Topic: Quadratic Functions & Parabolas.

18·01 a) $h(t) = 16t - t^2$

$60 = 16t - t^2$

$t^2 - 16t + 60 = 0$

$mn = 1 \times 60 = 60$

1	60
2	30
3	20
4	15
5	12
6	10 ✓

$t^2 - 6t - 10t + 60 = 0$

$t(t-6) - 10(t-6) = 0$

$(t-6)(t-10) = 0$

$t - 6 = 0 \quad t - 10 = 0$

$t = 6 \quad t = 10$

The rocket will first reach 60m when t = 6 seconds.

b) $h(t) = 16t - t^2$

$t^2 - 16t = 0$

$t(t-16) = 0$

$t = 0 \quad t - 16 = 0$

$t = 16$

maximum turning point occurs at t = 8 half way between roots.

$h(8) = 16(8) - (8)^2$

$= 64\text{m}$

∗ The rocket will not reach 70m

18·02

$y = ax^2$

substitute the coordinate point $(-3, 45)$ into equation ($-3 \to x$, $45 \to y$)

$45 = a(-3)^2$

$45 = 9a$

$a = 5$

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18.03 a) $y = (x-1)^2 - 16$

min tp $= (1, -16)$

x coordinate of the turning point $= +1$

y coordinate of the turning point $= -16$

b) Line of symmetry $x = 1$

c)
$$y = (x-1)(x-1) - 16$$
$$= x^2 - x - x + 1 - 16$$
$$= x^2 - 2x - 15$$

$mN = 1 \times 15 = 15$

1 15

$+3 -5$

$$x^2 + 3x - 5x - 15 = 0$$
$$x(x+3) - 5(x+3) = 0$$
$$(x+3)(x-5) = 0$$

$x + 3 = 0$ $x - 5 = 0$

$x = -3$ $x = +5$

$(-3, 0)$ $(5, 0)$

length AB $= 8$

18.04 a)
$$7 + 6x - x^2$$
$$-x^2 + 6x + 7$$
$$-(x^2 - 6x - 7)$$
$$-(x^2 + x - 7x - 7)$$
$$-(x(x+1) - 7(x+1))$$
$$-(x+1)(x-7)$$

$mN = 1 \times 7 = 7$

$+1 -7$

b) Roots occur at

$$-(x+1)(x-7) = 0$$

$x + 1 = 0$ $x - 7 = 0$

$x = -1$ $x = +7$

$(-1, 0)$ $(7, 0)$

c) x value of the turning point happens half way between the roots at $x = 3$

Substitute $x = 3$ into original function

$$7 + 6(3) - (3)^2$$
$$= 7 + 18 - 9$$
$$= +16$$

$TP = (3, 16)$

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18.05

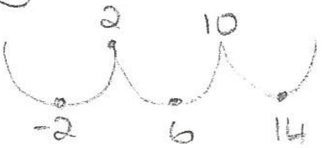
a) $y=(x+2)^2-16$

x coordinate of turning point $= -2$

y coordinate of turning point $= -16$

$P(-2,-16)$

b) The distance from $P \rightarrow R$ along x-axis is 4
This is the same from $R \rightarrow Q$. The y values for
P, Q, S are identical so $Q(6,-16)$



c) T.P of $S = (14,-16)$

$y=(x-14)^2-16$

18.06

a) $8x-x^2=0$

$x(8-x)=0$

$x=0 \quad 8-x=0$

$x=8$

$(0,0)$ $(8,0)$

b) The line of symmetry occurs half way between the roots at $x=4$

c) The turning point happens at $x=4$

$8x-x^2$

$=8(4)-(4)^2$

$=32-16$

$=16$

TP $(4,16)$

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18·07

a) $TP = (-3,-4) \rightarrow$ line of symmetry occurs at $x = -3$

b) $y = (x+3)^2 - 4$

c) c is the y-intercept and happens when $x=0$

$y = (0+3)^2 - 4$
$= 5$

$(0,5) \rightarrow$ y-intercept

18·08

a) Turning point $= (5,1)$ $y=(x-5)^2+1$ $a=-5$ $b=+1$

b) line of symmetry occurs at $x=5$

c) P has x coordinate 0
Q has x coordinate 10 } due to midpoint being $x=5$ at line of symmetry

* The y value of both coordinates can be found when the parabola cuts the y-axis at $x=0$

$y = (0-5)^2 + 1$
$= 25 + 1$
$= 26$

$P(0,26)$ $Q(10,26)$

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18.09

$y = -x^2$

Point $(-3, k)$
$\downarrow \quad \downarrow$
$x \quad y$

$k = -(-3)^2$

$k = -9$

* substitute point (x, y) into parabola equation

18.10

The turning point occurs at $(5, b)$
b ↓ unknown

a)

$y = (x-5)^2 + b$

$a = -5$

b) Due to symmetry $P(2,0)$ has x-coordinate 3 left of $x=5$ so Q must be 3 right of $x=5$ at $Q(8,0)$

c) We must use the roots to work backwards to find the equation & then complete the square.

