{cm}^2$

(b) s.f.(L) = 3; s.f. (A) = 9; $A = 864\text{mm}^2$

(c) s.f.(L) = 1·5; s.f. (A) = 2·25; $A = 90\text{mm}^2$

(d) s.f.(L) = 2·4; s.f. (A) = 5·76; $A = 288\text{cm}^2$

(e) s.f.(L) = 4; s.f. (A) = 16; $A = 352\text{cm}^2$

(f) s.f.(L) = 1·8; s.f. (A) = 3·24; $A = 388{\cdot}8\text{cm}^2$

2. **(a)** s.f.(L) = 0·5; s.f. (A) = 0·25; $A = 17{\cdot}5\text{cm}^2$

(b) s.f.(L) = 0·25; s.f. (A) = 0·0625; $A = 288\text{mm}^2$

(c) s.f.(L) = 0·8; s.f. (A) = 0·64; $A = 96\text{cm}^2$

(d) s.f.(L) = 0·75; s.f. (A) = 0·5625; $A = 225\text{mm}^2$

3. **(a)** 88 cm^2 **(b)** 166 mm^2 **(c)** 49cm^2 **(d)** 72 mm^2

SIMILARITY and VOLUME

1. **(a)** s.f.(L) = 2; s.f.(V) = 8; $V = 384\text{cm}^3$

(b) s.f.(L) = 3; s.f.(V) = 27; $V = 5832\text{mm}^3$

(c) s.f.(L) = 1·5; s.f.(V) = 3·375; $V = 243\text{mm}^3$

(d) s.f.(L) = 2·4; s.f.(V) = 13·824; $V = 276{\cdot}48\text{cm}^3$

(e) s.f.(L) = 4; s.f.(V) = 64; $V = 576\text{cm}^3$

(f) s.f.(L) = 1·4; s.f.(V) = 2·744; V = $1097{\cdot}6\text{cm}^3$

2. **(a)** s.f.(L) = 0·5; s.f.(V) = 0·125; $V = 46\text{cm}^3$

(b) s.f.(L) = 0·25; s.f.(V) = 0·015625; $V = 2{\cdot}25\text{mm}^3$

(c) s.f.(L) = 0·75; s.f.(V) = 0·421875; $V = 384{\cdot}75\text{cm}^3$

(d) s.f.(L) = 0·6; s.f.(V) = 0·216; $V = 49{\cdot}68\text{mm}^3$

3. **(a)** 1200 ml **(b)** 270 ml **(c)** 6·75 litres

SIMILARITY – EXAM QUESTIONS

1. no, will burn for 4 times the time **2.** priced correctly

3. rug is too small since 69·1 < 72 **4.** 7cm

4.1 WORKING with TRIGONOMETRIC FUNCTIONS – BASIC GRAPHS

1. Sine graph drawn **2.** Cosine graph drawn **3.** Tangent graph drawn

4.1 WORKING with TRIGONOMETRIC FUNCTIONS (1)

1. **(a)** $y = 5\sin x^o$ **(b)** $y = 3\sin x^o$ **(c)** $y = 0{\cdot}5\sin x^o$ **(d)** $y = 4\cos x^o$

(e) $y = 12\cos x^o$ **(f)** $y = \frac{1}{4}\cos x^o$ **(g)** $y = 1{\cdot}5\sin x^o$ **(h)** $y = 2{\cdot}7\cos x^o$

(i) $y = 3{\cdot}3\sin x^o$ **(j)** $y = -8\cos x^o$ **(k)** $y = -6\sin x^o$ **(l)** $y = -20\cos x^o$

(m) $y = -2{\cdot}8\sin x^o$ **(n)** $y = \frac{3}{4}\sin x^o$ **(o)** $y = 0{\cdot}6\cos x^o$

2. **(a)** $y = 3\sin 2x^o$ **(b)** $y = 5\sin 3x^o$ **(c)** $y = 2\cos 4x^o$ **(d)** $y = 10\cos 2x^o$

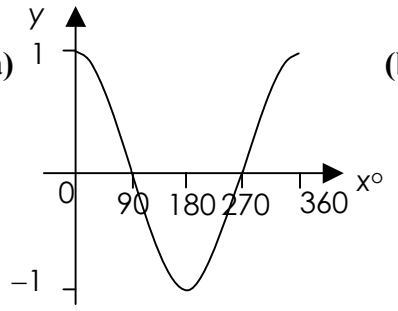
(e) $y = 7\sin 2x^o$ **(f)** $y = 4\cos 3x^o$ **(g)** $y = -6\sin 3x^o$ **(h)** $y = -5\cos 2x^o$

