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CFE National 5 - Pack 3

Unit : Applications (APP)

WORKSHEETS

- **Worksheets covering all the unit topics**
- **+ Answers**

INDEX **APPLICATIONS (APP)**

3.1 Working with percentages

- **Use reverse percentages to calculate an original quantity**
- **Appreciation including compound interest**
- **Depreciation**

3.2 Working with fractions

- **Operations and combinations of vulgar fractions including mixed numbers**

4.1 Comparing data sets using statistics

- **Compare data sets using calculated/determined:**
  - **Quartiles and interquartile range**
  - **Standard deviation**

4.2 Forming a linear model from a given set of data

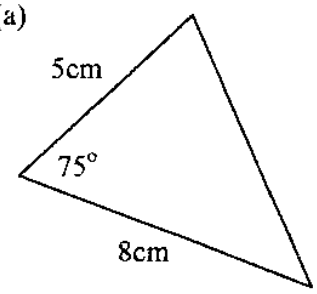
- **Determine the equation of a best – fitting straight line on a scattergraph and use it to estimate *y* given *x***

1.1 CALCULATING the AREA of a TRIANGLE using TRIGONOMETRY

The area of a triangle : $A = \frac{1}{2}ab\sin C$

1. Use trigonometry to calculate the area of each triangle below.

(a)



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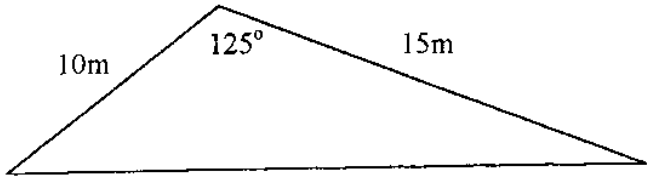

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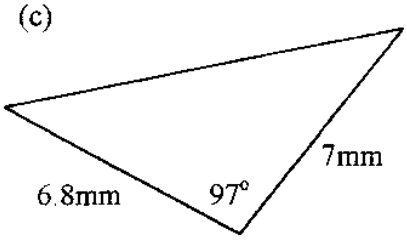

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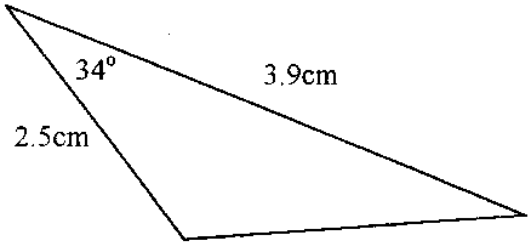

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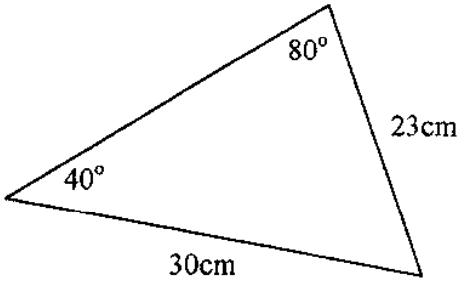

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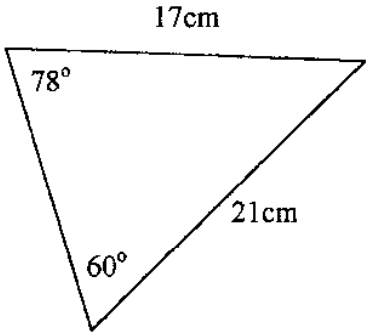
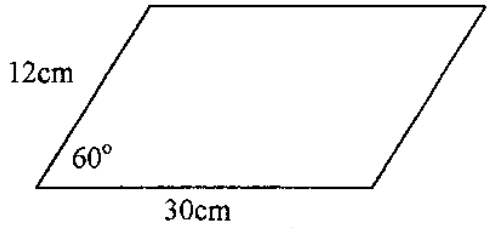
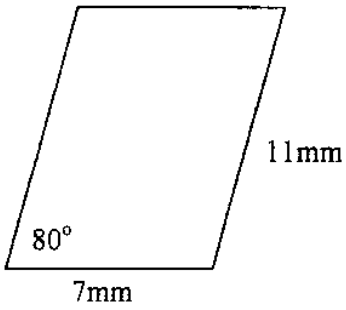


2. Calculate the area of each parallelogram below.

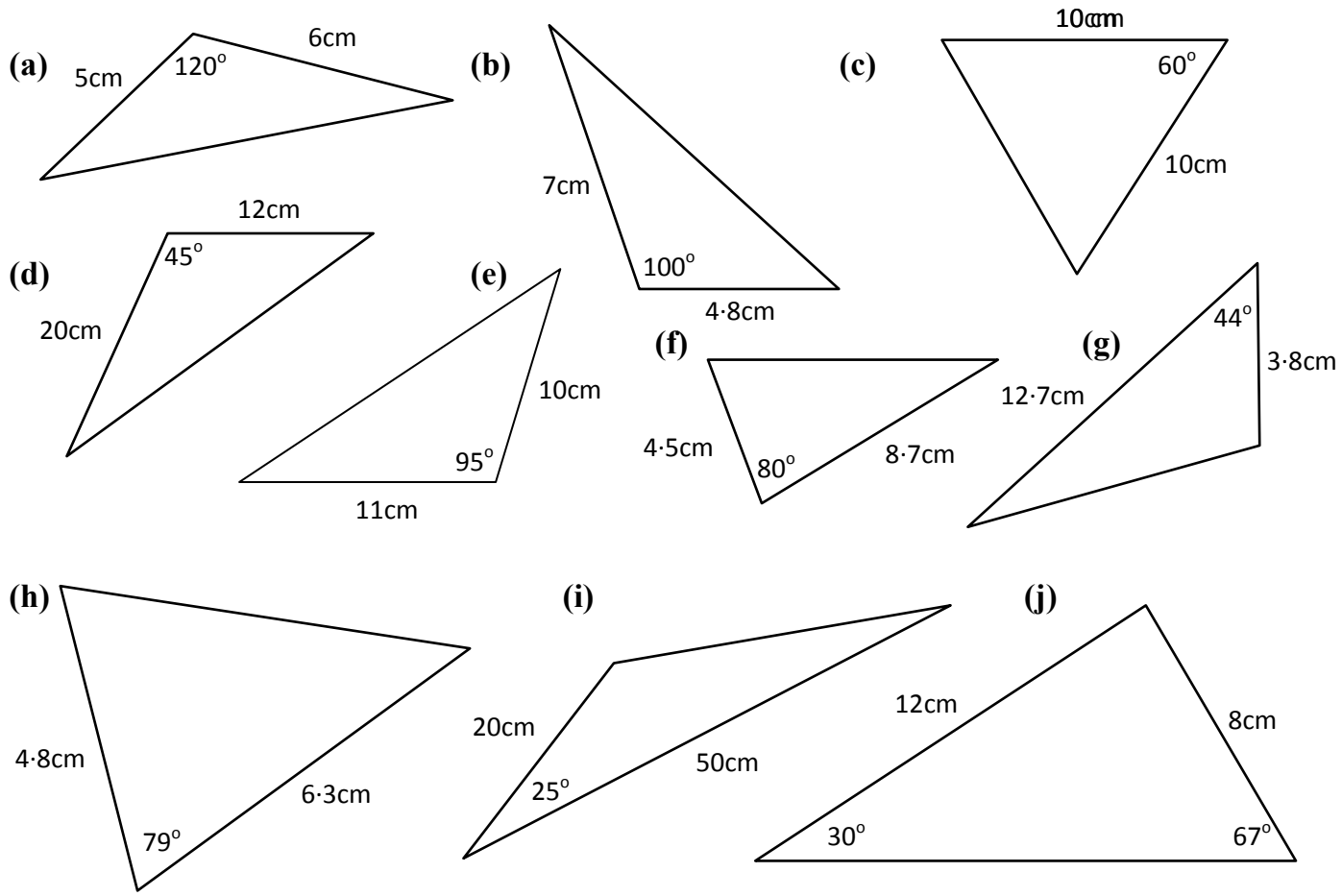
(a)



(b)



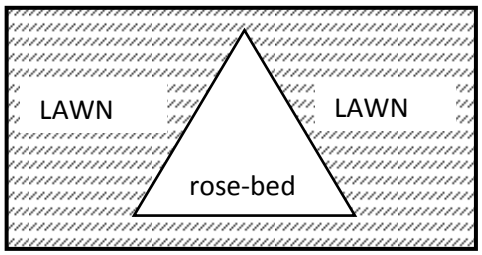
3. Find the area of the following triangles :



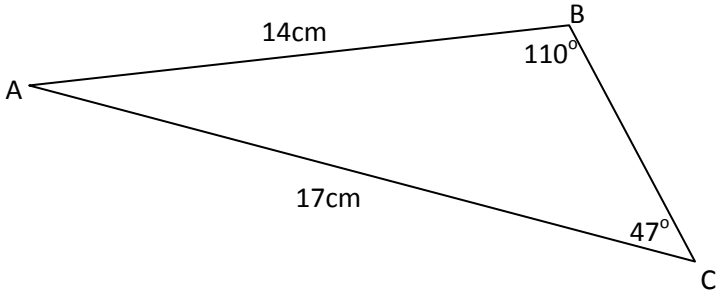
4. Mr. Fields is planting a rose-bed in his garden.

It is to be in the shape of an **equilateral** triangle of side 2m.

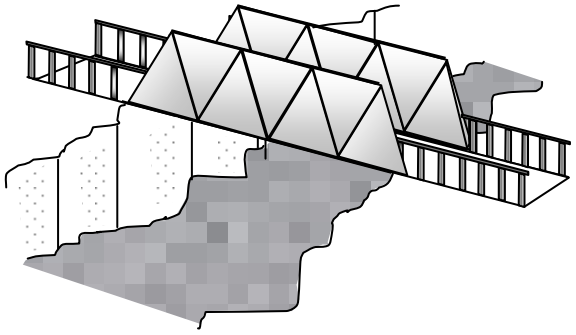
What area of lawn will he need to remove to plant his rose-bed?



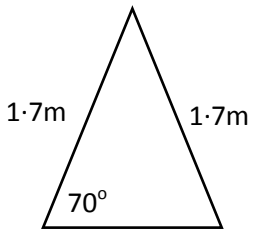
5. Calculate the area of triangle ABC where AB = 14cm, AC = 17cm, $\angle ABC = 110^o$ and $\angle BCA = 47^o$.



6. For safety reasons the sides of a footbridge are to be covered with triangular panels.



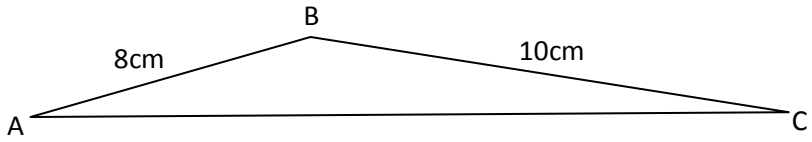
Each panel is an isosceles triangle as shown.



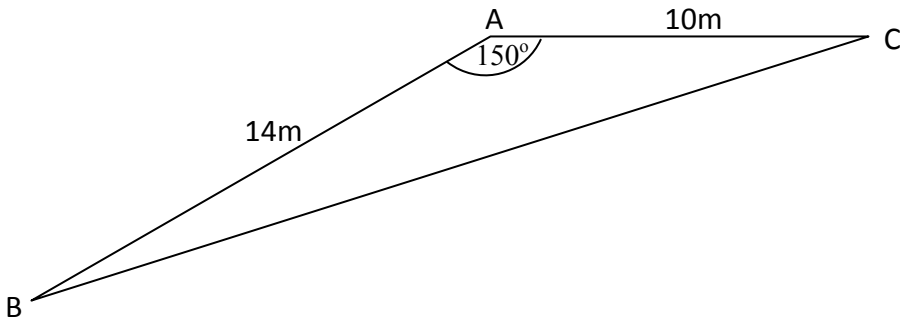
(a) Find the area of each panel.

(b) If there are 7 panels on each side of the bridge, find the total area of material required to cover the bridge.

7. Given that the area of this triangle is $20cm^2$, calculate the size of the **obtuse** angle ABC.



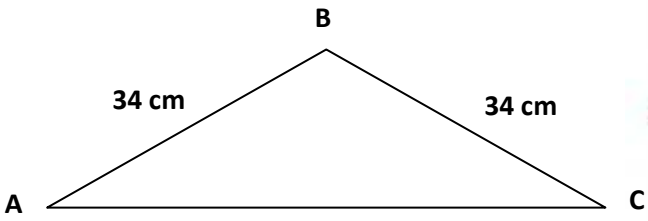
8. In triangle ABC, AB = 14m and AC = 10m. Angle BAC = 150^o.



Given that $\sin 150^o = 0{\cdot}5$, calculate the area of triangle ABC.

9. The area of a triangular flag is $429{\cdot}5cm^2$.

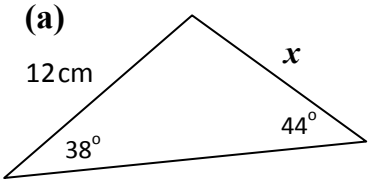
Calculate the size of the obtuse angle ABC.



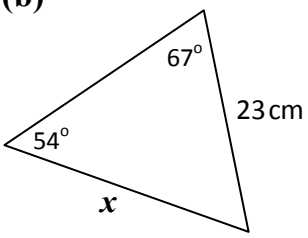
1.2 USING the SINE RULE to CALCULATE a SIDE

1. Use the *sine rule* to calculate the side marked ***x*** in each triangle below.

(a)



(b)



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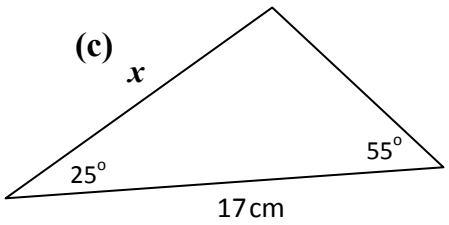

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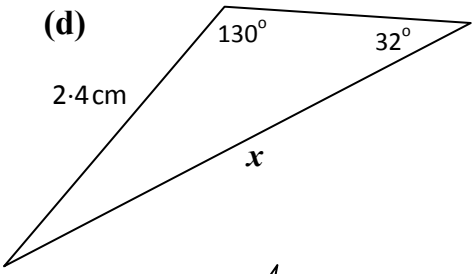

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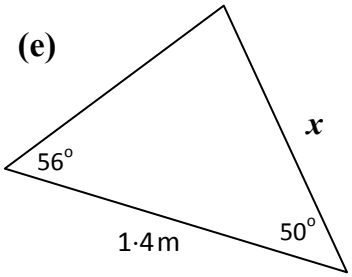

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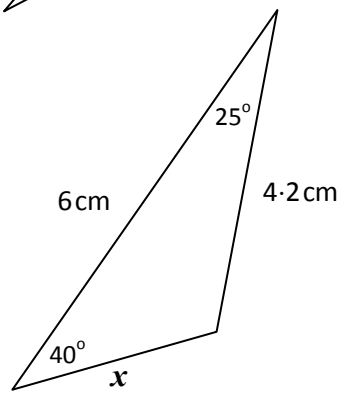

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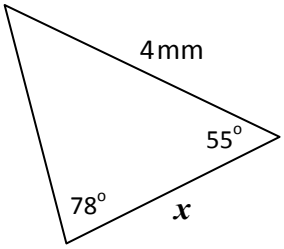

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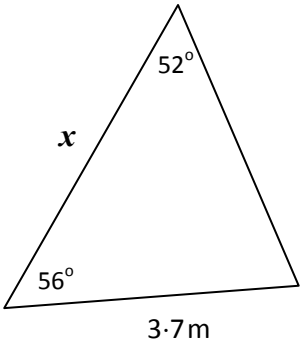
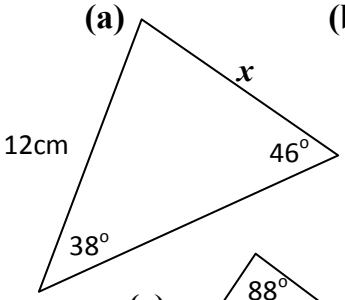


2. Use the sine rule to calculate the length of the side marked ***x*** in each of the triangles below.

(a)



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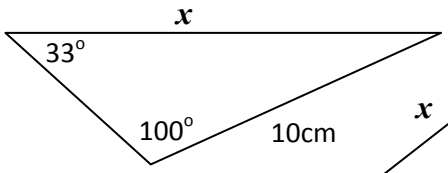

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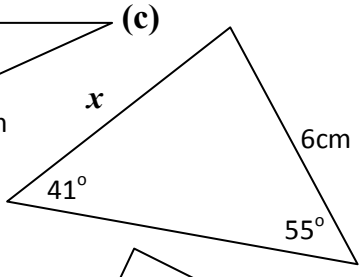

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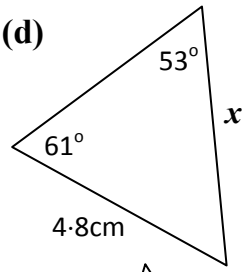

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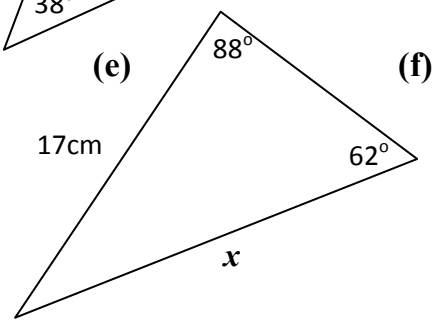

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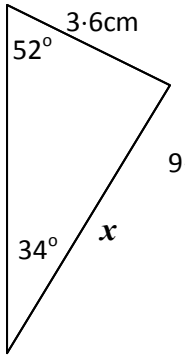
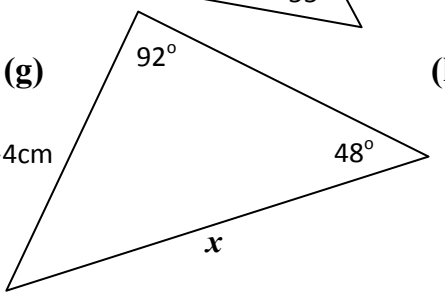
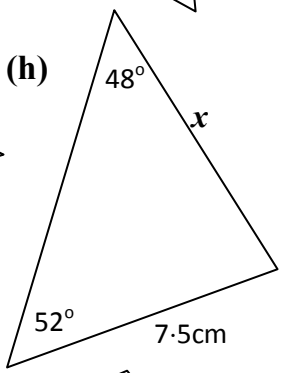


(g)



(h)



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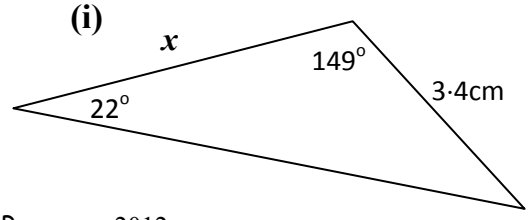

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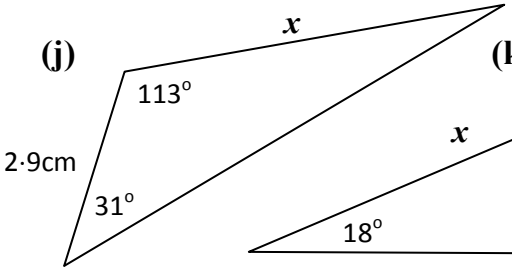

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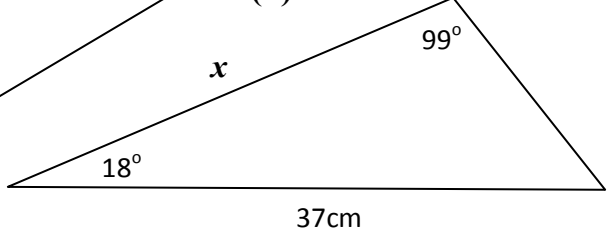
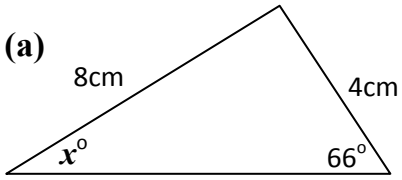


1.2 USING the SINE RULE to CALCULATE an ANGLE

1. Use the sine rule to calculate the length of the angle marked x^o in each of the triangles below.

(a)



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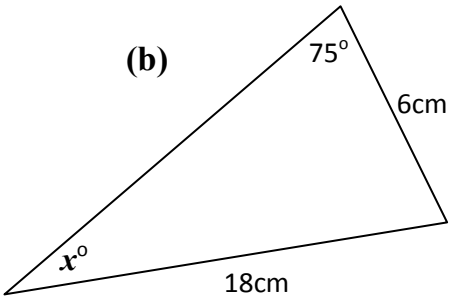

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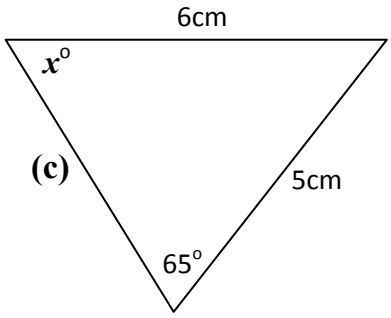

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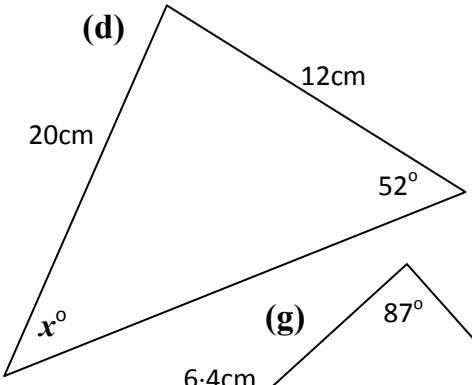

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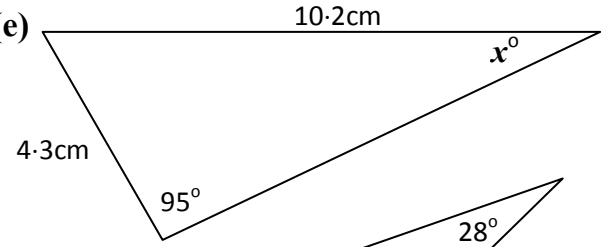

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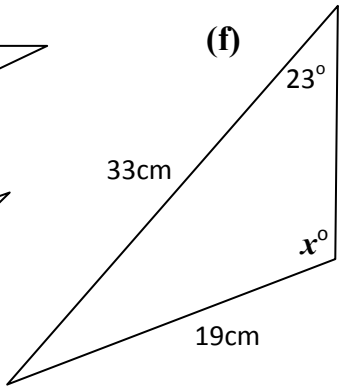

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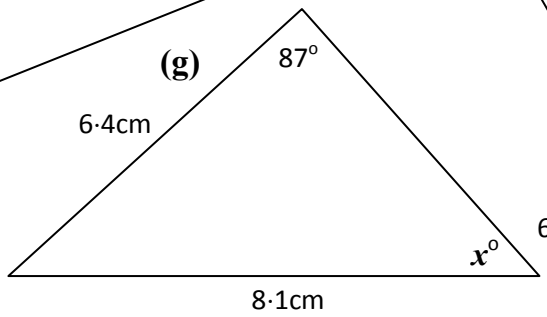

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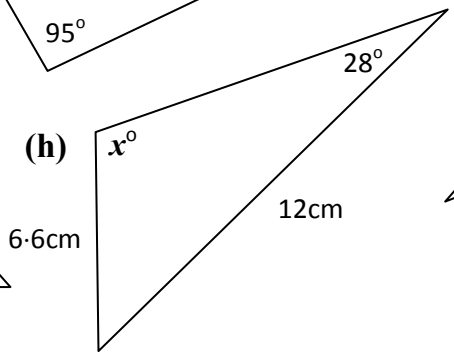
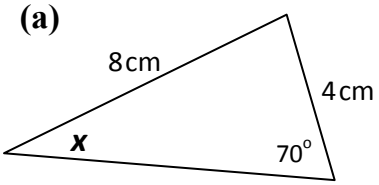


2. Use the *sine rule* to calculate the size of the angle marked ***x*** in each triangle below.

(a)



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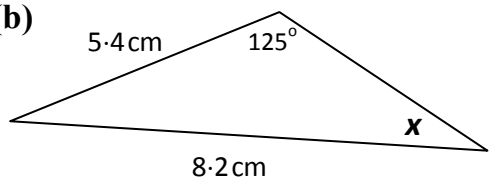

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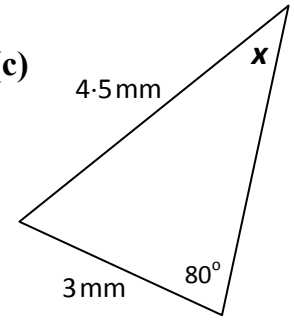
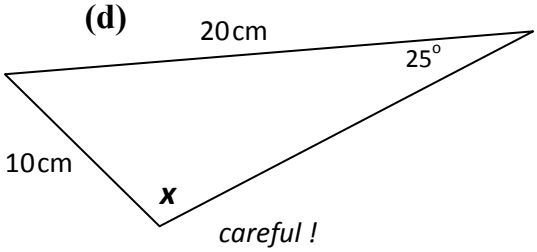
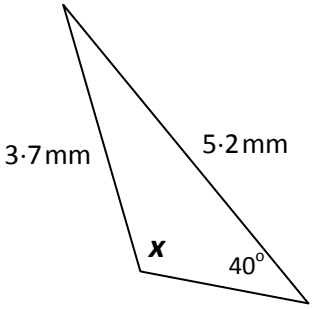


(d)



careful !

(e)



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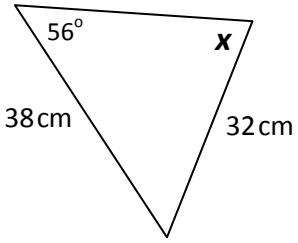
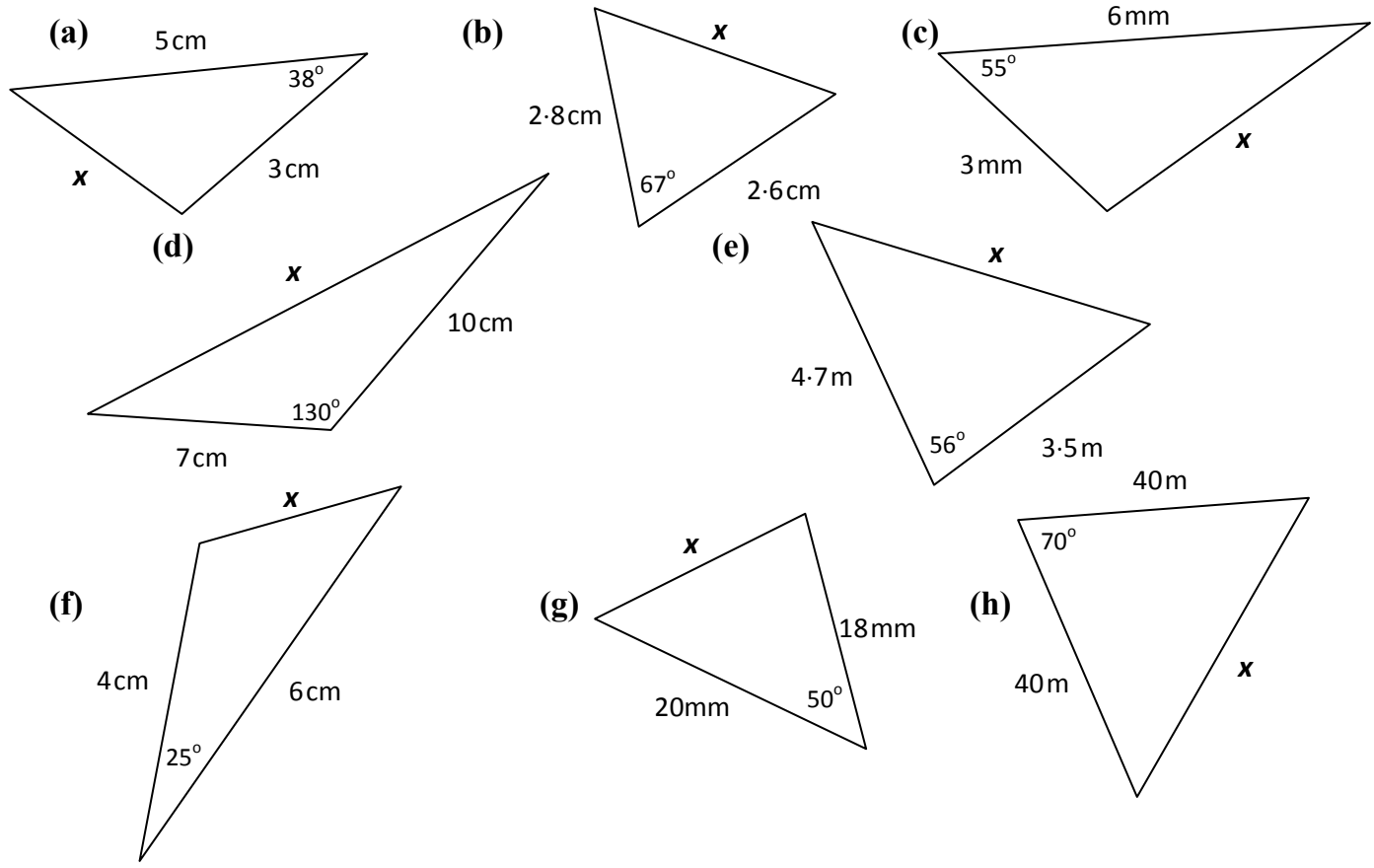
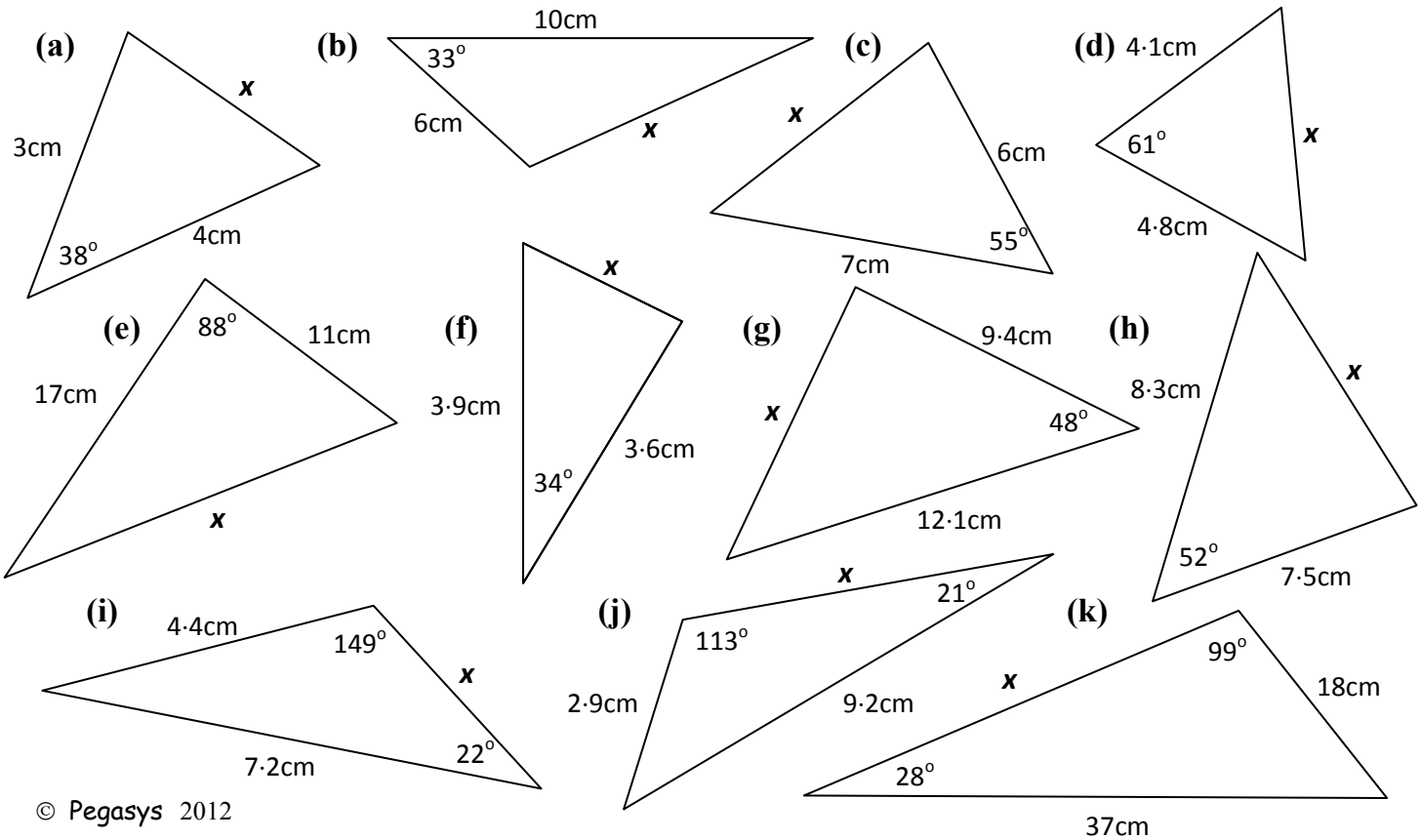


1.2 USING the COSINE RULE to CALCULATE a SIDE

1. Use the *cosine rule* to calculate the side marked ***x*** in each triangle below.



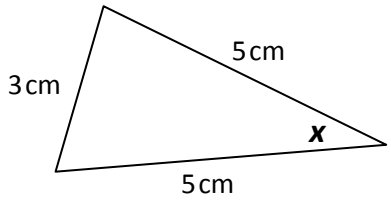
2. Use the cosine rule to calculate the length of the side marked ***x*** in each of the triangles below.



