

MATHEMATICS

NATIONAL 5 NOTES

\times

\pm

\div

$$A = \pi r^2$$

\leq

Σ

x

π

$$V = l b h$$

$\sqrt{\quad}$

$+$

$-$

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FORMULAE LIST

The roots of $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, $a \neq 0$

Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$ or $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

Area of a triangle: $\text{Area} = \frac{1}{2}ab \sin C$

Volume of a sphere: $\text{Volume} = \frac{4}{3}\pi r^3$

Volume of a cone: $\text{Volume} = \frac{1}{3}\pi r^2 h$

Volume of a pyramid: $\text{Volume} = \frac{1}{3}Ah$

Volume of a cylinder: $\text{Volume} = \pi r^2 h$

Standard deviation: $s = \sqrt{\frac{\sum(x - \bar{x})^2}{n-1}} = \sqrt{\frac{\sum x^2 - (\sum x)^2/n}{n-1}}$, where n is the sample size.

CHAPTER 1: FRACTIONS

Simplify (cancel down):

$$\frac{28}{36} = \frac{28 \div 4}{36 \div 4} = \frac{7}{9}$$

divide by **highest common factor**, $HCF(28,36) = 4$

Mixed numbers and improper fractions (top heavy):

$$\overbrace{4 \times 2 + 3 = 11} \rightarrow$$

$$2\frac{3}{4} = 2 + \frac{3}{4} = \frac{8}{4} + \frac{3}{4} = \frac{11}{4} \quad \text{'bottom' stays as 4}$$

$$\leftarrow \overbrace{11 \div 4 = 2 \text{ R}3}$$

Addition and Subtraction requires a common denominator:

$$\begin{aligned} & \frac{5}{6} - \frac{4}{9} && \text{least common multiple } LCM(6,9)=18 \\ & \frac{5 \times 3}{6 \times 3} = \frac{15}{18} && \frac{4 \times 2}{9 \times 2} = \frac{8}{18} && \text{both fractions now 18ths} \\ & = \frac{15}{18} - \frac{8}{18} \\ & = \frac{7}{18} && \text{subtract 'top' numbers, still 18ths} \end{aligned}$$

Multiplication and Division requires **no** mixed numbers:

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$$

$$\frac{3}{10} \times 2\frac{3}{4}$$

$$\frac{4}{5} \div \frac{7}{8}$$

$$= \frac{3}{10} \times \frac{11}{4} \quad \text{go 'top heavy'}$$

$$= \frac{4}{5} \times \frac{8}{7} \quad \text{multiply by reciprocal}$$

$$= \frac{33}{40}$$

$$= \frac{32}{35}$$

ORDER OF CALCULATION: \times and \div before $+$ or $-$
to change the order use brackets **BRACKETS FIRST!**

Carry out separate calculations:

(1) $7\frac{5}{6} - \frac{8}{9}$ of $2\frac{3}{4}$

multiply first

$$\begin{aligned} & \frac{8}{9} \times \frac{11}{4} \quad \text{of means multiply} \\ & = \frac{88}{36} \quad \text{fully simplify} \\ & = \frac{22}{9} \\ & = 2\frac{4}{9} \end{aligned}$$

subtraction last

$$\begin{aligned} & 7\frac{5}{6} - 2\frac{4}{9} \\ & = 7\frac{15}{18} - 2\frac{8}{18} \quad \text{common denominator} \\ & = 5\frac{7}{18} \quad \text{subtract whole numbers and} \\ & \quad \quad \quad \text{fractions separately} \end{aligned}$$

(2) $\left(\frac{5}{6} - \frac{1}{2}\right) \div 1\frac{3}{4}$

$$\begin{aligned} & \frac{5}{6} - \frac{1}{2} \quad \text{brackets first} \\ & = \frac{5}{6} - \frac{3}{6} \\ & = \frac{2}{6} \quad \text{fully simplify} \\ & = \frac{1}{3} \end{aligned}$$

$$\begin{aligned} & \frac{1}{3} \div 1\frac{3}{4} \\ & = \frac{1}{3} \div \frac{7}{4} \quad \text{'top-heavy' first} \\ & = \frac{1}{3} \times \frac{4}{7} \quad \text{multiply by reciprocal} \\ & = \frac{4}{21} \end{aligned}$$

CHAPTER 2: PERCENTAGES

PERCENTAGE CHANGE

	original value	changed value
INCREASE: growth, appreciation, compound interest	100%	$\xrightarrow{+a\%} (100 + a)\%$
DECREASE: decay, depreciation	100%	$\xrightarrow{-a\%} (100 - a)\%$

For example,

8% increase: $100\% \xrightarrow{+8\%} 108\% = 1.08$ multiply quantity by 1.08 for 8% increase

8% decrease: $100\% \xrightarrow{-8\%} 92\% = 0.92$ multiply quantity by 0.92 for 8% decrease

REVERSING PERCENTAGE CHANGE

Divide by the factor which produced the increase.

(1) Including VAT of 20%, a radio costs £96. Find the original cost exclusive of VAT.

<i>20% VAT added</i>	$£x \times 1.20 = £96$
$100\% \xrightarrow{+20\%} 120\% = 1.20$	$£x = £96 \div 1.20$
	$= £80$

non-calculator: $120\% \xrightarrow{\div 12} 10\% \xrightarrow{\times 10} 100\%$
 $£96 \div 12 = £8$ $£8 \times 10 = £80$

(2) A camera costs £120 after a discount of 25% is applied. Find the original cost.

<i>25% discount subtracted</i>	$£x \times 0.75 = £120$
$100\% \xrightarrow{-25\%} 075\% = 0.75$	$£x = £120 \div 0.75$
	$= £160$

non-calculator: $75\% = \frac{3}{4}$ $\frac{3}{4} \xrightarrow{\div 3} \frac{1}{4} \xrightarrow{\times 4} \frac{4}{4}$
 $£120 \div 3 = £40$ $£40 \times 4 = £160$

COMPOUND PERCENTAGE CHANGE

appreciation and depreciation

(1) A £240000 house appreciates in value by 5% in 2007, appreciates 10% in 2008 and depreciates by 15% in 2009. Calculate the value of the house at the end of 2009.

5% increase: $100\% + 5\% = 105\% = 1.05$
10% increase: $100\% + 10\% = 110\% = 1.10$
15% decrease: $100\% - 15\% = 85\% = 0.85$

$$\begin{aligned} &£240000 \times 1.05 \times 1.10 \times 0.85 \\ &= £235620 \end{aligned}$$

OR *evaluate year by year*

year 1

5% of £240000 = £12000
£240000 + £12000 = £252000

year 2

10% of £252000 = £25200
£252000 + £25200 = £277200

year 3

15% of £277200 = £41580
£277200 - £41580 = £235620

compound interest

(2) Calculate the compound interest on £12000 invested at 5% pa for 3 years.

$£12000 \times (1.05)^3$ *ie.* $\times 1.05 \times 1.05 \times 1.05$ *or evaluate year by year*
 $£12000 \times 1.157625$
 $= £13891.50$
compound interest = $£13891.50 - £12000 = £1891.50$

EXPRESSING CHANGE AS A PERCENTAGE

$$\% \text{ change} = \frac{\text{change}}{\text{start}} \times 100\%$$

A £15000 car is resold for £12000.
Find the percentage loss.

$$\begin{aligned} \text{loss} &= £15000 - £12000 = £3000 \\ \% \text{ loss} &= \frac{3000}{15000} \times 100\% = 20\% \end{aligned}$$

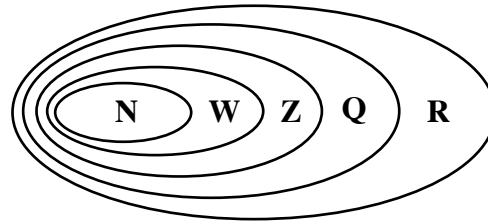
CHAPTER 3: SURDS

NUMBER SETS:

Natural numbers $N = \{1, 2, 3, \dots\}$

Whole numbers $W = \{0, 1, 2, 3, \dots\}$

Integers $Z = \{\dots -3, -2, -1, 0, 1, 2, 3, \dots\}$



Rational numbers, Q , can be written as a division of two integers.

Irrational numbers **cannot** be written as a division of two integers.