(i) $y = 3\sin \frac{1}{2}x^o$ **(j)** $y = 9\cos \frac{1}{2}x^o$ **(k)** $y = 20\cos \frac{1}{3}x^o$ **(l)** $y = 3\cos \frac{3}{2}x^o$

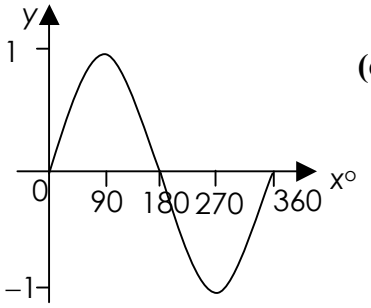
(m) $y = \tan x^o$ **(n)** $y = \tan 2x^o$ **(o)** $y = \tan \frac{1}{2}x^o$ **(p)** $y = \tan 4x^o$

3.

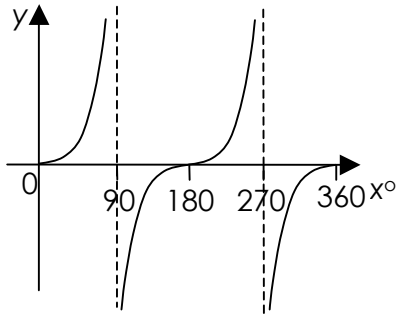
(a)



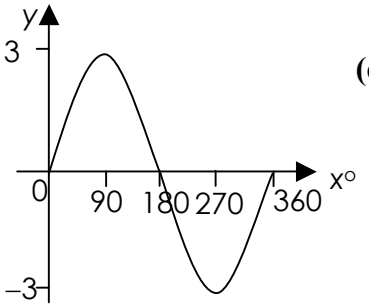
(b)



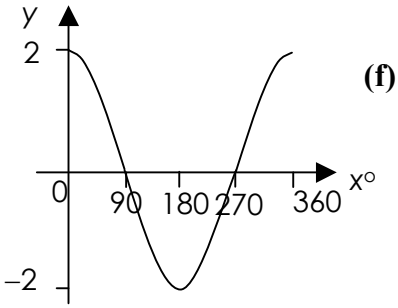
(c)



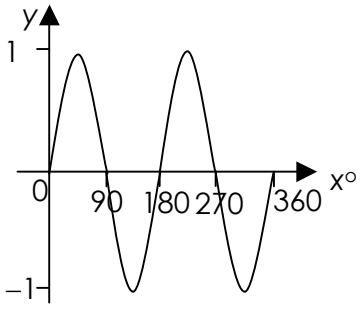
(d)



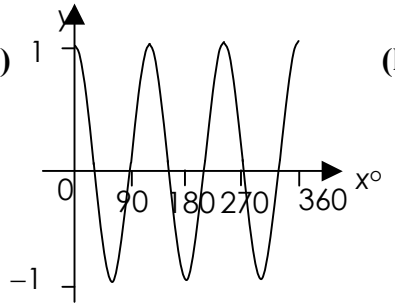
(e)



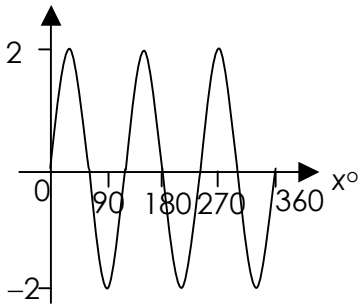
(f)



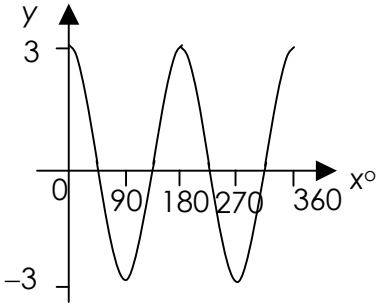
(g)



(h)



(i)



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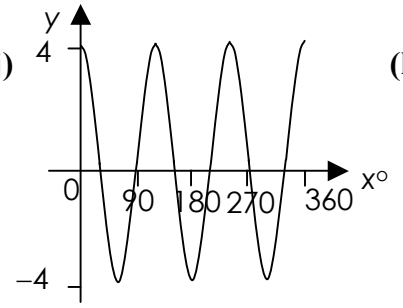