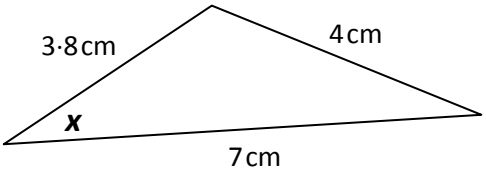
1.2 USING the COSINE RULE to CALCULATE an ANGLE

1. Use the 2nd form of the *cosine rule* to calculate the size of the angle marked ***x*** in each triangle below.

(a)



(b)



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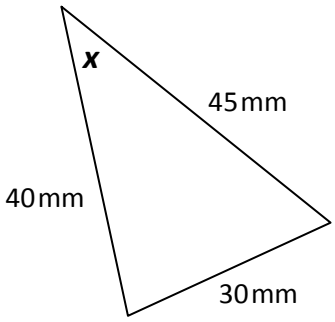
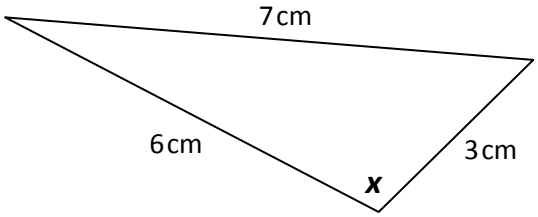
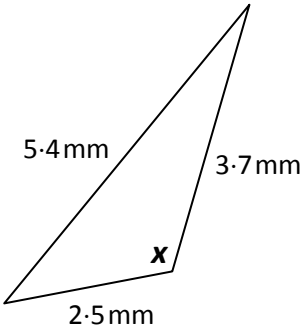


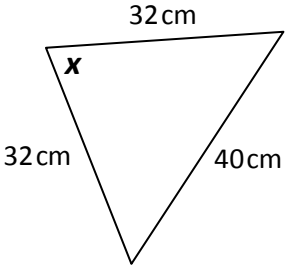
(d)



(e)

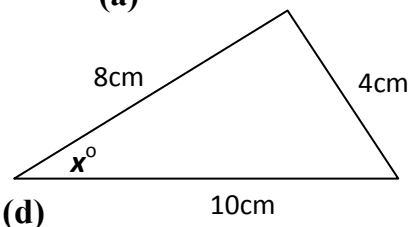


(f)

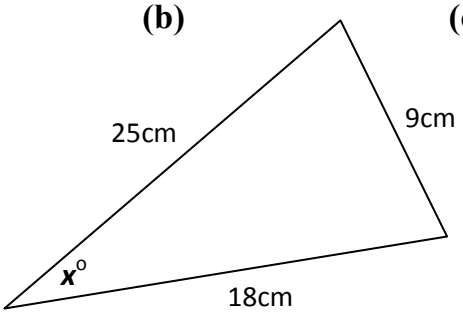


2. Use the cosine rule to calculate the angle marked x^o in each of the triangles below.

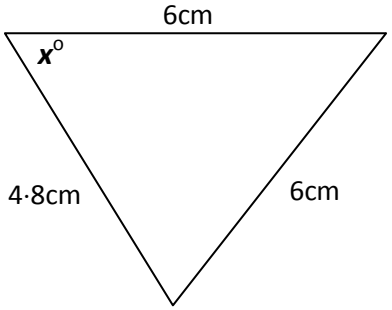
(a)



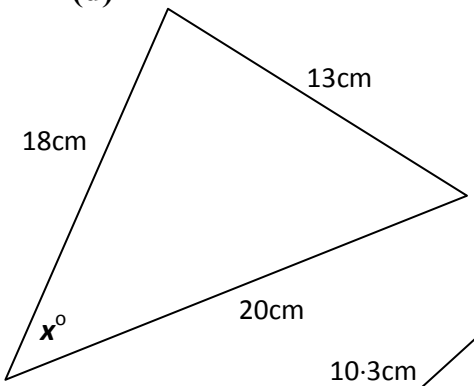
(b)



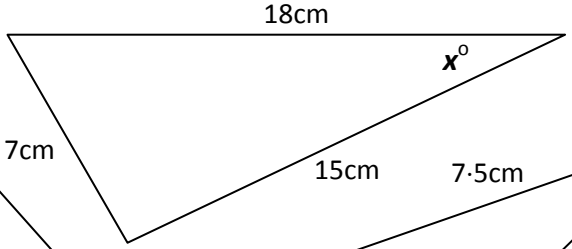
(c)



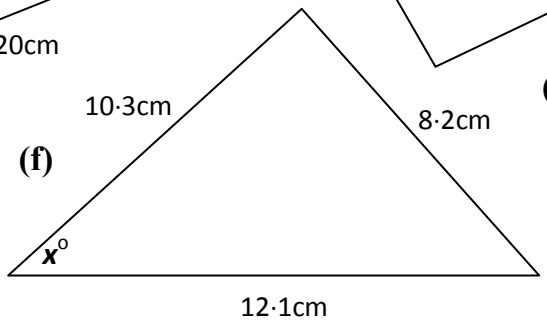
(d)



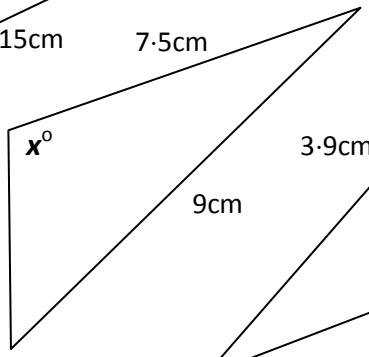
(e)



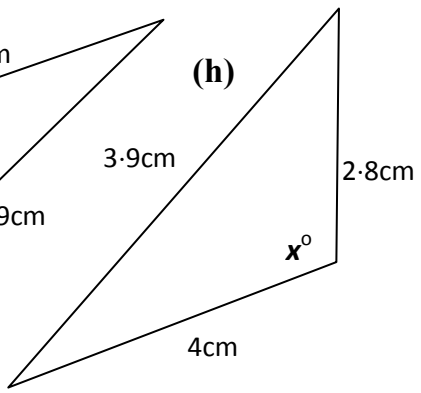
(f)



(g)



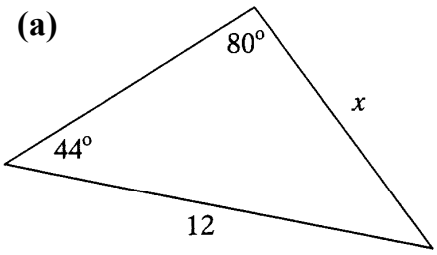
(h)



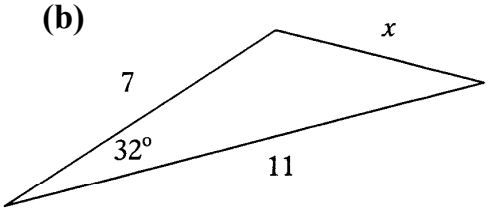
MIXED EXERCISE using TRIGONOMETRY RULES

1. Calculate the value of x in each triangle below.

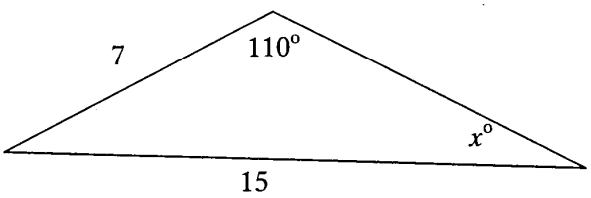
(a)



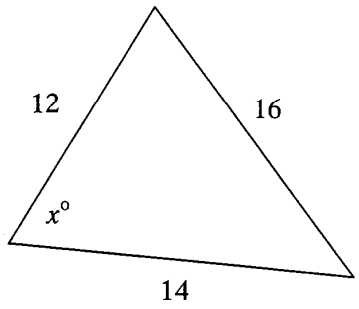
(b)



(c)



(d)

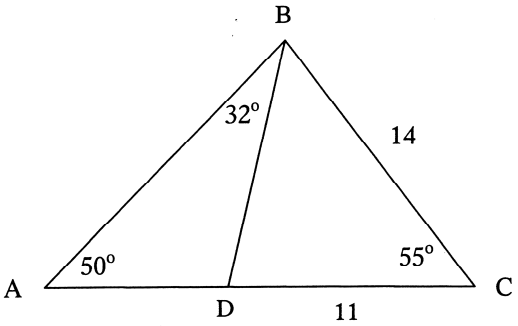


2. Calculate the area of the triangle with sides measuring 12 cm, 14 cm and 20 cm.

3. **(a)** Calculate the length of BD.

(b) Calculate the length of AD.

(c) Calculate the area of triangle ABC



4. From the framework opposite:

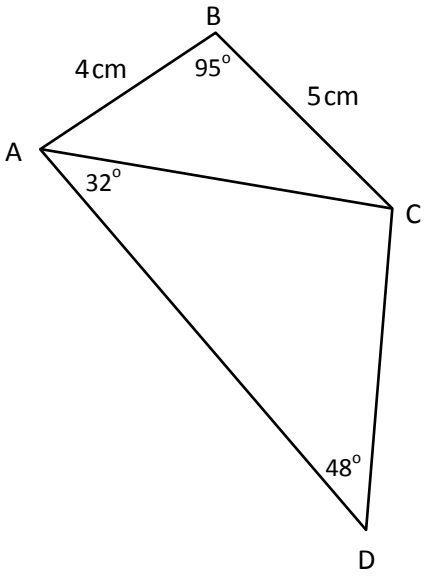
(a) Calculate the length of AC.

(b) Calculate the size of $\angle BAC$.

(c) Write down the size of $\angle ACD$.

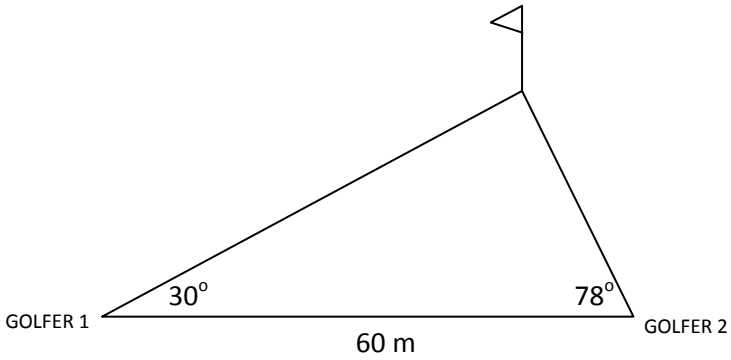
(d) Calculate the length of AD.

(e) Calculate the area of the quadrilateral ABCD.



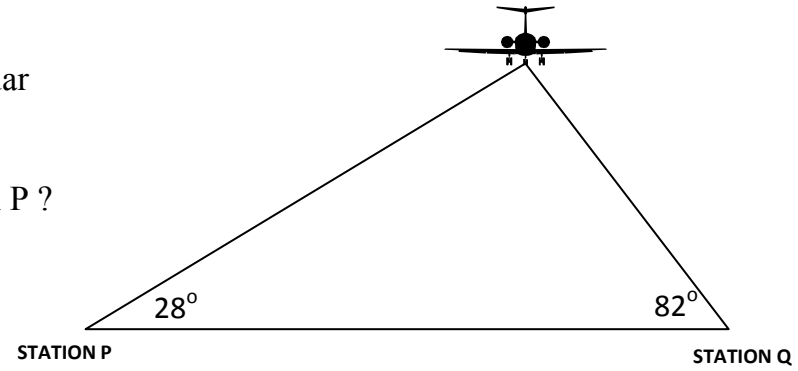
5. Two golfers are aiming for the green. The golfers are 60 m apart and the angles are as shown in the diagram.

What distance will each golfer have to hit the ball in order to reach the pin?

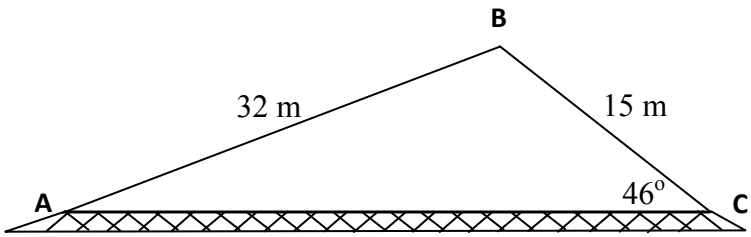


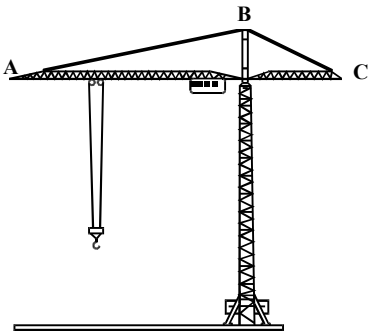
6. An aircraft is picked up by two radar stations, P and Q, 120 km apart.

How far is the aircraft from station P ?



7. A large crane is being used in the construction of a block of flats. The crossbeam is supported by two metal stays.

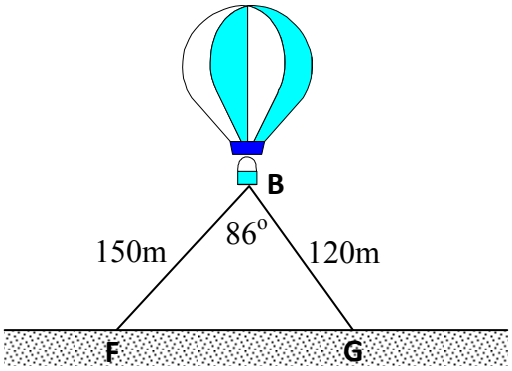




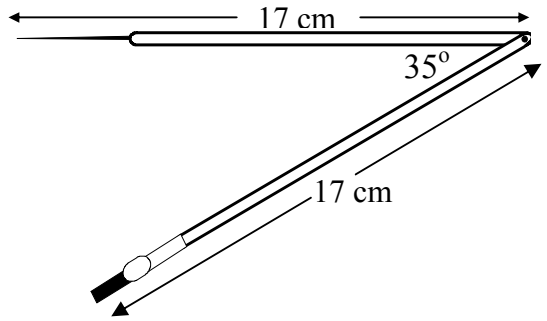
The length of AB is 32 m and the length of BC is 15 m. ∠BCA is 46°.
Calculate the size of ∠BAC and the length of the crossbeam AC.

8. A hot air balloon B is fixed to the ground at F and G by 2 ropes 120m and 150 m long.

If ∠FBG is 86°, how far apart are F and G?



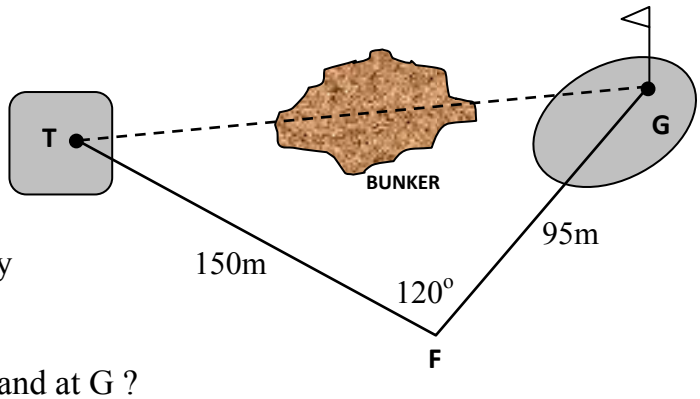
9.



A set of compasses is shown where the angle between the arms is set at 35°

Calculate the diameter of the circle which could be drawn with the arms in this position.

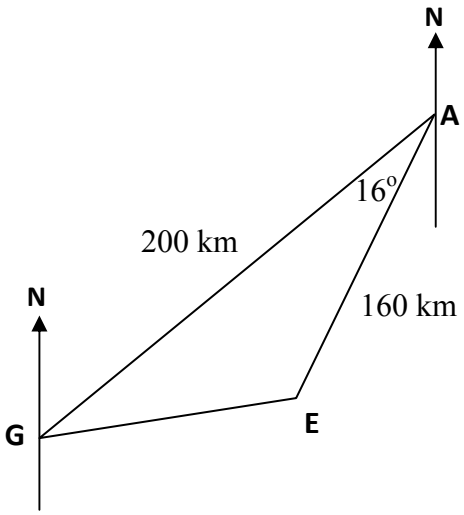
10. During a golf match, Ian discovers that he has forgotten his sand wedge, so to avoid the bunker he plays a shot from T to F and then from F to G.



His opponent Fred decides to play directly from T to G.

How far will Fred need to hit his shot to land at G ?

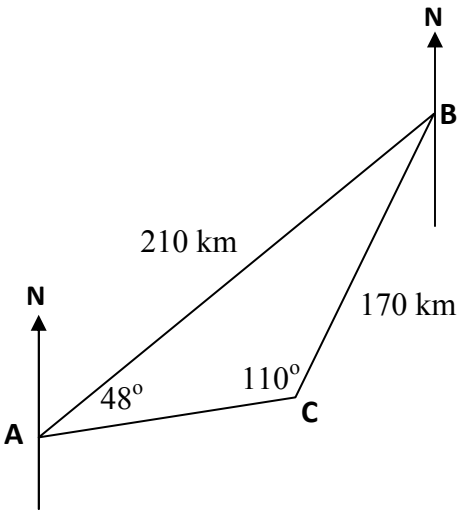
11.



The diagram shows the path of an aircraft from Glasgow to Aberdeen, a distance of 200 km and then from Aberdeen to Edinburgh, a distance of 160 km.

Calculate the distance from Glasgow to Edinburgh.

12.



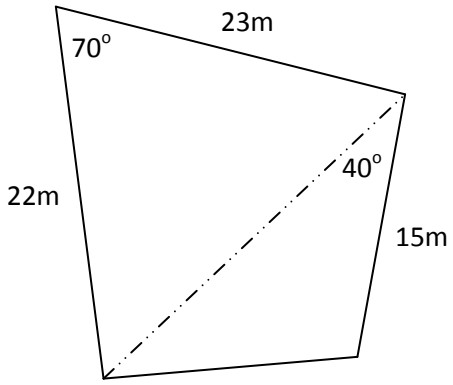
The diagram shows the path of an aircraft from A to B to C.

(a) Write down the size of ∠ABC.

(b) Calculate the distance AC.

EXAM QUESTIONS using TRIGOMOMETRY RULES

1. The sketch below shows a plot of land purchased to build a house on.

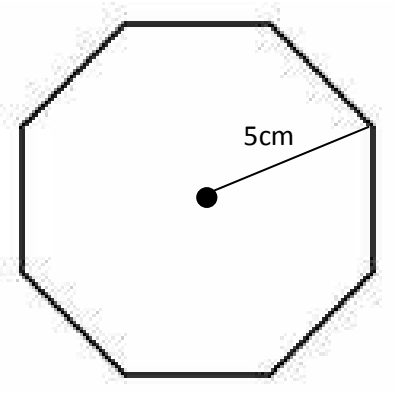


At present the land is valued £280 per square metre.

Calculate the value of the plot shown to the nearest £10.

2. The distance from the centre of a regular octagon to one of its vertexes is 5 cm.

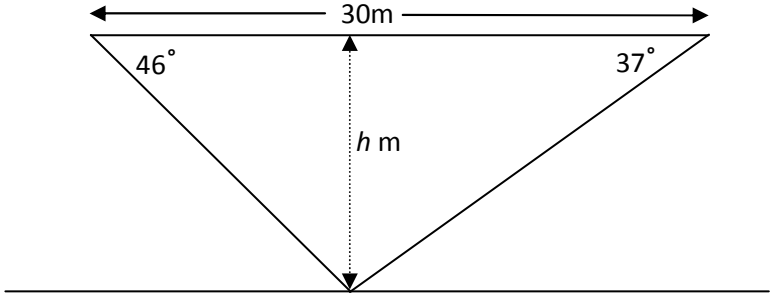
Calculate the area of the octagon.



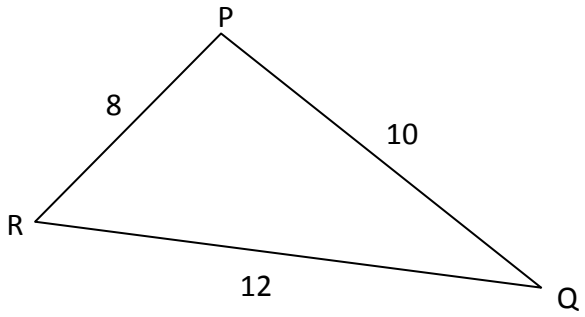
3. Two security cameras are positioned on a beam in a warehouse 30 metres apart.

One camera has an angle of depression of 37˚ and the other camera has an angle of depression of 46˚.

Calculate the height, *h* metres, of the beam above the ground.



4. Triangle PQR has sides with lengths, in centimetres, as shown.

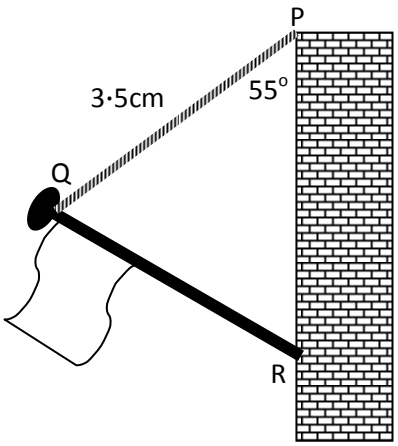


Show clearly that cos PQR = 0·75.

5. A flagpole is attached to a wall and is supported by a wire PQ as shown in the diagram.

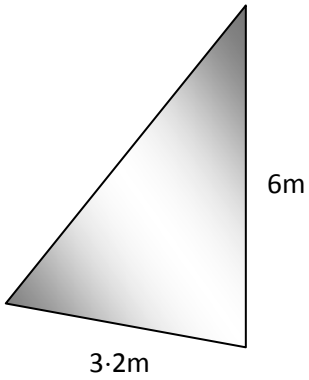
The wire is 3·5 metres long and makes an angle of 55° with the vertical wall.

Given that the point P is 4·5metres above R in the diagram, calculate the length of the flagpole.



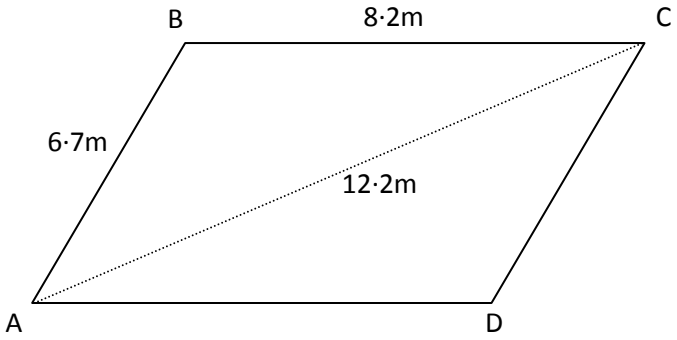
6. A triangular sail designed for a racing yacht is shown below.

Two of its edges measure 6 metres and 3·2 metres.



Given that the sail has a **perimeter** of 15·5 metres, calculate the **area** of the sail.

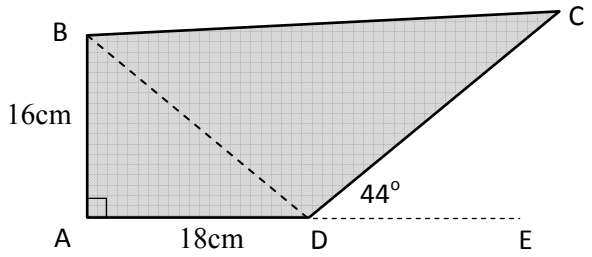
7. A sketch of Lee's garden is shown below.



(a) Calculate the size of angle ABC.

(b) Hence, or otherwise, calculate the area of the garden.

8. The diagram below shows a steel plate ABCD.



$AB = 16\,\text{cm},\ AD = 18\,\text{cm}$

$\angle DAB = 90^\circ$

(a) Calculate the length of BD correct to 1 decimal place.

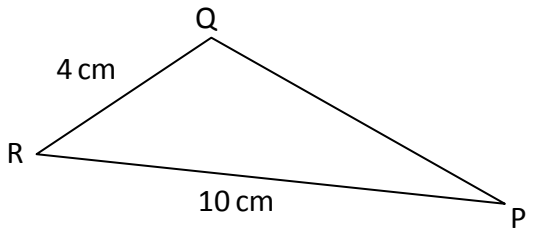
(b) Find the size of angle BDC correct to the nearest degree.

(c) Hence calculate the length of BC given that $DC = 25\,\text{cm}$.

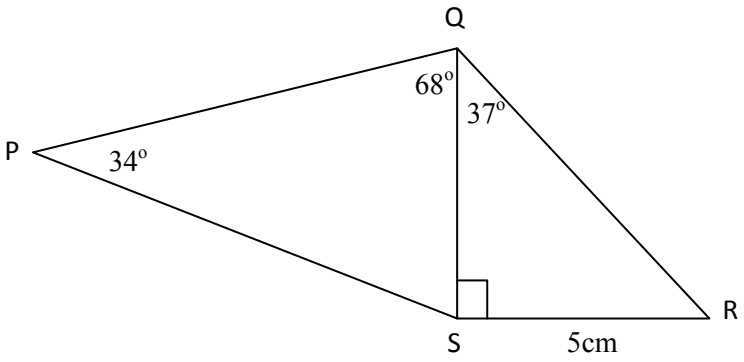
9. In triangle PQR, PR = 10 cm QR = 4 cm.

The perimeter of the triangle is 22 cm.

Find the size of angle PQR.

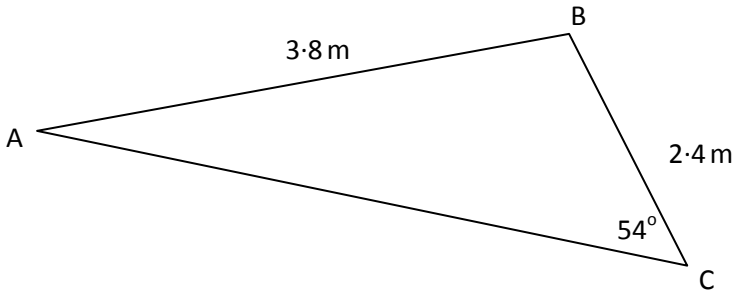


10. In the diagram shown SR = 5cm, angle SQR = 37^o, angle QPS = 34^o and angle PQS = 68^o.



Calculate the length of PS.

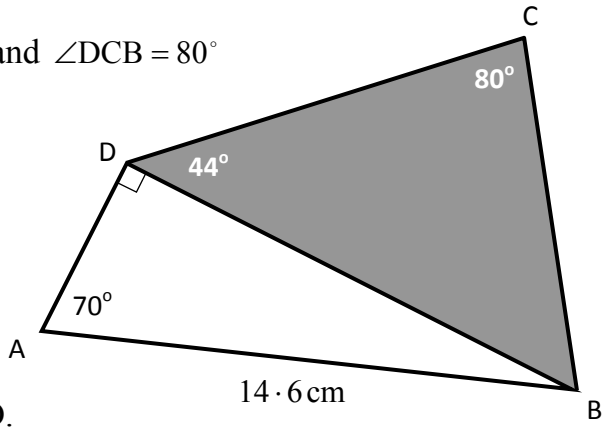
11. Calculate the size of angle BAC in this triangle.



12. In the diagram ABCD represents a steel framework with BCD being a triangular steel plate.

Angle ADB is a right angle.

$AB = 14 \cdot 6\,\text{cm},\ \angle BAC = 70^\circ,\ \angle BDC = 44^\circ$ and $\angle DCB = 80^\circ$

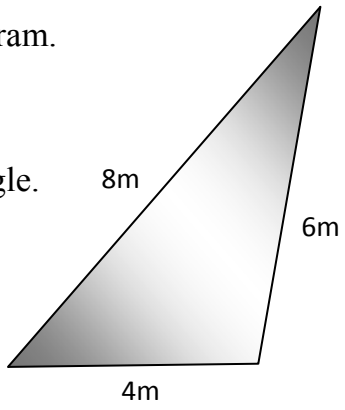


(a) Find the length of DB.

(b) Calculate the area of triangle BCD.

13. A triangular sail has measurements as shown in the diagram.

All lengths are in metres.

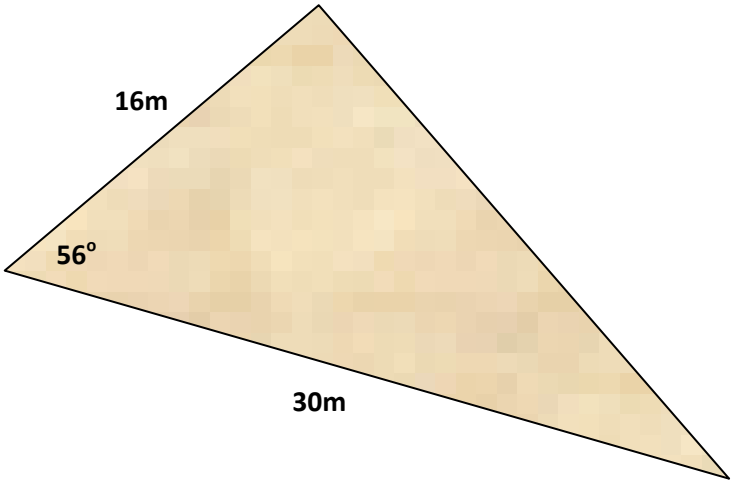


(a) Calculate the size of the largest angle in the triangle.

(b) Calculate the area of the sail in square metres.

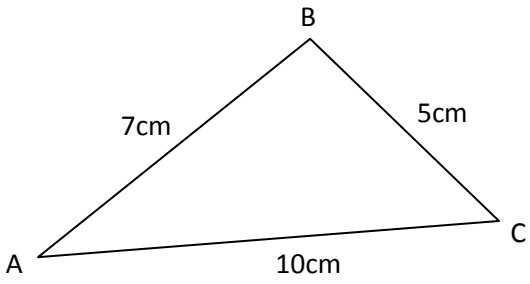
14. A building company has to fence off a triangular piece of waste ground.

The plan of the ground is shown below. All lengths are in metres.



If the fence costs £18.50 per metre to erect, how much will the company have to pay in total to fence off this piece of ground? {Fencing is priced in whole metres only}

15. **(a)** Calculate the value of cos ABC in this triangle.

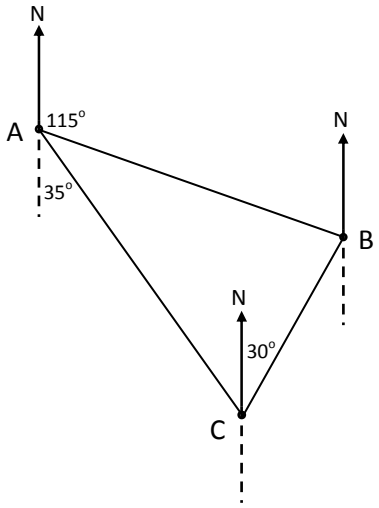


(b) Without actually calculating the size of the angle a pupil was able to say that angle ABC was obtuse.

By referring to your answer in (a), explain why the pupil was able to do this.

BEARINGS with TRIGONOMETRY RULES

1. **(a)** Copy the bearing diagram opposite fill in as many angles as you can.



(b) Now answer the following questions …..

What is the bearing of … **(i)** B from A

(ii) A from B

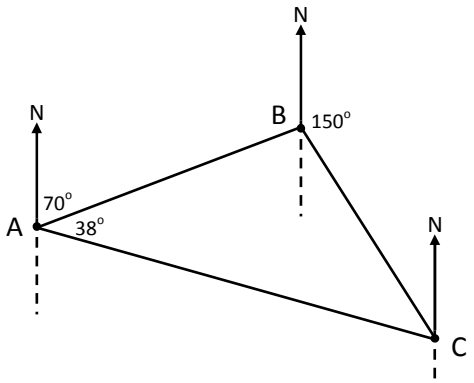
(iii) C from B

(iv) A from C

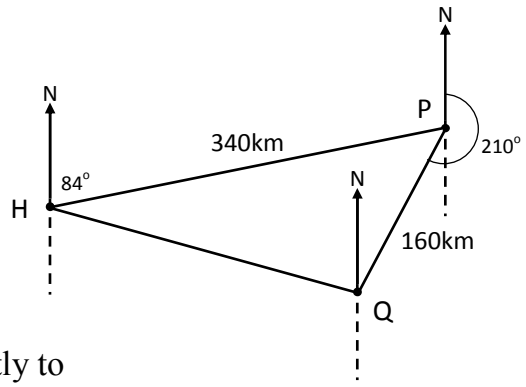
(v) C from A

(vi) B from C

2. Repeat question 1. for this bearing diagram.



3. A ship sails from harbour H on a bearing of 084° for 340km until it reaches point P. It then sails on a bearing of 210° for 160km until it reaches point Q.

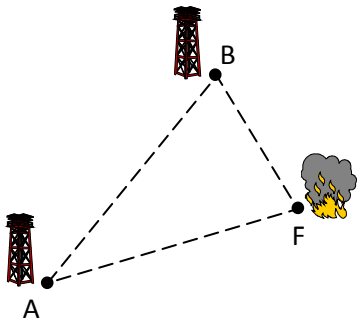


(a) Calculate the distance between point Q and the harbour.

(b) On what bearing must the ship sail to return directly to the harbour from Q?

4. A and B represent two forest look-out towers.

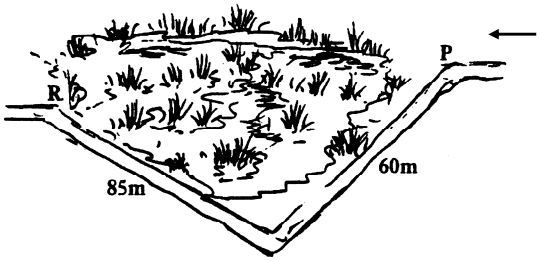
A is 5km and on a bearing of 220° from B.



A forest fire is sighted at F, on a bearing of 070° from A and 150° from B.

A fire-fighting helicopter leaves A for F. What distance does this helicopter have to travel to reach the fire?

5. A surveyor is walking due west when he comes to a marsh. To avoid the marsh he turns at P and walks for 60 metres on a bearing of 215° and then for 85 bearing of 290°.

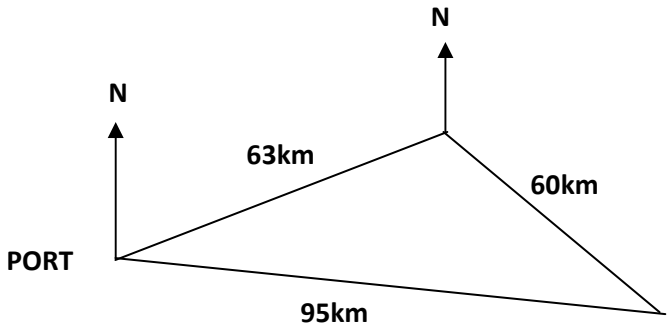


He then calculates the distance PR, the direct distance across the marsh. What answer should he get?

6. Two ships leave Liverpool at the same time. One of them travels north-west at an average speed of 10·5 km/h while the other travels at an average speed of 14 km/h on a bearing of 280°.

How far apart are these ships after 2 hours?

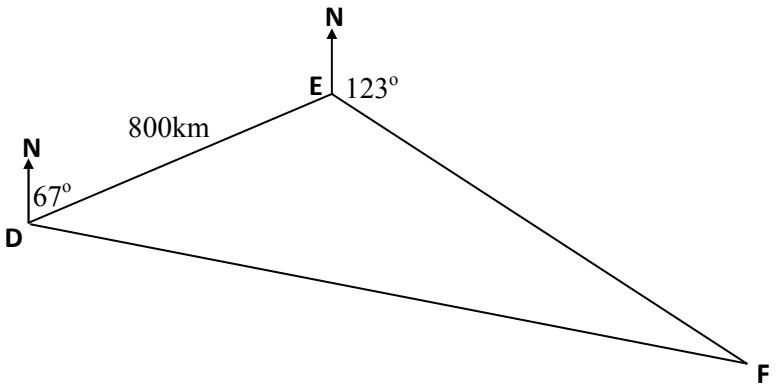
7. A ship leaves a port on a bearing of 073° and sails 63km. The ship then changes course and sails a further 60km on a bearing of 110° where it anchors. When it anchors it is 95km from the port. Calculate the bearing of the ship from the port at this point.



8. A ship's captain is plotting a course for the next voyage.

He knows that he has to sail from Port D to port E on a bearing of 067° for a distance of 800km and from there to Port F on a bearing of 123°.

His course is shown in the diagram below.

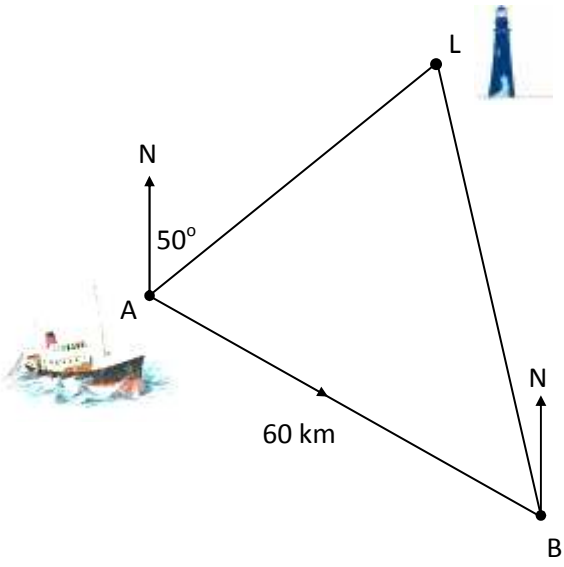


(a) Make a copy of the diagram and calculate the size of angle DEF.