Real numbers, R , are all rational and irrational numbers.

SURDS ARE IRRATIONAL ROOTS.

For example, $\sqrt{2}$, $\sqrt{\frac{5}{9}}$, $\sqrt[3]{16}$ are surds.

whereas $\sqrt{25}$, $\sqrt{\frac{4}{9}}$, $\sqrt[3]{-8}$ are **not** surds as they are 5 , $\frac{2}{3}$ and -2 respectively.

SIMPLIFYING ROOTS:

RULES: $\sqrt{mn} = \sqrt{m} \times \sqrt{n}$

$$\sqrt{\frac{m}{n}} = \frac{\sqrt{m}}{\sqrt{n}}$$

(1) Simplify $\sqrt{24} \times \sqrt{3}$

$$\begin{aligned} & \sqrt{24} \times \sqrt{3} \\ &= \sqrt{72} \end{aligned}$$

$$= \sqrt{36} \times \sqrt{2}$$

36 is the largest square number which is a factor of 72

$$= 6 \times \sqrt{2}$$

$$= 6\sqrt{2}$$

(2) Simplify $\sqrt{72} + \sqrt{48} - \sqrt{50}$

$$\begin{aligned} & \sqrt{72} + \sqrt{48} - \sqrt{50} \\ &= \sqrt{36} \times \sqrt{2} + \sqrt{16} \times \sqrt{3} - \sqrt{25} \times \sqrt{2} \end{aligned}$$

$$= 6\sqrt{2} + 4\sqrt{3} - 5\sqrt{2}$$

$$= 6\sqrt{2} - 5\sqrt{2} + 4\sqrt{3}$$

$$= \sqrt{2} + 4\sqrt{3}$$

(3) Remove the brackets and fully simplify:

(a) $(\sqrt{3} - \sqrt{2})^2$

$$= (\sqrt{3} - \sqrt{2})(\sqrt{3} - \sqrt{2})$$

$$= \sqrt{3}(\sqrt{3} - \sqrt{2}) - \sqrt{2}(\sqrt{3} - \sqrt{2})$$

$$= \sqrt{9} - \sqrt{6} - \sqrt{6} + \sqrt{4}$$

$$= 3 - \sqrt{6} - \sqrt{6} + 2$$

$$= 5 - 2\sqrt{6}$$

(b) $(3\sqrt{2} + 2)(3\sqrt{2} - 2)$

$$= (3\sqrt{2} + 2)(3\sqrt{2} - 2)$$

$$= 3\sqrt{2}(3\sqrt{2} - 2) + 2(3\sqrt{2} - 2)$$

$$= 9\sqrt{4} - 6\sqrt{2} + 6\sqrt{2} - 4$$

$$= 18 - 6\sqrt{2} + 6\sqrt{2} - 4$$

$$= 14$$

RATIONALISING DENOMINATORS:

Removing surds from the denominator.

Express with a rational denominator:

(1) $\frac{4}{\sqrt{6}}$

$$\frac{4}{\sqrt{6}}$$

$$= \frac{4 \times \sqrt{6}}{\sqrt{6} \times \sqrt{6}} \quad \text{multiply the 'top' and 'bottom'}$$

by the surd on the denominator

$$= \frac{4\sqrt{6}}{6}$$

$$= \frac{2\sqrt{6}}{3}$$

(2) $\frac{\sqrt{3}}{3\sqrt{2}}$

$$\frac{\sqrt{3}}{3\sqrt{2}}$$

$$= \frac{\sqrt{3} \times \sqrt{2}}{3\sqrt{2} \times \sqrt{2}}$$

$$= \frac{\sqrt{6}}{3 \times \sqrt{4}}$$

$$= \frac{\sqrt{6}}{6}$$

CHAPTER 4: INDICES

base \longrightarrow a^n \longleftarrow index or exponent

INDICES RULES: require the same base.

Examples:

$$a^m \times a^n = a^{m+n}$$

$$\frac{w^2 \times w^5}{w^3} = \frac{w^7}{w^3} = w^4$$

$$a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

$$(3^5)^2 = 3^{10}$$

$$(ab)^n = a^n b^n$$

$$(2a^3b)^2 = 2^2 a^6 b^2 = 4a^6 b^2$$

$$a^0 = 1$$

$$5^0 = 1$$

$$a^1 = a$$

$$5^1 = 5$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$$

$$8^{\frac{4}{3}} = (\sqrt[3]{8})^4 = 2^4 = 16$$

$$\frac{1}{a^p} = a^{-p}$$

$$8^{-\frac{4}{3}} = \frac{1}{8^{\frac{4}{3}}} = \frac{1}{16}$$

SCIENTIFIC NOTATION (STANDARD FORM)

Writing numbers in the form $a \times 10^n$

where $1 \leq a < 10$ *place the decimal point after the first non-zero digit*
and n is an integer *numbers ...-3, -2, -1, 0, 1, 2, 3...*

$$32800 = 3 \cdot 28 \times 10 \times 10 \times 10 \times 10 = \mathbf{3 \cdot 28 \times 10^4}$$

$$0 \cdot 000328 = 3 \cdot 28 \div 10 \div 10 \div 10 \div 10 = 3 \cdot 28 \div 10^4 = \mathbf{3 \cdot 28 \times 10^{-4}}$$

Notice for numbers starting $0 \cdot$ the power of 10 is negative (same as $\div 10$).

SIGNIFICANT FIGURES indicate the accuracy of a **measurement**.

For example, $3400 \text{ cm} = 34 \text{ m} = 0 \cdot 034 \text{ km}$
 same measurement, same accuracy, each 2 significant figures.

Count the number of figures used, but **not** zeros at the **end** of a whole number
or zeros at the **start** of a decimal.

rounding: $5713 \cdot 4$ 5700 to 2 significant figures
 $0 \cdot 057134$ $0 \cdot 057$ to 2 significant figures (note: $0 \cdot 057000$ wrong)

- (1) One **milligram** of hydrogen gas contains $2 \cdot 987 \times 10^{20}$ molecules.
Calculate, to 3 significant figures, the number of molecules in **5 grams** of hydrogen.

$$\begin{aligned} & 5000 \times 2 \cdot 987 \times 10^{20} \quad \textit{learn to enter standard form in the calculator using the} \\ & = 1 \cdot 4935 \times 10^{24} \quad \textit{appropriate button EE or EXP or x10^n \quad eg. 2 \cdot 987 EXP 20} \\ & \approx 1 \cdot 49 \times 10^{24} \text{ molecules} \end{aligned}$$

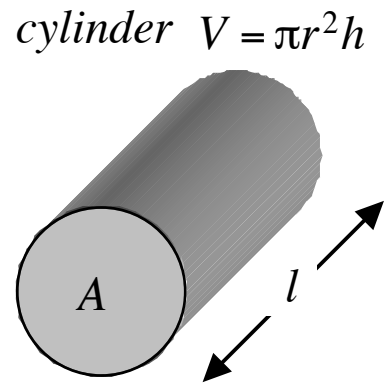
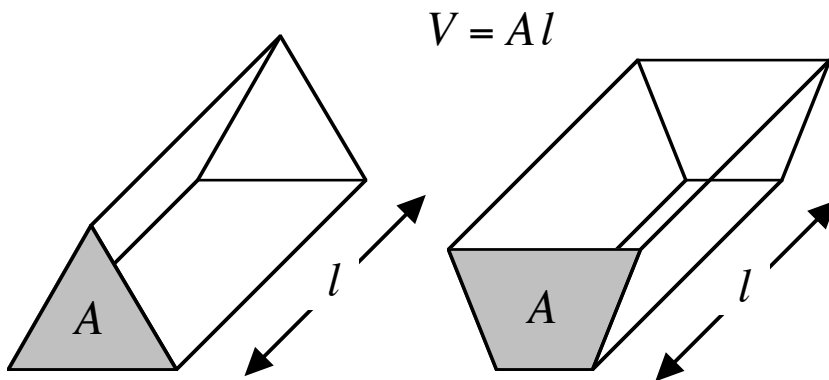
- (2) The total mass of argon in a flask is $4 \cdot 15 \times 10^{-2}$ grams.
The mass of a single atom of argon is $6 \cdot 63 \times 10^{-23}$ grams.