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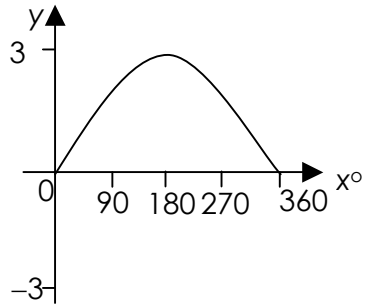
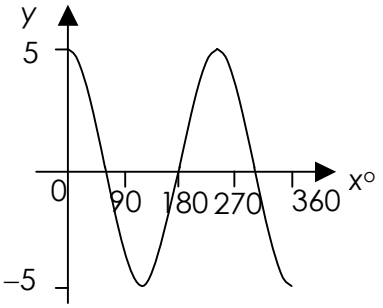
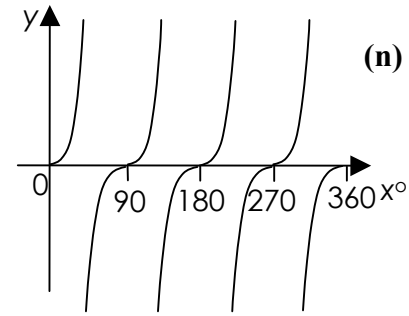


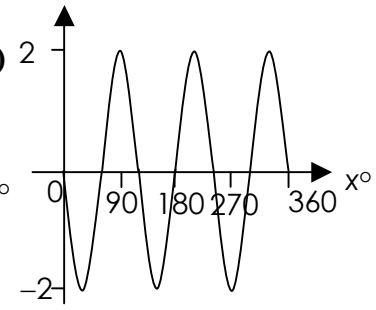
(l)



(m)



(n)



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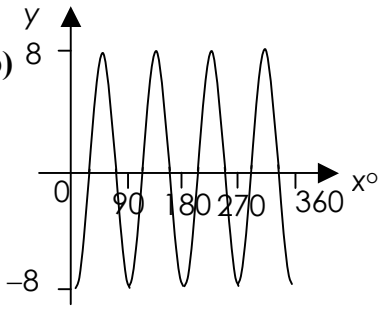


4.

(a) $y = \sin (x + 10)^{\circ}$ **(b)** $y = \sin (x - 40)^{\circ}$ **(c)** $y = \cos (x - 25)^{\circ}$

(d) $y = \cos (x + 30)^{\circ}$ **(e)** $y = \sin (x + 15)^{\circ}$ **(f)** $y = \cos (x - 30)^{\circ}$

(g) $y = \cos (x + 45)^{\circ}$ **(h)** $y = \sin (x - 37)^{\circ}$ **(i)** $y = \sin (x - 23)^{\circ}$

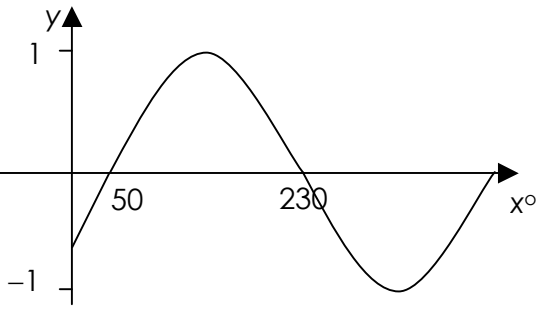
(j) $y = \cos (x - 18)^{\circ}$

5. **(a)** $y = \tan(x - 45)^\circ$ **(b)** $y = 3\sin(x - 30)^\circ$ **(c)** $y = 5\cos(x + 35)^\circ$

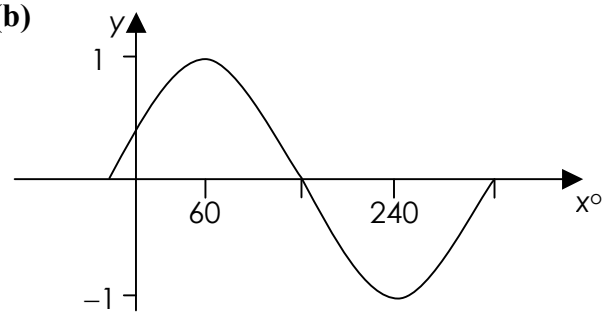
(d) $y = 2\sin(x + 25)^\circ$ **(e)** $y = 6\sin(x + 8)^\circ$ **(f)** $y = 1.5\cos(x - 25)^\circ$