(b) New instructions come through which inform the captain that he has to sail directly from Port D to Port F, a distance of 1750km.

Calculate the bearing on which the ship should sail in order to carry out these instructions. Give the bearing to the nearest degree.

9. A ship is at position A. Lighthouse L is on a bearing of 050° from the ship.



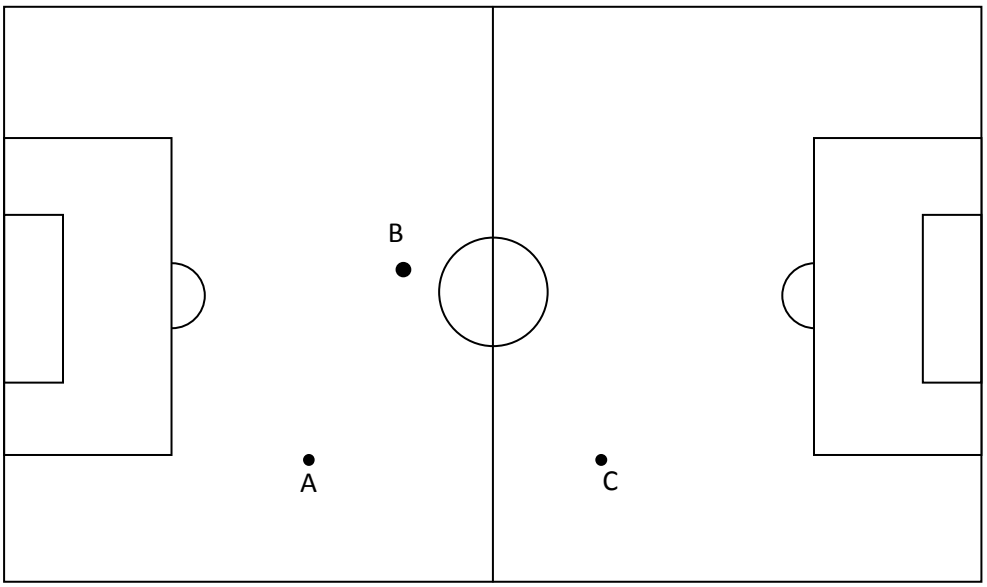
The ship then travels 60 kilometres on a bearing of 130° to position B.

From position B the captain now observes the lighthouse on a bearing of 340°.

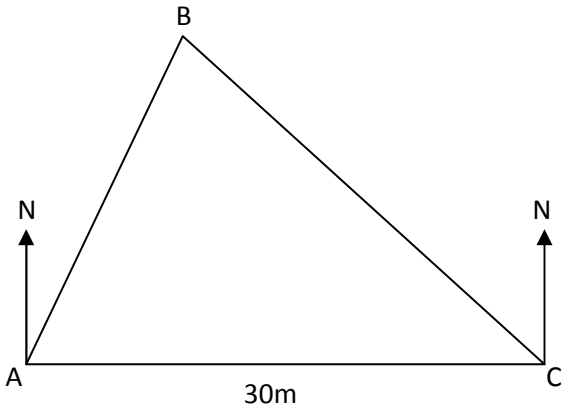
Calculate the distance between the ship and the lighthouse when the ship is at position B.

10. Two students, Ally and Cameron are playing football and at one point they are in the positions shown in the diagram. Ally (A) is 30m due west of Cameron (C).

They are both facing North.



The ball (B) is on a bearing of 026° from A and on a bearing of 312° from C.



(a) Make a copy of the above diagram and mark the sizes of the 3 angles in the triangle.

(b) Calculate how far Cameron is away from the ball.

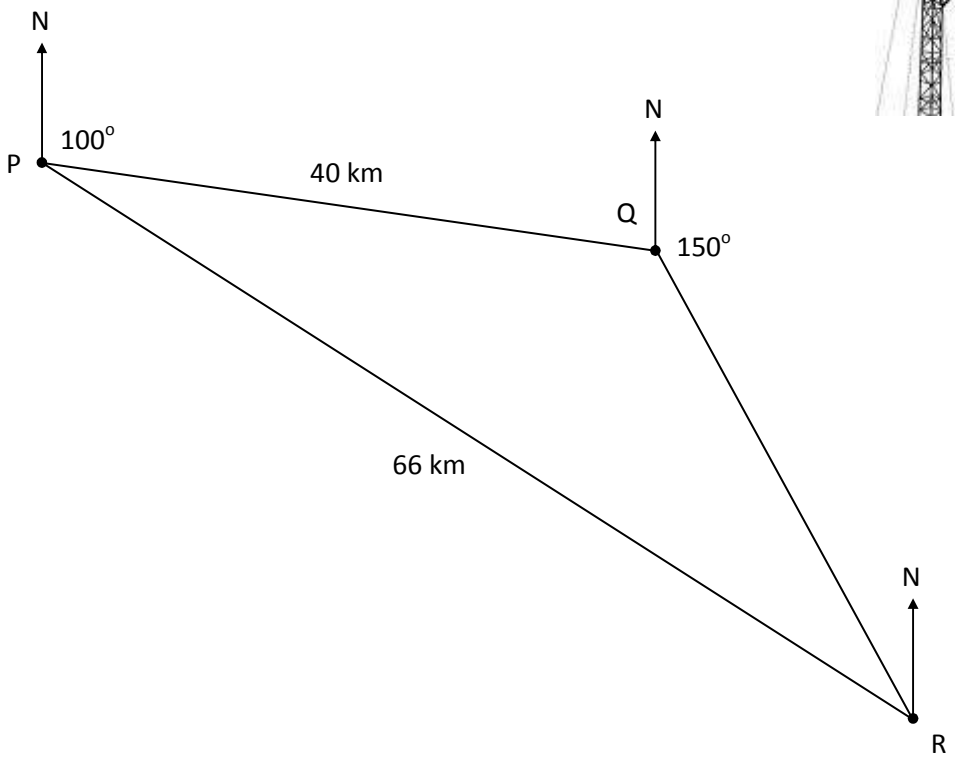
EXAM QUESTIONS involving BEARINGS and TRIGONOMETRY RULES

1. The diagram below, which is not drawn to scale, represents the positions of three mobile phone masts.

Mast Q is on a bearing of 100^o from mast P and is 40km away.

The bearing of mast R from mast Q is 150^o.

Masts P and R are 66km apart.



(a) Use the information in the diagram to establish the size of angle PQR.

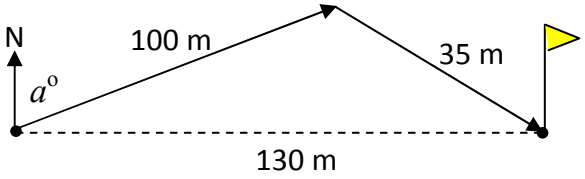
(b) Hence find the bearing of mast P **from** mast R.

2. A par 3 hole on a golf course the tee is a distance of 130 metres due west from the pin.

On his first shot, Bruce hits the ball 100 metres but not at the correct angle.

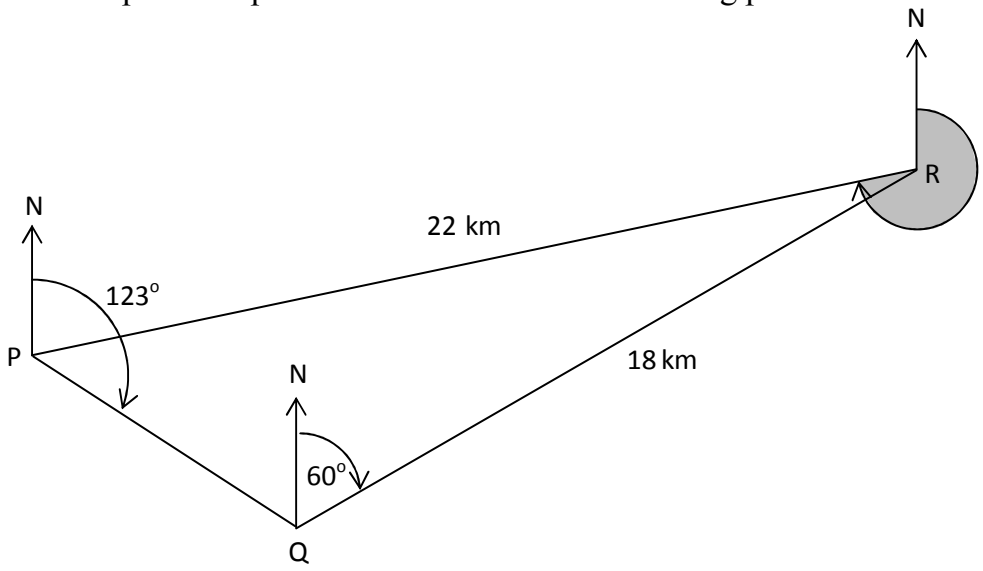
On his second shot he hits the ball 35 metres and gets it in the hole.

On what bearing, a^o, did he hit his first stroke?



3. A helicopter sets out from its base P and flies on a bearing of 123° to point Q where it changes course to 060° and flies 18 km to point R.

When the helicopter is at point R it is 22 km from its starting point.



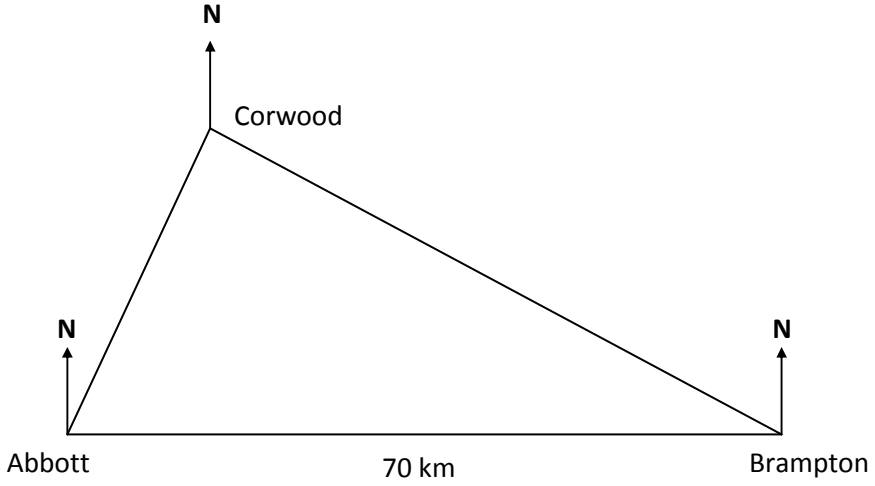
(a) Find the size of angle PQR.

(b) Calculate the bearing on which the helicopter must fly to return directly to its base i.e. the shaded angle in the diagram.

Give answers to the nearest whole number throughout your calculations.

4. Brampton is 70 kilometres due east of Abbott.

The bearing of Corwood from Abbott is 015° and from Brampton is 290°.



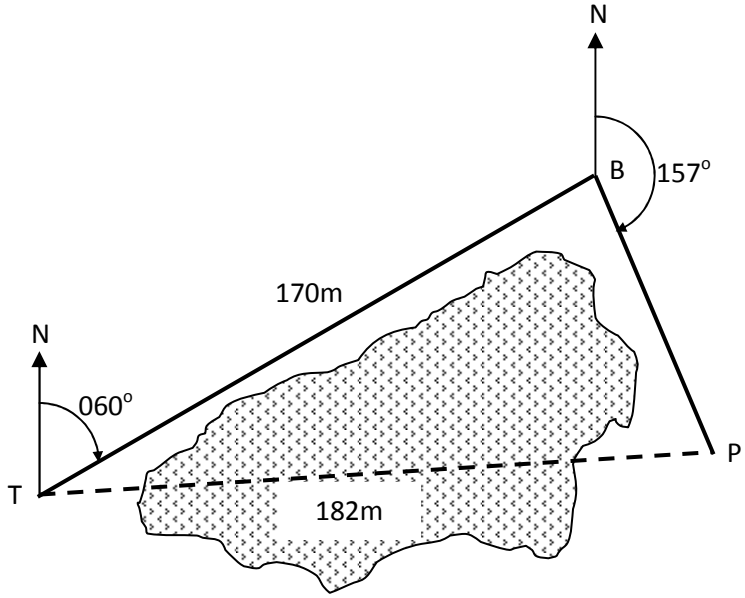
(a) Make a neat copy of the diagram and fill in all three angles inside the triangle.

(b) Calculate the distance between Corwood and Brampton, to the nearest kilometre.

5. The diagram shows part of a golf course where players have to get the ball from the tee (T) to the pin(P).

They can either play one stroke across the lake or play 1 stroke from T to B then another from B to P which avoids the lake.

Harry decides to take the 2 stroke option and hits his first shot on a bearing of 060° or a distance of 170metres. For his second shot he hits the ball on a bearing of 157° from B to P.

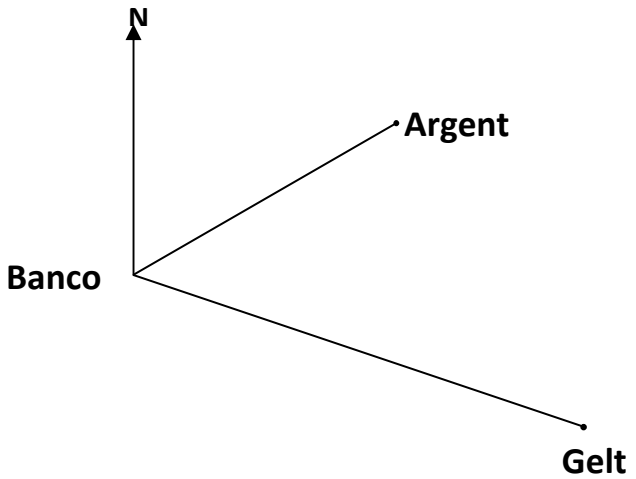


(a) Calculate the size of angle TBP.

The distance TP is 182 metres. David decided to attempt to hit his ball across the lake.

(b) Calculate the bearing on which he would have to hit the ball to achieve this.

6.



Two ships, the Argent and the Gelt leave port Banco at the same time.

The Argent follows a course of 045° for 20 km and the Gelt travels on a course of 108° for 30 km.

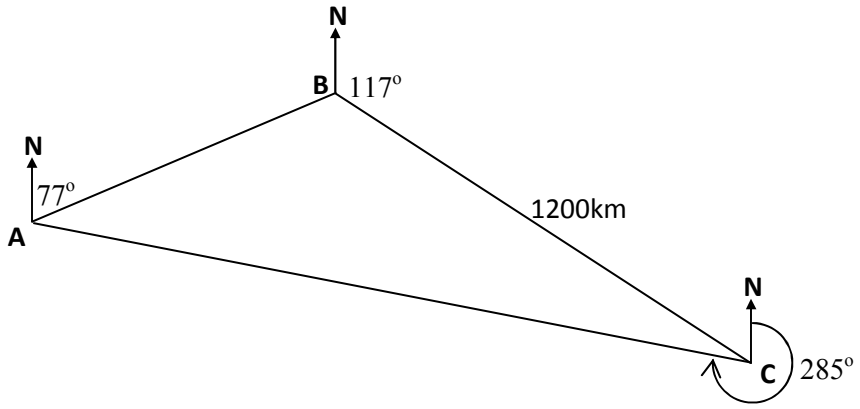
Calculate the distance between the two ships.

7. A ship's mate is planning the course for a voyage.

The course is shown in the diagram below.

He knows that he has to sail from Port A to Port B on a bearing of 077° and from there to Port C on a bearing of 117° for 1200 km.

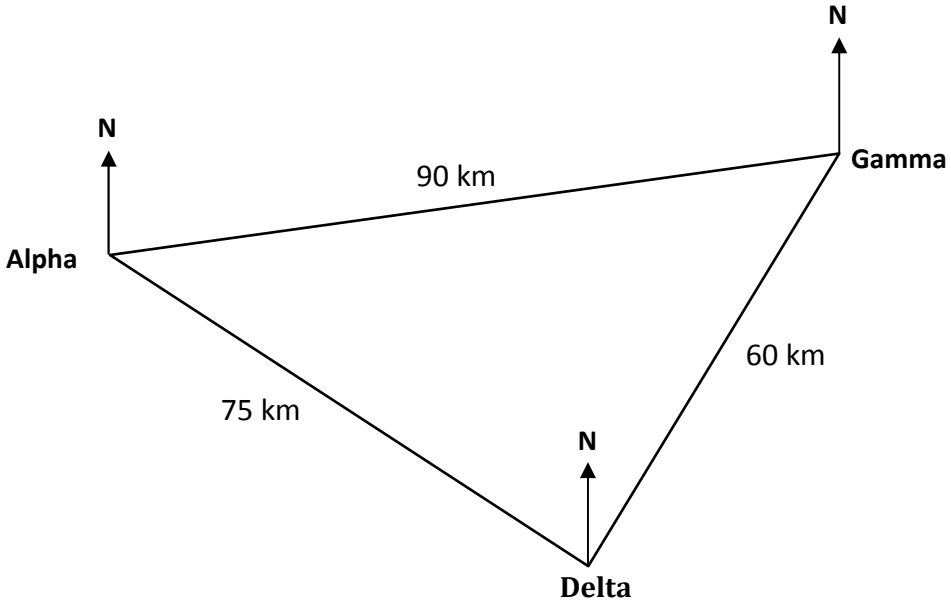
In order to return to port A the ship has to sail on a bearing of 285°.



Calculate how far the ship will have to sail to return to its starting point. i.e. the distance AC in the diagram.

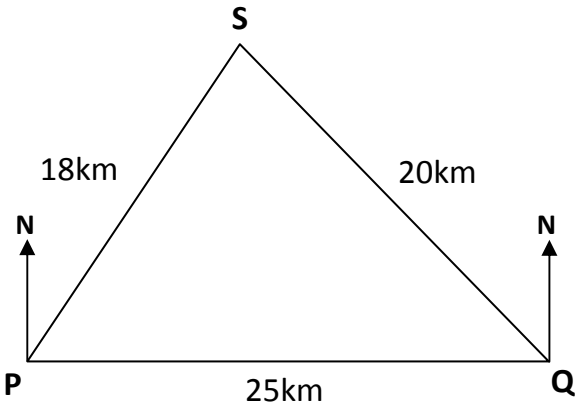
8. Three oil platforms, Alpha, Gamma and Delta are situated in the North Sea as shown in the diagram below.

The distances between the oil platforms are shown in the diagram.



If the bearing of Delta from Alpha is 125°, what is the bearing of Gamma from Alpha?

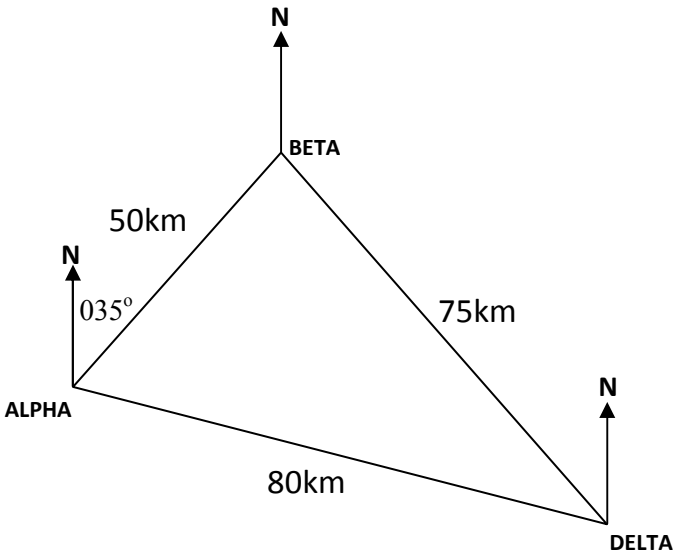
9. Two coastguard stations, P and Q, are 25 km apart. Q is due East of P. A ship, S, is at a distance of 18 km from P and 20 km from Q.



(a) Calculate the size of angle SPQ.

(b) Hence calculate the bearing of the ship S from station P.

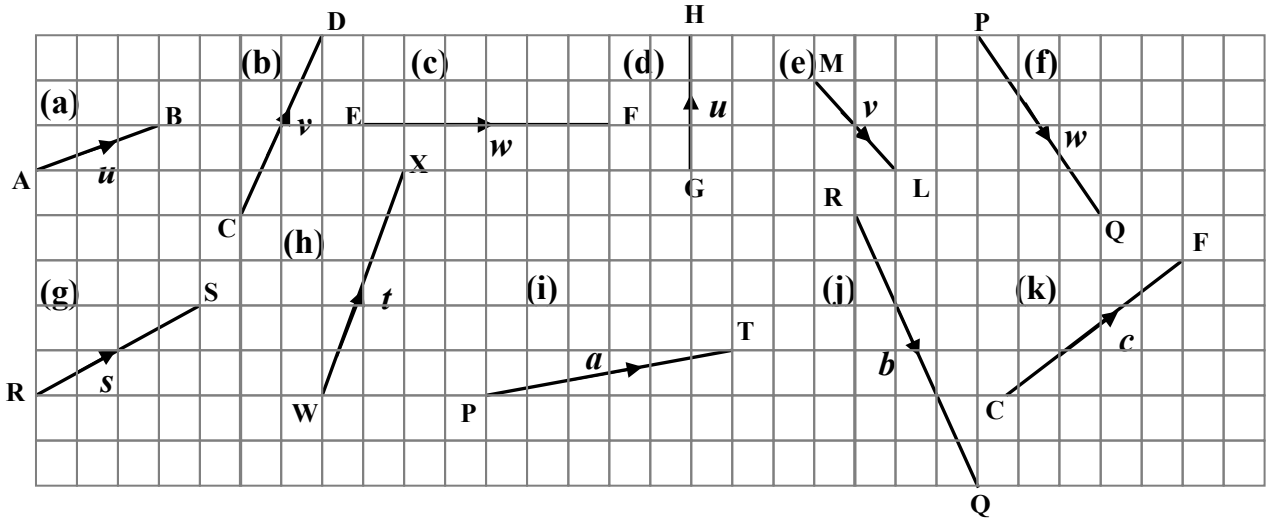
10. The diagram below shows the positions of three radar stations Alpha, Beta and Delta. The bearing of Beta from Alpha is 035^{o}.



Calculate the bearing of Delta from Alpha.

2.2 WORKING with 2D VECTORS

1. Name the following vectors in 2 ways and write down the components:



2. Draw representations of the following vectors on squared paper.

(a) $\boldsymbol{v} = \begin{pmatrix} 5 \\ 12 \end{pmatrix}$ **(b)** $\boldsymbol{w} = \begin{pmatrix} 3 \\ -6 \end{pmatrix}$ **(c)** $\boldsymbol{u} = \begin{pmatrix} -3 \\ 6 \end{pmatrix}$ **(d)** $\overrightarrow{AB} = \begin{pmatrix} 4 \\ -4 \end{pmatrix}$

(e) $\overrightarrow{CD} = \begin{pmatrix} -4 \\ -2 \end{pmatrix}$ **(f)** $\overrightarrow{EF} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}$ **(g)** $\boldsymbol{r} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$ **(h)** $\boldsymbol{p} = \begin{pmatrix} 0 \\ -3 \end{pmatrix}$

(i) $\boldsymbol{q} = \begin{pmatrix} -5 \\ -6 \end{pmatrix}$ **(j)** $\overrightarrow{XY} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$ **(k)** $\overrightarrow{PQ} = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$ **(l)** $\overrightarrow{ST} = \begin{pmatrix} -4 \\ 0 \end{pmatrix}$

3. Calculate the magnitude of each of the vectors in questions 1 and 2 above leaving your answers as surds in their simplest form.

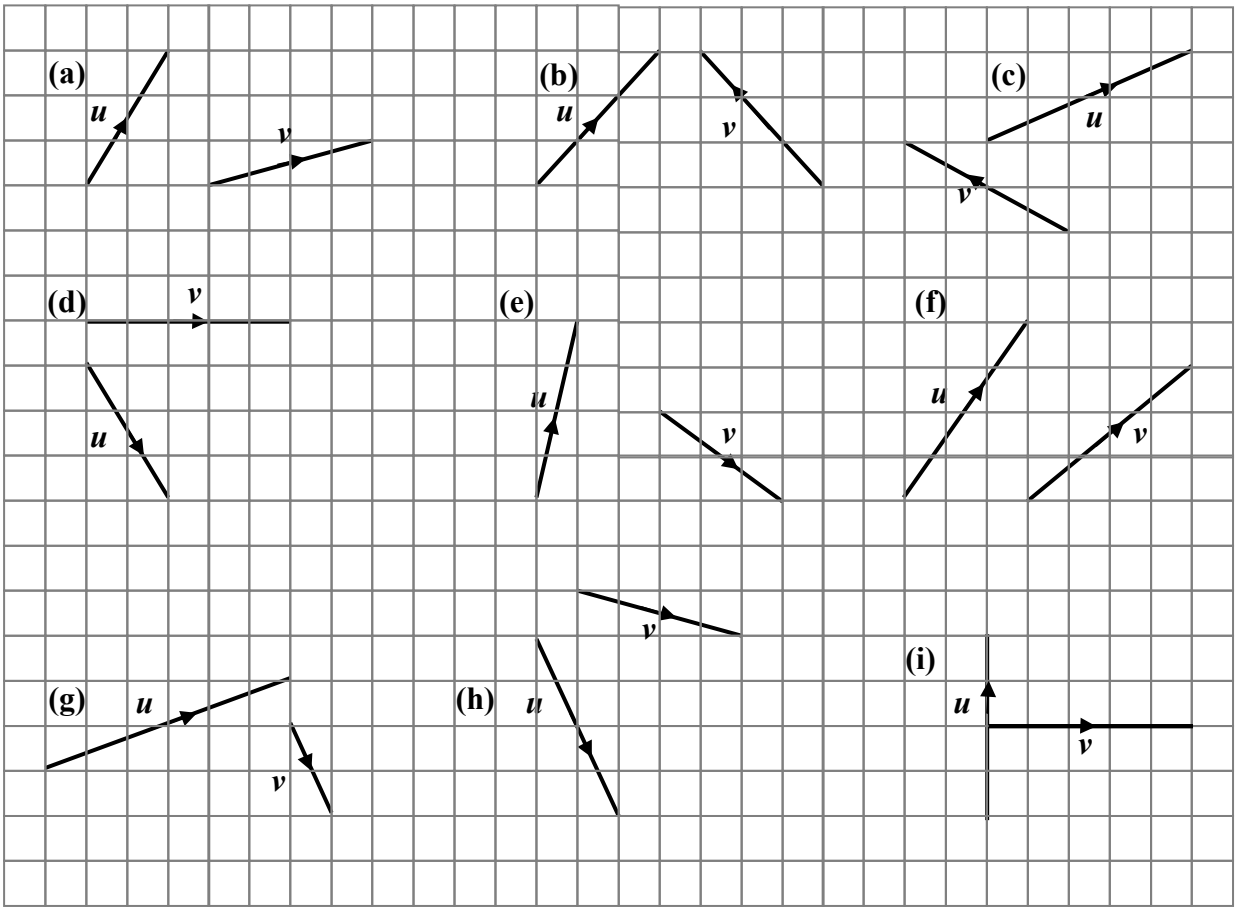
4. Find **(a)** $\left|\begin{pmatrix} 3 \\ 4 \end{pmatrix}\right|$ **(b)** $\left|\begin{pmatrix} 7 \\ 24 \end{pmatrix}\right|$ **(c)** $\left|\begin{pmatrix} 12 \\ 5 \end{pmatrix}\right|$

(d) $\left|\begin{pmatrix} -6 \\ -8 \end{pmatrix}\right|$ **(e)** $\left|\begin{pmatrix} -3 \\ 4 \end{pmatrix}\right|$ **(f)** $\left|\begin{pmatrix} 12 \\ -5 \end{pmatrix}\right|$

2.2 ADDITION of VECTORS using DIRECTED LINE SEGMENTS

1. **(i)** Draw diagrams on squared paper to illustrate $\boldsymbol{u} + \boldsymbol{v}$ for each pair of vectors given.

(ii) State the components of the resultant vector and calculate its magnitude leaving your answers as a surd in its simplest form



2. **(i)** Draw diagrams on squared to illustrate $\boldsymbol{a} + \boldsymbol{b}$ for each the following pairs of vectors.

(ii) State the components of the resultant vector and calculate its magnitude.

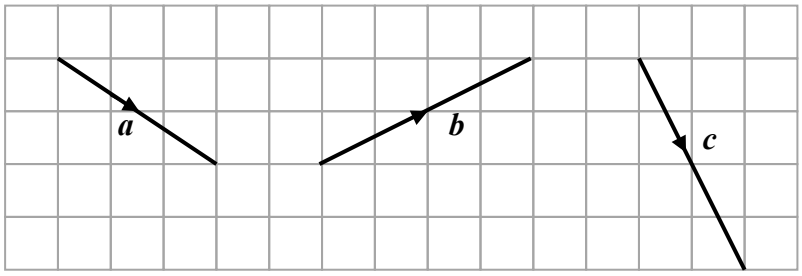
(a) $\boldsymbol{a} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$; $\boldsymbol{b} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$ **(b)** $\boldsymbol{a} = \begin{pmatrix} 4 \\ 7 \end{pmatrix}$; $\boldsymbol{b} = \begin{pmatrix} -9 \\ 3 \end{pmatrix}$

(c) $\boldsymbol{a} = \begin{pmatrix} -4 \\ -2 \end{pmatrix}$; $\boldsymbol{b} = \begin{pmatrix} 6 \\ -5 \end{pmatrix}$ **(d)** $\boldsymbol{a} = \begin{pmatrix} 0 \\ -5 \end{pmatrix}$; $\boldsymbol{b} = \begin{pmatrix} -3 \\ 3 \end{pmatrix}$

(e) $\boldsymbol{a} = \begin{pmatrix} -6 \\ -4 \end{pmatrix}$; $\boldsymbol{b} = \begin{pmatrix} -5 \\ 6 \end{pmatrix}$ **(f)** $\boldsymbol{a} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$; $\boldsymbol{b} = \begin{pmatrix} 0 \\ -3 \end{pmatrix}$

(g) $\boldsymbol{a} = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$; $\boldsymbol{b} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$ **(h)** $\boldsymbol{a} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$; $\boldsymbol{b} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$

3. The diagram shows 3 vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$.



(i) Draw diagrams on squared paper to represent:

(a) $\boldsymbol{a} + \boldsymbol{b}$ **(b)** $\boldsymbol{a} + \boldsymbol{c}$ **(c)** $\boldsymbol{b} + \boldsymbol{c}$ **(d)** $(\boldsymbol{a} + \boldsymbol{b}) + \boldsymbol{c}$

(e) $\boldsymbol{a} + (\boldsymbol{b} + \boldsymbol{c})$

(ii) For each resultant vector, state the components and calculate its magnitude correct to one decimal place.

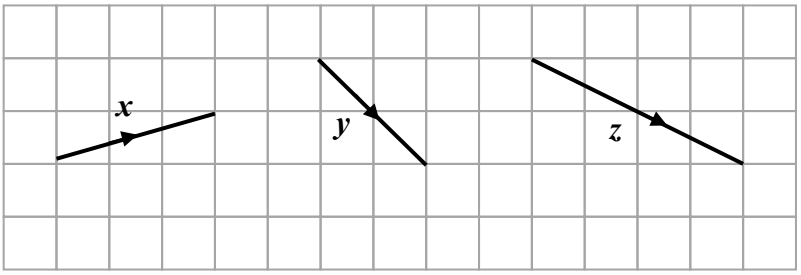
4. **(i)** For the vectors in question 3 draw representations of these vectors.

(a) $2\boldsymbol{a}$ **(b)** $3\boldsymbol{b}$ **(c)** $0{\cdot}5\boldsymbol{c}$ **(d)** $-2\boldsymbol{b}$

(e) $-4\boldsymbol{a}$ **(f)** $-\boldsymbol{c}$ **(g)** $3\boldsymbol{a} + 2\boldsymbol{b}$ **(h)** $\boldsymbol{c} + 4\boldsymbol{a}$

(ii) State the components of each of the vectors above and calculate the magnitude leaving answers as a surd in its simplest form.

5. The diagram shows 3 vectors $\boldsymbol{x}$, $\boldsymbol{y}$ and $\boldsymbol{z}$.



(i) Draw diagrams to represent:

(a) $\boldsymbol{x} + \boldsymbol{y}$ **(b)** $\boldsymbol{x} + \boldsymbol{z}$ **(c)** $\boldsymbol{y} + \boldsymbol{z}$ **(d)** $(\boldsymbol{x} + \boldsymbol{y}) + \boldsymbol{z}$

(e) $\boldsymbol{x} + (\boldsymbol{y} + \boldsymbol{z})$

(ii) Calculate, correct to one decimal place:

(a) $|\boldsymbol{x} + \boldsymbol{y}|$ **(b)** $|\boldsymbol{x} + \boldsymbol{z}|$ **(c)** $|\boldsymbol{y} + \boldsymbol{z}|$ **(d)** $|(\boldsymbol{x} + \boldsymbol{y}) + \boldsymbol{z}|$

(e) $|\boldsymbol{x} + (\boldsymbol{y} + \boldsymbol{z})|$

6. For the vectors in question 5, calculate:

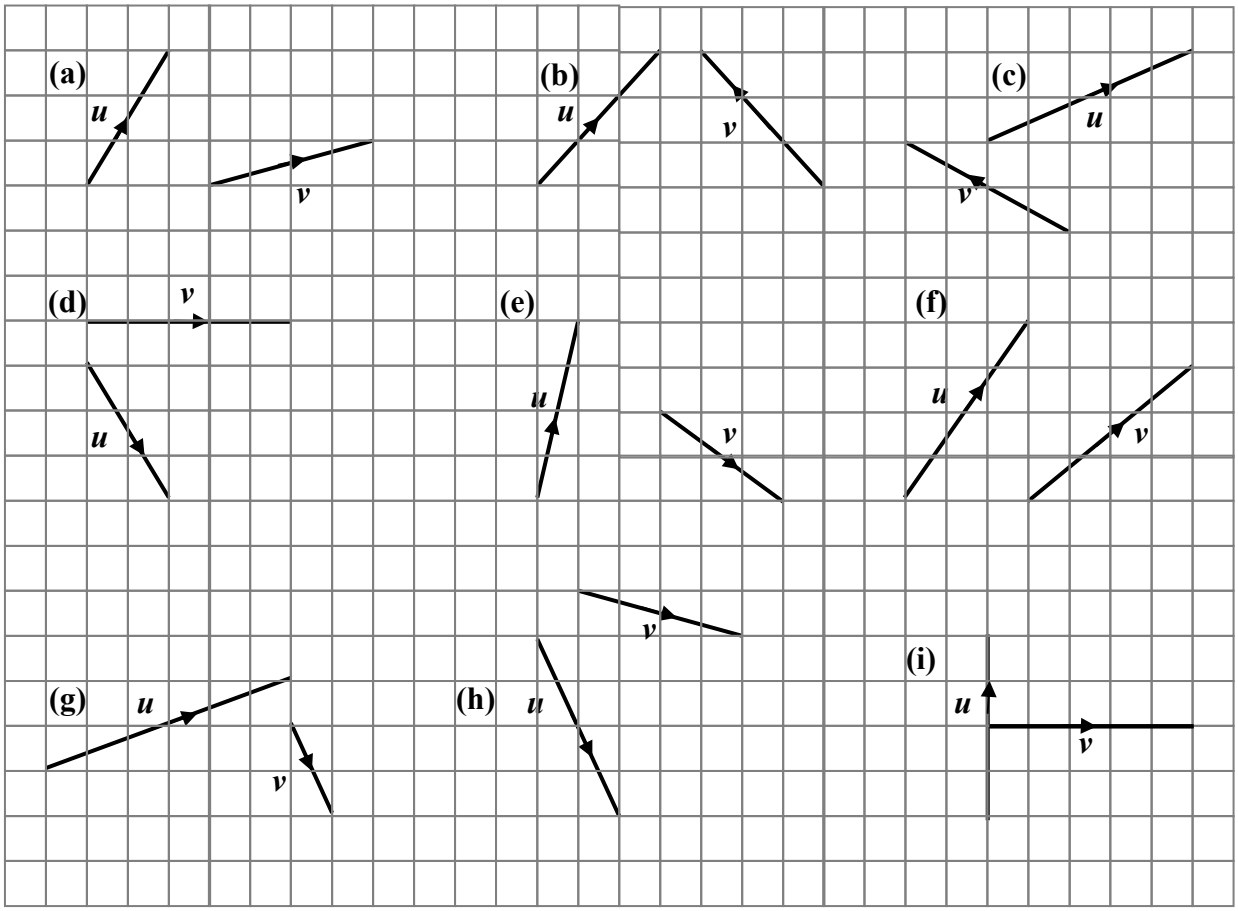
(a) $|2\boldsymbol{x}|$ **(b)** $|3\boldsymbol{y}|$ **(c)** $|0{\cdot}5\boldsymbol{z}|$ **(d)** $|-2\boldsymbol{y}|$

(e) $|-4\boldsymbol{x}|$ **(f)** $|-\boldsymbol{z}|$ **(g)** $|3\boldsymbol{x}+2\boldsymbol{y}|$ **(h)** $|4\boldsymbol{y}+3\boldsymbol{x}|$

2.2 SUBTRACTION of VECTORS using DIRECTED LINE SEGMENTS

1. **(i)** Draw diagrams on squared paper to illustrate $\boldsymbol{u} - \boldsymbol{v}$ for each pair of vectors given.