Find, correct to 3 significant figures, the number of argon atoms in the flask.

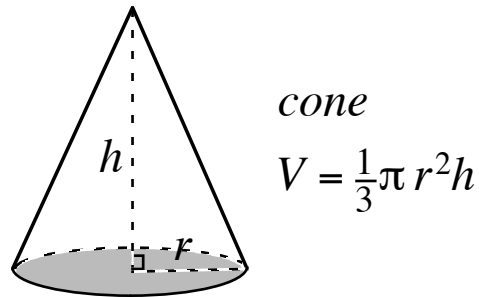
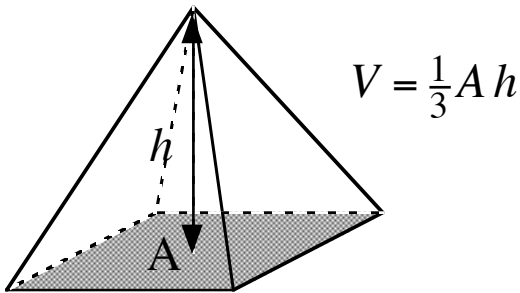
$$\begin{aligned} & \frac{4 \cdot 15 \times 10^{-2}}{6 \cdot 63 \times 10^{-23}} \quad \textit{use the (-) button for a minus \quad eg. 4 \cdot 15 EXP (-) 2} \\ & = 6 \cdot 259 \dots \times 10^{20} \quad \textit{divide 'top' by 'bottom' and write the unrounded answer} \\ & \approx 6 \cdot 26 \times 10^{20} \text{ atoms} \quad \textit{write the rounded answer} \end{aligned}$$

CHAPTER 5: VOLUMES OF SOLIDS

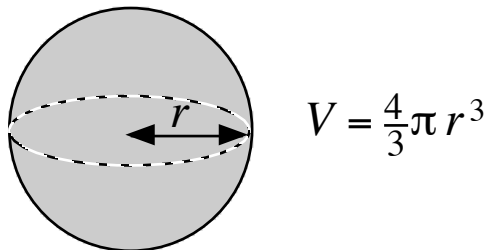
PRISMS a solid with the same cross-section throughout its length.
length l is at right-angles to the area A .



PYRAMIDS



SPHERE



EFFECT OF CHANGE

Describe the effect on the volume of a cylinder of:

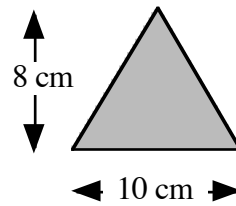
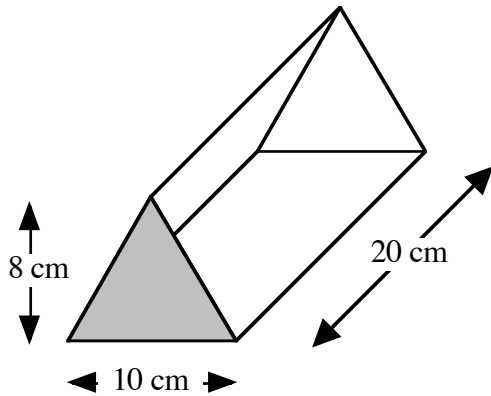
- (i) trebling the radius (ii) doubling the radius and halving the height

$$\begin{aligned} V &= \pi(3r)^2 h \\ &= \pi 9r^2 h \\ &= 9\pi r^2 h \\ &9 \text{ times bigger} \end{aligned}$$

$$\begin{aligned} V &= \pi(2r)^2 \left(\frac{1}{2}h\right) \\ &= \pi 4r^2 \times \frac{1}{2}h \\ &= 2\pi r^2 h \\ &2 \text{ times bigger} \end{aligned}$$

USING THE FORMULAE

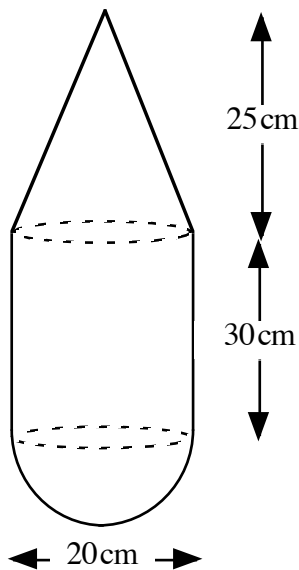
(1) Calculate the volume.



$$\begin{aligned} A &= \frac{1}{2}bh \\ &= 10 \times 8 \div 2 \\ &= 40 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} V &= Al \\ &= 40 \times 20 \\ &= 800 \text{ cm}^3 \end{aligned}$$

(2) Calculate the volume correct to **3 significant figures**.



$$\text{radius} = 20\text{cm} \div 2 = 10\text{cm}$$

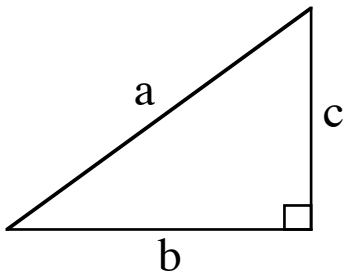
$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3} \times \pi \times 10 \times 10 \times 25 \\ &= 2617.993... \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi \times 10 \times 10 \times 30 \\ &= 9424.777... \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \div 2 \\ &= \frac{4}{3} \times \pi \times 10 \times 10 \times 10 \div 2 \\ &= 2094.395... \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} \text{total volume} &= 2617.993... + 9424.777... + 2094.395... \\ &= 14137.166... \\ &\approx 14100 \text{ cm}^3 \end{aligned}$$

CHAPTER 6: PYTHAGORAS' THEOREM

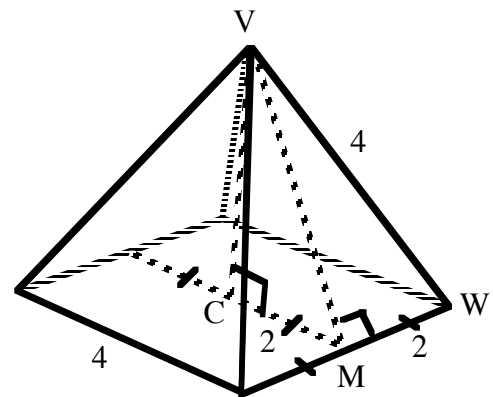
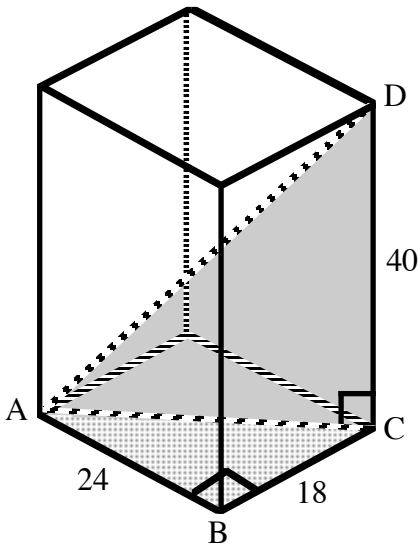


Side length **a** units is opposite the right angle. It is the longest side, the **hypotenuse**.

For **right-angled triangles**:

$$a^2 = b^2 + c^2$$

3D Identify right-angled triangles in 3D diagrams.



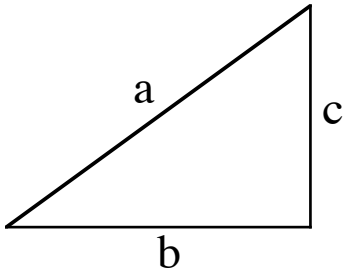
face diagonal $AC^2 = 24^2 + 18^2$
 $= 576 + 324$
 $= 900$
 $AC = \sqrt{900}$
 $= 30$

slant height $VM^2 = 4^2 - 2^2$
 $= 16 - 4$
 $= 12$
 $VM = \sqrt{12}$
 $= 2\sqrt{3}$

space diagonal $AD^2 = 40^2 + 30^2$
 $= 1600 + 900$
 $= 2500$
 $AD = \sqrt{2500}$
 $= 50$

height $VC^2 = (\sqrt{12})^2 - 2^2$
 $= 12 - 4$
 $= 8$
 $VC = \sqrt{8}$
 $= 2\sqrt{2}$

CONVERSE OF PYTHAGORAS' THEOREM

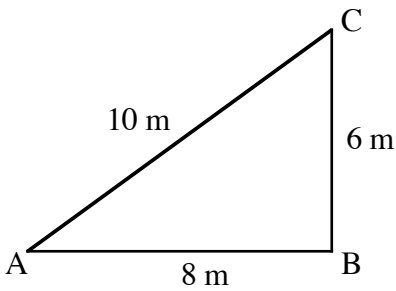


Longest side, length **a** units, is opposite the biggest angle.