(g) $y = 5\cos(x - 20)^\circ$ **(h)** $y = 4\sin(x + 75)^\circ$

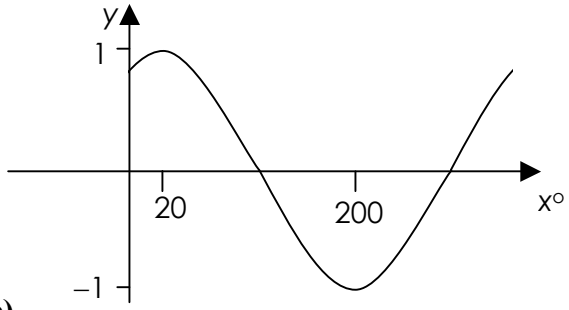
6. **(a)**



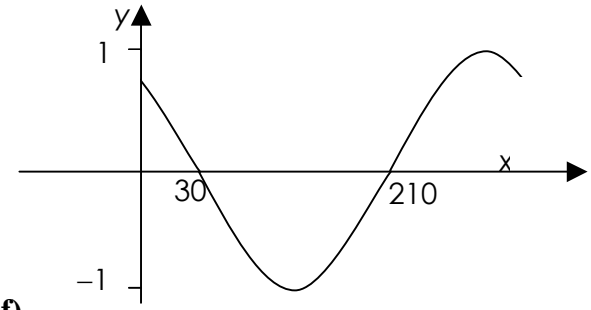
(b)



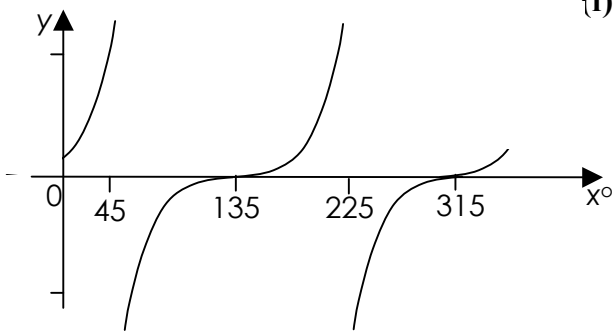
(c)



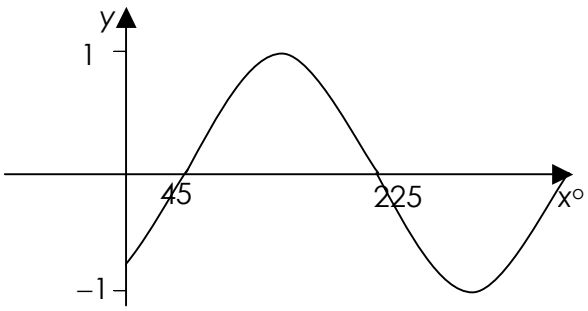
(d)



(e)



(f)



4.1 WORKING with TRIGONOMETRIC FUNCTIONS (2)

1. **(a)** 5; $y = 5\sin x^\circ + 2$ **(b)** 3; $y = 3\sin x^\circ - 1$ **(c)** 0·5; $y = 0{\cdot}5\sin x^\circ - 0{\cdot}5$

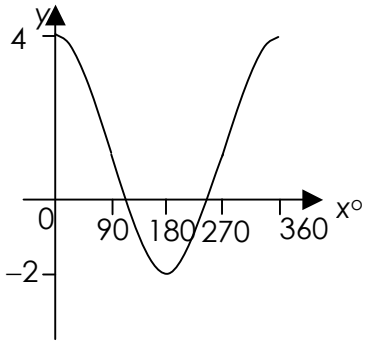
(d) 4; $y = 4\cos x^\circ + 1$ **(e)** 12; $y = 12\cos x^\circ - 3$ **(f)** 4; $y = 4\cos 2x^\circ - 1$

(g) 1·5; $y = 1{\cdot}5\sin x^\circ - 0{\cdot}5$ **(h)** 2·7; $y = 2{\cdot}7\cos 3x^\circ - 0{\cdot}2$

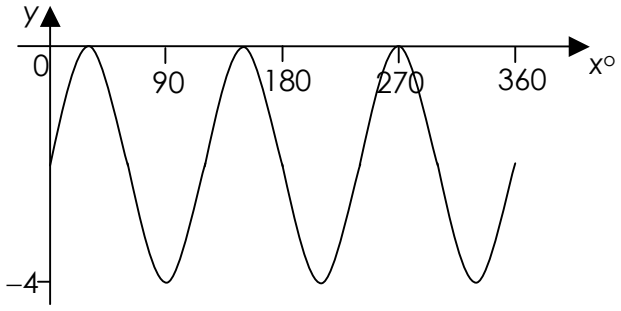
(i) 3·3; $y = 3{\cdot}3\sin 2x^\circ - 0{\cdot}8$ **(j)** 8; $y = -8\cos x^\circ + 4$