(ii) State the components of the resultant vector and calculate its magnitude leaving your answers as surds in their simplest form.



2. **(i)** Draw diagrams on squared to illustrate $\boldsymbol{a} - \boldsymbol{b}$ for each the following pairs of vectors.

(ii) State the components of the resultant vector and calculate its magnitude correct to one decimal place.

(a) $\boldsymbol{a} = \begin{pmatrix} 9 \\ 7 \end{pmatrix}; \ \boldsymbol{b} = \begin{pmatrix} 8 \\ 4 \end{pmatrix}$ **(b)** $\boldsymbol{a} = \begin{pmatrix} -4 \\ -7 \end{pmatrix}; \ \boldsymbol{b} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$

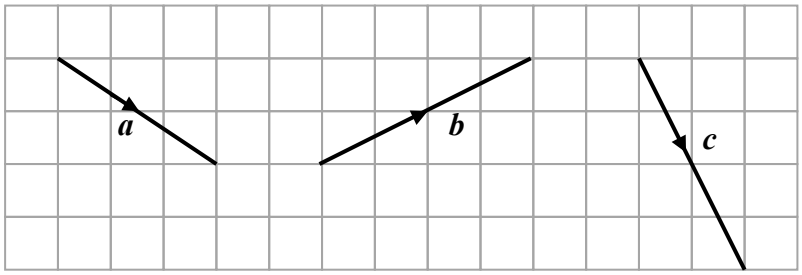
(c) $\boldsymbol{a} = \begin{pmatrix} 5 \\ 8 \end{pmatrix}; \ \boldsymbol{b} = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$ **(d)** $\boldsymbol{a} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}; \ \boldsymbol{b} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

(e) $\boldsymbol{a} = \begin{pmatrix} -2 \\ -4 \end{pmatrix}; \ \boldsymbol{b} = \begin{pmatrix} -2 \\ -6 \end{pmatrix}$ **(f)** $\boldsymbol{a} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}; \ \boldsymbol{b} = \begin{pmatrix} -4 \\ 0 \end{pmatrix}$

(g) $\boldsymbol{a} = \begin{pmatrix} 0 \\ 7 \end{pmatrix}; \ \boldsymbol{b} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$ **(h)** $\boldsymbol{a} = \begin{pmatrix} 0 \\ -6 \end{pmatrix}; \ \boldsymbol{b} = \begin{pmatrix} -4 \\ 5 \end{pmatrix}$

(i) $\boldsymbol{a} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}; \ \boldsymbol{b} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$ **(j)** $\boldsymbol{a} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}; \ \boldsymbol{b} = \begin{pmatrix} -4 \\ 0 \end{pmatrix}$

3. The diagram shows 3 vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$.



(i) Draw diagrams on squared paper to represent:

(a) $\boldsymbol{a} - \boldsymbol{b}$ **(b)** $\boldsymbol{a} - \boldsymbol{c}$ **(c)** $\boldsymbol{b} - \boldsymbol{c}$ **(d)** $(\boldsymbol{a} + \boldsymbol{b}) - \boldsymbol{c}$

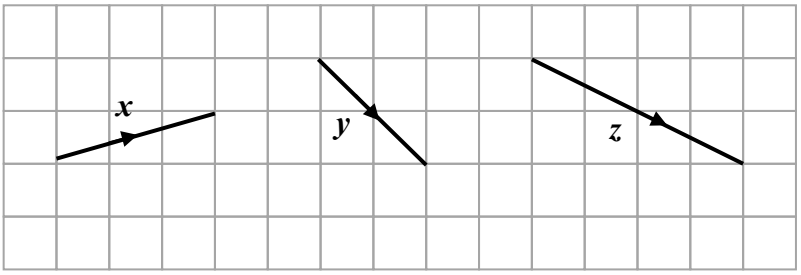
(e) $\boldsymbol{a} - (\boldsymbol{b} - \boldsymbol{c})$

(ii) Calculate, correct to two decimal places:

(a) $|\boldsymbol{a} - \boldsymbol{b}|$ **(b)** $|\boldsymbol{a} - \boldsymbol{c}|$ **(c)** $|\boldsymbol{b} - \boldsymbol{c}|$ **(d)** $|(\boldsymbol{a} + \boldsymbol{b}) - \boldsymbol{c}|$

(e) $|\boldsymbol{a} - (\boldsymbol{b} - \boldsymbol{c})|$

4. The diagram shows 3 vectors $\boldsymbol{x}$, $\boldsymbol{y}$ and $\boldsymbol{z}$.



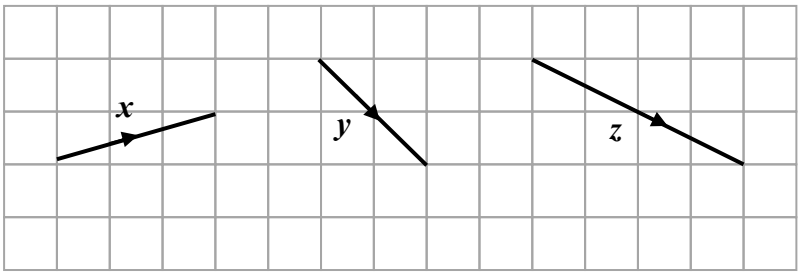
(i) Draw diagrams to represent:

(a) $\boldsymbol{x} - \boldsymbol{y}$ **(b)** $\boldsymbol{x} - \boldsymbol{z}$ **(c)** $\boldsymbol{y} - \boldsymbol{z}$ **(d)** $(\boldsymbol{x} - \boldsymbol{y}) - \boldsymbol{z}$

(e) $\boldsymbol{x} - (\boldsymbol{y} - \boldsymbol{z})$

(ii) For each resultant vector, state the components and calculate its magnitude correct to one decimal place.

5. The diagram shows 3 vectors $\boldsymbol{x}$, $\boldsymbol{y}$ and $\boldsymbol{z}$.



(i) Draw diagrams on squared paper to show:

(a) $2\boldsymbol{x} + \boldsymbol{y}$ **(b)** $3\boldsymbol{z} + 2\boldsymbol{y}$ **(c)** $3\boldsymbol{x} + \boldsymbol{z}$ **(d)** $2\boldsymbol{z} + 4\boldsymbol{x}$

(e) $3\boldsymbol{x} - 4\boldsymbol{y}$ **(f)** $3\boldsymbol{x} - \boldsymbol{z}$ **(g)** $3\boldsymbol{y} - 2\boldsymbol{x}$ **(h)** $-3\boldsymbol{y} - 2\boldsymbol{z}$ (careful!)

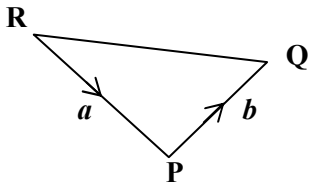
(ii) State the components of each resultant vector above and calculate its magnitude correct to 3 significant figures.

VECTOR JOURNEYS in 2D Part 1

1. Express each of the following displacements in terms of vectors $\boldsymbol{a}$ and $\boldsymbol{b}$.

(a) $\overrightarrow{PQ}$ **(b)** $\overrightarrow{QP}$ **(c)** $\overrightarrow{PR}$

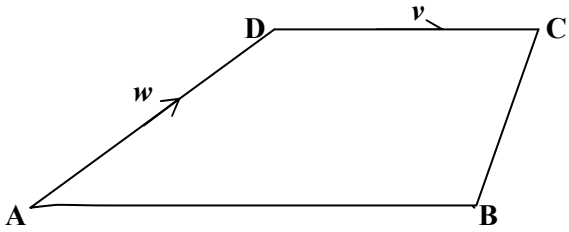
(d) $\overrightarrow{RQ}$ **(e)** $\overrightarrow{QR}$



2. In the diagram $\overrightarrow{AB} = 2\overrightarrow{DC}$. Express each of the following displacements in terms of vectors $\boldsymbol{v}$ and $\boldsymbol{w}$.

(a) $\overrightarrow{CD}$ **(b)** $\overrightarrow{CA}$ **(c)** $\overrightarrow{AB}$

(d) $\overrightarrow{CB}$ **(e)** $\overrightarrow{BD}$

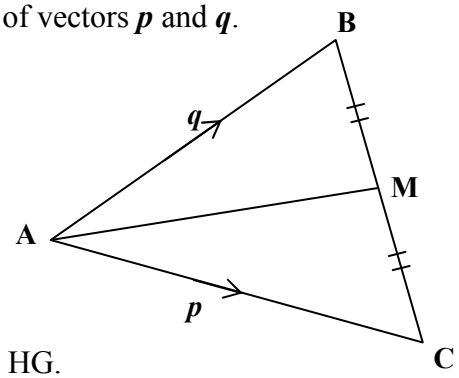


3. In the diagram 'M' is the mid – point of BC.

Express each of the following displacements in terms of vectors $\boldsymbol{p}$ and $\boldsymbol{q}$.

(a) $\overrightarrow{CB}$ **(b)** $\overrightarrow{BC}$ **(c)** $\overrightarrow{BM}$

(d) $\overrightarrow{AM}$

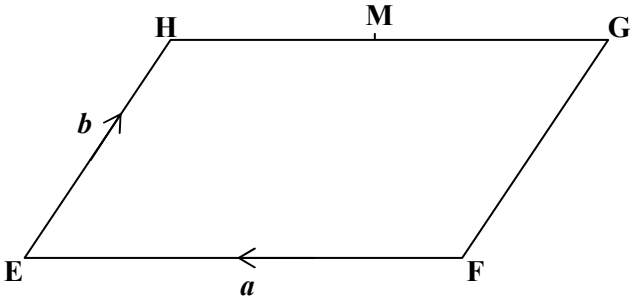


4. EFGH is a parallelogram. 'M' is the mid point of side HG.

Express each of the following displacements in terms of vectors $\boldsymbol{a}$ and $\boldsymbol{b}$.

(a) $\overrightarrow{FG}$ **(b)** $\overrightarrow{GH}$ **(c)** $\overrightarrow{GM}$

(d) $\overrightarrow{FM}$

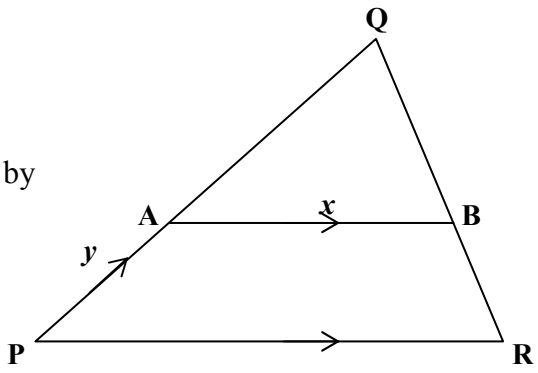


5. In the diagram AB is parallel to PR.

PA = 1 cm and PQ = 3 cm

Find in terms of $\boldsymbol{x}$ and/or $\boldsymbol{y}$ the vectors represented by

(a) $\overrightarrow{AQ}$ **(b)** $\overrightarrow{QB}$



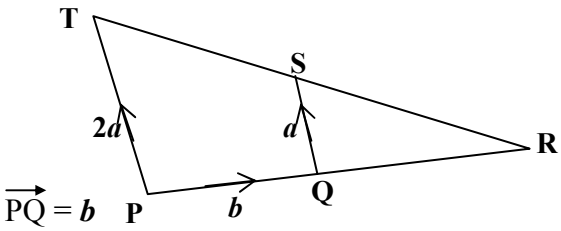
VECTOR JOURNEYS in 2D Part 2

1. **(a)** Express in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

(i) $\overrightarrow{PS}$ **(ii)** $\overrightarrow{ST}$

(b) If $\overrightarrow{QR} = \frac{3}{2}\overrightarrow{PQ}$, show that RS can be expressed as

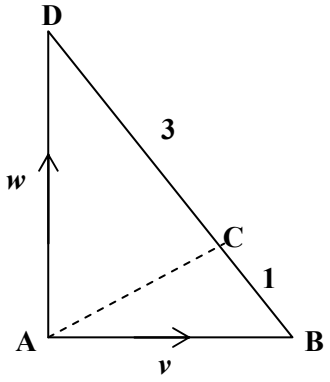
$\frac{1}{2}(2\boldsymbol{a} - 3\boldsymbol{b})$



2. Express in terms of vectors $\boldsymbol{v}$ and $\boldsymbol{w}$.

(a) $\overrightarrow{BD}$ **(b)** $\overrightarrow{BC}$ **(c)** $\overrightarrow{AC}$

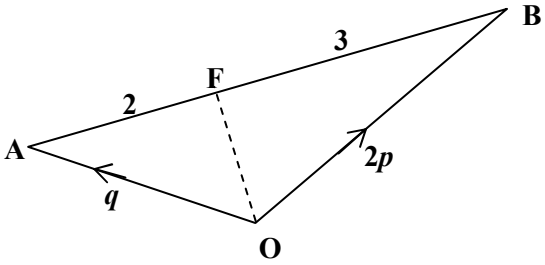
If $\boldsymbol{v} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}$ and $\boldsymbol{w} = \begin{pmatrix} 0 \\ 12 \end{pmatrix}$, find the components of the displacement $\overrightarrow{AC}$.



3. Express in terms of $\boldsymbol{p}$ and $\boldsymbol{q}$.

(a) $\overrightarrow{AB}$ **(b)** $\overrightarrow{AF}$ **(c)** $\overrightarrow{OF}$

If $\boldsymbol{p} = \begin{pmatrix} -10 \\ 5 \end{pmatrix}$ and $\boldsymbol{q} = \begin{pmatrix} 5 \\ 5 \end{pmatrix}$ find the components of $\overrightarrow{OF}$ and hence its magnitude correct to 1 decimal place.

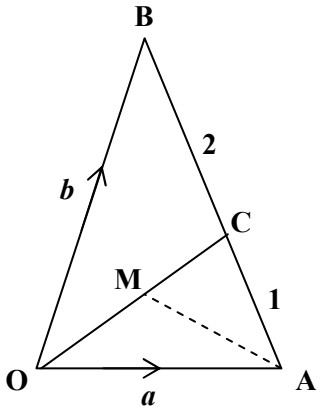


4. **(a)** Express in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$:-

(i) $\overrightarrow{AB}$ **(ii)** $\overrightarrow{AC}$ **(iii)** $\overrightarrow{OC}$

(b) If M is the mid-point of OC show that:-

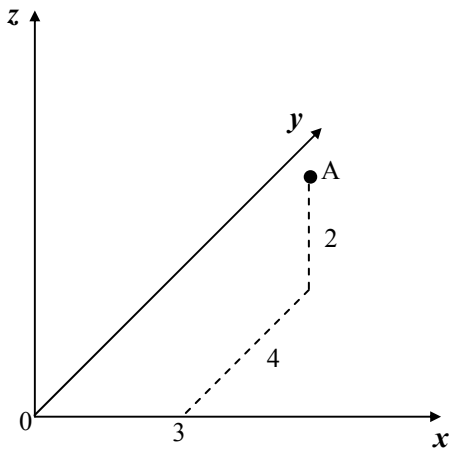
$\overrightarrow{AM} = \frac{1}{6}\boldsymbol{b} - \frac{2}{3}\boldsymbol{a} = \frac{1}{6}(\boldsymbol{b} - 4\boldsymbol{a})$



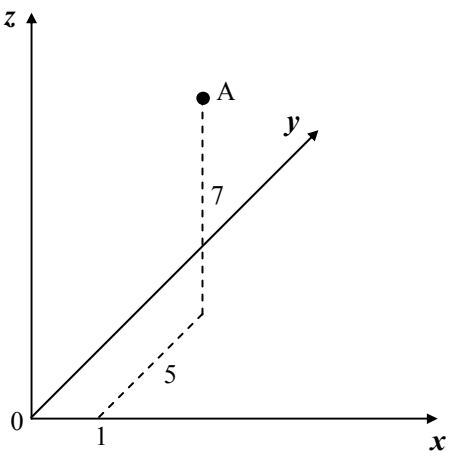
2.2 WORKING with 3D COORDINATES and VECTORS

1. For each diagram, write down the coordinates of the point A and the components of the vector $\overrightarrow{OA}$.

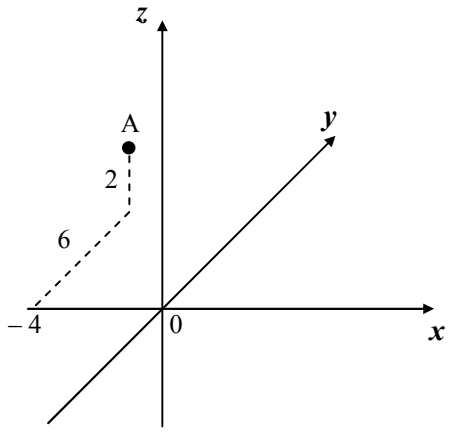
(a)



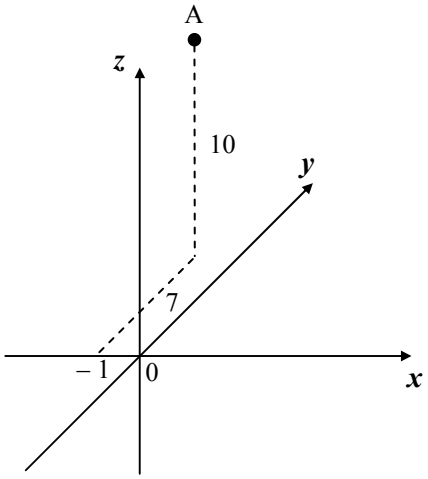
(b)



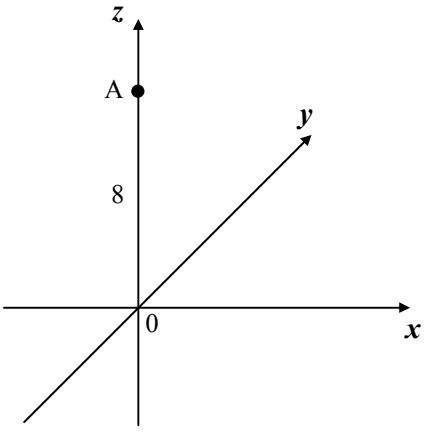
(c)



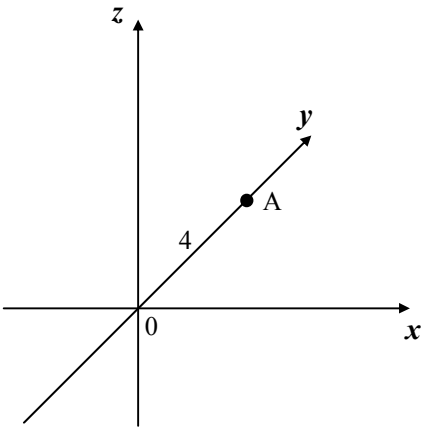
(d)



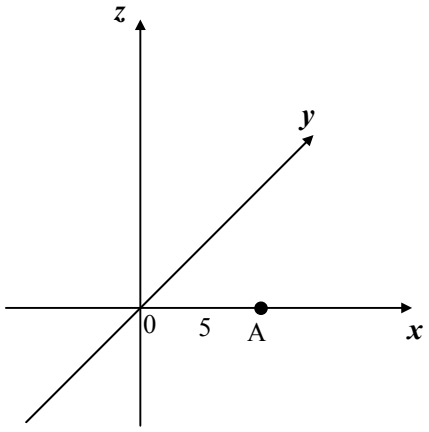
(e)



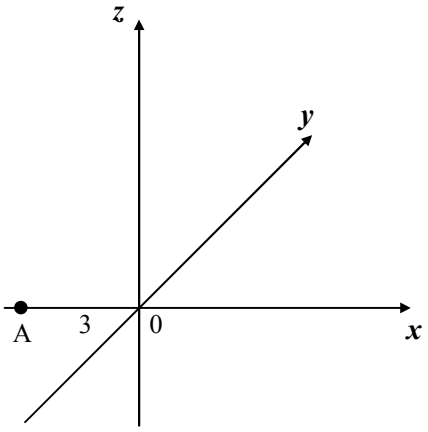
(f)



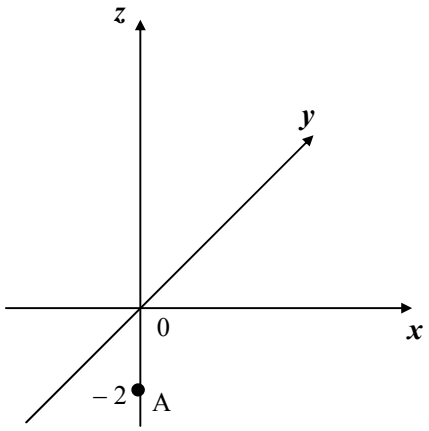
(g)



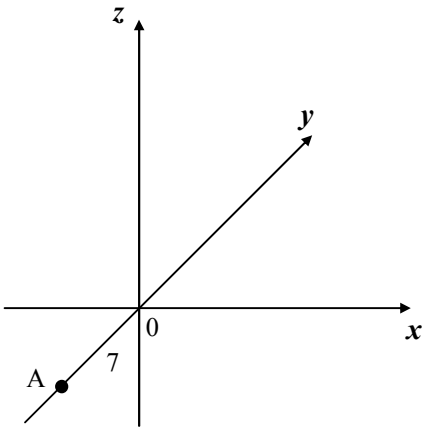
(h)



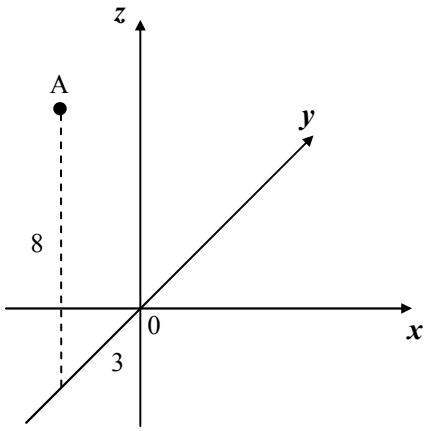
(i)



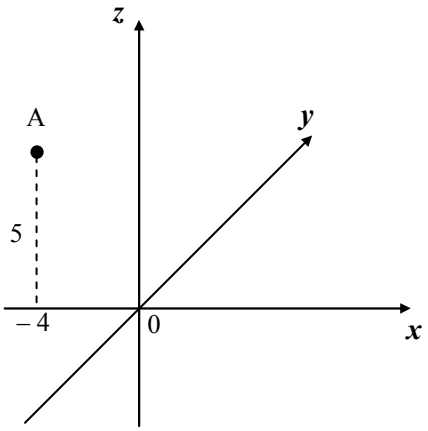
(j)



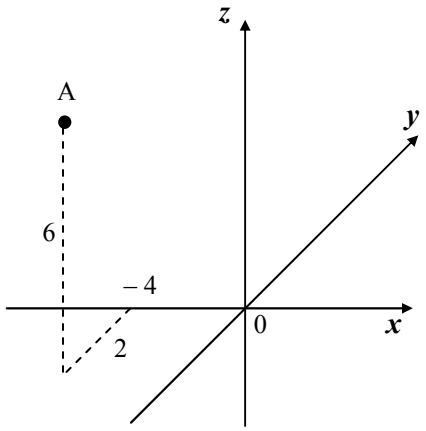
(k)



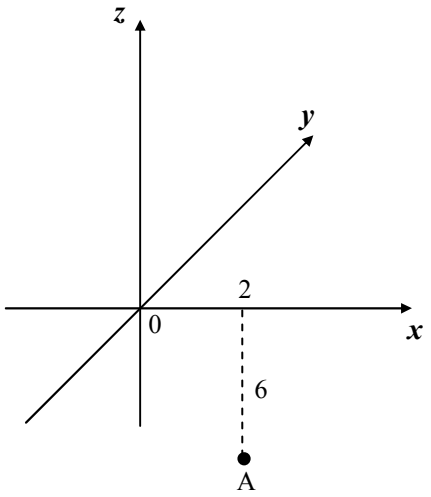
(l)



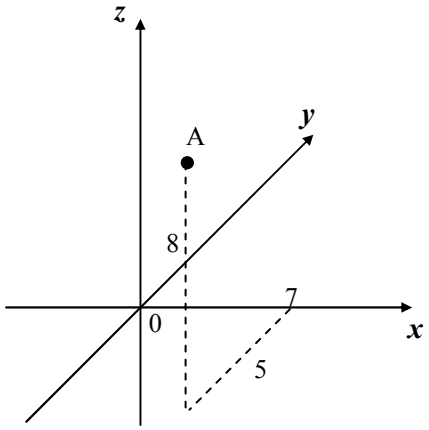
(m)



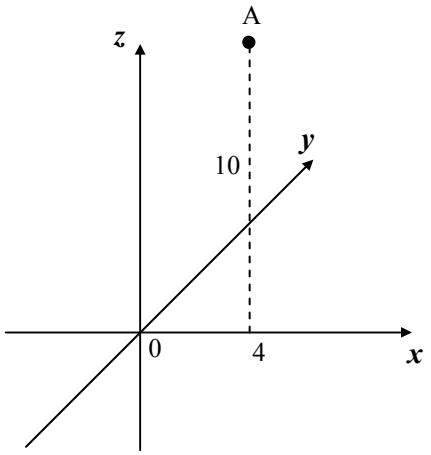
(n)



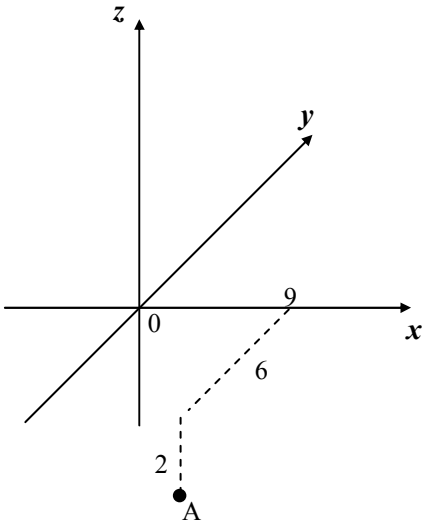
(o)



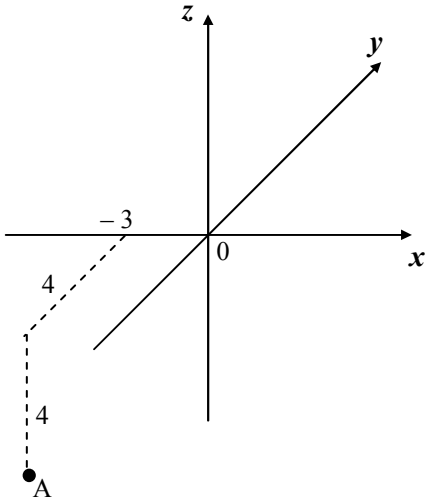
(p)



(q)

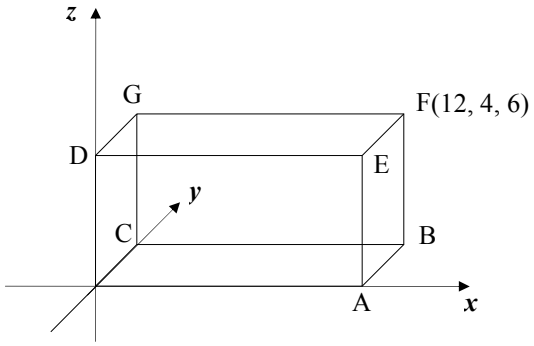


(r)

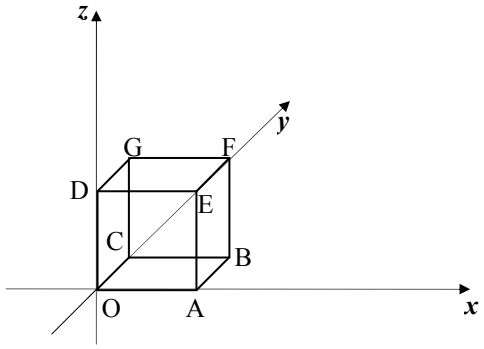


2. Calculate the magnitude of each of the vectors in question 1 correct to one decimal place.

3. State the coordinates of each vertex of the cuboid shown in the diagram.

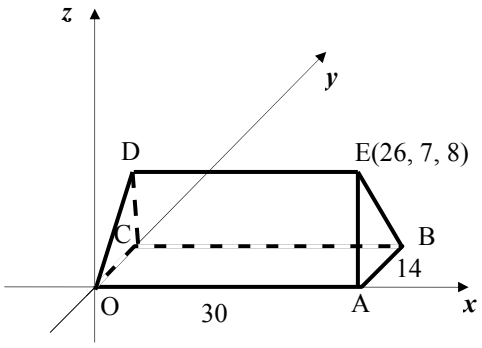


4. A cube of side 6 units is placed on coordinate axes as shown in the diagram. Write down the coordinates of each vertex of the cube.

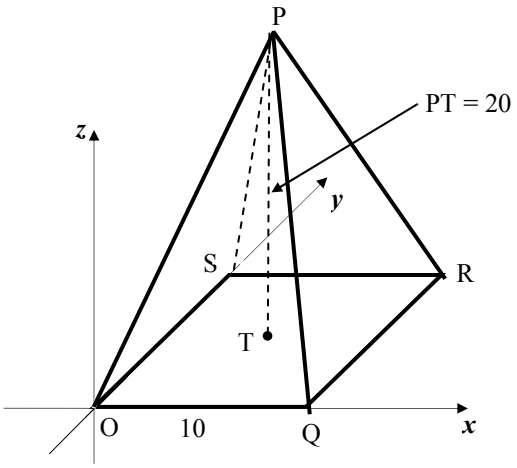


5. This shape is made up from 2 congruent trapezia and 2 congruent isosceles triangles.

From the information given in the diagram, write down the coordinates of each corner of the shape.



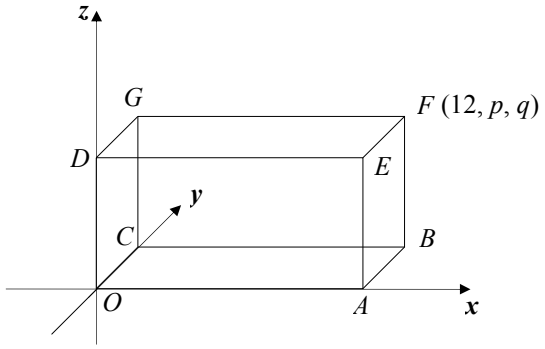
6. State the coordinates of each vertex of the **square based** pyramid shown in the diagram.



7. A cuboid is placed on coordinate axes as shown.

The dimensions of the cuboid are in the ratio $OA : AB : BF = 4 : 1 : 2$

The point F has coordinates $(12, p, q)$ as shown.



Establish the values of p and q and write down the coordinates of all the vertices of the cuboid.

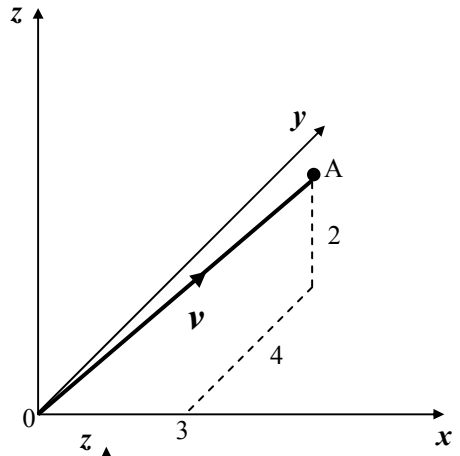
8. Write in component form **(a)** $\boldsymbol{v} = 2\boldsymbol{i} + 3\boldsymbol{j} - 4\boldsymbol{k}$ **(b)** $\boldsymbol{w} = 3\boldsymbol{i} - 6\boldsymbol{j} + 2\boldsymbol{k}$

(c) $\boldsymbol{u} = 6\boldsymbol{i} - 3\boldsymbol{k}$ **(d)** $\boldsymbol{a} = -3\boldsymbol{j} - 4\boldsymbol{k}$

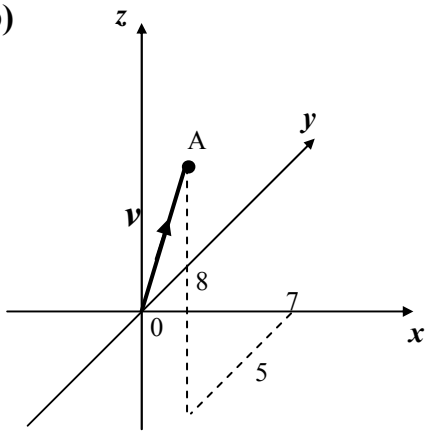
(e) $\boldsymbol{b} = 7\boldsymbol{i} - 2\mathbf{j}$ **(f)** $\boldsymbol{c} = 6\boldsymbol{j}$

9. For each of these diagrams express $\boldsymbol{v}$ in terms of $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$.

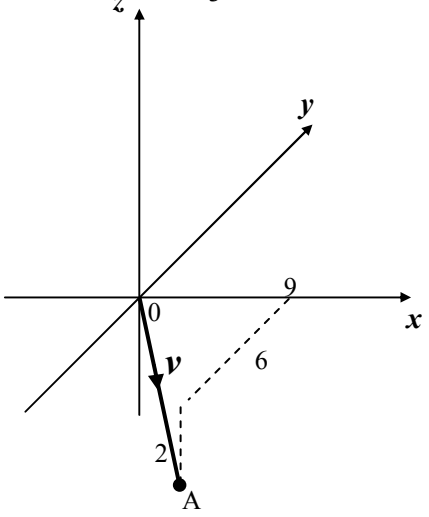
(a)



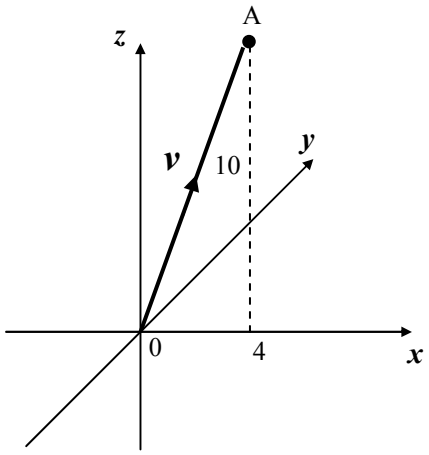
(b)



(c)



(d)



2.3 USING VECTOR COMPONENTS

Adding or subtracting 2 or 3 dimensional vectors using components.