If $a^2 = b^2 + c^2$

then the triangle is right-angled

Show that $\triangle ABC$ is right angled.



$$AB^2 + BC^2 = 8^2 + 6^2 = 100$$

$$AC^2 = 10^2 = 100$$

$$\text{since } AB^2 + BC^2 = AC^2$$

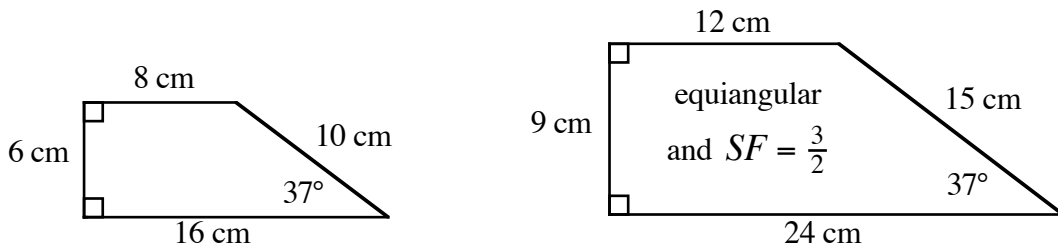
by the Converse of Pyth. Thm.

$\triangle ABC$ is right - angled at B (ie. $\angle ABC = 90^\circ$)

CHAPTER 7: SIMILAR SHAPES

Shapes are similar if they are enlargement or reductions of each other.

- and
- (1) the angles remain unchanged - the shapes are **equiangular**.
 - (2) the sides are enlarged or reduced by some **scale factor (SF)**.



Triangles are special:

Enlarge or reduce sides by some scale factor and the two triangles will be equiangular.

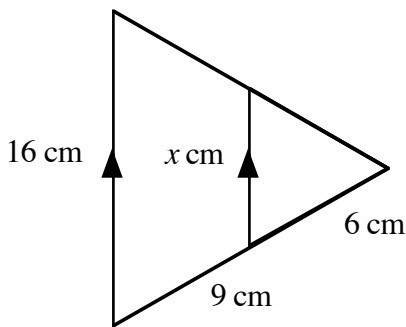
If triangles are equiangular then they are similar.

SCALING LENGTH

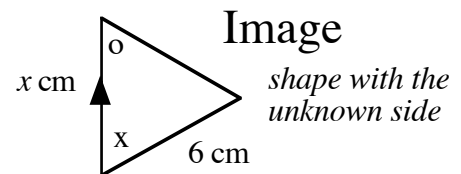
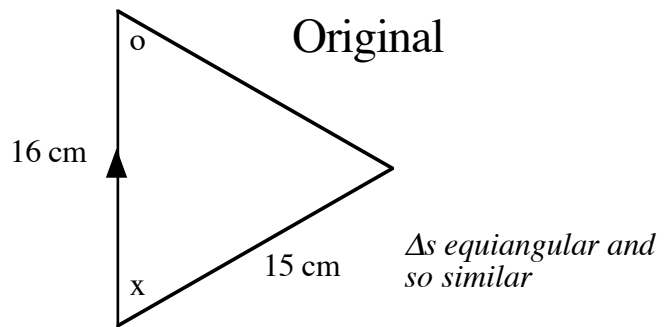
length scale factor, $SF = \frac{\text{image side}}{\text{original side}}$

enlargement if $SF > 1$

reduction if $0 < SF < 1$



Find the value of x .



$$SF = \frac{\text{image}}{\text{original}} = \frac{6}{15} = \frac{2}{5} \quad 0 < SF < 1 \text{ as expected for a reduction}$$

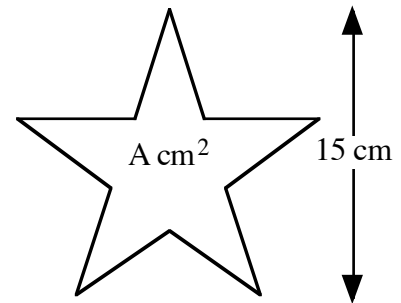
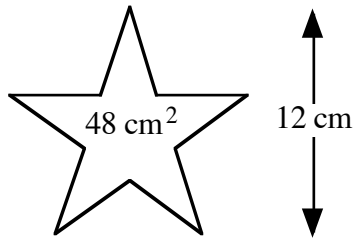
$$x = \frac{2}{5} \times 16 = 6.4 \quad \text{smaller than 16 as expected for a reduction}$$

SCALING AREA

for a 2D shape both length and breadth must be scaled.

$$\text{length SF} = n$$

$$\text{area SF} = n^2$$



Given that the two shapes shown are similar, find the area of the larger shape.

$$\text{length SF} = \frac{\text{image}}{\text{original}} = \frac{15}{12} = \frac{5}{4}$$

$SF > 1$ as expected for an enlargement

$$\text{area SF} = \frac{5}{4} \times \frac{5}{4} = \frac{25}{16}$$

$$A = \frac{25}{16} \times 48 = 75$$

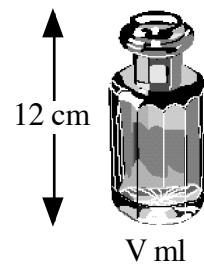
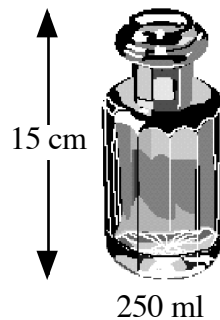
bigger than 48 as expected for an enlargement

SCALING VOLUME

for a 3D shape length, breadth and height must be scaled.

$$\text{length SF} = n$$

$$\text{volume SF} = n^3$$



Given that the two solids shown are similar find the volume of the smaller solid.

$$\text{length SF} = \frac{\text{image}}{\text{original}} = \frac{12}{15} = \frac{4}{5}$$

$0 < SF < 1$ as expected for a reduction

$$\text{volume SF} = \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} = \frac{64}{125}$$

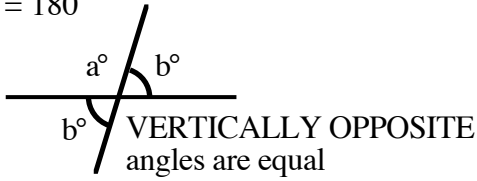
$$V = \frac{64}{125} \times 250 = 128$$

smaller than 250 as expected for a reduction

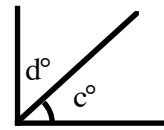
CHAPTER 8: ANGLES and SHAPE

LINES

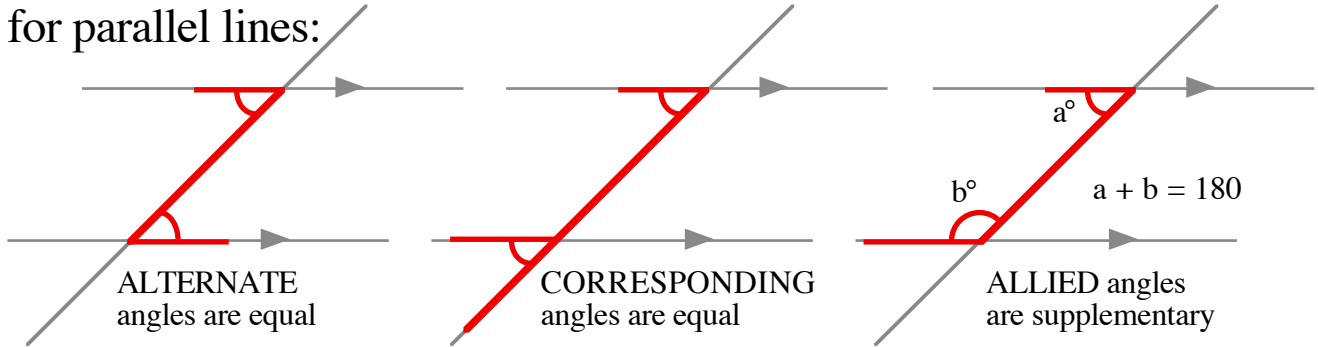
SUPPLEMENTARY angles
 $a + b = 180$



COMPLEMENTARY angles
 $c + d = 90$

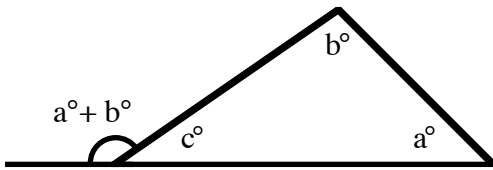


for parallel lines:

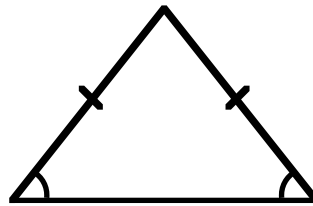


TRIANGLES

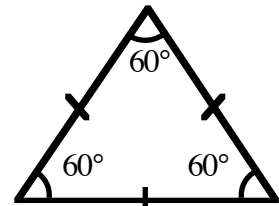
angle sum $a^\circ + b^\circ + c^\circ = 180^\circ$



EXTERIOR ANGLE is the sum of the opposite INTERIOR ANGLES



ISOSCELES

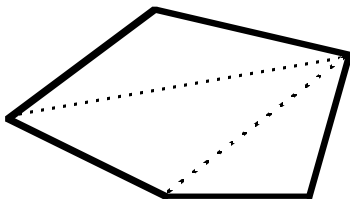


EQUILATERAL

POLYGONS

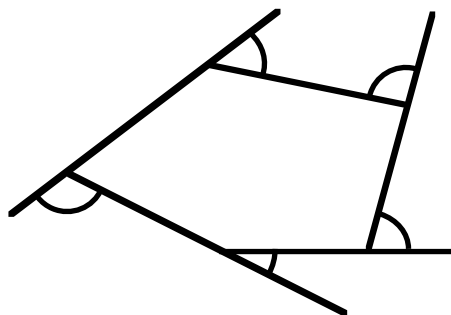
INTERIOR ANGLES sum to $(n-2) \times 180^\circ$ where n sides

EXTERIOR ANGLES sum to 360°



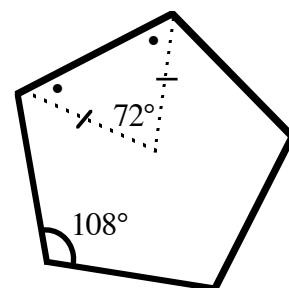
pentagon

interior sum $3 \times 180^\circ = 540^\circ$



exterior sum 360°

all sides and angles equal

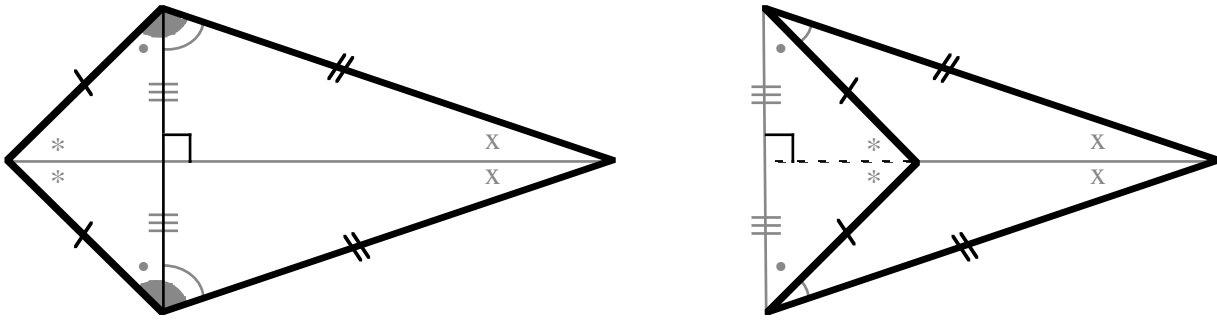


REGULAR pentagon

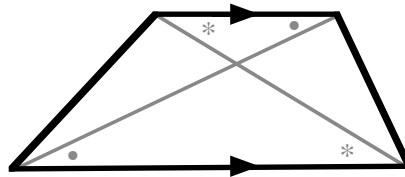
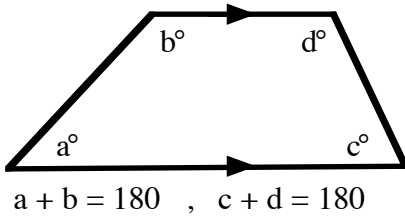
interior angle $540^\circ \div 5 = 108^\circ$

QUADRILATERALS

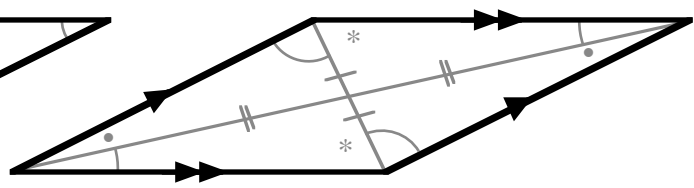
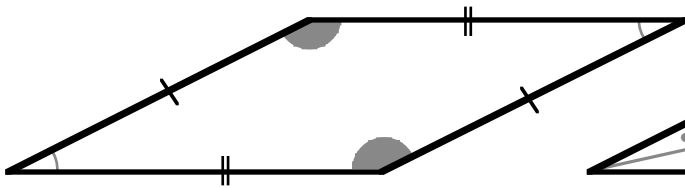
KITE



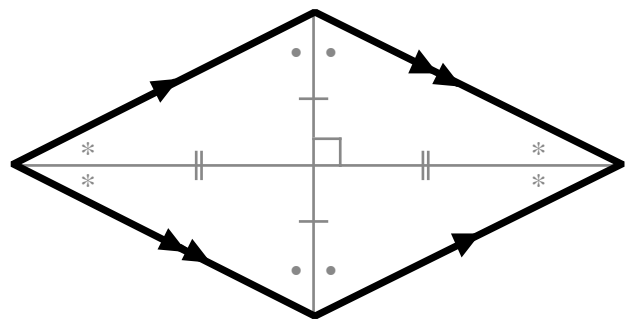
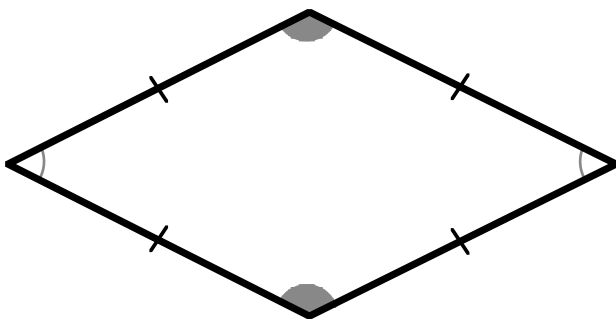
TRAPEZIUM



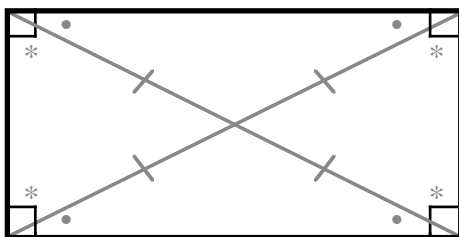
PARALLELOGRAM a trapezium



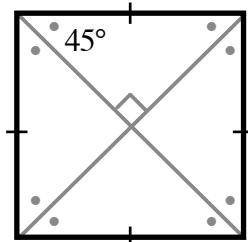
RHOMBUS a kite and parallelogram



RECTANGLE a parallelogram



SQUARE a rectangle and rhombus.