(k) 6; $y = -6\sin 3x^\circ + 3$ **(l)** 20; $y = -20\cos x^\circ - 5$

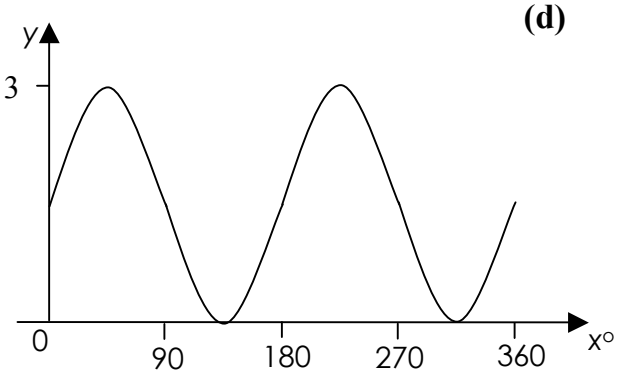
2. **(a)**



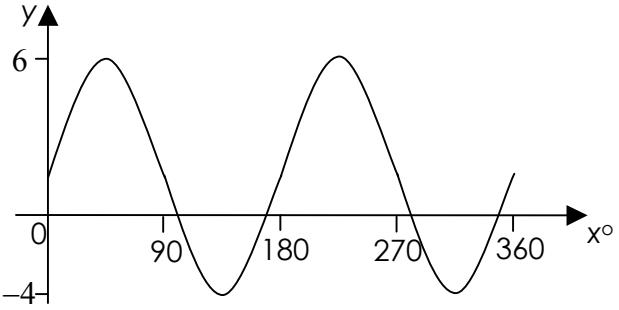
(b)



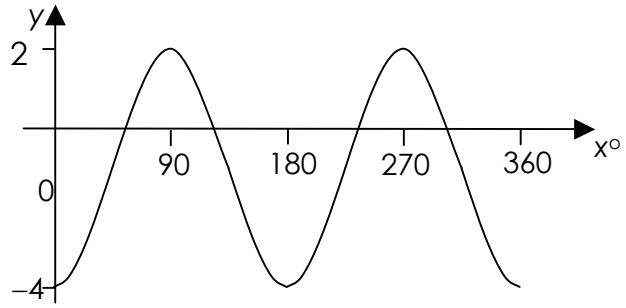
(c)



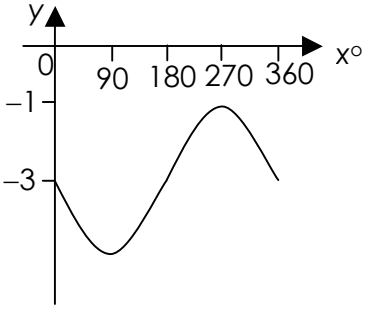
(d)



(e)



(f)



4.2 WORKING with TRIGONOMETRIC RELATIONSHIPS in DEGREES
Sine, cosine and tangent of angles 0 – 360°

1.

	$0 < x < 90$	$90 < x < 180$	$180 < x < 270$	$270 < x < 360$
$\sin x^o$	+	+	−	−
$\cos x^o$	+	−	−	+
$\tan x^o$	+	−	+	−

2. **(a)** + **(b)** − **(c)** − **(d)** + **(e)** + **(f)** +

(g) + **(h)** + **(i)** − **(j)** − **(k)** − **(l)** −

3. **(a)** $\cos 22^o$ **(b)** $-\tan 57^o$ **(c)** $-\sin 45^o$ **(d)** $\sin 15^o$

(e) $\tan 16^o$ **(f)** $-\cos 5^o$ **(g)** $\tan 66^o$ **(h)** $\sin 48^o$

(i) $-\cos 10^o$ **(j)** $-\sin 38^o$ **(k)** $-\cos 20^o$ **(l)** $-\tan 50^o$

4.2 WORKING with TRIGONOMETRIC RELATIONSHIPS in DEGREES EXACT VALUES

1. **(a)** $\frac{1}{2}$ **(b)** $\frac{1}{2}$ **(c)** $-\frac{1}{2}$ **(d)** $-\frac{1}{2}$

(e) $\frac{\sqrt{3}}{2}$ **(f)** $-\frac{\sqrt{3}}{2}$ **(g)** $-\frac{\sqrt{3}}{2}$ **(h)** $\frac{\sqrt{3}}{2}$

(i) $\frac{1}{\sqrt{3}}$ **(j)** $-\frac{1}{\sqrt{3}}$ **(k)** $\frac{1}{\sqrt{3}}$ **(l)** $-\frac{1}{\sqrt{3}}$

2. **(a)** $\frac{\sqrt{3}}{2}$ **(b)** $\frac{\sqrt{3}}{2}$ **(c)** $-\frac{\sqrt{3}}{2}$ **(d)** $-\frac{\sqrt{3}}{2}$

(e) $\frac{1}{2}$ **(f)** $-\frac{1}{2}$ **(g)** $-\frac{1}{2}$ **(h)** $\frac{1}{2}$