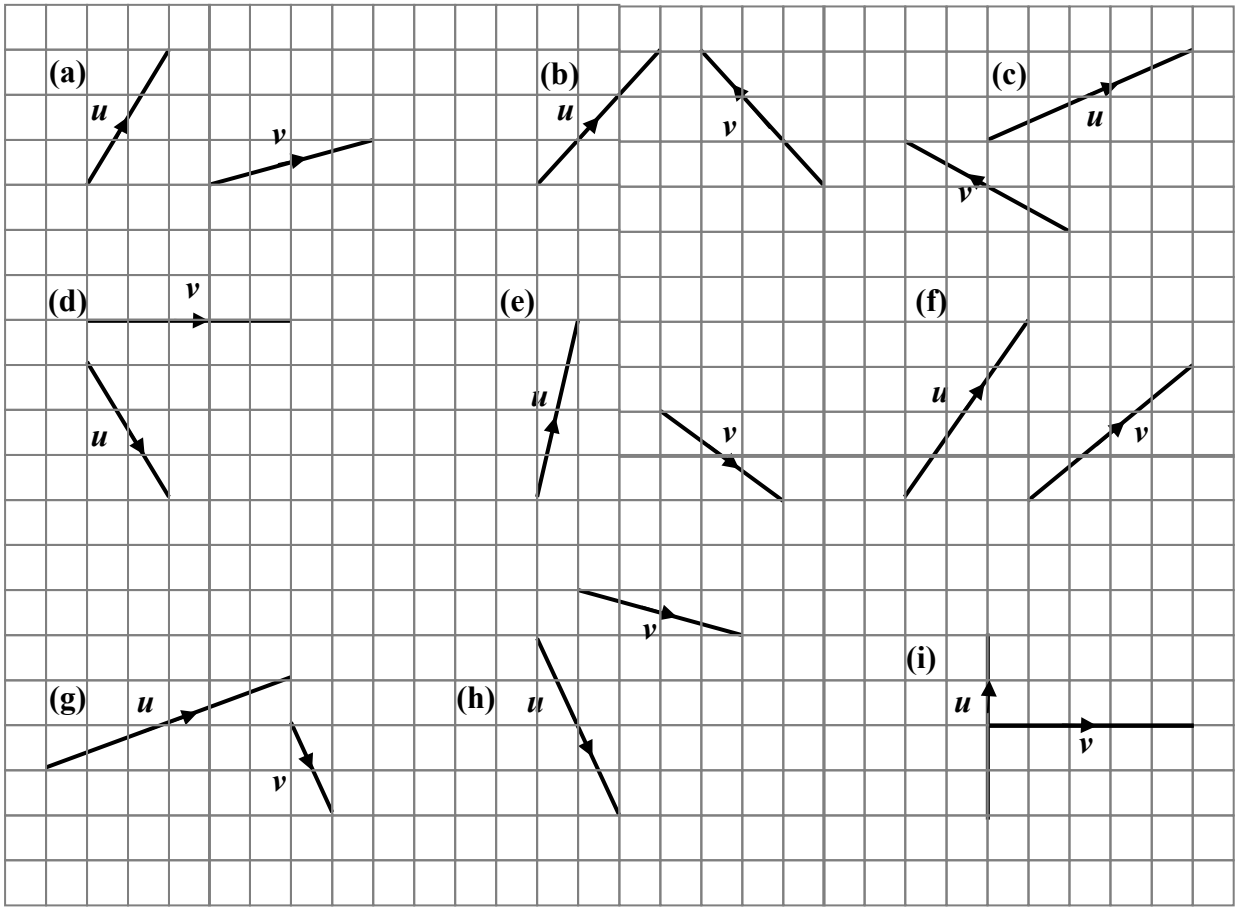
1. For each pair of vectors: **(i)** Write down the components of $\boldsymbol{u}$ and $\boldsymbol{v}$.

(ii) Find the components of the resultant vector $\boldsymbol{u} + \boldsymbol{v}$

(iii) Find the components of the resultant vector $\boldsymbol{v} - \boldsymbol{u}$

(iv) Find the components of the resultant vector $2\boldsymbol{v} + 3\boldsymbol{u}$

(v) Find the components of the resultant vector $3\boldsymbol{v} - 4\boldsymbol{u}$



2. $\boldsymbol{u}$, $\boldsymbol{v}$ and $\boldsymbol{w}$ are 3 vectors with components $\begin{pmatrix}2\\3\end{pmatrix}, \begin{pmatrix}-4\\5\end{pmatrix}$ and $\begin{pmatrix}-1\\-3\end{pmatrix}$ respectively.

Find the components of the following: **(a)** $2\boldsymbol{u} + 3\boldsymbol{v}$ **(b)** $3\boldsymbol{u} - 6\mathbf{v}$

(c) $3\boldsymbol{w} + 2\boldsymbol{v}$ **(d)** $4\boldsymbol{u} - 2\boldsymbol{w}$ **(e)** $-3\boldsymbol{u} - 4\boldsymbol{v}$ **(f)** $3\boldsymbol{w} - 4\boldsymbol{u}$

(g) $3\boldsymbol{u} - 6\boldsymbol{v} + 2\boldsymbol{w}$ **(h)** $2\boldsymbol{u} + 3\boldsymbol{v} - 4\boldsymbol{w}$ **(i)** $3\boldsymbol{u} - 2\boldsymbol{v} + \boldsymbol{w}$

3. Calculate the magnitude of each of these vectors giving answers to one decimal place:

(a) $\boldsymbol{p} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$ **(b)** $\boldsymbol{v} = \begin{pmatrix} 3 \\ 4 \\ -7 \end{pmatrix}$ **(c)** $\boldsymbol{r} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}$ **(d)** $\boldsymbol{t} = \begin{pmatrix} -3 \\ 0 \\ 4 \end{pmatrix}$

(e) $\boldsymbol{u} = \begin{pmatrix} 6 \\ -1 \\ -4 \end{pmatrix}$ **(f)** $\boldsymbol{q} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ **(g)** $\boldsymbol{a} = \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$ **(h)** $\boldsymbol{b} = \begin{pmatrix} 5 \\ -12 \\ 0 \end{pmatrix}$

4. $\boldsymbol{u}$, $\boldsymbol{v}$ and $\boldsymbol{w}$ are 3 vectors with components $\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$, $\begin{pmatrix} 4 \\ 8 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 5 \\ -1 \end{pmatrix}$ respectively.

(i) Find the components of the following:

(a) $2\boldsymbol{u} + 3\boldsymbol{v}$ **(b)** $3\boldsymbol{u} - 6\boldsymbol{v}$ **(c)** $3\boldsymbol{w} + 2\boldsymbol{v}$ **(d)** $4\boldsymbol{u} - 2\boldsymbol{w}$

(e) $-3\boldsymbol{u} - 4\boldsymbol{v}$ **(f)** $3\boldsymbol{w} - 4\boldsymbol{u}$ **(g)** $3\boldsymbol{u} - 6\boldsymbol{v} + 2\boldsymbol{w}$ **(h)** $2\boldsymbol{u} + 3\boldsymbol{v} - 4\boldsymbol{w}$

(ii) Calculate the magnitude of each resultant vector above giving answers to 1 decimal place.

5. **(i)** If $\boldsymbol{p} = 4\boldsymbol{i} + 2\boldsymbol{j} - 5\boldsymbol{k}$ and $\boldsymbol{q} = \boldsymbol{i} - 3\boldsymbol{j} + \boldsymbol{k}$, express the following in component form:

(a) $\boldsymbol{p} + \boldsymbol{q}$ **(b)** $\boldsymbol{p} - \boldsymbol{q}$ **(c)** $\boldsymbol{q} - 2\boldsymbol{p}$ **(d)** $3\boldsymbol{p} + \boldsymbol{q}$

(e) $3\boldsymbol{p} - 2\boldsymbol{q}$ **(f)** $2\boldsymbol{q} - 3\boldsymbol{p}$ **(g)** $3\boldsymbol{p} + 4\boldsymbol{q}$ **(h)** $-2\boldsymbol{q} - 2\boldsymbol{p}$

(ii) Calculate the magnitude of each resultant vector above giving answers to 1 decimal place.

6. Calculate the magnitude of these vectors , leaving you answer a surd in its in simplest form.

(a) $\boldsymbol{u} = \begin{pmatrix} -5 \\ 3 \\ 2 \end{pmatrix}$ **(b)** $AB = \begin{pmatrix} -1 \\ 1 \\ 5 \end{pmatrix}$ **(c)** $\boldsymbol{t} = 3\boldsymbol{i} - 2\boldsymbol{j} + 5\boldsymbol{k}$

(d) $\boldsymbol{t}$ where point T has coordinates$(\sqrt{3}, \sqrt{5}, 2\sqrt{2})$ **(e)** $\boldsymbol{v} = \sqrt{3}\boldsymbol{k} + \boldsymbol{j} - 7\boldsymbol{i}$

7. Given that $\boldsymbol{v} = 2\boldsymbol{k} - 3\boldsymbol{i} + 4\boldsymbol{k}$, $\boldsymbol{u} = 5\boldsymbol{i} + a\boldsymbol{j} - \boldsymbol{k}$ have the same magnitude, calculate the value of a if $a > 0$.

8. A skater is suspended by three wires with forces $\begin{pmatrix}2\\3\\4\end{pmatrix}, \begin{pmatrix}4\\8\\0\end{pmatrix}$ and $\begin{pmatrix}-2\\5\\-1\end{pmatrix}$ acting on them.

Calculate the resultant force and its magnitude correct to 3 significant figures where necessary.

9. If $\boldsymbol{u} = \begin{pmatrix}-4\\1\\3\end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix}2\\2\\-5\end{pmatrix}$, solve each vector equation for $\boldsymbol{x}$.

(a) $\boldsymbol{u} + \boldsymbol{x} = \boldsymbol{v}$ **(b)** $2\boldsymbol{u} + \boldsymbol{x} = 2\boldsymbol{v}$ **(c)** $2\boldsymbol{x} + 3\boldsymbol{v} = 4\boldsymbol{u} - \boldsymbol{x}$

10. **(i)** If $\boldsymbol{r} = \begin{pmatrix}2\\6\\-3\end{pmatrix}$, $\boldsymbol{s} = \begin{pmatrix}6\\6\\-1\end{pmatrix}$ and $\boldsymbol{t} = \begin{pmatrix}-4\\0\\1\end{pmatrix}$, express these in component form:

(a) $2\boldsymbol{r} + \boldsymbol{s}$ **(b)** $3\boldsymbol{t} - 2\boldsymbol{s}$ **(c)** $(\boldsymbol{r} - \boldsymbol{s}) + \boldsymbol{t}$ **(d)** $\boldsymbol{r} - (\boldsymbol{s} + \boldsymbol{t})$

(ii) Find: **(a)** $|2\boldsymbol{r} + \boldsymbol{s}|$ **(b)** $|3\boldsymbol{t} - 2\boldsymbol{s}|$ **(c)** $|(\boldsymbol{r} - \boldsymbol{s}) + \boldsymbol{t}|$ **(d)** $|\boldsymbol{r} - (\boldsymbol{s} + \boldsymbol{t})|$

11. Two forces are represented by the vectors $F_1 = 2\underset{\sim}{i} + \underset{\sim}{j} - 3\underset{\sim}{k}$ and $F_2 = \underset{\sim}{i} + 4\underset{\sim}{k}$.

Find the magnitude of the resultant force $F_1 + F_2$.

12. Two vectors are defined as $V_1 = 4\underset{\sim}{i} + \underset{\sim}{j} + \sqrt{8}\underset{\sim}{k}$ and $V_2 = 8\underset{\sim}{i} + \sqrt{24}\underset{\sim}{j} + a\sqrt{3}\underset{\sim}{k}$ where a is a constant and <u>all</u> coefficients of $\underset{\sim}{i}$, $\underset{\sim}{j}$ and $\underset{\sim}{k}$ are greater than zero.

Given that $|V_2| = 2|V_1|$, calculate the value of a.

13. Vector $\boldsymbol{a}$ has components $\boldsymbol{a} = \begin{pmatrix}3\\-2\\k\end{pmatrix}$. If $|\boldsymbol{a}| = 4$, calculate the value(s) of k.

14. Calculate the length of vector $\boldsymbol{a}$ defined as $\boldsymbol{a} = 4\boldsymbol{i} + 2\sqrt{3}\boldsymbol{j} - 2\sqrt{2}\boldsymbol{k}$.

15. Vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are defined by $\boldsymbol{a} = \boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k}$ and $\boldsymbol{b} = 3\boldsymbol{i} - \boldsymbol{j}$. Find the components of $2\boldsymbol{a} - \boldsymbol{b}$ and calculate its magnitude.

3.1 WORKING with PERCENTAGES

Use reverse percentages to calculate an original quantity.

1. These amounts have been reduced by 15%. What was the original amount?

(a)	£85	**(b)**	212·5 mm	**(c)**	£63.75
(d)	25·5 litres	**(e)**	357 miles	**(f)**	435·2 m
(g)	1 275 km	**(h)**	£4 462.50	**(i)**	10 200 m
(j)	605·2 cm	**(k)**	£658.75	**(l)**	76·5 kg

2. These amounts have been increased by 22%. What was the original amount?

(a)	£26.84	**(b)**	£54.90	**(c)**	£87.84
(d)	103·7 ml	**(e)**	£21.35	**(f)**	122 cm
(g)	3 111 m	**(h)**	10 370 km	**(i)**	68·32 m
(j)	£13 664	**(k)**	118·95 litres	**(l)**	£7 564

3. A shop is having a sale. There is '20% OFF'. Calculate the original cost of these items.

(a)

£32

(b)

£52

(c)

£20

(d)

£340

(e)

£5.60

(f)

£239.20

(g)

£1 640

(h)

£20.80

(i)

£21.20

4. A company gave their workers a 7% wage rise. Calculate how much each of these people were earning **each year before** the increase.

(a)	Irene	£13 375 per year	**(b)**	Billy	£19 324.20 per year
(c)	Peter	£26 322 per year	**(d)**	Isobel	£40 060.80 per year
(e)	Stewart	£481.50 per week	**(f)**	Jackie	£1 820 per month
(g)	Alan	£75 per week	**(h)**	Anne	£1 200 per month

EXAM QUESTIONS

1. A gym's membership has increased by 17% over the past year. It now has 585 members.

How many members did it have a year ago?

2. The number of school pupils not wearing school uniform has decreased by 72% since the start of last year. There are now 42 pupils not wearing school uniform.

How many pupils were not wearing school uniform at the start of last year?

3. My house has increased in value by 15% in the last two years. It is now worth £230 000.

How much was it worth 2 years ago?

4. I bought a new car in September of last year. By this September the car had

depreciated by 20% and was now worth £9600.

How much did I pay for the car last September?

5. Jane bought a painting in an auction. Unfortunately the painting depreciated in value by 7% and is now worth £4185.

How much was the painting worth when it was bought?

6. An antique chair has increased in value by 34% since it was bought. It is now worth £3 484.

What was it worth when it was bought?

APPRECIATION and DEPRECIATION

1. For each of the investments below, calculate

(i) the amount due at the end of the term

(ii) the total interest

	Bank/ Building Society	Amount Invested (£)	Rate of interest (per year)	Number of Years
(a)	Hamilton Bank	2000	8 %	2
(b)	Allied Friendly	5000	6 %	3
(c)	Northern Hill	4800	7 %	2
(d)	Highland Bank	3500	7·5 %	3
(e)	Church National	1600	5·5 %	4
(f)	Southern Rock	1750	11 %	3
(g)	London Savings Bank	20 000	6%	3
(h)	Bath & Eastern	18 000	8·5%	2
(i)	Royal Bank of Britain	50 000	9%	3
(j)	Bingford & Bradley	400	4·8%	2

2. At the beginning of the year, Mr. Bradford borrows £5000 from the bank. The rate of compound interest is 8%. He agrees to pay back £108 per month.

Calculate how much he still owes at the end of the second year.

3. The Smiths buy a house for £60,000. If it appreciates in value at the rate of 9% per year, how much will it be worth in 5 years time?

4. Amanda wins some money and decides to spend £200 on some jewellery. If it appreciates at the rate of 2% per year, how much will the jewellery be worth 3 years from now?

5. In 1990 the world population was estimated to be 5300 million, and was increasing at the rate of 1·7% per annum.

What will the population be in the year 2000? (answer to 2 significant figures)

6. Peter buys a car for £3000. If it depreciates at the rate of 20% per annum, how much will he be able to sell it for in 3 years time?

7. Brian buys a new car costing £12600. It depreciates in value by 30% in the first year and by 20% each year after that.

How much will he be able to trade it in for in 3 years time

8. Each year a factory's machinery depreciates by 25% of its value at the beginning of the year. The initial value of the machinery was £360 000.

(a) What was the value of the machinery after 1 year

(b) The machinery was to be scrapped at the end of the year when its value fell below half its original value. After how many years should the machinery be scrapped?

EXAM QUESTIONS

1. Joseph invests £4500 in a bank that pays 6·4% interest per annum.

If Joseph does not touch the money in the bank, how much interest will he have gained after 3 years?

Give your answer to the nearest penny.

2. Jane bought a painting in an auction for £32 250.

Unfortunately the painting depreciated in value by 7% each year.

Calculate how much the painting was worth after 2 years.

Give your answer to 3 significant figures.

3. **Non calculator**

Last year (2008) a company made a profit of £1 000 000. This year (2009) it expects to increase its profit by 20% and by 2010 to have increased it by a further 25%.

Calculate the profit the company expects to make in 2010.

4. A patient in hospital is given 200mg of a drug at 0900. 12% of the amount of the drug at the beginning of each hour is lost, through natural body processes, by the end of that hour.

How many mg of the drug will be **lost** by 1200?

5. Holly buys an antique watch costing £1200. The watch appreciates in value by 3·7% per annum.

How much will the watch be worth in 4 years time?

Give your answer to the nearest pound.

6. A local council recycles 28 000 tonnes of glass each year. After a publicity campaign they expect to increase the amount of glass recycled by 12% each year.

(a) How much glass do they expect to recycle in 3 years time?

Give your answer correct to **3 significant figures**.

(b) The council aim to double the amount of glass recycled in 6 years.

If this rate is maintained, will the council meet their target?

Give a reason for your answer.

7. **Non calculator**

Arthur's new car cost him £15 000. The value of it will depreciate by 20% each year.

How much will Arthur's car be worth when he trades it in for a new one in 2 years time?

8. Barry bought a house last year costing £115 000. This year it is valued at £110 400.

(a) Calculate the percentage decrease in the value of the house.

(b) If the value of the house continues to decrease at this rate what will the house be worth in a further 3 years time?

Give your answer to 3 significant figures.

9. Marcus invested £3000 in a bank which paid 2·5% interest per year.

(a) Calculate how much money Marcus would have in his account after 3 years.

(b) How long would it take for Marcus' money to increase by 12%?

10. In 2007 a company made a profit of £45 000. Over the next three years its profit dropped by 3% each year due to increased manufacturing costs.

Calculate, correct to 3 significant figures, the company's profit in 2010.

11. The value of an industrial machine is expected to decrease each year by 14·2% of its value at the beginning of the year.

If it was valued at £15500 at the **beginning** of 2011, what will its expected value be at the **end** of 2013? **Give your answer correct to the nearest pound**.

12. The membership of the 'Watch your Weight' slimming club is 40 000 and is increasing at the rate of 4% per month.

The membership of 'World of Slimming' is 70 000 but is decreasing at the rate of 9% per month.

(a) Calculate the membership of the 'Watch your Weight' club after 3 months, giving your answer correct to 4 significant figures.

(b) How many months will it take for the membership of the 'Watch your Weight' club to be more than the 'World of Slimming'?

13. A woman had a Body Mass Index (BMI) of 30. After following a healthy eating plan she managed to reduce her BMI to 27·6 in 1 month.

(a) Calculate the percentage reduction in her BMI.

(b) If she managed to continue to reduce her BMI by the same percentage in each of the next 3 months, what was her BMI then? Give your answer correct to 3 significant figures.

14. The value of an antique chair increased in value by 12½ % each year.

The chair was bought for £4800. What was its value at the end of 3 years?

15. Non Calculator

Charlene's house is valued at £120 000 and is expected to appreciate at the rate of 10% per annum for the next three years.

If this happens, what will the house be valued at in three years time?

16. Three years ago I bought a new car which cost £10 500. An offer from the garage at the time stated:

"Keep the car for 3 years, return it to us and we will refund half the original cost"

The car depreciated in value by 20% during the first year and by 15% in subsequent years.

By calculating the value of the car after 3 years decide whether the garage's offer, in this case, was a good one or not. Give a reason for your answer.

17. A piece of jewellery was bought for £2580 two years ago. Its present value is 65% of its original price.

(a) What is its present day value?

An expert estimates that it will increase in value at a rate of 12% per annum over the next few years.

(b) How many years will it take for the jewellery to regain its original value?

18. Bill invested £10 000 in the Dodgy Building Society but his money lost 5% per annum over the first 2 years.

At the end of this time he decided to move his money to the Goody Building Society which guaranteed that his money would gain 6% per annum over the next 2 years.

How much did Bill gain or lose over the four years?

19. Chocolate fountains have become very popular at parties.

It takes a minimum of 900g of melted chocolate to operate a fountain properly.

On one occasion 2kg of melted chocolate was added to the fountain.

23% of the remaining chocolate was used every 20 minutes.

Was there still enough chocolate left to operate the fountain properly one hour later?

You must show all working and give a reason for your answer.

20. In 2008 the Portable Phone Company announced that their profits were £850 000. In the next 3 years their profits increased by 4·2% each year. How much profit did the company make in 2011? **Give your answer to the nearest thousand.**

3.2 WORKING with FRACTIONS

Operations and combinations of vulgar fractions including mixed numbers.

ADDITION and SUBTRACTION

1. Express each sum as a fraction in its simplest form.

(a) $\frac{1}{5}+\frac{3}{5}$ **(b)** $\frac{2}{5}+\frac{1}{10}$ **(c)** $\frac{3}{4}+\frac{1}{8}$ **(d)** $\frac{1}{6}+\frac{2}{3}$

(e) $\frac{1}{9}+\frac{2}{3}$ **(f)** $\frac{1}{3}+\frac{1}{4}$ **(g)** $\frac{3}{5}+\frac{1}{4}$ **(h)** $\frac{1}{4}+\frac{1}{6}$

(i) $\frac{1}{3}+\frac{5}{8}$ **(j)** $\frac{1}{2}+\frac{2}{5}$ **(k)** $\frac{3}{4}+\frac{1}{6}$ **(l)** $\frac{1}{2}+\frac{3}{7}$

(m) $\frac{2}{7}+\frac{1}{8}$ **(n)** $\frac{1}{5}+\frac{3}{8}$ **(o)** $\frac{2}{9}+\frac{3}{7}$ **(p)** $\frac{1}{6}+\frac{3}{5}$

2. Express each sum as a fraction in its simplest form.

(a) $\frac{2}{5}+\frac{4}{5}$ **(b)** $\frac{2}{5}+\frac{9}{10}$ **(c)** $\frac{3}{4}+\frac{5}{8}$ **(d)** $\frac{5}{6}+\frac{2}{3}$

(e) $\frac{5}{9}+\frac{2}{3}$ **(f)** $\frac{2}{3}+\frac{3}{4}$ **(g)** $\frac{4}{5}+\frac{1}{4}$ **(h)** $\frac{3}{4}+\frac{5}{6}$

(i) $\frac{2}{3}+\frac{7}{8}$ **(j)** $\frac{1}{2}+\frac{4}{5}$ **(k)** $\frac{3}{7}+\frac{5}{6}$ **(l)** $\frac{5}{7}+\frac{5}{9}$

(m) $\frac{2}{7}+\frac{7}{8}$ **(n)** $\frac{4}{5}+\frac{3}{8}$ **(o)** $\frac{7}{9}+\frac{3}{7}$ **(p)** $\frac{5}{8}+\frac{4}{5}$

(q) $\frac{1}{2}+\frac{1}{2}+\frac{3}{4}$ **(r)** $\frac{1}{2}+\frac{3}{5}+\frac{1}{10}$ **(s)** $\frac{1}{2}+\frac{5}{8}+\frac{3}{16}$ **(t)** $\frac{2}{3}+\frac{1}{2}+\frac{1}{4}$

(u) $\frac{5}{6}+\frac{1}{4}+\frac{1}{3}$ **(v)** $\frac{7}{12}+\frac{1}{3}+\frac{3}{4}$ **(w)** $\frac{1}{6}+\frac{1}{7}+\frac{3}{8}$ **(x)** $\frac{3}{4}+\frac{2}{3}+\frac{2}{5}$

3. Express each difference as a fraction in its simplest form.

(a) $\frac{3}{4}-\frac{1}{4}$ **(b)** $\frac{1}{2}-\frac{1}{6}$ **(c)** $\frac{5}{6}-\frac{2}{3}$ **(d)** $\frac{11}{12}-\frac{5}{6}$

(e) $\frac{11}{12}-\frac{2}{3}$ **(f)** $\frac{1}{2}-\frac{1}{16}$ **(g)** $\frac{2}{3}-\frac{1}{4}$ **(h)** $\frac{1}{2}-\frac{2}{5}$

(i) $\frac{7}{8}-\frac{3}{16}$ **(j)** $\frac{4}{5}-\frac{1}{2}$ **(k)** $\frac{3}{4}-\frac{1}{2}$ **(l)** $\frac{7}{12}-\frac{1}{3}$

(m) $\frac{5}{8}-\frac{2}{5}$ **(n)** $\frac{5}{6}-\frac{3}{5}$ **(o)** $\frac{7}{9}-\frac{3}{7}$ **(p)** $\frac{7}{8}-\frac{2}{3}$

4. Express each sum as a fraction in its simplest form.

(a) $1\frac{1}{2}+1\frac{1}{4}$ **(b)** $1\frac{1}{2}+1\frac{3}{4}$ **(c)** $2\frac{3}{8}+1\frac{1}{4}$ **(d)** $3\frac{1}{2}+1\frac{5}{6}$

(e) $3\frac{5}{8}+2\frac{1}{4}$ **(f)** $5\frac{2}{3}+2\frac{3}{4}$ **(g)** $1\frac{3}{5}+1\frac{3}{5}$ **(h)** $2\frac{3}{8}+2\frac{5}{6}$

(i) $5\frac{3}{4}+2\frac{3}{8}$ **(j)** $6\frac{1}{3}+2\frac{7}{12}$ **(k)** $3\frac{1}{2}+\frac{5}{6}$ **(l)** $4\frac{1}{8}+\frac{3}{16}$

(m) $2\frac{7}{10}+\frac{2}{5}$ **(n)** $4\frac{2}{3}+3\frac{1}{12}$ **(o)** $1\frac{11}{16}+2\frac{3}{8}$ **(p)** $5\frac{7}{9}+\frac{2}{3}$

(q) $3\frac{3}{4}+2\frac{5}{12}$ **(r)** $5\frac{2}{3}+2\frac{1}{2}$ **(s)** $2\frac{7}{8}+1\frac{1}{12}$ **(t)** $5\frac{9}{16}+8\frac{5}{8}$

5. Express each difference as a fraction in its simplest form.

(a) $3-\frac{2}{3}$ **(b)** $4-\frac{7}{12}$ **(c)** $3-\frac{7}{8}$ **(d)** $1-\frac{2}{9}$

(e) $1-\frac{1}{2}$ **(f)** $3-\frac{3}{4}$ **(g)** $2-\frac{1}{4}$ **(h)** $2-\frac{5}{6}$

6. Express each difference as a fraction in its simplest form.

(a) $3\frac{3}{4}-1\frac{1}{2}$ **(b)** $6\frac{7}{8}-4\frac{1}{3}$ **(c)** $2\frac{4}{5}-1\frac{1}{4}$ **(d)** $4\frac{7}{12}-1\frac{1}{3}$

(e) $5\frac{4}{5}-1\frac{3}{4}$ **(f)** $6\frac{11}{12}-1\frac{5}{6}$ **(g)** $4\frac{2}{3}-1\frac{1}{7}$ **(h)** $3\frac{3}{4}-1\frac{1}{6}$

(i) $5\frac{1}{3}-1\frac{1}{8}$ **(j)** $5\frac{5}{8}-2\frac{1}{2}$ **(k)** $8\frac{7}{12}-4\frac{1}{3}$ **(l)** $4\frac{9}{10}-2\frac{4}{5}$

(m) $9\frac{1}{3}-4\frac{1}{3}$ **(n)** $8\frac{5}{6}-1\frac{1}{12}$ **(o)** $8\frac{2}{5}-1\frac{3}{10}$ **(p)** $5\frac{5}{9}-4\frac{1}{3}$

7. Express each difference as a fraction in its simplest form.

(a) $4-2\frac{3}{4}$ **(b)** $7-3\frac{4}{7}$ **(c)** $5-2\frac{2}{3}$ **(d)** $10-5\frac{2}{3}$

(e) $5\frac{2}{7}-4\frac{2}{5}$ **(f)** $7\frac{7}{12}-3\frac{5}{6}$ **(g)** $4\frac{1}{12}-1\frac{1}{3}$ **(h)** $5\frac{3}{5}-1\frac{3}{4}$

(i) $3\frac{1}{4}-1\frac{5}{6}$ **(j)** $5\frac{1}{8}-2\frac{1}{2}$ **(k)** $8\frac{1}{4}-4\frac{1}{3}$ **(l)** $4\frac{3}{10}-2\frac{4}{5}$

(m) $9\frac{1}{3}-4\frac{2}{3}$ **(n)** $2\frac{2}{5}-1\frac{3}{4}$ **(o)** $8\frac{2}{5}-1\frac{7}{10}$ **(p)** $5\frac{1}{9}-4\frac{2}{3}$

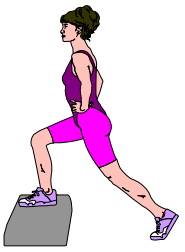
8. Tom walked for $1\frac{5}{8}$ kilometres and then walked another $2\frac{3}{5}$ km.

How far did he walk in total?

9. A rectangle has length $3\frac{5}{7}$ cm and breadth $1\frac{2}{5}$ cm. Calculate its perimeter.

10. Siobhan likes to go to the gym. Last week she went on Monday, Tuesday, Thursday, Friday, Saturday and Sunday. The table below shows the number of hours she trained on each of the six days.

MON	TUES	THURS	FRI	SAT	SUN
$1\frac{1}{2}$	$1\frac{1}{4}$	$\frac{3}{4}$	$1\frac{2}{5}$	$\frac{4}{5}$	$1\frac{3}{10}$



How many hours in total did she spend in the gym last week?

11. Billy is a long distance lorry driver. One day he drove for $2\frac{1}{2}$ hours, had a break and then drove for another $3\frac{2}{3}$ hours.

How long did he drive in total?

12. Peter is walking to school. When he reaches half way he meets Mike. They walk a third of the way together when they meet Anne. They walk the rest of the way together. What fraction of the journey is this?

13. A group of friends went to a burger bar. $\frac{2}{5}$ of them bought a burger, $\frac{1}{3}$ bought chips and the rest bought cola/ What fraction of the group bought cola?

14. At a school $\frac{1}{5}$ of the time is spent in **Mathematics** classes and $\frac{3}{20}$ is spent in **English**.

(a) What fraction of the time is spent in Maths and English altogether?

(b) If $\frac{1}{15}$ of the time is spent in **PE**, what fraction of the time is spent on all the other subjects apart from the three subjects already mentioned?

MULTIPLICATION and DIVISION

1. Express each product as a fraction in its simplest form:

(a)	$\frac{1}{4}\times\frac{4}{7}$	**(b)**	$\frac{1}{3}\times\frac{3}{10}$	**(c)**	$\frac{1}{2}\times\frac{4}{7}$	**(d)**	$\frac{2}{3}\times\frac{1}{8}$
(e)	$\frac{4}{5}\times\frac{1}{16}$	**(f)**	$\frac{6}{7}\times\frac{2}{3}$	**(g)**	$\frac{3}{5}\times\frac{10}{21}$	**(h)**	$\frac{3}{8}\times\frac{4}{21}$
(i)	$\frac{21}{32}\times\frac{4}{7}$	**(j)**	$\frac{1}{9}\times\frac{12}{13}$	**(k)**	$\frac{5}{16}\times\frac{6}{25}$	**(l)**	$\frac{5}{7}\times\frac{14}{15}$
(m)	$\frac{7}{9}\times\frac{12}{35}$	**(n)**	$\frac{12}{13}\times\frac{39}{48}$	**(o)**	$\frac{2}{3}\times\frac{5}{9}$	**(p)**	$\frac{2}{9}\times\frac{3}{8}$
(q)	$\frac{1}{2}\times\frac{3}{5}$	**(r)**	$\frac{3}{8}\times\frac{2}{3}$	**(s)**	$\frac{5}{14}\times\frac{8}{15}$	**(t)**	$\frac{7}{10}\times\frac{5}{14}$

2. Express each product as a fraction in its simplest form:

(a) $1\frac{1}{4}\times 1\frac{1}{3}$ **(b)** $1\frac{1}{4}\times 1\frac{2}{3}$ **(c)** $2\frac{1}{2}\times 2\frac{1}{2}$ **(d)** $1\frac{3}{4}\times 1\frac{2}{3}$

(e) $3\frac{1}{4}\times 1\frac{1}{5}$ **(f)** $1\frac{1}{3}\times 2\frac{2}{3}$ **(g)** $1\frac{1}{15}\times 2\frac{1}{2}$ **(h)** $3\frac{3}{4}\times 1\frac{1}{5}$

(i) $2\frac{1}{2}\times 5$ **(j)** $7\frac{1}{2}\times 4$ **(k)** $2\frac{1}{7}\times 1\frac{1}{3}$ **(l)** $2\frac{5}{8}\times 3\frac{2}{7}$

(m) $4\frac{4}{7}\times 2\frac{5}{8}$ **(n)** $3\frac{3}{5}\times 3\frac{1}{3}$ **(o)** $1\frac{1}{5}\times 3\frac{1}{3}$ **(p)** $2\frac{1}{4}\times 3\frac{1}{2}$

(q) $2\frac{3}{4}\times 3\frac{1}{2}$ **(r)** $1\frac{4}{9}\times 3\frac{2}{3}$ **(s)** $5\frac{3}{5}\times 3\frac{3}{4}$ **(t)** $1\frac{1}{7}\times 2\frac{4}{5}$

3. Express as a single fraction:

(a) $\frac{1}{4}\div\frac{1}{3}$ **(b)** $\frac{2}{5}\div\frac{2}{7}$ **(c)** $\frac{4}{5}\div\frac{3}{4}$ **(d)** $\frac{3}{7}\div\frac{2}{5}$

(e) $\frac{5}{12}\div\frac{5}{3}$ **(f)** $\frac{5}{9}\div\frac{1}{3}$ **(g)** $\frac{2}{5}\div\frac{9}{10}$ **(h)** $\frac{3}{7}\div\frac{11}{14}$

(i) $\frac{4}{9}\div\frac{2}{3}$ **(j)** $\frac{2}{5}\div\frac{4}{5}$ **(k)** $\frac{24}{35}\div\frac{20}{21}$ **(l)** $\frac{6}{25}\div\frac{9}{20}$

(m) $\frac{8}{21}\div\frac{9}{14}$ **(n)** $\frac{10}{21}\div\frac{19}{14}$ **(o)** $\frac{20}{33}\div\frac{15}{44}$ **(p)** $\frac{3}{5}\div\frac{3}{4}$

(q) $\frac{8}{15}\div\frac{2}{3}$ **(r)** $\frac{11}{36}\div\frac{22}{24}$ **(s)** $\frac{10}{33}\div\frac{25}{36}$ **(t)** $\frac{4}{5}\div\frac{2}{15}$

4. Express as a single fraction:

(a) $5\div 1\frac{1}{4}$ **(b)** $7\frac{1}{2}\div 2\frac{1}{2}$ **(c)** $3\frac{1}{2}\div 1\frac{3}{4}$ **(d)** $1\frac{1}{10}\div 1\frac{1}{5}$

(e) $1\frac{3}{8}\div 2\frac{1}{4}$ **(f)** $2\frac{6}{7}\div 1\frac{1}{14}$ **(g)** $2\frac{2}{3}\div 1\frac{7}{9}$ **(h)** $1\frac{5}{12}\div 3\frac{3}{16}$

(i) $3\frac{3}{5}\div 2\frac{1}{4}$ **(j)** $1\frac{11}{24}\div\frac{14}{15}$ **(k)** $3\frac{11}{15}\div 1\frac{7}{25}$ **(l)** $1\frac{9}{35}\div\frac{8}{15}$

(m) $1\frac{7}{20}\div 4\frac{4}{5}$ **(n)** $4\frac{4}{9}\div 2\frac{1}{12}$ **(o)** $2\frac{11}{12}\div 3\frac{1}{9}$ **(p)** $6\frac{2}{3}\div 2\frac{1}{2}$

(q) $5\frac{2}{5}\div 6\frac{2}{5}$ **(r)** $1\frac{1}{2}\div 1\frac{3}{7}$ **(s)** $4\frac{1}{5}\div 3\frac{1}{2}$ **(t)** $1\frac{2}{3}\div 2\frac{2}{9}$

5. Express as a single fraction:

(a) $\frac{3}{4}\times\frac{2}{9}\times\frac{3}{8}$ **(b)** $\frac{2}{11}$ of $5\frac{1}{2}\times\frac{1}{6}$ **(c)** $\frac{1}{3}\times\frac{3}{5}\times2\frac{1}{7}$

(d) $3\frac{1}{3}\times1\frac{1}{8}\times8\frac{1}{3}$ **(e)** $16\times4\frac{3}{8}\times\frac{3}{5}$ **(f)** $6\frac{3}{4}\times2\frac{1}{9}\times\frac{2}{19}$

(g) $(3\frac{1}{3}\times1\frac{1}{2})\div7\frac{1}{2}$ **(h)** $(\frac{11}{14}$ of $2\frac{4}{5})\div\frac{3}{4}$ **(i)** $(\frac{1}{11}$ of $5\frac{1}{2})\div6\frac{2}{3}$

(j) $18\div(\frac{9}{10}$ of $4\frac{2}{7})$ **(k)** $6\frac{1}{4}\div(7\frac{1}{5}$ of $\frac{5}{18})$ **(l)** $\frac{4}{13}\div(1\frac{3}{11}$ of $6\frac{2}{7})$

(m) $(5\frac{5}{8}\div3\frac{3}{4})\times\frac{1}{12}$ **(n)** $(\frac{13}{16}\div\frac{26}{49})\times\frac{8}{21}$ **(o)** $\frac{5}{24}\div(\frac{3}{8}$ of $\frac{7}{9})$

(p) $9\frac{1}{4}\div(17\frac{1}{2}$ of $10\frac{4}{7})$ **(q)** $(8\frac{1}{3}\times1\frac{1}{2})\div\frac{3}{4}$ **(r)** $(\frac{13}{30}\div3\frac{7}{15})\times\frac{8}{9}$

6. A sack of potatoes weighs 11 kgs.

(a) How many bags each weighing $1\frac{3}{4}$ kgs can be filled from the bag?