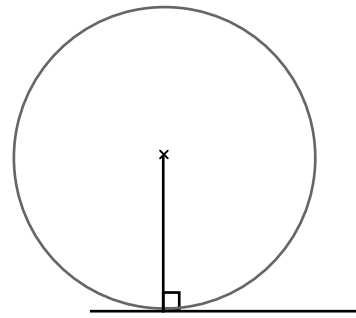
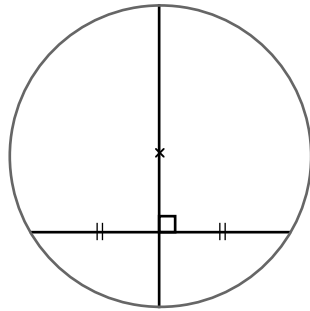
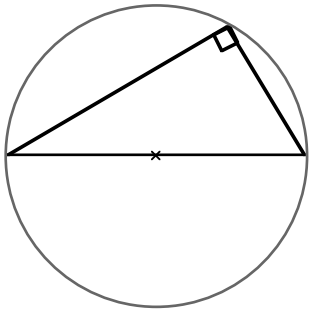


CHAPTER 9: PROPERTIES OF THE CIRCLE

angle in a semicircle is a right-angle.

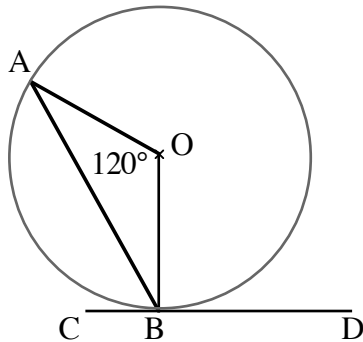
the perpendicular bisector of a chord is a diameter.

a tangent and the radius drawn to the point of contact form a right-angle.



ANGLES

(1)



radius $OA = OB$ so $\triangle AOB$ is isosceles and \triangle angle sum 180° :

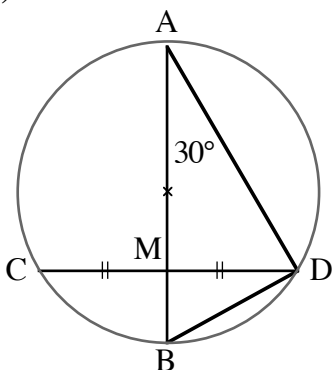
$$\angle OBA = (180^\circ - 120^\circ) \div 2 = 30^\circ$$

tangent CD and radius OB : $\angle OBC = 90^\circ$

Calculate the size of angle ABC.

$$\angle ABC = 90^\circ - 30^\circ = 60^\circ$$

(2)



diameter AB bisects chord CD : $\angle AMD = 90^\circ$

$\triangle AMD$ angle sum 180° :

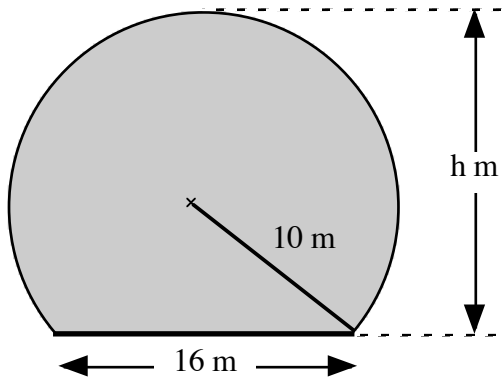
$$\angle ADM = 180^\circ - 90^\circ - 30^\circ = 60^\circ$$

angle in a semicircle : $\angle ADB = 90^\circ$

Calculate the size of angle BDC.

$$\angle BDC = 90^\circ - 60^\circ = 30^\circ$$

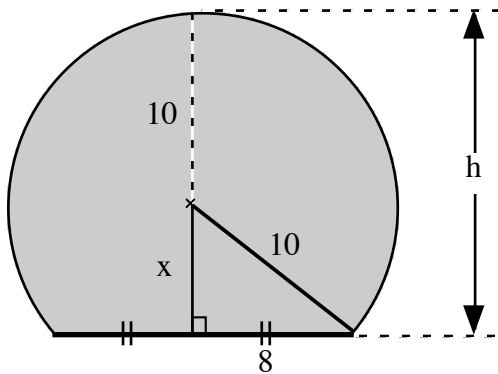
PYTHAGORAS' THEOREM



A circular road tunnel, radius 10 metres, is cut through a hill.

The road has a width 16 metres.

Find the height of the tunnel.



*the diameter drawn is the perpendicular bisector of the chord:
 Δ is right-angled so can apply Pyth. Thm.*

$$\begin{aligned}x^2 &= 10^2 - 8^2 \\ &= 100 - 64 \\ &= 36\end{aligned}$$

$$x = \sqrt{36}$$

$$x = 6$$

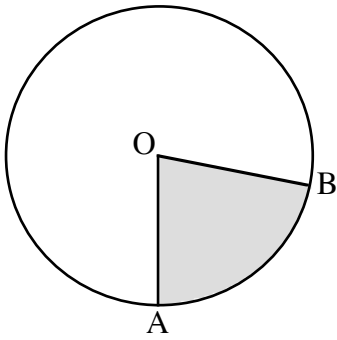
$$h = x + 10$$

$$= 6 + 10$$

$$h = 16$$

height 16 metres

CHAPTER 10: ARCS and SECTORS

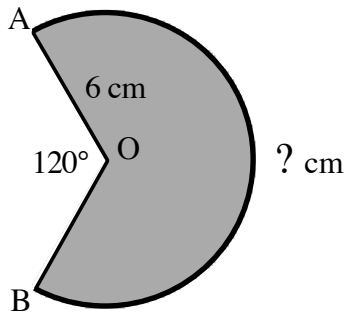


$$\frac{\angle AOB}{360^\circ} = \frac{\text{arc } AB}{\pi d} = \frac{\text{area of sector } AOB}{\pi r^2}$$

Choose the appropriate pair of ratios based on:

- (i) the ratio which includes the quantity to be found
- (ii) the ratio for which both quantities are known (or can be found).

(1) Find the **exact** length of **major** arc AB.



$$\frac{\angle AOB}{360^\circ} = \frac{\text{arc } AB}{\pi d}$$

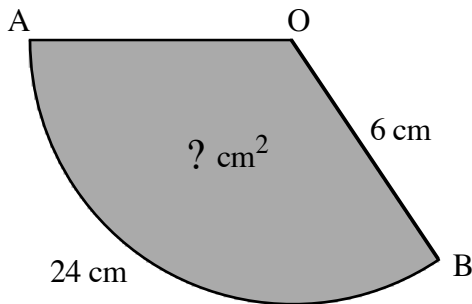
$$\frac{240^\circ}{360^\circ} = \frac{\text{arc } AB}{\pi \times 12}$$

$$\begin{aligned} \text{arc } AB &= \frac{240^\circ}{360^\circ} \times \pi \times 12 \\ &= 8\pi \text{ cm} \quad (25.132\dots) \end{aligned}$$

diameter $d = 2 \times 6 \text{ cm} = 12 \text{ cm}$

reflex $\angle AOB = 360^\circ - 120^\circ = 240^\circ$

(2) Find the **exact** area of sector AOB.



$$\frac{\text{arc } AB}{\pi d} = \frac{\text{area of sector } AOB}{\pi r^2}$$

$$\frac{24}{\pi \times 12} = \frac{\text{area of sector } AOB}{\pi \times 6 \times 6}$$

$$\begin{aligned} \text{area of sector } AOB &= \frac{24}{\pi \times 12} \times \pi \times 6 \times 6 \\ &= 72 \text{ cm}^2 \end{aligned}$$

CHAPTER 11: ALGEBRAIC EXPRESSIONS

REMOVING BRACKETS

SINGLE BRACKETS

$$(1) \quad 3x(2x - y + 7) \\ = 6x^2 - 3xy + 21x$$

$$\begin{aligned} 3x \times 2x &= 6x^2 \\ 3x \times -y &= -3xy \\ 3x \times +7 &= +21x \end{aligned}$$

$$(2) \quad -2(3t + 5) \\ = -6t - 10$$

$$\begin{aligned} -2 \times 3t &= -6t \\ -2 \times +5 &= -10 \end{aligned}$$

$$(3) \quad -3w(w^2 - 4) \\ = -3w^3 + 12w$$

$$\begin{aligned} -3w \times w^2 &= -3w^3 \\ -3w \times -4 &= +12w \end{aligned}$$