(i) $\sqrt{3}$ **(j)** $-\sqrt{3}$ **(k)** $\sqrt{3}$ **(l)** $-\sqrt{3}$

3. **(a)** $\frac{1}{\sqrt{2}}$ **(b)** $\frac{1}{\sqrt{2}}$ **(c)** $-\frac{1}{\sqrt{2}}$ **(d)** $-\frac{1}{\sqrt{2}}$

(e) $\frac{1}{\sqrt{2}}$ **(f)** $-\frac{1}{\sqrt{2}}$ **(g)** $-\frac{1}{\sqrt{2}}$ **(h)** $\frac{1}{\sqrt{2}}$

(i) 1 **(j)** -1 **(k)** 1 **(l)** -1

4.2 WORKING with TRIGONOMETRIC RELATIONSHIPS in DEGREES

PERIOD

1. **(a)** 12 **(b)** 40 **(c)** 120^o **(d)** 90^o
(e) 90^o **(f)** 180^o **(g)** 360^o **(h)** 180^o
(i) 240^o **(j)** 720^o

2. **(a)** 180^o **(b)** 90^o **(c)** 180^o **(d)** 60^o
(e) 90^o **(f)** 120^o **(g)** 240^o **(h)** 80^o
(i) 360^o **(j)** 45^o **(k)** 30^o **(l)** 30^o
(m) 10^o **(n)** 40^o **(o)** 12^o **(p)** 24^o
(q) 36^o **(r)** 45^o

3. **(a)** 720^o **(b)** 540^o **(c)** 1440^o **(d)** 900^o
(e) 2160^o **(f)** 540^o **(g)** 180^o **(h)** 120^o
(i) 90^o **(j)** 180^o **(k)** 180^o **(l)** 90^o

SOLVING BASIC EQUATIONS

1. **(a)** $30^o, 150^o$ **(b)** $30^o, 330^o$ **(c)** $45^o, 225^o$
(d) $120^o, 240^o$ **(e)** $150^o, 330^o$ **(f)** $240^o, 300^o$
(g) $60^o, 240^o$ **(h)** $45^o, 135^o$ **(i)** $45^o, 315^o$
(j) $225^o, 315^o$ **(k)** $150^o, 210^o$ **(l)** $120^o, 300^o$

2. **(a)** $18{\cdot}2^o, 161{\cdot}8^o$ **(b)** $64{\cdot}8^o, 295{\cdot}2^o$ **(c)** $79^o, 259^o$
(d) $95^o, 265^o$ **(e)** $139^o, 319^o$ **(f)** $191^o, 349^o$
(g) $85^o, 265^o$ **(h)** $44^o, 136^o$ **(i)** $41^o, 319^o$
(j) $201^o, 339^o$ **(k)** $133^o, 227^o$ **(l)** $165^o, 345^o$

3. **(a)** $30^o, 150^o$ **(b)** $48{\cdot}2^o, 311{\cdot}8^o$ **(c)** $59^o, 239^o$
(d) $120^o, 240^o$ **(e)** $104^o, 284^o$ **(f)** $228{\cdot}6^o, 311{\cdot}4^o$
(g) $78^o, 258^o$ **(h)** $23{\cdot}6^o, 156{\cdot}4^o$ **(i)** $80{\cdot}4^o, 279{\cdot}6^o$
(j) $202^o, 338^o$ **(k)** $144{\cdot}9^o, 215{\cdot}1^o$ **(l)** $138^o, 318^o$

4. **(a)** 90^o **(b)** 180^o **(c)** $45^o, 225^o$
(d) $210^o, 330^o$ **(e)** $60^o, 300^o$ **(f)** $26{\cdot}6^o, 206{\cdot}6^o$
(g) $41{\cdot}4^o, 318{\cdot}6^o$ **(h)** $41{\cdot}8^o, 138{\cdot}2^o$ **(i)** $113{\cdot}6^o, 246{\cdot}4^o$
(j) $33{\cdot}7^o, 213{\cdot}7^o$ **(k)** $109{\cdot}5^o, 250{\cdot}5^o$ **(l)** $205{\cdot}4^o, 334{\cdot}6^o$

5. **(a)** $104{\cdot}5^o, 255{\cdot}5^o$ **(b)** $44{\cdot}4^o, 135{\cdot}6^o$ **(c)** $84{\cdot}3^o, 264{\cdot}3^o$

(d) $60^o, 300^o$ **(e)** $45^o, 225^o$ **(f)** $126{\cdot}9^o, 233{\cdot}1^o$

(g) $23{\cdot}6^o, 156{\cdot}4^o$ **(h)** $120^o, 240^o$ **(i)** $53{\cdot}1^o, 126{\cdot}9^o$