(b) What weight of potatoes would be left over?

7. A twenty – one metre length of fabric is cut into $\frac{5}{8}$ metre pieces.

(a) How many pieces can be cut?

(b) What length of fabric would be left over?

8. A triangle has base $2\frac{3}{4}$ cm and height $3\frac{2}{5}$ cm. Calculate its area.

9. A rectangle measures $5\frac{1}{4}$ metres by $5\frac{2}{3}$ metres. Calculate its area.

MIXED QUESTIONS on FRACTIONS

1. Anne mixed $1\frac{2}{3}$ kgs of flour with $1\frac{1}{4}$ kgs of sugar.

What is the total of flour and sugar?

2. Brendan ran $6\frac{3}{4}$ km of a 10 km run. How far did he still have to run?

3. At a Christmas party, David drank $1\frac{3}{4}$ litres of fruit punch. Simon drank $\frac{7}{8}$ of a litre and John drank $1\frac{1}{3}$ litres.

(a) How much fruit punch did they drink altogether?

(b) If the bowl held 8 litres altogether, how much was left over?

4. A garden is rectangular in shape and measures $7\frac{4}{5}$ metres by $10\frac{2}{3}$ metres.

(a) Calculate the perimeter of the garden.

(b) Find its area.

5. Billy is a long distance lorry driver. One day he had to drive to Birmingham. He drove for $2\frac{1}{2}$ hours at a speed of 76km/h and then for $3\frac{2}{3}$ hours at a speed of 81km/h before arriving at his destination.

(a) How far did he drive during the first part of his journey?

(b) How far did he drive during the second part?

(c) How far did he travel altogether?

(d) How many hours did it take him in total?

(e) Calculate Billy's average speed for the whole journey.

6. Laura has applied to join the RAF and has to sit an 'Entrance Test'. Part of it includes some problems with fractions. Here are the answers that Laura worked out.

(a) $\frac{5}{6}+\frac{3}{7}$ **(b)** $3\frac{3}{8}-\frac{5}{16}$ **(c)** $\frac{11}{15}\times\frac{3}{22}$ **(d)** $\frac{34}{45}\div\frac{85}{102}$

$\frac{8}{13}$	$3\frac{1}{16}$	$\frac{1}{10}$	$\frac{17}{27}$

(e) A plank of wood $3\frac{3}{4}$ metres long is cut up into 5 equal pieces.

How long is each piece?

$\frac{3}{4}$ metre

(f) Each cow in a herd of 25 produces $4\frac{2}{3}$ litres of milk.

How much milk is this in total?

$116\frac{2}{3}$ litres

(i) How many did Laura get correct out of the six questions?

(ii) Write down the correct answers for the ones that Laura got wrong.

COMPARING DATA SETS

QUARTILES AND INTERQUARTILE RANGE

1. For each of the data sets below find the median, lower quartile, upper quartile and interquartile range.

(a)	2	4	4	6	7	8	10	14	15			
(b)	29	30	32	33	34	37	40					
(c)	17	19	20	22	23	25	26					
(d)	0	0	0	1	1	2	2	2	3	3	4	
(e)	1·8	1·8	2·8	2·9	4·0	4·0	4·0	4·7	5·1	5·2	5·3	
(f)	0·13	0·18	0·18	0·19	0·25	0·26	0·29	0·29	0·30	0·31	0·33	0·39
(g)	133	136	136	138	140	141	143	145				
(h)	371	375	376	379	380	384	385	387	389	390		
(i)	57	58	58	60	63	67	67	69	82	85	86	90
(j)	11	11	11	12	13	14	15	15	16	18	20	

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2. For each of the data sets below find the median, lower quartile, upper quartile and interquartile range.

(a)	47	56	58	48	60	65	50	52	61	53	63	
(b)	12	20	27	15	35	16	26	34	38	24	26	
(c)	149	165	154	167	170	179	151	168	158			
(d)	1	8	3	1	2	5	3	1	4	3	2	
(e)	108	114	132	95	144	120	116	125	172	188	155	160
(f)	65	74	59	43	63	52	48	63	67	85	92	48
(g)	190	165	174	187	166	172	184	190	166	183	180	
(h)	325	363	347	359	314	329	364	372	301	317	346	
(i)	0·5	1·3	0·4	1·0	0·9	1·4	0·8	0·9	1·1	0·6		
(j)	10	13	11	11	20	10	10	14	50	10	11	10

COMPARING DATA SETS

STANDARD DEVIATION

1. Calculate the mean and standard deviation for the following sets of data.

(a)	20	21	19	22	21	20	19	20	21	20	
(b)	303	299	306	298	304	307	299	302	305	299	300
(c)	15·3	14·9	15·1	15·2	14·8	14·7	15·1	14·8	15·0	15·0	
(d)	87	89	84	88	89	87	86	87	86	87	
(e)	48	73	29	82	54	43	95	41	92	71	
(f)	4·4	4·6	4·8	4·0	4·2	4·3	4·5	4·7	4·9	4·1	
(g)	0·2	0·3	0·4	0·2	0·2	0·0	0·4	0·1	0·2	0·3	
(h)	40	40	39	38	38	40	40	42	40	39	

2. A third year pupil conducting an experiment with a die got the following results

6	1	1	4	4	2	2	6	5	6
1	1	1	5	1	4	2	3	4	6
1	4	4	1	5	4	4	3	6	2
5	3	5	6	3	2	6	5	5	2
3	1	4	5	2	4	1	4	4	3

(a) Show these results in a frequency table

(b) Use your table to calculate the mean and standard deviation.

3. A company that manufactures shoelaces spot checks the length (in cm) of the laces.

Here are the results for two different production lines.

Line A	26·8	27·2	26·5	27·0	27·3	27·5	26·1	26·4	27·9	27·3
Line B	26·8	26·7	27·1	27·0	26·9	27·0	27·3	26·9	27·0	27·3

Calculate the mean and standard deviation and comment on any differences between line A and line B.

4. The running times, in minutes, of films shown on television over a week are as follows.

110	95	135	70	100	125	140	105	95	105
95	95	110	90	110	100	125	105	90	120

Calculate the mean and standard deviation.

5. The temperatures, in ^{o}C, at a seaside resort were recorded at noon over a 10-day period.

19	20	19	17	21	18	19	24	25	28

Calculate the mean and standard deviation.

6. John James plays golf with his brother Joe each month. They keep a note of their scores.

John	74	73	74	73	71	73	72	75	73	73	72	73
Joe	68	74	70	67	80	81	69	68	79	67	70	71

Calculate the mean and standard deviation and comment on John's and Joe's performance over the year.

7. The weekly takings in small store, to the nearest £, for a week in December and March are shown below

December	2131	2893	2429	3519	4096	4810
March	1727	2148	1825	2397	2901	3114

Calculate the mean and standard deviation and comment on any differences.

8. Two sixth year classes take part in a Sponsored Fast for Famine Relief. The number of hours each pupil lasted are shown below.

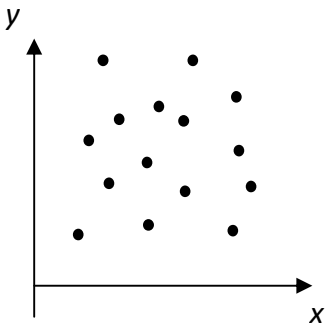
6C1	20	22	21	20	22	20	22	20	20	24	21	22	23	22	22	23
6C2	15	20	24	23	22	24	18	24	22	23	24	17	20	24	24	20

Calculate the mean and standard deviation for each class and comment on how well each class did.

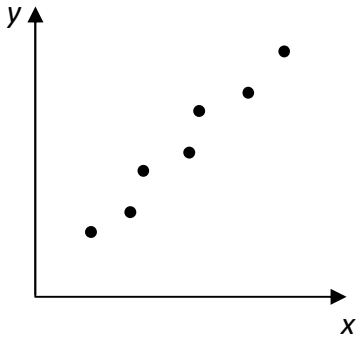
4.2 FORMING a LINEAR MODEL from a given SET of DATA

1. Using the words positive, negative or no relation, describe the correlation in each of the diagrams below.

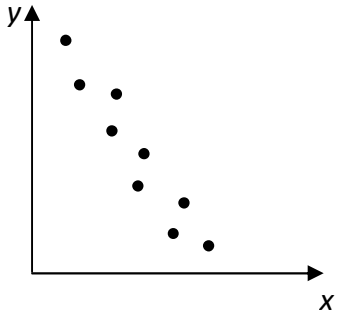
(a)



(b)

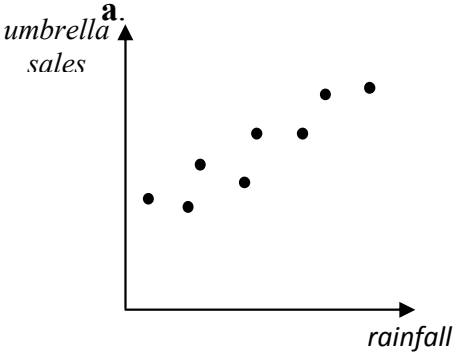


(c)

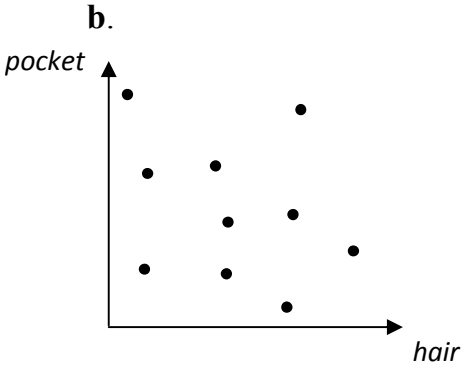


2. What do the diagrams tell you about the correlation between the two variables involved ?

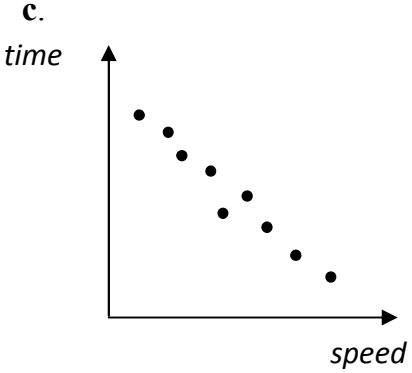
a.



b.



c.



3. A random survey of 20 pupils gave the following results

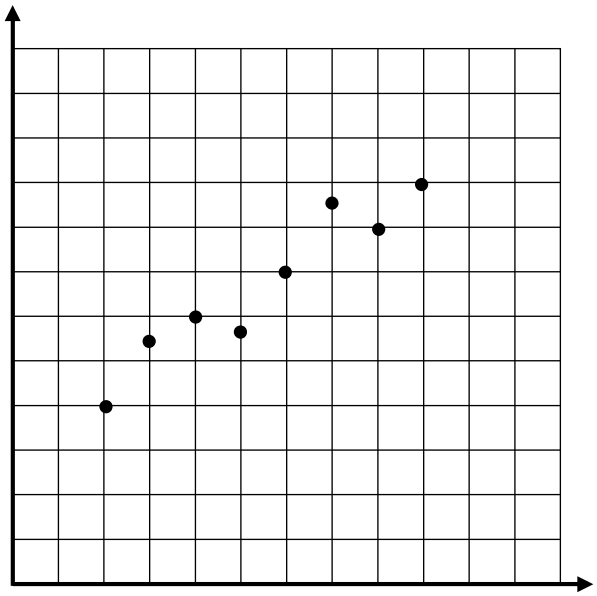
Pupil	1	2	3	4	5	6	7	8	9	10
Age	16	17	14	17	14	12	12	16	18	15
Height(cm)	182	199	171	200	183	159	170	179	198	180
Weight (kg)	71	78	69	66	54	60	46	72	76	63
Cash carried (£)	4·23	10·90	25·50	1·43	2·98	6·24	3·18	0·72	1·98	0·25

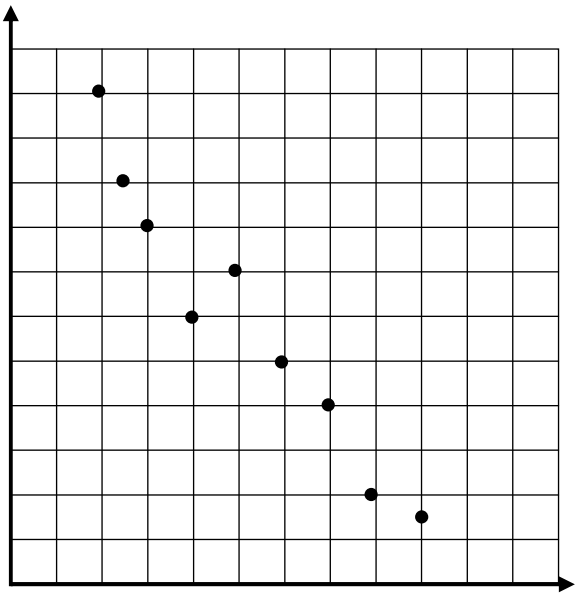
Pupil	11	12	13	14	15	16	17	18	19	20
Age	18	18	17	16	11	11	13	12	14	14
Height (cm)	190	179	187	169	160	151	150	171	170	182
Weight (kg)	68	75	77	76	49	41	55	53	60	67
Cash carried	12·06	4·31	2·38	12·30	2·15	4·12	2·71	0·40	1·80	3·10

Draw a scatter diagram to find out if there is a correlation between

(a) age and height
(b) height and weight
(c) age and weight
(d) age and amount of cash carried.

4. Copy these graphs and use your ruler to draw what you think is the line of best fit.





5. For the following sets of data, draw a scatter diagram and find the equation of the line of best fit.

(a)

x	1	2	3	4	5
y	5	7	8	10	12

(b)

x	1	2	3	4	5
y	2	2·5	2·5	3·5	3

(c)

x	6	7	8	9	10
y	1	2	4	4·5	6

(d)

x	1	2	3	4	5
y	8	6	5	4	2

(e)

x	1	2	3	4	5
y	8	10	8	5	3

(f)

x	5	6	7	8	9
y	6	5·5	5·4	5·5	5

6. The height of a plant measured over five days is shown below.

Days (D)	1	2	3	4	5
Height (H)	1·6	1·9	2·5	3·4	3·5

(a) Plot the points and draw the best fitting straight line through them
(b) Work out the equation of the line.
(c) Use your line to estimate the height after 1½ days.

7. The table shows the results of an experiment.

x	1	2	3	4	5	6
y	9·2	12·0	18·3	19·0	25·1	30·2

Plot the points, draw a best fitting straight line and find its equation.

8. The results below show the length of a spring when a force is applied.

Force (F)	1	2	3	4	5	6
Length (l)	3·0	3·9	4·8	5·9	6·9	8·1

(a) Plot the points and draw the best fitting straight line through them.
(b) Find the equation of the line.
(c) Use your graph to estimate the length when a force of 4·5 is applied.

9. The following table gives the temperature of a bottle of water as it cools.

Time, min (T)	1	3	5	7	9
Temperature (^oC)	66	61	57	53	50

(a) Plot the points and draw the best fitting straight line through them.
(b) Find the equation of the line.
(c) Use your graph to estimate the temperature after 2½ minutes.

10. The following table shows the speed of a car accelerating from rest.

Time (secs)	0	2	6	8	12	16
Speed (mph)	0	14	44	56	82	110

(a) Plot the points and draw the best fitting straight line through them.
(b) Find the equation of the line.
(c) Use your graph to estimate the speed after 10 seconds.

11. A restaurant manager finds that the cost of running his restaurant depends on the number of meals served.

Number of meals	10	20	30	40	50	60
Cost in £	188	192	220	216	232	248

(a) Plot the points and draw the best fitting straight line through them.
(b) Find the equation of the line.
(c) Use your equation to estimate the cost when 35 meals are served.

12. The results of an experiment are shown in the table below.

V	0	0·35	0·6	0·95	1·2	1·3
R	0·60	0·48	0·33	0·18	0·11	0·05

(a) Plot the points and draw the best fitting straight line through them.
(b) Find the equation of the line.
(c) Use your graph to estimate ***R*** when ***V*** is 0.8.

EXAM QUESTIONS

MEAN and STANDARD DEVIATION

1. The weights of 6 plums are

40·5g 37·8g 42·1g 35·9g 46·3g 41·6g

(a) Calculate the mean and standard deviation.

The weights of 6 apples are

140·5g 137·8g 142·1g 135·9g 146·3g 141·6g

(b) **Write down** the mean and standard deviation.

2. During a recent rowing competition the times, in minutes, recorded for a 2000 metre race were

$7 \cdot 2$ $7 \cdot 3$ $7 \cdot 3$ $7 \cdot 5$ $7 \cdot 6$ $8 \cdot 4$

(a) Calculate the mean and standard deviation of these times. Give both answers correct to 2 decimal places.

(b) In the next race the mean time was $7 \cdot 76$ and the standard deviation was $0 \cdot 49$.

Make two valid comments about this race compared to the one in part (a).

3. 6 friends joined "Super Slimmers", a weight loss class. Their weights were recorded and the results are shown below.

65kg 72kg 74kg 81kg 90kg 98kg

(a) Calculate the mean and standard deviation of the weights.

After 6 weeks the mean weight was 74kg and the standard deviation was 8·6

(b) Compare the mean and standard deviation of the friend's weights.

4. Stewart and Jenni complete a crossword puzzle every day. Here are the times (in minutes) that Stewart took to complete it each day for a week.

63 71 68 59 69 75 57

(a) Calculate the mean and standard deviation for Stewart's times.

Every day Jenni took exactly 5 minutes longer than Stewart to complete the puzzle.

(b) Write down Jenni's mean and standard deviation.

5. The number of hours spent studying by a group of 6 student nurses over a week were

20 23 14 21 27 24

(a) Calculate the mean and standard deviation of this data.

(b) A group of student teachers had a mean of 21·5 and a standard deviation of 6.

Make two valid comments to compare the study times of the 2 groups of students.

6. Barbara is looking for a new 'A-Pod' and searches for the best deal.

The costs of the 'A-Pod' are shown below.

£175 £185 £115 £87 £150 £230

(a) Calculate the mean and standard deviation of the above data.

(b) A leading competitor, the 'E-Pod', has a mean price of £170 and a standard deviation of 26·7. Make **two** valid comparisons between the 2 products.

7. In Bramley's Toy Shop there are 6 styles of teddy bear. The price of each is sh

£19 £25 £17 £32 £20 £22

(a) Calculate the mean and standard deviation of these prices.

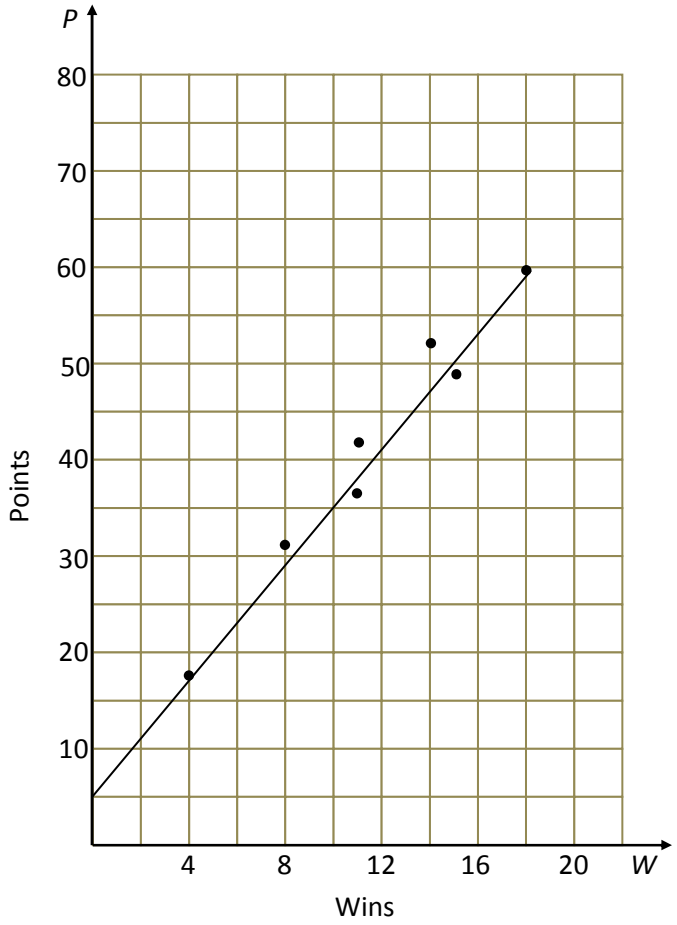
In the same shop the prices of the dolls have a mean of £22.50 and a standard deviation of 2·3. .

(b) Compare the two sets of data making particular reference to the spread of the prices.

EXAM QUESTIONS

FORMING a LINEAR MODEL from a given SET of DATA

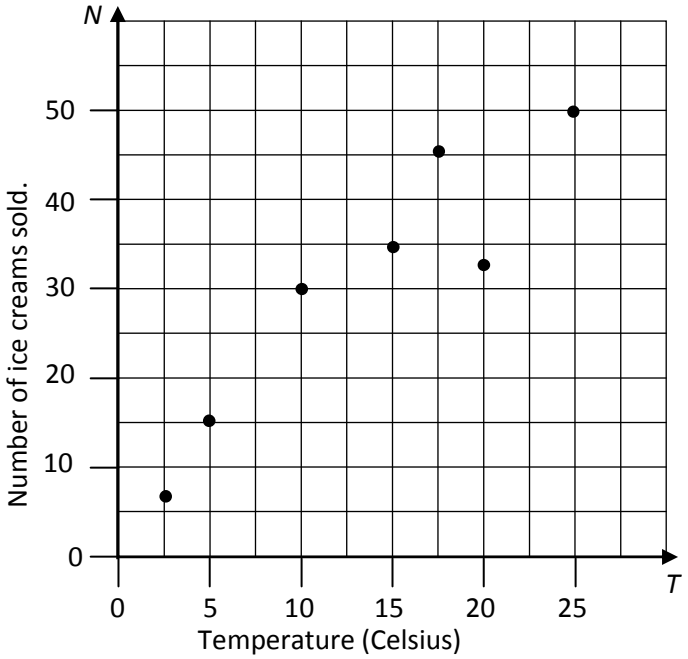
1. A selection of the number of games won and the total points gained by teams in the Scottish Premier League were plotted on this scattergraph and the line of best fit was drawn.



(a) Find the equation of the line of best fit.

(b) Use your equation to calculate the points gained by a team who won 27 matches.

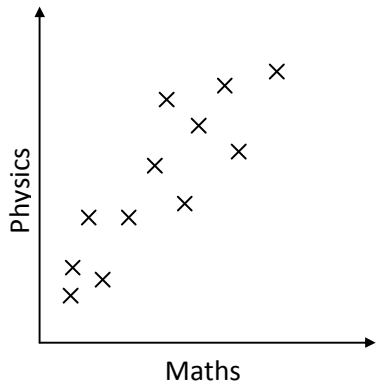
2. The graph below shows the temperature and sales of ice cream for one week during the summer.



(a) Make a copy of the graph and draw the line of best fit on it.

(b) Find the equation of the best-fit line.

3. The scattergraph shows the marks gained in Physics and Maths by a group of college students.



Which of the following statements best describes the correlation between the 2 sets of marks?

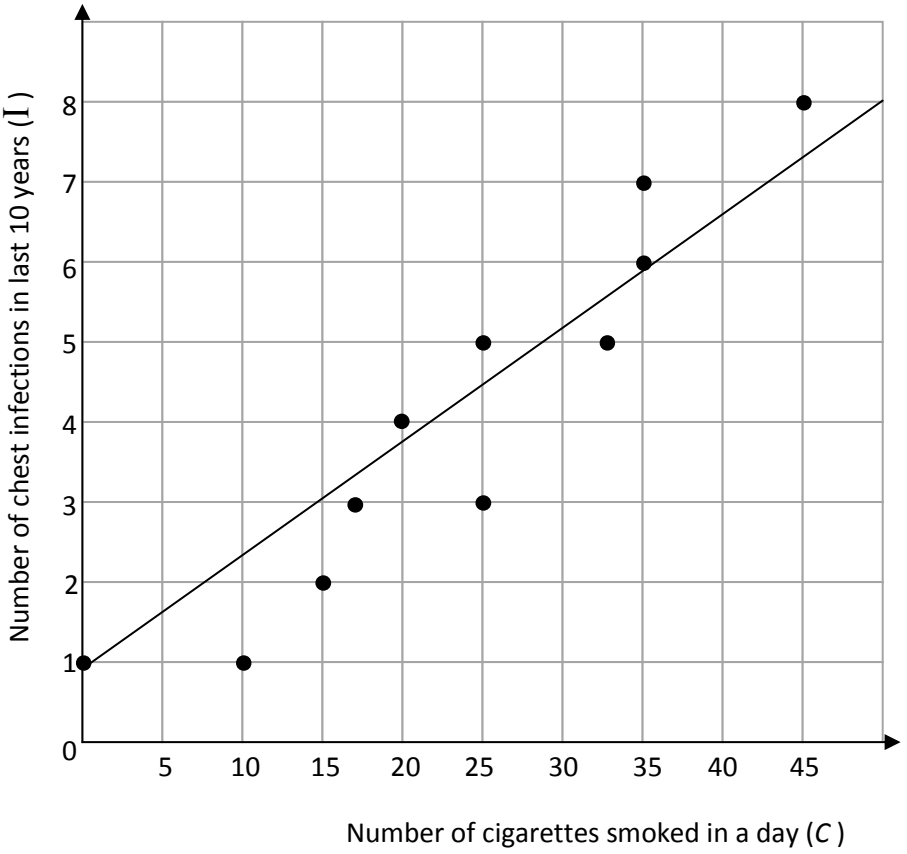
A strong positive correlation

B strong negative correlation

C weak positive correlation

D weak negative correlation

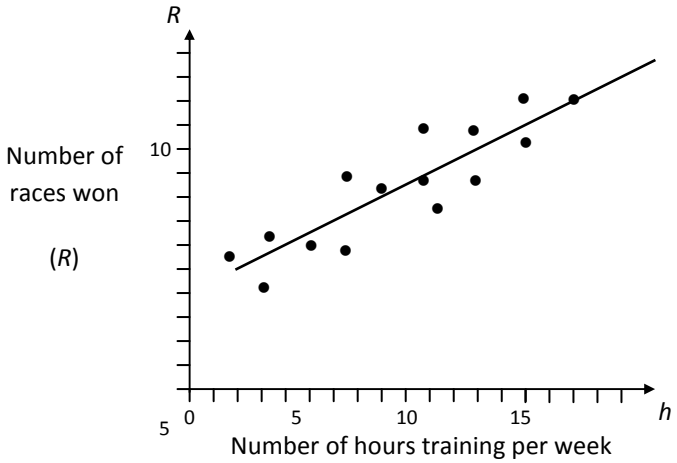
4. A group of smokers were asked how many cigarettes they smoked in a day and how many chest infections they had suffered in the last ten years. The results are shown in the scattergraph with the line of best fit drawn.



(a) Comment on the correlation between the 2 sets of data.

(b) Find the equation of the line of best fit.

5. The graph below shows the relationship between the number of hours (h) a swimmer trains per week and the number of races (R) they have won.



A best fitting straight line has been drawn.

(a) Use information from the graph to find the equation of this line of best fit.

(b) Use the equation to predict how many races a swimmer who trains 22 hours per week should win.