$x = 2 \quad x = 8$
$x - 2 = 0 \quad x - 8 = 0$
$(x-2)(x-8) = 0$
$x^2 - 8x - 2x + 16 = 0$
$x^2 - 10x + 16 = 0$

$y = (x-5)^2 + 16 - 25$

$y = (x-5)^2 - 9$

$a = +5 \quad b = -9$

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18·11

a) $x^2 - 4x - 21$

$mN = 1 \times 21 = 21$

1 21

+3 −7 ✓

$x^2 + 3x | -7x - 21$

$x(x+3) - 7(x+3)$

$(x+3)(x-7)$

b) Roots occur

at $x+3=0$ $x-7=0$

$x=-3$ $x=+7$

c) $x^2 - 4x - 21$

$(x-2)^2 - 21 - 4$

$(-2)^2 = 4$

$(x-2)^2 - 25$

Turning point $(+2, -25)$

18·12

a) Roots occur at $x-2=0$ $x-4=0$

$x=2$ $x=4$

$(2,0)$ $(4,0)$

b) A ⇒ y-intercept B, C ⇒ Roots

A occurs when $x=0$ ⟶ $x^2 - 6x + 8$

$(0)^2 - 6(0) + 8$

$= 8$

A $(0,8)$

B $(2,0)$

C $(4,0)$

c) Line of symmetry is half way between the roots at $x=3$

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18·13

a) $y = k(x-a)(x-b)$ Roots → $(-1,0)(3,0)$

$a = -1$ $b = -3$

Be careful with symbol

$x = -1$ $x = 3$

$(x+1) = 0$ $(x-3) = 0$

$(x+1)(x-3) = 0$

b) $y = k(x+1)(x-3)$

substitute a coordinate point on the graph into the equation and solve for k.

$(0, -6)$
x y

$-6 = k(0+1)(0-3)$

$-6 = k(1)(-3)$

$-6 = -3k$

$k = 2$

c) $y = 2(x+1)(x-3)$

Turning point occurs at $x = 1$ → mid point of roots.

$y = 2(1+1)(1-3)$

$= 2 \times 2 \times -2$

$= -8$

$(1, -8)$

18·14

a) A = y intercept and occurs at $x = 0$

$y = 4x^2 + 4x - 3$

$= 4(0)^2 + 4(0) - 3$

$= -3$

$\underline{A(0, -3)}$

b) B and C are the roots ⟶

$y = 4x^2 + 4x - 3 = 0$ $m \times n = 4 \times 3$
$= 12$

$y = 4x^2 - 2x + 6x - 3 = 0$

1	12
-2	+6 ✓
3	4

$y = 2x(2x-1) + 3(2x-1) = 0$

$y = (2x-1)(2x+3) = 0$

$2x - 1 = 0$ $2x + 3 = 0$

$2x = 1$ $2x = -3$

$x = 1/2$ $x = -3/2$

$(1/2, 0)$ $(-3/2, 0)$

c) min tp occurs mid way between the roots

at $x = \dfrac{1/2 + (-3/2)}{2} = -1/2$

$y = 4(\frac{-1}{2})^2 + 4(\frac{-1}{2}) - 3$

$= 4(\frac{1}{4}) - \frac{4}{2} - 3$

$= 1 - 2 - 3$

$= -4$

$\underline{\underline{(\frac{-1}{2}, -4)}}$

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18.15

$y=x^2$ moved sideways in right direction by 2 and upwards by 3. This produces graph

$y=(x-2)^2+3$

$y=x^2$ has now been moved in right direction by 1 and downwards by 4. This produces graph

$y=(x-1)^2-4$ turning point $(1,-4)$

18.16

a) $y=(x-8)(2-x)$

Q and R are the roots that occur at $x-8=0$ $2-x=0$

$x=8$ $x=2$

$R(8,0)$ $Q(2,0)$

b) The height occurs at the maximum turning point at $x=5$

$(5,9)$

$y=(5-8)(2-5)$

$=-3\times-3$

$=9$

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18.17

a) $h(t) = -2t(t-14)$

The time of flight is calculated from finding the roots

$-2t(t-14)=0$ * 14 seconds

$-2t=0 \quad t-14=0$

$t=0 \quad t=14$

b) The maximum turning point occurs at $x=7$ mid way between the roots.

$-2(7)(7-14)$

$= -2\times 7\times -7$

$= 98$ metres

18.18

a) $y=36-(x-2)^2$ turning point $(+2,+36)$

$y=-(x-2)^2+36$

b) Line of symmetry occurs at $x=2$

c) The line $y=20$ cuts the parabola $y=36-(x-2)^2$

$36-(x-2)^2=20$

$36-(x-2)(x-2)=20$

$36-(x^2-4x+4)=20$

$36-x^2+4x-4-20=0$

$x^2-4x-12=0$

$mn=12$

1 12
+2 -6 ✓
3 4

$x^2+2x-6x-12=0$

$x(x+2)-6(x+2)=0$

$(x+2)(x-6)=0$

$x+2=0 \quad x-6=0$

$x=-2 \quad x=6$

$R(-2,20) \quad S(6,20)$

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18.19

a) $y = (x+a)^2 + b$ min t.p $= (2,-4)$

$y = (x-2)^2 - 4$

(i) $a = -2$ (ii) $b = -4$

b) Line of symmetry $\Rightarrow x = -2$