(j) $118{\cdot}1^o, 241{\cdot}9^o$ **(k)** $45{\cdot}6^o, 134{\cdot}4^o$ **(l)** $33{\cdot}2^o, 146{\cdot}8^o$

IDENTITIES INVOLVING $\cos^2x + \sin^2x = 1$ and $\tan x = \sin x/\cos x$

1. **(a)** $3(\cos^2x + \sin^2x) = 3 \times 1 = 3$ **(b)** $\sin^2x$ **(c)** $\cancel{\cos A} \times \frac{\sin A}{\cancel{\cos A}} = \sin A$

(d) $5(1 - \sin^2B) = 5\cos^2B$ **(e)** $\frac{4\sin a^o}{4\cos a^o} = \frac{\sin a°}{\cos a°} = \tan a°$

(f) $\frac{4\tan x^o}{2\cos x^o} = \frac{2\tan x°}{\cos x°} = \frac{2\frac{\sin x°}{\cos x°}}{\cos x} = 2\frac{\sin x°}{\cos x°} \times \frac{1}{\cos x°} = \frac{2\sin x°}{\cos^2 x}$

(g) $\frac{(1-\sin^2 x)}{2\cos x} = \frac{\cos^2 x}{2\cos x} = \frac{\cos x}{2}$ **(h)** $\frac{8-8\cos^2 x}{2\sin x} = \frac{8(1-\cos^2 x)}{2\sin x} = \frac{8\sin^2 x}{2\sin x} = 4\sin x$

(i) $\frac{3\sin x\cos x}{6\tan x} = \frac{3\sin x\cos x}{6\frac{\sin x}{\cos x}} = 3\sin x\cos x \times \frac{\cos x}{6\sin x} = \frac{\cos^2 x}{2}$

(j) $4\sin^2A + 3\cos^2A - 3 = 3(\sin^2 A + \cos^2 A) + \sin^2 A - 3 = 3 + \sin^2 A - 3 = \sin^2 A$

(k) $4\cos^2B - 2\sin^2B + 2 = 4\cos^2 B - 2(1-\cos^2 B) + 2 = 4\cos^2 B - 2 + \cos^2 B - 2 = 6\cos^2 B$

(l) $(\cos x + \sin x)^2 - 2\sin x\cos x$
$= \cos^2 x + 2\cos x\sin x + \sin^2 x - 2\sin x\cos x = \cos^2 x + \sin^2 x = 1$

(m) $\tan^2a(1 - \sin^2a) = \frac{\sin^2\alpha}{\cos^2\alpha} \times \cos^2\alpha = \sin^2\alpha$

2. **(a)** $3(\cos^2A + \sin^2A) - \cos^2A = 3(1) - \cos^2A = 3 - \cos^2A$

(b) $\frac{1}{\tan x} + \tan x = \frac{\cos x}{\sin x} + \frac{\sin x}{\cos x} = \frac{\cos^2 x + \sin^2 x}{\sin x\cos x} = \frac{1}{\sin x\cos x}$

(c) $(2\cos B + 3\sin B)^2 + (3\cos B - 2\sin B)^2$
$= 4\cos^2 B + 12\cos B\sin B + 9\sin^2 B + 9\cos^2 B - 12\cos B\sin B + 4\sin^2 B$
$= 13\cos^2 B + 13\sin^2 B = 13(\cos^2 B + \sin^2 B) = 13$

(d) $(1 + \sin x)(1 - \sin x) = 1 - \sin^2 x = \cos^2x$

(e) $\sin\theta.\tan\theta = \sin\theta \times \frac{\sin\theta}{\cos\theta} = \frac{\sin^2\theta}{\cos\theta} = \frac{1-\cos^2\theta}{\cos\theta}$

3. **(a)** $\sin x^\circ = \frac{1}{2}$; $\cos x^\circ = \frac{\sqrt{3}}{2}$; $\tan x^\circ = \frac{1}{\sqrt{3}}$.

(b) $\sin^2 x^\circ + \cos^2 x^\circ = (\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2 = \frac{1}{4} + \frac{3}{4} = 1$.

(c) $\frac{\sin x^\circ}{\cos x^\circ} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{2} \times \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}} = \tan x^\circ$.

4. **(a)** $\sin a^\circ = \frac{2}{\sqrt{5}}$**;** $\tan a^\circ = 2$

(b) $\cos^2 a^\circ = (\frac{1}{\sqrt{5}})^2 = \frac{1}{5}$; $1 - (\frac{2}{\sqrt{5}})^2 = 1 - \frac{4}{5} = \frac{1}{5} = \cos^2 x$

(c) $\frac{(\frac{2}{\sqrt{5}})^2}{(\frac{1}{\sqrt{5}})^2} = \frac{\frac{4}{5}}{\frac{1}{5}} = \frac{4}{5} \times \frac{5}{1} = 4$; $\tan^2 a^\circ = 2^2 = 4$.