ANSWERS

CALCULATING the AREA of a TRIANGLE using TRIGONOMETRY

1. **(a)** $19{\cdot}3cm^2$ **(b)** $61{\cdot}4m^2$ **(c)** $23{\cdot}6mm^2$
(d) $2{\cdot}7cm^2$ **(e)** $298{\cdot}8cm^2$ **(f)** 119·4cm

2. **(a)** $311{\cdot}8cm^2$ (b) $75{\cdot}8cm^2$

3. **(a)** 13 cm^2 **(b)** 16·5 cm^2 **(c)** 43·3 cm^2 **(d)** 84·9 cm^2
(e) 54·8 cm^2 **(f)** 19·3 cm^2 **(g)** 16·8 cm^2 **(h)** 14·8 cm^2
(i) 211·3 cm^2 **(j)** 47·6 cm^2

4. 1·7 m^2 **5.** 46·5cm²

6. **(a)** 0·93 m^2 **(b)** 13 m^2 **7.** 150^o **8.** 35m²

9. 132^o

1.2 USING the SINE RULE to CALCULATE a SIDE

1. **(a)** 10·6cm **(b)** 26·2cm **(c)** 14·1cm **(d)** 3·5cm
(e) 1·2m **(f)** 2·8cm **(g)** 3mm **(h)** 4·5m

2. **(a)** 10·3 cm **(b)** 18·1 cm **(c)** 7·5 cm **(d)** 5·3 cm
(e) 19·2 cm **(f)** 5·1 cm **(g)** 12·6 cm **(h)** 8·0 cm
(i) 4·7 cm **(j)** 2·5 cm **(k)** 33·4 cm

1.2 USING the SINE RULE to CALCULATE an ANGLE

1. **(a)** $27{\cdot}2^o$ **(b)** $18{\cdot}8^o$ **(c)** $49{\cdot}0^o$ **(d)** $28{\cdot}2^o$
(e) $24{\cdot}8^o$ **(f)** $137{\cdot}3^o$ **(g)** $52{\cdot}1^o$ **(h)** 121^o

2. **(a)** 28^o **(b)** $32{\cdot}6^o$ **(c)** 41^o
(d) $122{\cdot}3^o$ **(e)** $115{\cdot}4^o$ **(f)** $79{\cdot}9^o$

1.2 USING the COSINE RULE to CALCULATE a SIDE

1. **(a)** 3·2cm **(b)** 3cm **(c)** 4·9mm
(d) 15·5cm **(e)** 4m **(f)** 2·9cm
(g) 16·2mm **(h)** 45·9m

2. **(a)** 2·5 cm **(b)** 5·9 cm **(c)** 6·1 cm **(d)** 4·6 cm
(e) 19·9 cm **(f)** 2·2 cm **(g)** 9·1 cm **(h)** 7 cm
(i) 2·9 cm **(j)** 7·5 cm **(k)** 29·9 cm

1.2 USING the COSINE RULE to CALCULATE an ANGLE

1. **(a)** $34{\cdot}9^o$ **(b)** $26{\cdot}9^o$ **(c)** $40{\cdot}8^o$
(d) $96{\cdot}4^o$ **(e)** $119{\cdot}9^o$ **(f)** $77{\cdot}4^o$

2. **(a)** $22{\cdot}3^o$ **(b)** $15{\cdot}3^o$ **(c)** $66{\cdot}4^o$ **(d)** $39{\cdot}6^o$
(e) $22{\cdot}2^o$ **(f)** $42{\cdot}0^o$ **(g)** $98{\cdot}4^o$ **(h)** $67{\cdot}3^o$

MIXED EXERCISE using TRIGONOMETRY RULES

1. **(a)** 8·5 **(b)** 6·3 **(c)** 26^o **(d)** $75{\cdot}5^o$

2. $82{\cdot}6cm^2$

3. **(a)** 11·8 **(b)** 8·2 **(c)** 110

4. **(a)** 6·7cm **(b)** 48^o **(c)** 100^o **(d)** 8·9cm **(e)** $25{\cdot}8cm^2$

5. Golfer 1: 61·7m Golfer 2: 31·5m **6.** 126·km

7. $19{\cdot}7^o$; 40·5m **8.** 185·4m **9.** 20·4cm

10. 214m **11.** 63·9km **12.** **(a)** 22^o **(b)** 82·5km

EXAM QUESTIONS using TRIGONOMETRY RULES

1. £101 390 **2.** $70{\cdot}7cm^2$ **3.** 13m **4.** Proof

5. 3·8m **6.** $9{\cdot}46m^2$ **7.** **(a)** 110^o **(b)** $51{\cdot}6m^2$

8. **(a)** 24·1cm **(b)** 94^o **(c)** 36cm

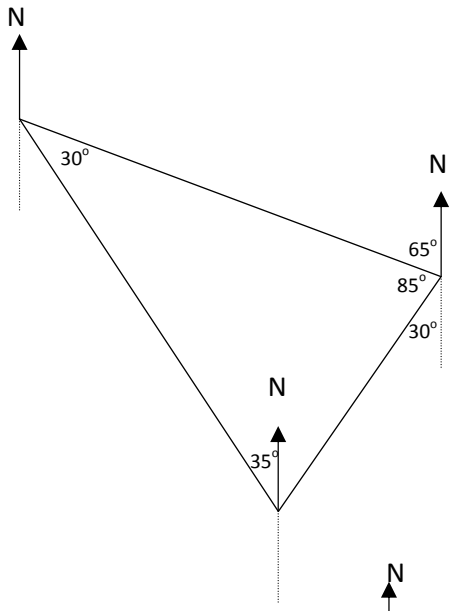
9. 108^o **10.** 11cm **11.** $30{\cdot}7^o$

12. **(a)** 13·7cm **(b)** $54{\cdot}7cm^2$ **13.** **(a)** $104{\cdot}5^o$ **(b)** ` $11{\cdot}6m^2$

14. £1313.50 **15.** **(a)** – 13/35 [or equivalent] (b) cosine is negative

BEARINGS with TRIGONOMETRY RULES

1. **(a)**



(b) **(i)** 115^o

(ii) 295^o

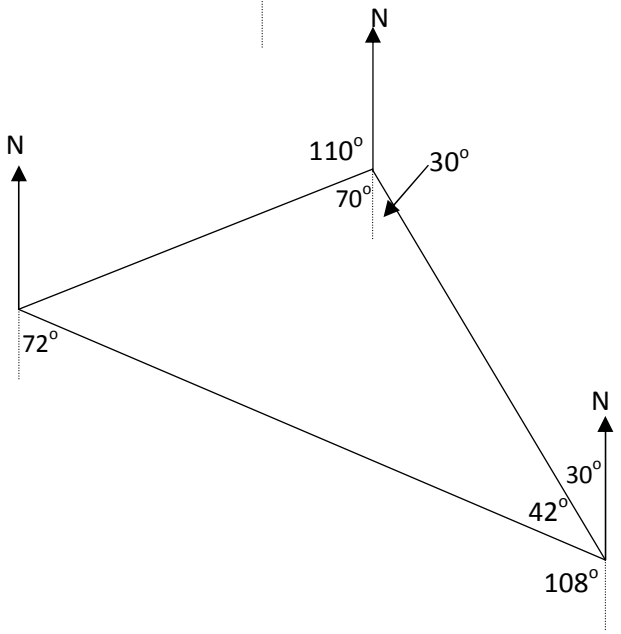
(iii) 210^o

(iv) 325^o

(v) 145^o

(vi) 030^o

2. **(a)**



(i) 070^o

(ii) 250^o

(iii) 150^o

(iv) 288^o

(v) 108^o

(vi) 330

3. **(a)** 278km **(b)** 292^o **4.** 4·77km

5. 116·0m **6.** 16·2km **7.** 249^o

8. **(a)** 124^o **(b)** 101^o **9.** 62·9km

10. **(a)** A: 64^o B: 74^o C: 42^o **(b)** 28m

EXAM QUESTIONS involving BEARINGS and TRIGONOMETRY RULES

1. **(a)** 130^o **(b)** 302^o **2.** 081^o

3. **(a)** 117^o **(b)** 256^o

4. **(a)** Abbott: 75^o Brampton: 20^o Corwood: 85^o **(b)** 68km

5. **(a)** 83^o **(b)** 089^o **6.** 27·5km

7. 1 643km **8.** 084^o **9.** **(a)** 52^o **(b)** 038^o

10. 101^o

2.2 WORKING with 2D VECTORS

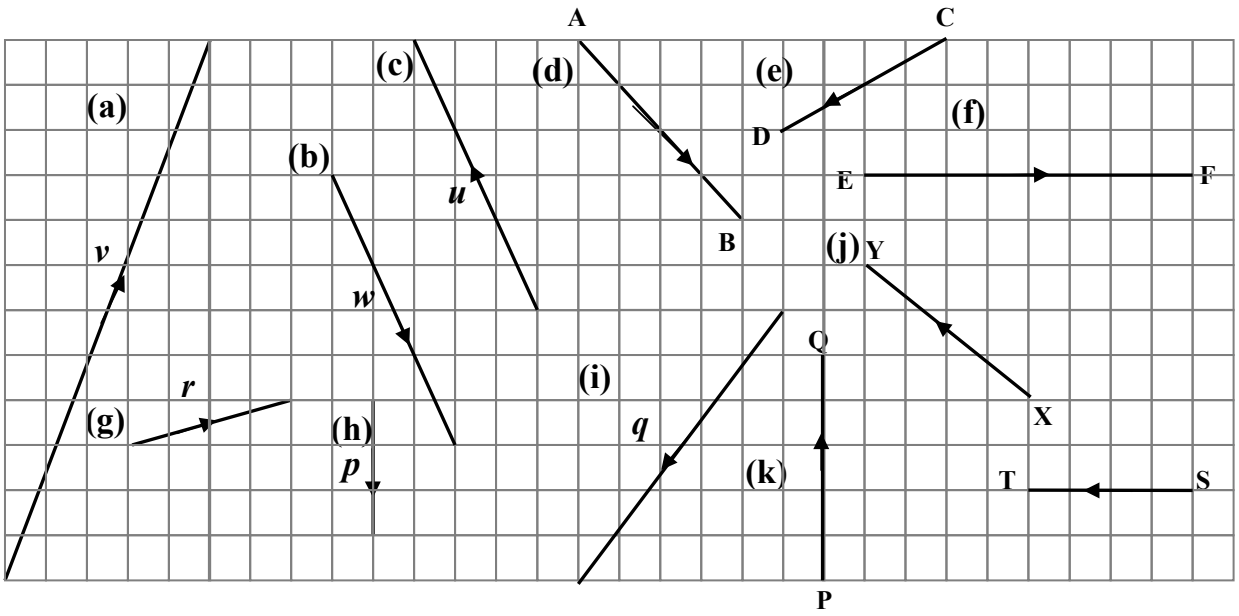
1. **(a)** $\overrightarrow{AB} = \boldsymbol{u} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ **(b)** $\overrightarrow{CD} = \boldsymbol{v} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$ **(c)** $\overrightarrow{EF} = \boldsymbol{w} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$

(d) $\overrightarrow{GH} = \boldsymbol{u} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$ **(e)** $\overrightarrow{ML} = \boldsymbol{v} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$ **(f)** $\overrightarrow{PQ} = \boldsymbol{w} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$

(g) $\overrightarrow{RS} = \boldsymbol{s} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ **(h)** $\overrightarrow{WX} = \boldsymbol{t} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ **(i)** $\overrightarrow{PT} = \boldsymbol{a} = \begin{pmatrix} 6 \\ 1 \end{pmatrix}$

(j) $\overrightarrow{RQ} = \boldsymbol{b} = \begin{pmatrix} 3 \\ -6 \end{pmatrix}$ **(k)** $\overrightarrow{CF} = \boldsymbol{c} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

2.



3. For question 1

(a)	$\sqrt{10}$	**(b)**	$2\sqrt{5}$	**(c)**	6	**(d)**	3
(e)	$2\sqrt{2}$	**(f)**	5	**(g)**	$2\sqrt{5}$	**(h)**	$\sqrt{29}$
(i)	$\sqrt{37}$	**(j)**	$3\sqrt{5}$	**(k)**	5		

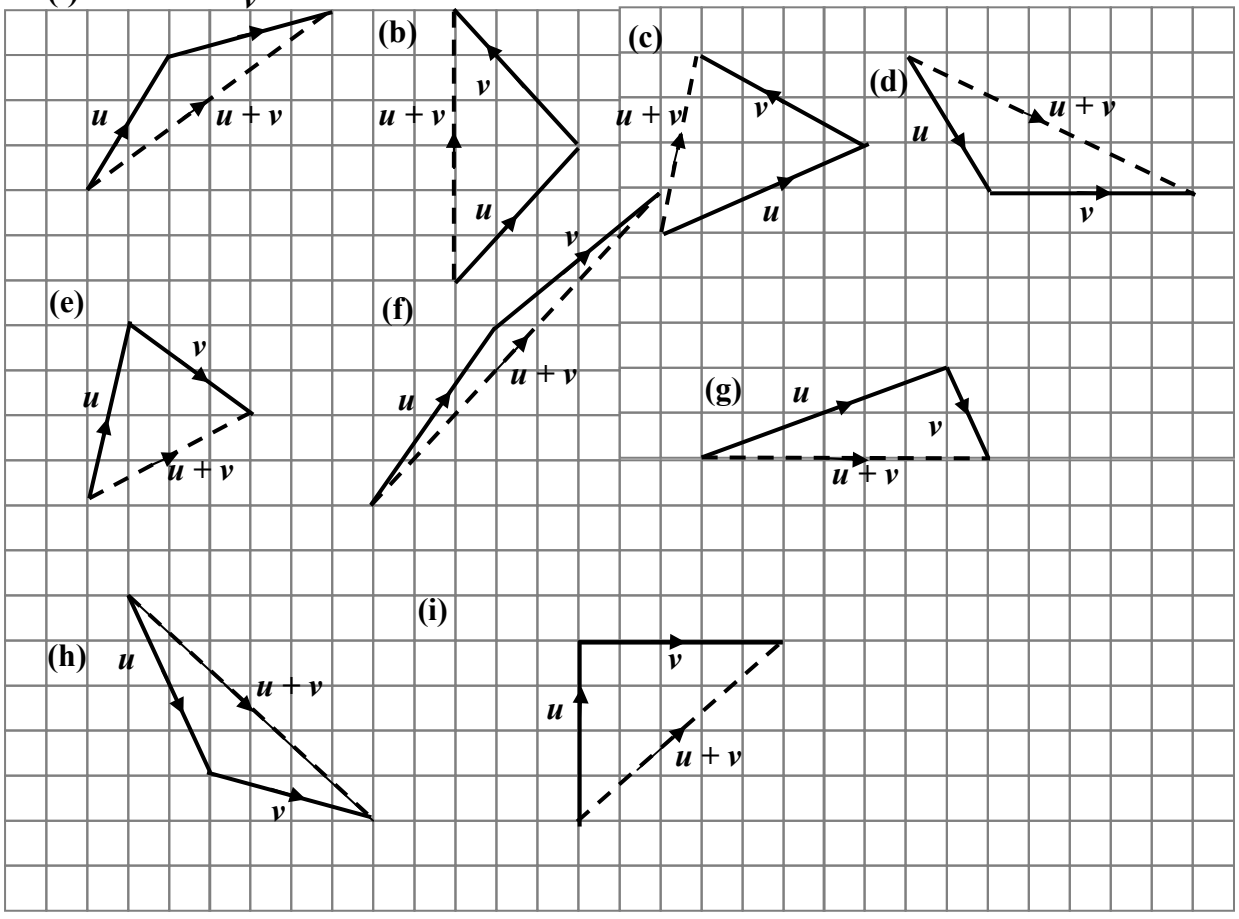
For question 2

(a)	13	**(b)**	$3\sqrt{5}$	**(c)**	$3\sqrt{5}$	**(d)**	$4\sqrt{2}$
(e)	$2\sqrt{5}$	**(f)**	8	**(g)**	$\sqrt{17}$	**(h)**	3
(i)	$\sqrt{61}$	**(j)**	5	**(k)**	5	**(l)**	4

4. **(a)** 5 **(b)** 25 **(c)** 13 **(d)** 10 **(e)** 5 **(f)** 13

2.2 ADDITION of VECTORS using DIRECTED LINE SEGMENTS

1. **(i)**

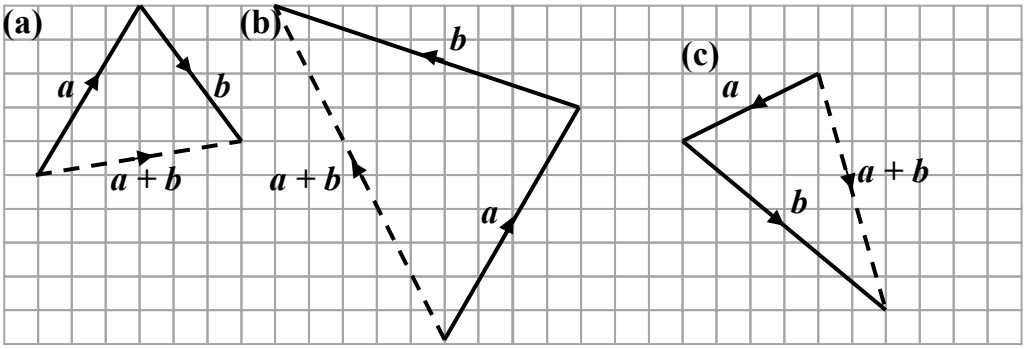


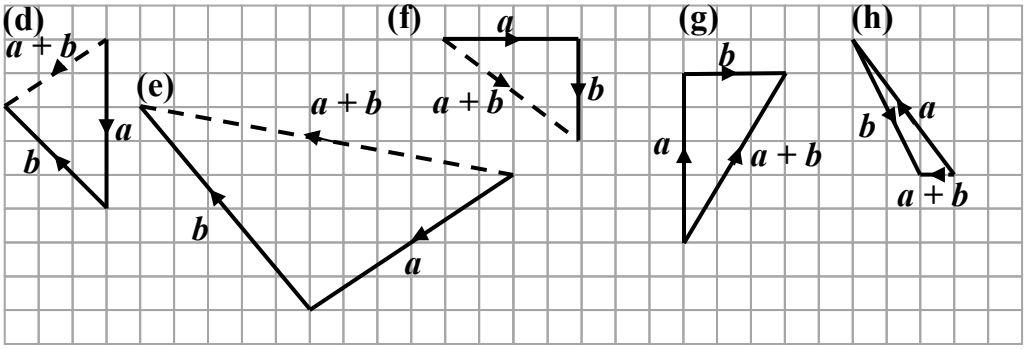
(ii) **(a)** $\begin{pmatrix}6\\4\end{pmatrix}$; $2\sqrt{13}$ **(b)** $\begin{pmatrix}0\\6\end{pmatrix}$; 6 **(c)** $\begin{pmatrix}1\\4\end{pmatrix}$; $\sqrt{17}$

(d) $\begin{pmatrix}7\\-3\end{pmatrix}$; $\sqrt{58}$ **(e)** $\begin{pmatrix}4\\2\end{pmatrix}$; $2\sqrt{5}$ **(f)** $\begin{pmatrix}7\\7\end{pmatrix}$; $7\sqrt{2}$

(g) $\begin{pmatrix}7\\0\end{pmatrix}$; 7 **(h)** $\begin{pmatrix}6\\-5\end{pmatrix}$; $\sqrt{61}$ **(i)** $\begin{pmatrix}5\\4\end{pmatrix}$; $\sqrt{41}$

2. **(i)**

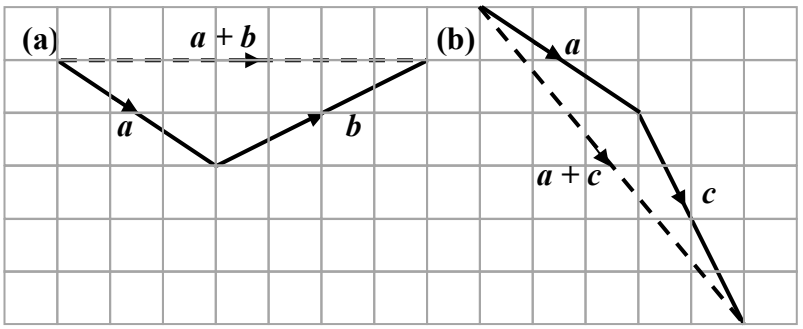


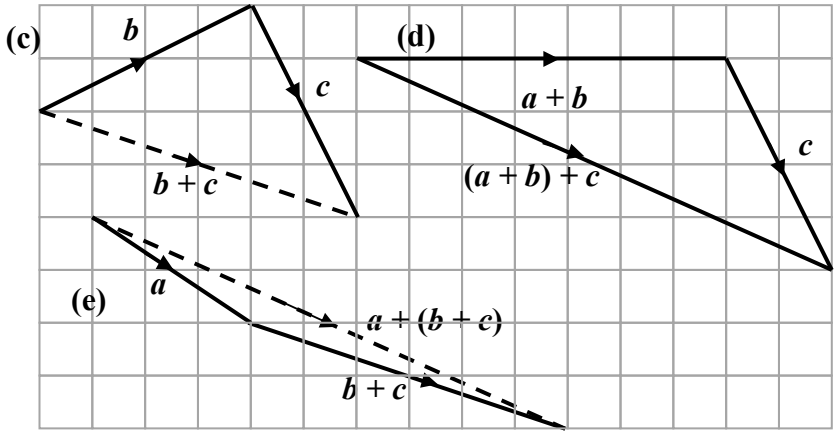


(ii) **(a)** $\begin{pmatrix}6\\1\end{pmatrix}$; $\sqrt{37}$ **(b)** $\begin{pmatrix}-5\\10\end{pmatrix}$; $5\sqrt{5}$ **(c)** $\begin{pmatrix}2\\-7\end{pmatrix}$; $\sqrt{53}$ **(d)** $\begin{pmatrix}-3\\-2\end{pmatrix}$; $\sqrt{13}$

(e) $\begin{pmatrix}-11\\2\end{pmatrix}$; $5\sqrt{5}$ **(f)** $\boldsymbol{a} = \begin{pmatrix}4\\-3\end{pmatrix}$; 5 **(g)** $\begin{pmatrix}3\\5\end{pmatrix}$; $\sqrt{34}$ **(h)** $\begin{pmatrix}-1\\0\end{pmatrix}$; 1

3. **(i)**

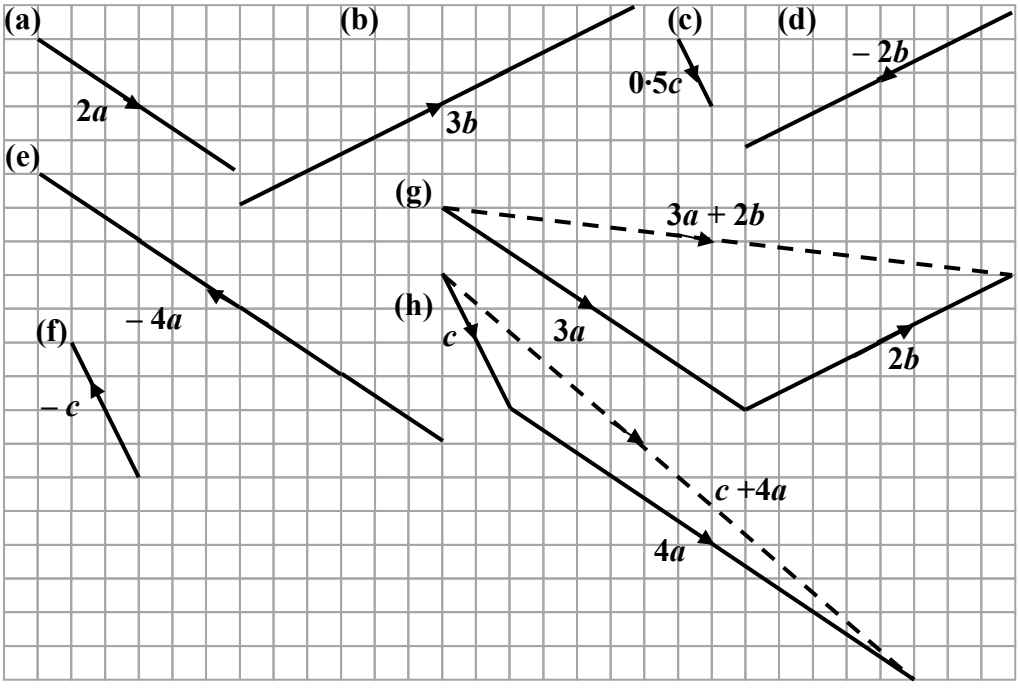




(ii) **(a)** $\begin{pmatrix}7\\0\end{pmatrix}$; 7·0 **(b)** $\begin{pmatrix}5\\-6\end{pmatrix}$; 7·8 **(c)** $\begin{pmatrix}6\\-2\end{pmatrix}$; 6·3

(d) $\begin{pmatrix}9\\-4\end{pmatrix}$; 9·8 **(e)** $\begin{pmatrix}9\\-4\end{pmatrix}$; 9·8

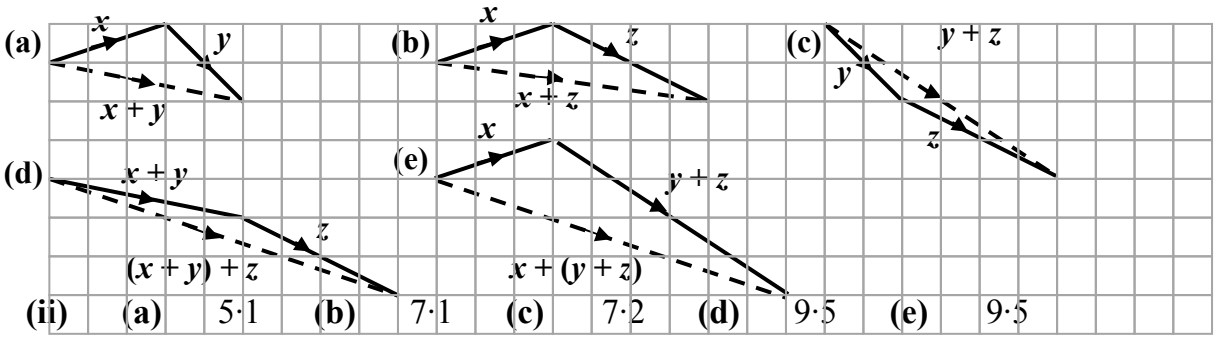
4. **(i)**



. **(ii)** **(a)** $\begin{pmatrix}6\\-4\end{pmatrix}$; $2\sqrt{13}$ **(b)** $\begin{pmatrix}12\\6\end{pmatrix}$; $6\sqrt{5}$ **(c)** $\begin{pmatrix}1\\-2\end{pmatrix}$; $\sqrt{5}$ **(d)** $\begin{pmatrix}-8\\-4\end{pmatrix}$; $4\sqrt{5}$

(e) $\begin{pmatrix}-12\\8\end{pmatrix}$; $4\sqrt{13}$ **(f)** $\begin{pmatrix}-2\\4\end{pmatrix}$; $2\sqrt{5}$ **(g)** $\begin{pmatrix}17\\-2\end{pmatrix}$; $\sqrt{293}$ **(h)** $\begin{pmatrix}14\\-12\end{pmatrix}$; $2\sqrt{85}$

5. (i)



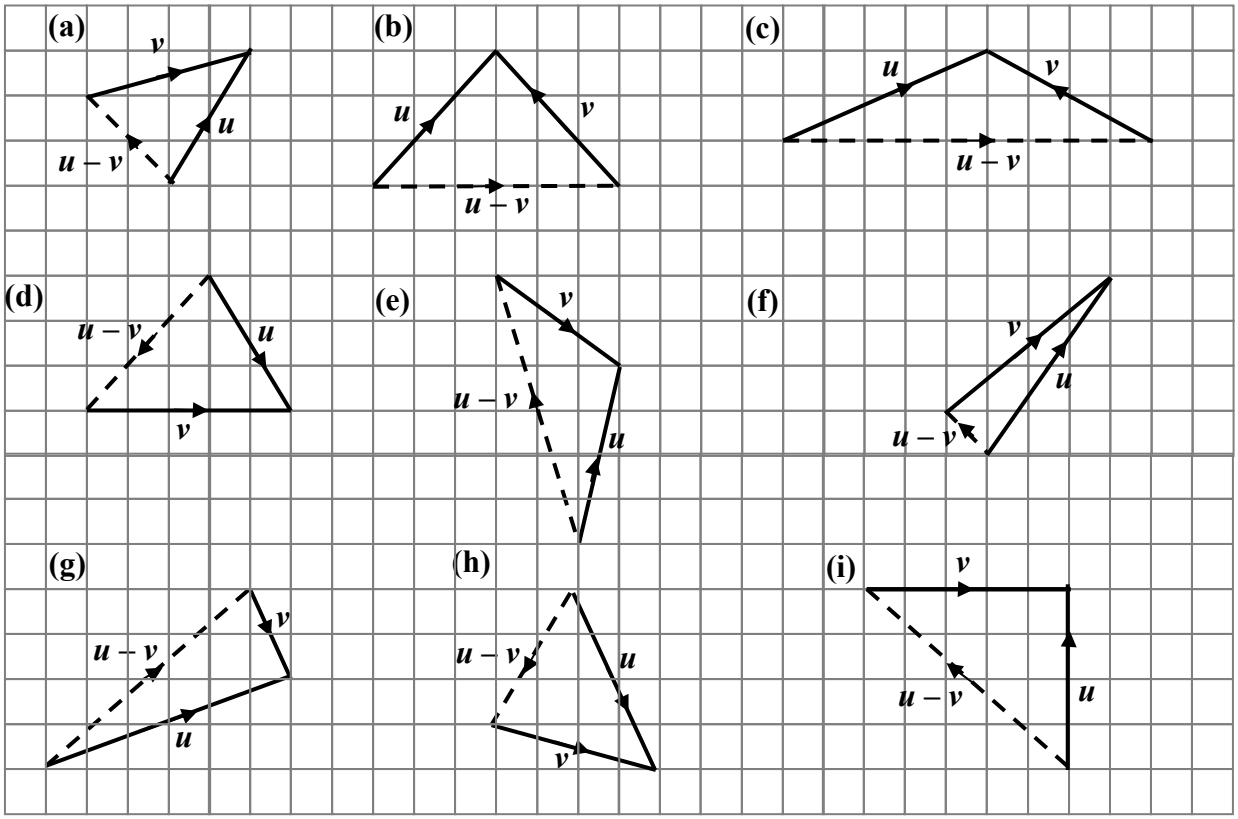
(ii) **(a)** 5·1 **(b)** 7·1 **(c)** 7·2 **(d)** 9·5 **(e)** 9·5

6. **(a)** 6·3 **(b)** 8·5 **(c)** 2·2 **(d)** 5·7

(e) 12·6 **(f)** 4·5 **(g)** 13·0 **(h)** 17·7

2.2 SUBTRACTION of VECTORS using DIRECTED LINE SEGMENTS

1. **(i)**

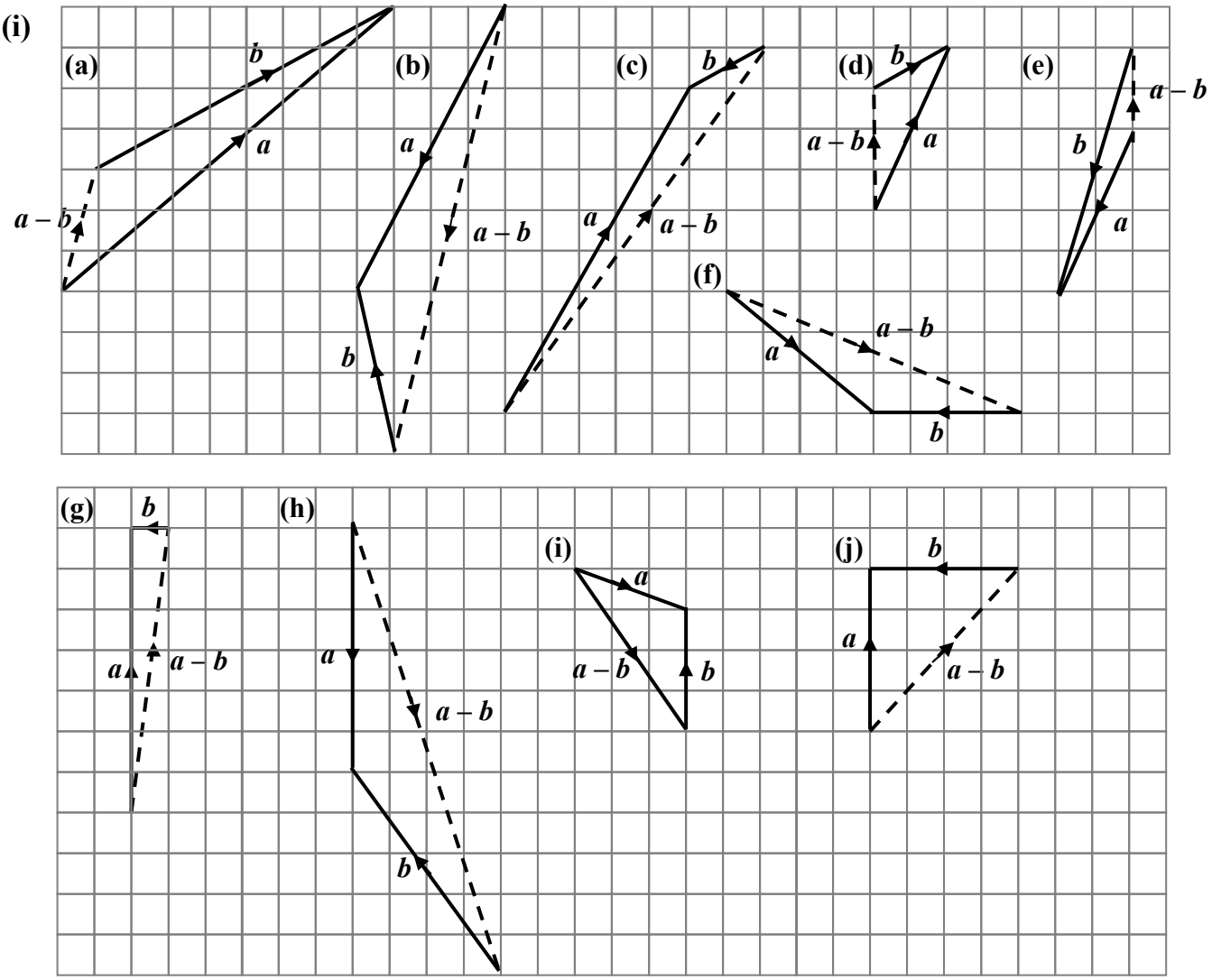


(ii) **(a)** $\begin{pmatrix}-2\\2\end{pmatrix}$; $2\sqrt{2}$ **(b)** $\begin{pmatrix}6\\0\end{pmatrix}$; 6 **(c)** $\begin{pmatrix}9\\0\end{pmatrix}$; 9 **(d)** $\begin{pmatrix}-3\\-3\end{pmatrix}$; $3\sqrt{2}$

(e) $\begin{pmatrix}-2\\6\end{pmatrix}$; $2\sqrt{10}$ **(f)** $\begin{pmatrix}-1\\1\end{pmatrix}$; $\sqrt{2}$ **(g)** $\begin{pmatrix}5\\4\end{pmatrix}$; $\sqrt{41}$ **(h)** $\begin{pmatrix}-2\\-3\end{pmatrix}$; $\sqrt{13}$

(i) $\begin{pmatrix}-5\\4\end{pmatrix}$; $\sqrt{41}$

2. **(i)**

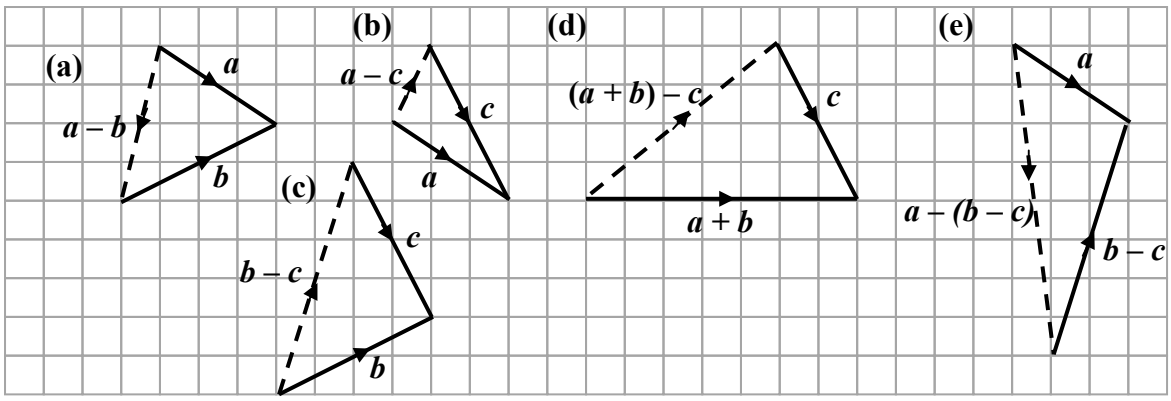


(ii) **(a)** $\begin{pmatrix}1\\3\end{pmatrix}$; 3·1 **(b)** $\begin{pmatrix}-3\\-11\end{pmatrix}$; 11·4 **(c)** $\begin{pmatrix}7\\9\end{pmatrix}$; 11·4 **(d)** $\begin{pmatrix}0\\3\end{pmatrix}$; 3

(e) $\begin{pmatrix}0\\2\end{pmatrix}$; 2 **(f)** $\begin{pmatrix}8\\-3\end{pmatrix}$; 8·5 **(g)** $\begin{pmatrix}1\\7\end{pmatrix}$; 7·1 **(h)** $\begin{pmatrix}4\\-11\end{pmatrix}$; 11·7

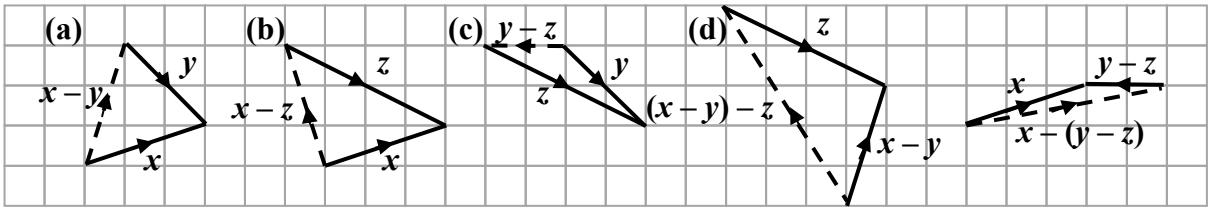
(i) $\begin{pmatrix}3\\-4\end{pmatrix}$; 5 **(j)** $\begin{pmatrix}4\\4\end{pmatrix}$; 5·7