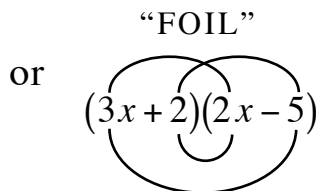
Fully simplify:

$$(4) \quad 2t(3 - t) + 5t^2 \\ = 6t - 2t^2 + 5t^2 \\ = 6t + 3t^2$$

$$(5) \quad 5 - 3(n - 2) \\ = 5 - 3n + 6 \\ = 5 + 6 - 3n \\ = 11 - 3n$$

DOUBLE BRACKETS

$$(1) \quad (3x + 2)(2x - 5) \\ = 3x(2x - 5) + 2(2x - 5) \\ = 6x^2 - 15x + 4x - 10 \\ = 6x^2 - 11x - 10$$



$$(2) \quad (2t - 3)^2 \\ = (2t - 3)(2t - 3) \\ = 2t(2t - 3) - 3(2t - 3) \\ = 4t^2 - 6t - 6t + 9 \\ = 4t^2 - 12t + 9$$

$$(3) \quad (w + 2)(w^2 - 3w + 5) \\ = w(w^2 - 3w + 5) + 2(w^2 - 3w + 5) \\ = w^3 - 3w^2 + 5w + 2w^2 - 6w + 10 \\ = w^3 - 3w^2 + 2w^2 + 5w - 6w + 10 \\ = w^3 - w^2 - w + 10$$

FACTORSATION

COMMON FACTORS

$$ab + ac = a(b + c)$$

Highest Common Factors are used to write expressions in **fully** factorised form.

Factorise **fully**: $4a - 2a^2$

$$= 2a(2 - a)$$

$$2a \times 2 - 2a \times a \text{ using } HCF(4a, 2a^2) = 2a$$

NOTE: the following answers are factorised but not **fully** factorised:

$$2(2a - a^2)$$

$$a(4 - 2a)$$

DIFFERENCE OF TWO SQUARES

$$a^2 - b^2 = (a + b)(a - b)$$

Factorise **fully**:

$$(1) 4x^2 - 9$$

$$(2) 4x^2 - 36$$

$$= (2x)^2 - 3^2$$

$$= 4(x^2 - 9)$$

common factor first

$$= (2x + 3)(2x - 3)$$

$$= 4(x + 3)(x - 3)$$

NOTE: $(2x + 6)(2x - 6)$ is factorised but not **fully** factorised.

TRINOMIALS

$$ax^2 + bx + c, a = 1 \quad \text{ie. } 1x^2$$

$x^2 + bx + c = (x + ?)(x + ?)$ The missing numbers are: **a pair of factors of c**
that **sum to b**

Factorise **fully**:

(1) $x^2 + 5x + 6$

(2) $x^2 - 5x + 6$

(3) $x^2 - 5x - 6$

$$1 \times 6 = 2 \times 3 = 6$$

$$-1, -6 \text{ or } -2, -3$$

$$-1, 6 \text{ or } 1, -6 \text{ or } -2, 3 \text{ or } 2, -3$$

$$2 + 3 = 5$$

$$-2 + (-3) = -5$$

$$1 + (-6) = -5$$

use +2 and +3

use -2 and -3

use +1 and -6

$$= (x + 2)(x + 3)$$

$$= (x - 2)(x - 3)$$

$$= (x + 1)(x - 6)$$

$$ax^2 + bx + c, a \neq 1 \quad \text{ie. not } 1x^2$$

Try out the possible combinations of the factors which could be in the brackets.

Factorise $3t^2 - 10t - 8$

$3 \times (-8) = -24$ pairs of factors 1,24 or 2,12 or 3,8 or 4,6 one factor is negative

$$\overbrace{3t^2 - 10t - 8}$$

$$-12 + 2 = -10$$

factors sum to -10

$$-12t + 2t = -10t$$

try combinations so that $-12t$ and $2t$ are obtained

factor pairs: $3t \times t$ for $3t^2$

1×8 or 2×4 for 8, one factor negative

$$\begin{array}{cc} 3t & +1 \\ & \swarrow \searrow \\ t & -8 \\ 1t & -24t \end{array}$$

$$\begin{array}{cc} 3t & -8 \\ & \swarrow \searrow \\ t & +1 \\ -8t & 3t \end{array}$$

$$\begin{array}{cc} 3t & +2 \\ & \swarrow \searrow \\ t & -4 \\ 2t & -12t \end{array} \quad \checkmark$$

$$\begin{array}{cc} 3t & +4 \\ & \swarrow \searrow \\ t & -2 \\ 4t & -6t \end{array}$$

$$\underline{\underline{(3t + 2)(t - 4)}}$$

ALGEBRAIC FRACTIONS

SIMPLIFYING: fully factorise 'top' and 'bottom' and 'cancel' common factors

MULTIPLY/DIVIDE:

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$$

ADD/SUBTRACT: a common denominator is required.

$$(1) \quad \frac{x^2 - 9}{x^2 + 2x - 3}$$

$$= \frac{(x-3)(x+3)}{(x-1)(x+3)} \quad \text{fully factorise}$$

$$= \frac{x-3}{x-1} \quad \text{cancel common factors}$$

$$(2) \quad \frac{4}{ab} \div \frac{8a}{b^2}$$

$$= \frac{4}{ab} \times \frac{b^2}{8a}$$

$$= \frac{4 \times b^2}{ab \times 8a}$$

$$= \frac{4 \times b \times b}{8 \times a \times a \times b}$$

$$= \frac{1 \times b}{2 \times a \times a}$$

$$= \frac{b}{2a^2}$$

$$(3) \quad \frac{x}{2} - \frac{x-3}{3}$$

$$= \frac{3x}{6} - \frac{2(x-3)}{6}$$

$$= \frac{3x - 2(x-3)}{6}$$

$$= \frac{3x - 2x + 6}{6} \quad \text{notice sign change}$$

$$= \frac{x+6}{6}$$

$$(4) \quad \frac{1}{x} + \frac{3}{x(x-3)}$$

$$= \frac{1(x-3)}{x(x-3)} + \frac{3}{x(x-3)}$$

$$= \frac{x-3+3}{x(x-3)}$$

$$= \frac{x}{x(x-3)} \quad \text{can simplify}$$

$$= \frac{1}{x-3}$$

CHAPTER 12: EQUATIONS and INEQUALITIES

Simplify by following the rules:

addition and subtraction

$$\begin{array}{l} x + a = b \\ x = b - a \end{array} \qquad \begin{array}{l} x - a = b \\ x = b + a \end{array}$$

multiplication and division

$$\begin{array}{l} ax = b \\ x = \frac{b}{a} \end{array} \qquad \begin{array}{l} \frac{x}{a} = b \\ x = ab \end{array}$$

(1) Solve: $5x - 4 = 2x - 19$

$$\begin{array}{l} 3x - 4 = -19 \\ 3x = -15 \\ x = -5 \end{array} \qquad \begin{array}{l} \textit{subtracted } 2x \textit{ from each side} \\ \textit{added } 4 \textit{ to each side} \\ \textit{divided each side by } 3 \end{array}$$

WITH BRACKETS remove first and simplify

(2) Solve: $(4x + 3)(x - 2) = (2x - 3)^2$

$$\begin{array}{l} 4x^2 - 5x - 6 = 4x^2 - 12x + 9 \\ -5x - 6 = -12x + 9 \\ 7x - 6 = +9 \\ 7x = 15 \\ x = \frac{15}{7} \end{array} \qquad \begin{array}{l} \textit{removed brackets, fully simplifying} \\ \textit{subtracted } 4x^2 \textit{ from each side} \\ \textit{added } 12x \textit{ to each side} \\ \textit{added } 6 \textit{ to each side} \\ \textit{divided each side by } 7 \end{array}$$

WITH FRACTIONS remove first, multiplying by the LCM of the denominators

(3) Solve $\frac{1}{2}(x + 3) + \frac{1}{3}x = 1$

$$\begin{array}{l} \frac{3}{6}(x + 3) + \frac{2}{6}x = \frac{6}{6} \\ 3(x + 3) + 2x = 6 \\ 3x + 9 + 2x = 6 \\ 5x = -3 \\ x = -\frac{3}{5} \end{array} \qquad \begin{array}{l} \textit{write with common denominators} \\ \textit{both sides } \times 6 \textit{ to remove fractions} \end{array}$$