(d) $2[3(\frac{2}{\sqrt{5}}) + 4(\frac{1}{\sqrt{5}})] = 2(\frac{6}{\sqrt{5}} + \frac{4}{\sqrt{5}}) = \frac{20}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{20\sqrt{5}}{5} = 4\sqrt{5}$.

5. **(a)** $3\cos^2 a + 3\sin^2 a = 3(\cos^2 a + \sin^2 a) = 3$

(b) $(\cos x + \sin x)^2 = \cos^2 x + 2\sin x \cos x + \sin^2 x = 1 + 2\sin x \cos x$

(c) $(\cos x + \sin x)(\cos x - \sin x) = \cos^2 x - \sin^2 x = \cos^2 x - (1 - \cos^2 x)$
$= \cos^2 x - 1 + \cos^2 x - 2\cos^2 x - 1$

(d) $\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\sin^2 x + \cos^2 x}{\cos x \sin x} = \frac{1}{\cos x \sin x}$

(e) $\tan^2 p - \tan^2 p \sin^2 p = \tan^2 p(1 - \sin^2 p) = \frac{\sin^2 p}{\cos^2 p} \times \cos^2 p = \sin^2 p$

(f) $\cos^4 x - \sin^4 x = (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)$
$= \cos^2 x - (1 - \cos^2 x) = \cos^2 x - 1 + \cos^2 x = 2\cos^2 x - 1$

(g) $3\sin^2 \theta + 2\cos^2 \theta = 2(\sin^2 \theta + \cos^2 \theta) + \sin^2 \theta = 2 + \sin^2 \theta$

(h) $\tan \alpha + \frac{1}{\tan \alpha} = \frac{\sin \alpha}{\cos \alpha} + \frac{\cos \alpha}{\sin \alpha} = \frac{\sin^2 \alpha + \cos^2 \alpha}{\sin \alpha \cos \alpha} = \frac{1}{\sin \alpha \cos \alpha}$

6. $(2\cos x + 5\sin x)^2 \ + \ (5\cos x - 2\sin x)^2$

$= 4\cos^2 x + 20\sin x\cos x + 25\sin^2 x + 25\cos^2 x - 20\sin x\cos x + 4\sin^2 x$

$= \ 29\cos^2 x + 29\sin^2 x = 29(\cos^2 x + \sin^2 x) = 29$

7. **(a)** $pq \ = (\cos\theta + \sin\theta)(\cos\theta - \sin\theta) = \cos^2\theta - \sin^2\theta$

$= \cos^2\theta - (1 - \cos^2\theta) = 2\cos^2\theta \ - \ 1$

(b) $\frac{1}{2}(1 \ - \ pq) = \frac{1}{2}[1 - (2\cos^2\theta - 1)] = \frac{1}{2}(2 - 2\cos^2\theta) = 1 - \cos^2\theta = \sin^2\theta$

4.2 WORKING with TRIGONOMETRIC RELATIONSHIPS in DEGREES

EXAM QUESTIONS

1. $53{\cdot}1^o$, $126{\cdot}9^o$ **2.** Graph drawn **3.** $\tan^2 x^o$

4. $a = -5$; $b = 2$ **5.** $221{\cdot}8^o$, $318{\cdot}2^o$ **6.** $a = 3$; $b = 2$

7. $53{\cdot}1^o$, $223{\cdot}1^o$ **8.** $48{\cdot}6^o$, $131{\cdot}4^o$ **9.** **(a)** 4 **(b)** P(240^o, –2)

10. $a = 3$; $b = 4$ **11.** Graph draw **12.** Graph drawn

13. 1 **14.** $221{\cdot}8^o$ **15.** $135{\cdot}6^o$, $224{\cdot}4^o$

16. any angle between 180^o and 270^o **17.** Graph drawn

18. $194{\cdot}5^o$, $345{\cdot}5^o$ **19.** $1 + 2\sin x^o\cos x^o$ **20.** Proof

21. $113{\cdot}6^o$, $246{\cdot}4^o$ **22.** C and D **23.** $104{\cdot}5^o$, $255{\cdot}5^o$

24. $143{\cdot}1^o$, $323{\cdot}1^o$ **25.** $a = 2$; $b = 30$ **26.** $\cos x^o\sin x^o$

27. $107{\cdot}5^o$, $252{\cdot}5^o$ **28.** Proof **29.** $y = 3\cos(x + 30^o)$