3. **(i)**



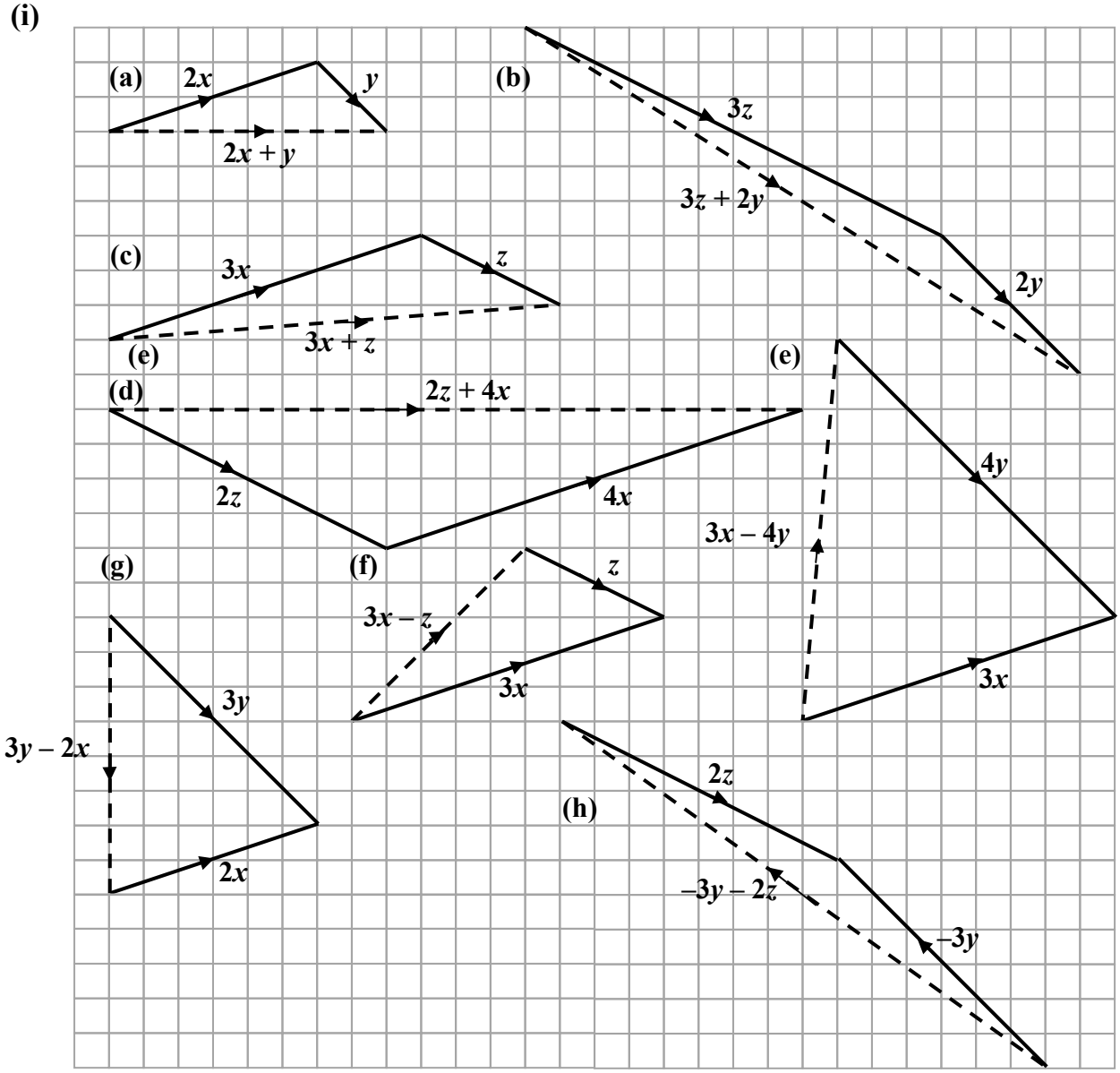
(ii) **(a)** 4·12 **(b)** 2·24 **(c)** 6·32 **(d)** 6·40 **(e)** 8·06

4. **(i)**



(ii) **(a)** $\begin{pmatrix}1\\3\end{pmatrix}$; 3·2 **(b)** $\begin{pmatrix}-1\\3\end{pmatrix}$; 3·2 **(c)** $\begin{pmatrix}-2\\0\end{pmatrix}$; 2 **(d)** $\begin{pmatrix}-3\\5\end{pmatrix}$; 5·8 **(e)** $\begin{pmatrix}5\\1\end{pmatrix}$; 5·1

5. **(i)**



(ii) **(a)** $\begin{pmatrix} 8 \\ 0 \end{pmatrix}$; 8·00 **(b)** $\begin{pmatrix} 16 \\ -10 \end{pmatrix}$; 18·9 **(c)** $\begin{pmatrix} 13 \\ 1 \end{pmatrix}$; 13·0

(d) $\begin{pmatrix} 20 \\ 0 \end{pmatrix}$; 20 **(e)** $\begin{pmatrix} 1 \\ 11 \end{pmatrix}$; 11·0 **(f)** $\begin{pmatrix} 5 \\ 5 \end{pmatrix}$; 7·07

(g) $\begin{pmatrix} 0 \\ -8 \end{pmatrix}$; 8·00 **(h)** $\begin{pmatrix} -14 \\ 10 \end{pmatrix}$; 17·2

VECTOR JOURNEYS in 2D Part 1

1. **(a)** $\boldsymbol{b}$ **(b)** $-\boldsymbol{b}$ **(c)** $-\boldsymbol{a}$ **(d)** $\boldsymbol{a}+\boldsymbol{b}$ **(e)** $-(\boldsymbol{a}+\boldsymbol{b})$

2. **(a)** $-\boldsymbol{v}$ **(b)** $-\boldsymbol{v}-\boldsymbol{w}$ **(c)** $2\boldsymbol{v}$ **(d)** $\boldsymbol{v}-\boldsymbol{w}$ **(e)** $\boldsymbol{w}-2\boldsymbol{v}$

3. **(a)** $\boldsymbol{q}-\boldsymbol{p}$ **(b)** $\boldsymbol{p}-\boldsymbol{q}$ **(c)** $\frac{1}{2}(\boldsymbol{p}-\boldsymbol{q})$ **(d)** $\frac{1}{2}(\boldsymbol{p}+\boldsymbol{q})$

4. **(a)** $\boldsymbol{b}$ **(b)** $\boldsymbol{a}$ **(c)** $\frac{1}{2}\boldsymbol{a}$ **(d)** $\boldsymbol{b}+\frac{1}{2}\boldsymbol{a}$

5. **(a)** $2\boldsymbol{y}$ **(b)** $\boldsymbol{x}-2\boldsymbol{y}$

VECTOR JOURNEYS in 2D Part 2

1. **(a)** **(i)** $\boldsymbol{b}+\boldsymbol{a}$ **(ii)** $\boldsymbol{a}-\boldsymbol{b}$ **(b)** proof

2. **(a)** $\boldsymbol{w}-\boldsymbol{v}$ **(b)** $\frac{1}{4}(\boldsymbol{w}-\boldsymbol{v})$ **(c)** $\frac{1}{4}(\boldsymbol{w}+3\boldsymbol{v})$; $\begin{pmatrix}6\\3\end{pmatrix}$

3. **(a)** $2\boldsymbol{p}-\boldsymbol{q}$ **(b)** $\frac{2}{5}(2\boldsymbol{p}-\boldsymbol{q})$ **(c)** $\frac{1}{5}(4\boldsymbol{p}+3\boldsymbol{q})$ $\begin{pmatrix}-5\\7\end{pmatrix}$; 8·6

4. **(a)** **(i)** $\boldsymbol{b}-\boldsymbol{a}$ **(ii)** $\frac{1}{3}(\boldsymbol{b}-\boldsymbol{a})$ **(iii)** $\frac{1}{3}(2\boldsymbol{a}+\boldsymbol{b})$ **(b)** Proof

2.2 WORKING with 3D COORDINATES and VECTORS

1. **(a)** $(3, 4, 2); \begin{pmatrix}3\\4\\2\end{pmatrix}$ **(b)** $(1, 5, 7); \begin{pmatrix}1\\5\\7\end{pmatrix}$ **(c)** $(-4, 6, 2); \begin{pmatrix}-4\\6\\2\end{pmatrix}$

(d) $(-1, 7, 10); \begin{pmatrix}-1\\7\\10\end{pmatrix}$ **(e)** $(0, 0, 8); \begin{pmatrix}0\\0\\8\end{pmatrix}$ **(f)** $(0, 4, 0); \begin{pmatrix}0\\4\\0\end{pmatrix}$

(g) $(5, 0, 0); \begin{pmatrix}5\\0\\0\end{pmatrix}$ **(h)** $(-3, 0, 0); \begin{pmatrix}-3\\0\\0\end{pmatrix}$ **(i)** $(0, 0, -2); \begin{pmatrix}0\\0\\-2\end{pmatrix}$

(j) $(0, -7, 0); \begin{pmatrix}0\\-7\\0\end{pmatrix}$ **(k)** $(0, -3, 8); \begin{pmatrix}0\\-3\\8\end{pmatrix}$ **(l)** $(-4, 0, 5); \begin{pmatrix}-4\\0\\5\end{pmatrix}$

(m) $(-4, -2, 6); \begin{pmatrix} -4 \\ -2 \\ 6 \end{pmatrix}$ **(n)** $(2, 0, -6); \begin{pmatrix} 2 \\ 0 \\ -6 \end{pmatrix}$ **(o)** $(7, -5, 8); \begin{pmatrix} 7 \\ -5 \\ 8 \end{pmatrix}$

(p) $(4, 0, 10); \begin{pmatrix} 4 \\ 0 \\ 10 \end{pmatrix}$ **(q)** $(9, -6, -2); \begin{pmatrix} 9 \\ -6 \\ -2 \end{pmatrix}$ **(r)** $(-3, -4, -4); \begin{pmatrix} -3 \\ -4 \\ -4 \end{pmatrix}$

2. **(a)** 5·4 **(b)** 8·7 **(c)** 7·5 **(d)** 12·2 **(e)** 8 **(f)** 4

(g) 5 **(h)** 3 **(i)** 2 **(j)** 7 **(k)** 8·5 **(l)** 6·4

(m) 7·5 **(n)** 6·3 **(o)** 11·7 **(p)** 10·8 **(q)** 11 **(q)** 6·4

3. O(0, 0, 0); A(12, 0, 0); B(12, 4, 0); C(0, 4, 0);

D(0, 0, 6); E(12, 0, 6); F(12, 4, 6); G(0, 4, 6)

4. O(0, 0, 0); A(6, 0, 0); B(6, 6, 0); C(0, 6, 0);

D(0, 0, 6); E(6, 0, 6); F(6, 6, 6); G(0, 6, 6)

5. O(0, 0, 0); A(30, 0, 0); B(30, 14, 0); C(0, 14, 0);

D(4, 7, 8); E(26, 7, 8)

6. O(0, 0, 0); P(5, 5, 20); Q(10, 0, 0); R(10, 10, 0); S(0, 10, 0)

7. $p = 3$; $q = 6$

O(0, 0, 0); A(12, 0, 0); B(12, 3, 0); C(0, 3, 0);

D(0, 0, 6); E(12, 0, 6); F(12, 3, 6); G(0, 3, 6)

8. **(a)** $\begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix}$ **(b)** $\begin{pmatrix} 3 \\ -6 \\ 2 \end{pmatrix}$ **(c)** $\begin{pmatrix} 6 \\ 0 \\ -3 \end{pmatrix}$ **(d)** $\begin{pmatrix} 0 \\ -3 \\ -4 \end{pmatrix}$ **(e)** $\begin{pmatrix} 7 \\ -2 \\ 0 \end{pmatrix}$ **(f)** $\begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix}$

9. **(a)** $\boldsymbol{v} = 3\boldsymbol{i} + 4\boldsymbol{j} + 2\boldsymbol{k}$ **(b)** $\boldsymbol{v} = 7\boldsymbol{i} - 5\boldsymbol{j} + 8\boldsymbol{k}$

(c) $\boldsymbol{v} = 9\boldsymbol{i} - 6\boldsymbol{j} - 2\boldsymbol{k}$ **(d)** $\boldsymbol{v} = 4\boldsymbol{i} + 10\boldsymbol{k}$

2.3 USING VECTOR COMPONENTS

Adding or subtracting 2 or 3 dimensional vectors using components.

1. **(a)** **(i)** $\boldsymbol{u} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \boldsymbol{v} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$ **(ii)** $\begin{pmatrix} 6 \\ 4 \end{pmatrix}$ **(iii)** $\begin{pmatrix} 2 \\ -2 \end{pmatrix}$

(iv) $\begin{pmatrix} 14 \\ 11 \end{pmatrix}$ **(v)** $\begin{pmatrix} 4 \\ -9 \end{pmatrix}$

(b) **(i)** $\boldsymbol{u} = \begin{pmatrix} 3 \\ 3 \end{pmatrix} \boldsymbol{v} = \begin{pmatrix} -3 \\ 3 \end{pmatrix}$ **(ii)** $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$ **(iii)** $\begin{pmatrix} -6 \\ 0 \end{pmatrix}$ **(iv)** $\begin{pmatrix} 3 \\ 15 \end{pmatrix}$ **(v)** $\begin{pmatrix} -21 \\ -3 \end{pmatrix}$

(c) **(i)** $\boldsymbol{u} = \begin{pmatrix} 5 \\ 2 \end{pmatrix} \boldsymbol{v} = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$ **(ii)** $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$ **(iii)** $\begin{pmatrix} -9 \\ 0 \end{pmatrix}$ **(iv)** $\begin{pmatrix} 7 \\ 10 \end{pmatrix}$ **(v)** $\begin{pmatrix} -32 \\ -2 \end{pmatrix}$

(d) **(i)** $\boldsymbol{u} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} \boldsymbol{v} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$ **(ii)** $\begin{pmatrix} 7 \\ -3 \end{pmatrix}$ **(iii)** $\begin{pmatrix} 3 \\ 3 \end{pmatrix}$ **(iv)** $\begin{pmatrix} 16 \\ -9 \end{pmatrix}$ **(v)** $\begin{pmatrix} 7 \\ 12 \end{pmatrix}$

(e) **(i)** $\boldsymbol{u} = \begin{pmatrix} 1 \\ 4 \end{pmatrix} \boldsymbol{v} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ **(ii)** $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$ **(iii)** $\begin{pmatrix} 2 \\ -6 \end{pmatrix}$ **(iv)** $\begin{pmatrix} 9 \\ 8 \end{pmatrix}$ **(v)** $\begin{pmatrix} 5 \\ -22 \end{pmatrix}$

(f) **(i)** $\boldsymbol{u} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \boldsymbol{v} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ **(ii)** $\begin{pmatrix} 7 \\ 7 \end{pmatrix}$ **(iii)** $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ **(iv)** $\begin{pmatrix} 17 \\ 18 \end{pmatrix}$ **(v)** $\begin{pmatrix} 0 \\ -7 \end{pmatrix}$

(g) **(i)** $\boldsymbol{u} = \begin{pmatrix} 6 \\ 2 \end{pmatrix} \boldsymbol{v} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ **(ii)** $\begin{pmatrix} 7 \\ 0 \end{pmatrix}$ **(iii)** $\begin{pmatrix} -5 \\ -4 \end{pmatrix}$ **(iv)** $\begin{pmatrix} 20 \\ 2 \end{pmatrix}$ **(v)** $\begin{pmatrix} -21 \\ -14 \end{pmatrix}$

(h) **(i)** $\boldsymbol{u} = \begin{pmatrix} 2 \\ -4 \end{pmatrix} \boldsymbol{v} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$ **(ii)** $\begin{pmatrix} 6 \\ -5 \end{pmatrix}$ **(iii)** $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ **(iv)** $\begin{pmatrix} 14 \\ -14 \end{pmatrix}$ **(v)** $\begin{pmatrix} 4 \\ 13 \end{pmatrix}$

(i) **(i)** $\boldsymbol{u} = \begin{pmatrix} 0 \\ 4 \end{pmatrix} \boldsymbol{v} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$ **(ii)** $\begin{pmatrix} 5 \\ 4 \end{pmatrix}$ **(iii)** $\begin{pmatrix} 5 \\ -4 \end{pmatrix}$ **(iv)** $\begin{pmatrix} 10 \\ 12 \end{pmatrix}$ **(v)** $\begin{pmatrix} 15 \\ -16 \end{pmatrix}$

2. **(a)** $\begin{pmatrix} -8 \\ 21 \end{pmatrix}$ **(b)** $\begin{pmatrix} 30 \\ -21 \end{pmatrix}$ **(c)** $\begin{pmatrix} -11 \\ 1 \end{pmatrix}$ **(d)** $\begin{pmatrix} 10 \\ 18 \end{pmatrix}$

(e) $\begin{pmatrix} 10 \\ -29 \end{pmatrix}$ **(f)** $\begin{pmatrix} -11 \\ -21 \end{pmatrix}$ **(g)** $\begin{pmatrix} 28 \\ -27 \end{pmatrix}$ **(h)** $\begin{pmatrix} -4 \\ 33 \end{pmatrix}$ **(i)** $\begin{pmatrix} 13 \\ -4 \end{pmatrix}$

3. **(a)** 5·4 **(b)** 8·6 **(c)** 3·7 **(d)** 5

(e) 7·3 **(f)** 1·7 **(g)** 3 **(e)** 13

4. **(i)** **(a)** $\begin{pmatrix}16\\30\\8\end{pmatrix}$ **(b)** $\begin{pmatrix}-18\\-39\\12\end{pmatrix}$ **(c)** $\begin{pmatrix}2\\31\\-3\end{pmatrix}$ **(d)** $\begin{pmatrix}12\\2\\18\end{pmatrix}$

(e) $\begin{pmatrix}-22\\-41\\-12\end{pmatrix}$ **(f)** $\begin{pmatrix}-14\\3\\-19\end{pmatrix}$ **(g)** $\begin{pmatrix}-22\\-29\\10\end{pmatrix}$ **(h)** $\begin{pmatrix}24\\10\\12\end{pmatrix}$

(ii) **(a)** 34·9 **(b)** 44·6 **(c)** 31·2 **(d)** 21·7

(e) 48·1 **(f)** 23·8 **(g)** 37·7 **(h)** 28·6

5. **(i)** **(a)** $\begin{pmatrix}5\\-1\\-4\end{pmatrix}$ **(b)** $\begin{pmatrix}3\\5\\-6\end{pmatrix}$ **(c)** $\begin{pmatrix}-7\\-7\\11\end{pmatrix}$ **(d)** $\begin{pmatrix}13\\3\\-14\end{pmatrix}$

(e) $\begin{pmatrix}10\\12\\-17\end{pmatrix}$ **(f)** $\begin{pmatrix}-10\\-12\\17\end{pmatrix}$ **(g)** $\begin{pmatrix}16\\-6\\-11\end{pmatrix}$ **(h)** $\begin{pmatrix}-10\\2\\8\end{pmatrix}$

(ii) **(a)** 6·5 **(b)** 8·4 **(c)** 14·8 **(d)** 19·3

(e) 23·1 **(f)** 23·1 **(g)** 20·3 **(h)** 13·0

6. **(a)** $\sqrt{38}$ **(b)** $3\sqrt{3}$ **(c)** $\sqrt{38}$ **(d)** 4 **(e)** $\sqrt{53}$

7. $a = \sqrt{3}$

8. $\begin{pmatrix}4\\16\\3\end{pmatrix}$; 16·8

9. **(a)** $\begin{pmatrix}6\\1\\-8\end{pmatrix}$ **(b)** $\begin{pmatrix}12\\2\\-16\end{pmatrix}$ **(c)** $\begin{pmatrix}\frac{-22}{3}\\\frac{-2}{3}\\-1\end{pmatrix}$

10. **(i)** **(a)** $\begin{pmatrix}10\\18\\-7\end{pmatrix}$ **(b)** $\begin{pmatrix}-24\\-12\\5\end{pmatrix}$ **(c)** $\begin{pmatrix}-8\\0\\-1\end{pmatrix}$ **(d)** $\begin{pmatrix}0\\0\\-3\end{pmatrix}$

(ii) **(a)** 21·7 **(b)** 27·3 **(c)** 8·1 **(d)** 3

11. $\begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$; $\sqrt{11}$

12. $a = 2$

13. $k = \pm\sqrt{3}$

14. 6

15. $\begin{pmatrix} -1 \\ 3 \\ 4 \end{pmatrix}$; $\sqrt{26}$ or 5·1

3.1 WORKING with PERCENTAGES

Use reverse percentages to calculate an original quantity.

1.	**(a)**	£100	**(b)**	250mm		**(c)**	£75	
	(d)	30 litres	**(e)**	420 miles		**(f)**	512 m	
	(g)	1 500 km	**(h)**	£5 250		**(i)**	12 000m	
	(j)	712 cm	**(k)**	£775		**(l)**	90 kg	

2.	**(a)**	£22	**(b)**	£45		**(c)**	£72	
	(d)	85 ml	**(e)**	£17.50		**(f)**	100 cm	
	(g)	2 550 m	**(h)**	8 500 km		**(i)**	56 m	
	(j)	£11 200	**(k)**	97·5 litres		**(l)**	£6 200	

3.	**(a)**	£40	**(b)**	£65		**(c)**	£25	
	(d)	£425	**(e)**	£7		**(f)**	£299	
	(g)	£2 050	**(h)**	£26		**(i)**	£26.50	

4.	**(a)**	£12 500	**(b)**	£18 060		**(c)**	£24 600	
	(d)	£37 440	**(e)**	£23 400		**(f)**	£20 411.22	
	(g)	£3 644.86	**(h)**	£13 457.94				

EXAM QUESTIONS

1. 500 **2.** 150 **3.** £200 000

4. £12 000 **5.** £4 500 **6.** £2 600

APPRECIATION and DEPRECIATION

1. **(a)** £2332.80, £332.80 **(b)** £5955.08, £955.08 **(c)** £5495.52, £695.52

(d) £4348.04, £848.04 **(e)** £1982.12, £382.12 **(f)** 2393.35, 643.35

(g) £23820.32, 3820.32 **(h)** £21190.05, 3190.05 **(i)** £64751.45, £14751.45

(j) £439.32, £39.32

2. £3240 **3.** £92317 **4.** £212.24

5. 6300 million **6.** £1536 **7.** £5644.80

8. **(a)** £270 000 **(b)** after 3 years

EXAM QUESTIONS APPRECIATION and DEPRECIATION

1. £920.48 **2.** £27 900 **3.** £1 500 000

4. 136·3mg **5.** £1 388

6. **(a)** 39 300 tonnes **(b)** just falls short of doubling

7. £9 600 **8.** **(a)** 4% **(b)** £97 700

9. **(a)** £3 230.67 **(b)** 5 years **10.** £41 100

11. £9 790 **12.** **(a)** 44 990 **(b)** 5 months

13. **(a)** 8% **(b)** 21·5 **14.** £6 834.38 **15.** £159 720

16. Car is valued at more than half the original value.

17. **(a)** £1 677 **(b)** 4 years

18. Gained £140.49 **19.** Yes, since 913g > 900g **20.** £962 000

3.2 WORKING with FRACTIONS

Operations and combinations of vulgar fractions including mixed numbers.

ADDITION and SUBTRACTION

1. **(a)** $\frac{4}{5}$ **(b)** $\frac{1}{2}$ **(c)** $\frac{7}{8}$ **(d)** $\frac{5}{6}$ **(e)** $\frac{7}{9}$ **(f)** $\frac{7}{12}$

(g) $\frac{17}{20}$ **(h)** $\frac{5}{12}$ **(i)** $\frac{23}{24}$ **(j)** $\frac{9}{10}$ **(k)** $\frac{11}{12}$ **(l)** $\frac{13}{14}$

(m) $\frac{23}{56}$ **(n)** $\frac{23}{40}$ **(o)** $\frac{41}{63}$ **(p)** $\frac{23}{30}$

2. **(a)** $1\frac{1}{5}$ **(b)** $1\frac{3}{10}$ **(c)** $1\frac{3}{8}$ **(d)** $1\frac{1}{2}$ **(e)** $1\frac{2}{9}$ **(f)** $1\frac{5}{12}$

(g) $1\frac{1}{20}$ **(h)** $1\frac{7}{12}$ **(i)** $1\frac{13}{24}$ **(j)** $1\frac{3}{10}$ **(k)** $1\frac{11}{42}$ **(l)** $1\frac{17}{80}$

(m) $1\frac{9}{56}$ **(n)** $1\frac{7}{40}$ **(o)** $1\frac{13}{63}$ **(p)** $1\frac{17}{40}$ **(q)** $1\frac{3}{4}$ **(r)** $1\frac{1}{5}$

(s) $1\frac{5}{16}$ **(t)** $1\frac{5}{12}$ **(u)** $1\frac{5}{12}$ **(v)** $1\frac{2}{3}$ **(w)** $1\frac{5}{12}$ **(x)** $1\frac{49}{60}$

3. **(a)** $\frac{1}{2}$ **(b)** $\frac{1}{3}$ **(c)** $\frac{1}{6}$ **(d)** $\frac{1}{12}$ **(e)** $\frac{1}{4}$ **(f)** $\frac{7}{16}$

(g) $\frac{5}{12}$ **(h)** $\frac{1}{10}$ **(i)** $\frac{11}{16}$ **(j)** $\frac{3}{10}$ **(k)** $\frac{1}{4}$ **(l)** $\frac{1}{4}$

(m) $\frac{9}{40}$ **(n)** $\frac{7}{30}$ **(o)** $\frac{22}{63}$ **(p)** $\frac{5}{24}$

4. **(a)** $2\frac{3}{4}$ **(b)** $3\frac{1}{4}$ **(c)** $3\frac{5}{8}$ **(d)** $5\frac{1}{3}$ **(e)** $5\frac{7}{8}$ **(f)** $8\frac{5}{12}$

(g) $3\frac{1}{5}$ **(h)** $5\frac{5}{24}$ **(i)** $8\frac{1}{8}$ **(j)** $8\frac{11}{12}$ **(k)** $4\frac{1}{3}$ **(l)** $4\frac{5}{16}$

(m) $3\frac{1}{10}$ **(n)** $7\frac{3}{4}$ **(o)** $4\frac{1}{16}$ **(p)** $6\frac{4}{9}$ **(q)** $6\frac{1}{6}$ **(r)** $8\frac{1}{6}$

(s) $3\frac{23}{24}$ **(t)** $14\frac{3}{16}$

5. **(a)** $2\frac{1}{3}$ **(b)** $3\frac{5}{12}$ **(c)** $2\frac{1}{8}$ **(d)** $\frac{7}{9}$

(e) $\frac{1}{2}$ **(f)** $2\frac{1}{4}$ **(g)** $1\frac{3}{4}$ **(h)** $1\frac{1}{6}$

6. **(a)** $2\frac{1}{4}$ **(b)** $2\frac{13}{24}$ **(c)** $1\frac{11}{20}$ **(d)** $3\frac{1}{4}$ **(e)** $4\frac{1}{20}$ **(f)** $5\frac{1}{12}$

(g) $3\frac{11}{21}$ **(h)** $2\frac{7}{12}$ **(i)** $4\frac{5}{24}$ **(j)** $3\frac{1}{8}$ **(k)** $4\frac{1}{4}$ **(l)** $2\frac{1}{10}$

(m) 5 **(n)** $7\frac{3}{4}$ **(o)** $7\frac{1}{10}$ **(p)** $1\frac{2}{9}$

7. **(a)** $1\frac{1}{4}$ **(b)** $3\frac{3}{7}$ **(c)** $2\frac{1}{3}$ **(d)** $4\frac{1}{3}$ **(e)** $\frac{31}{35}$ **(f)** $3\frac{3}{4}$

(g) $2\frac{3}{4}$ **(h)** $3\frac{17}{20}$ **(i)** $1\frac{5}{12}$ **(j)** $2\frac{5}{8}$ **(k)** $3\frac{11}{12}$ **(l)** $1\frac{1}{2}$

(m) $4\frac{2}{3}$ **(n)** $\frac{13}{20}$ **(o)** $6\frac{7}{10}$ **(p)** $\frac{4}{9}$

8. $4\frac{9}{40}$ km **9.** $10\frac{8}{35}$ cm **10.** 7 hours

11. $6\frac{1}{6}$ hours **12.** $\frac{1}{6}$ **13.** $\frac{4}{15}$

14. **(a)** $\frac{7}{20}$ **(b)** $\frac{7}{12}$

MULTIPLICATION and DIVISION

1. **(a)** $\frac{1}{7}$ **(b)** $\frac{1}{10}$ **(c)** $\frac{2}{7}$ **(d)** $\frac{1}{12}$ **(e)** $\frac{1}{20}$ **(f)** $\frac{4}{7}$

(g) $\frac{2}{7}$ **(h)** $\frac{1}{14}$ **(i)** $\frac{3}{8}$ **(j)** $\frac{4}{39}$ **(k)** $\frac{3}{40}$ **(l)** $\frac{2}{3}$

(m) $\frac{4}{15}$ **(n)** $\frac{3}{4}$ **(o)** $\frac{10}{27}$ **(p)** $\frac{1}{12}$ **(q)** $\frac{3}{10}$ **(r)** $\frac{1}{4}$

(s) $\frac{4}{21}$ **(t)** $\frac{1}{4}$

2. **(a)** $1\frac{2}{3}$ **(b)** $2\frac{1}{12}$ **(c)** $6\frac{1}{4}$ **(d)** $2\frac{11}{12}$ **(e)** $3\frac{9}{10}$ **(f)** $3\frac{5}{9}$

(g) $2\frac{2}{3}$ **(h)** $4\frac{1}{2}$ **(i)** $12\frac{1}{2}$ **(j)** 30 **(k)** $2\frac{6}{7}$ **(l)** $8\frac{5}{8}$

(m) 12 **(n)** 14 **(o)** 4 **(p)** $7\frac{7}{8}$ **(q)** $9\frac{5}{8}$ **(r)** $5\frac{8}{27}$

(s) 21 **(t)** $3\frac{1}{5}$

3. **(a)** $\frac{3}{4}$ **(b)** $1\frac{2}{5}$ **(c)** $1\frac{1}{15}$ **(d)** $1\frac{1}{14}$ **(e)** $\frac{1}{4}$ **(f)** $1\frac{2}{3}$

(g) $\frac{4}{9}$ **(h)** $\frac{6}{11}$ **(i)** $\frac{2}{3}$ **(j)** $\frac{1}{2}$ **(k)** $\frac{18}{25}$ **(l)** $\frac{8}{15}$

(m) $\frac{16}{27}$ **(n)** $\frac{20}{57}$ **(o)** $1\frac{7}{9}$ **(p)** $\frac{4}{5}$ **(q)** $\frac{4}{5}$ **(r)** $\frac{1}{3}$

(s) $\frac{24}{55}$ **(t)** 6

4. **(a)** 4 **(b)** 3 **(c)** 2 **(d)** $\frac{11}{12}$ **(e)** $\frac{11}{18}$ **(f)** $2\frac{2}{3}$

(g) $1\frac{1}{2}$ **(h)** $\frac{4}{9}$ **(i)** $1\frac{3}{5}$ **(j)** $1\frac{9}{16}$ **(k)** $2\frac{11}{12}$ **(l)** $2\frac{5}{14}$

(m) $\frac{9}{32}$ **(n)** $2\frac{2}{15}$ **(o)** $\frac{15}{16}$ **(p)** $1\frac{1}{3}$ **(q)** $\frac{27}{32}$ **(r)** $\frac{9}{20}$

(s) $1\frac{1}{5}$ **(t)** $\frac{3}{4}$

5. **(a)** $\frac{1}{16}$ **(b)** $\frac{1}{6}$ **(c)** $\frac{3}{7}$ **(d)** $31\frac{1}{4}$ **(e)** 42 **(f)** $1\frac{1}{2}$

(g) $\frac{7}{10}$ **(h)** $2\frac{14}{15}$ **(i)** $\frac{3}{40}$ **(j)** $4\frac{2}{3}$ **(k)** $3\frac{1}{8}$ **(l)** $\frac{1}{26}$

(m) $\frac{1}{8}$ **(n)** $\frac{7}{12}$ **(o)** $\frac{5}{7}$ **(p)** $\frac{1}{20}$ **(q)** $16\frac{2}{3}$ **(r)** $\frac{1}{9}$

6. **(a)** 6 **(b)** $\frac{1}{2}$ kg **7.** **(a)** 33 **(b)** $\frac{3}{8}$ metre

8. $4\frac{27}{40}$ cm² **9.** $29\frac{3}{4}$ cm²

MIXED QUESTIONS on FRACTIONS

1. $2\frac{11}{12}$ kgs **2.** $3\frac{1}{4}$ km **3.** **(a)** $3\frac{23}{24}$ litres **(b)** $4\frac{1}{24}$ litres

4. **(a)** $36\frac{14}{15}$ metres **(b)** $83\frac{1}{5}$ m²

5. **(a)** 190 km **(b)** 297 km **(c)** 487 km

(d) $6\frac{1}{6}$ hours **(e)** $78\frac{36}{37}$ km/h

6. **(i)** 4 questions correct **(ii)** **(a)** $1\frac{11}{42}$ **(d)** $\frac{68}{75}$

COMPARING DATA SETS
QUARTILES and INTERQUARTILE RANGE

1.

	median	Q1	Q3	SIR
(a)	7	4	12	8
(b)	33	30	37	7
(c)	22	19	25	6
(d)	2	0	3	3
(e)	4·0	2·8	5·1	2·3
(f)	0·275	0·185	0·305	0·12
(g)	139	136	142	6
(h)	382	376	387	11
(i)	67	59	83·5	24·5
(j)	14	11	16	5

2.

	median	Q1	Q3	SIR
(a)	56	50	61	11
(b)	26	16	34	18
(c)	165	152·5	169	16·5
(d)	3	1	4	3
(e)	128·5	115	157·5	42.5
(f)	63	50	70·5	20.5
(g)	180	166	187	21
(h)	346	317	363	46
(i)	0·9	0·6	1·1	0·5
(j)	11	10	13·5	3·5

STANDARD DEVIATION

1.

	(a)	**(b)**	**(c)**	**(d)**	**(e)**	**(f)**	**(g)**	**(h)**
mean	20·3	302	14·99	87	62·8	4·45	0·23	39·6
SD	0·95	3·19	0·19	1·49	22·9	0·30	0·13	1·17

2. 3·44, 1·72

3. line A 27, 0·55; line B 27,0·19; line B more consistent

4. 106, 16·7

5. 21, 3·6

6. John 73, 1·04 ; Joe 72, 5·20 — Joe has lower mean score but John has better overall performance (lower standard deviation)

7. Dec 3313, 1025; Mar 2352, 565 — December has higher mean takings but March has less variation in takings

8. 6C1 21·5,1·26 ; 6C2 21·5, 2·88 — Same average but 6C1 has lower SD so less spread out.

4.2 FORMING a LINEAR MODEL from a given SET of DATA

1. **(a)** no relation **(b)** positive **(c)** negative

2. **(a)** positive correlation (more rain – more people buy umbrellas)
(b) no relation
(c) negative correlation (the faster you go, the less time it takes)

3. **(a)** yes **(b)** yes, but not strong **(c)** yes **(d)** no

4. student's best fit lines

5. Answers will vary depending on where line is drawn
(a) $y = 1{\cdot}67x + 3.3$ **(b)** $y = 0{\cdot}4x + 1.5$ **(c)** $y = 1{\cdot}2x - 6$
(d) $y = -1{\cdot}5x + 9$ **(e)** $y = -1{\cdot}5x + 12$ **(f)** $y = -0{\cdot}25x + 7$

6. H = 0·6D + 0·7, 1·6 **7**. $y = 3{\cdot}8x + 6$

8. $l = 0{\cdot}9F + 2{\cdot}2$, 6·25 **9**. C = –2T + 67, 62°C

10. S = 7T, 70 mph **11**. C = 1·1m + 177, £215.50

12. R = –0·35V + 0·61, 0·3

EXAM QUESTIONS

MEAN and STANDARD DEVIATION

1. **(a)** 40·7g, 3·6 **(b)** 140·7g, 3·6

2. **(a)** 7·55; 0·44 **(b)** slightly higher mean so slower times on average in 2nd race
higher SD so times are less consistent than 1st race

3. **(a)** 80kg, 12·2 **(b)** on average weight is less and less spread out

4. **(a)** 66; 6·56 **(b)** 71; 6·56

5. **(a)** 21·5; 4·42 **(b)** On average study times same but teachers are more varied

6. **(a)** £157, 51·3 **(b)** on average E-Pod more expensive and less spread out

7. **(a)** £22·50, 5·4 **(b)** prices of dolls are less spread out than teddies

FORMING a LINEAR MODEL from a given SET of DATA

1. **(a)** $P = 3W + 5$ **(b)** 6 points

2. Answers depend on line drawn

3. A – strong positive correlation.

4. **(a)** strong positive correlation **(b)** $I = 1/7C + 1$

5. **(a)** $R = \frac{1}{2}h + 4$ **(b)** 15