INEQUALITIES

Follow the rules for equations, except:

multiply or divide by a **negative** number, reverse the direction of the inequality sign

$$-ax > b \qquad \frac{x}{-a} > b$$

$$x < \frac{b}{-a} \qquad x < -ab$$

$$(1) \quad 8 + 3x > 2$$

$$+3x > -6$$

$$x > \frac{-6}{+3} \quad \text{divided each side by } +3 \\ \text{notice sign unchanged}$$

$$x > -2$$

$$(2) \quad 8 - 3x > 2$$

$$-3x > -6 \quad \text{subtracted 8 from each side}$$

$$x < \frac{-6}{-3} \quad \text{divided each side by } -3 \\ \text{notice sign reversed}$$

$$x < 2 \quad \text{simplified}$$

RESTRICTIONS ON SOLUTIONS

$$(1) \quad x \leq \frac{5}{2} \quad \text{where } x \text{ is a whole number}$$

$$x = 0, 1, 2$$

$$(2) \quad -2 \leq x < 2 \quad \text{where } x \text{ is an integer}$$

$$x = -2, -1, 0, 1$$

TRANSPOSING FORMULAE (CHANGE OF SUBJECT)

Follow the rules for equations to isolate the **target letter**.

addition and subtraction

$$\begin{array}{l} x + a = b \\ x = b - a \end{array} \qquad \begin{array}{l} x - a = b \\ x = b + a \end{array}$$

multiplication and division

$$\begin{array}{l} ax = b \\ x = \frac{b}{a} \end{array} \qquad \begin{array}{l} \frac{x}{a} = b \\ x = ab \end{array}$$

powers and roots

$$\begin{array}{l} x^2 = a \\ x = \sqrt{a} \end{array} \qquad \begin{array}{l} \sqrt{x} = a \\ x = a^2 \end{array}$$

$F = 3r^2 + p$ Change the subject of the formula to r.

$$\begin{array}{ccccccc} r & \xrightarrow{\text{square}} & r^2 & \xrightarrow{\times 3} & 3r^2 & \xrightarrow{+p} & F \\ \sqrt{\frac{F-p}{3}} & \xleftarrow{\sqrt{\quad}} & \frac{F-p}{3} & \xleftarrow{\div 3} & F-p & \xleftarrow{-p} & F \end{array}$$

inverse operations in reverse order

$$F = 3r^2 + p$$

$$F - p = 3r^2 \qquad \text{subtract } p \text{ from each side}$$

$$\frac{F-p}{3} = r^2 \qquad \text{divide each side by 3}$$

$$\sqrt{\frac{F-p}{3}} = r \qquad \text{square root both sides}$$

$$r = \sqrt{\frac{F-p}{3}} \qquad \text{subject of formula now } r$$

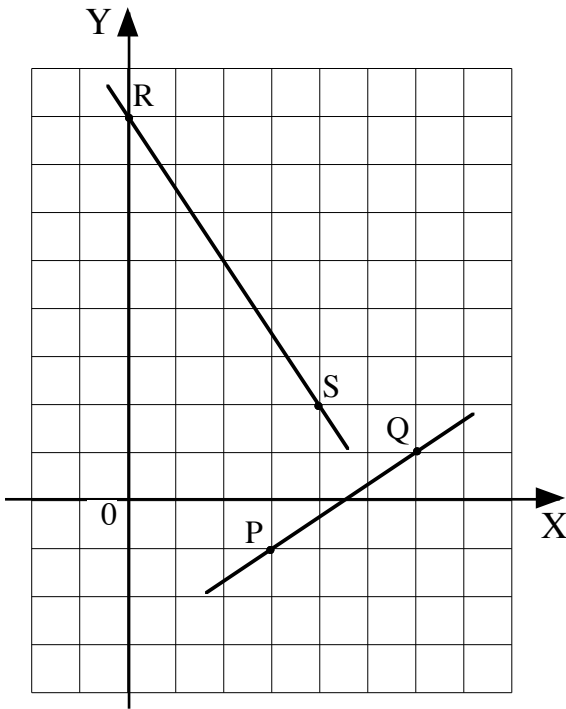
CHAPTER 13: STRAIGHT LINE

GRADIENT The slope of a line is given by the ratio:

$$m = \frac{\text{vertical change}}{\text{horizontal change}}$$

Using coordinates, the **gradient formula** is

$$m_{AB} = \frac{y_B - y_A}{x_B - x_A}$$



$$R(0,8) \quad , \quad S(4,2)$$

$$m_{RS} = \frac{y_S - y_R}{x_S - x_R} = \frac{2 - 8}{4 - 0} = \frac{-6}{4} = -\frac{3}{2}$$

$$P(3,-1) \quad , \quad Q(6,1)$$

$$m_{PQ} = \frac{y_Q - y_P}{x_Q - x_P} = \frac{1 - (-1)}{6 - 3} = \frac{2}{3}$$

note: same result for $\frac{-1 - 1}{3 - 6} = \frac{-2}{-3} = \frac{2}{3}$

EQUATION OF A STRAIGHT LINE

gradient m , y-intercept C units ie. meets the y-axis at $(0,C)$

$$y = mx + C$$

gradient m , through the point (a,b)

$$y - b = m(x - a)$$

equation of line RS:

$$m_{RS} = -\frac{3}{2} \quad y = mx + C$$

$$R(0,8) \quad C = 8 \quad y = -\frac{3}{2}x + 8$$

equation of line PQ:

$$m_{PQ} = \frac{2}{3} \quad y - b = m(x - a)$$

$$\begin{matrix} a & b \\ Q(6, 1) \end{matrix} \quad y - 1 = \frac{2}{3}(x - 6)$$

or use point P

$$3y - 3 = 2(x - 6)$$

$$3y - 3 = 2x - 12$$

$$3y = 2x - 9$$

Rearrange the equation to $y = mx + C$ for the gradient and y-intercept.

$$3x + 2y - 16 = 0$$

$$2y = -3x + 16 \quad \text{isolate } y \text{ - term}$$

$$y = -\frac{3}{2}x + 8 \quad \text{obtain } 1y =$$

$$y = mx + C \quad \text{compare to the general equation}$$

$$m = -\frac{3}{2}, \quad C = 8 \quad \text{meets the } Y \text{ - axis at } (0,8)$$

RATE OF CHANGE The gradient is the rate of change.

distance/time graphs: the gradient is the speed.

speed/time graphs: the gradient is the acceleration.

The steeper the graph, the greater the rate of change.

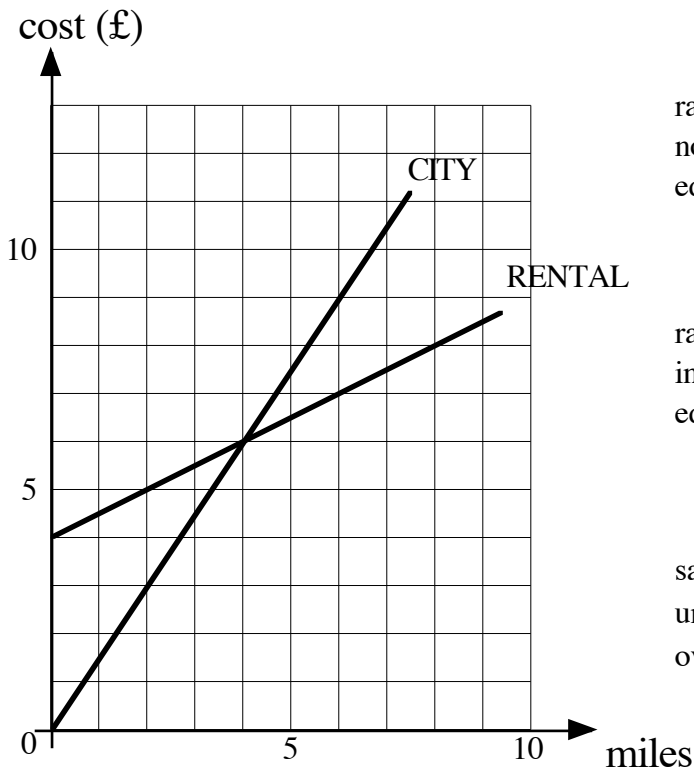
A positive gradient is an increase, a negative gradient is a decrease.

Gradient zero (horizontal line), no change.

RENTAL TAXIS requires a £4 payment plus a charge of 50p per mile.

CITY TAXIS charges £1.50 per mile.

Advise on the better buy.

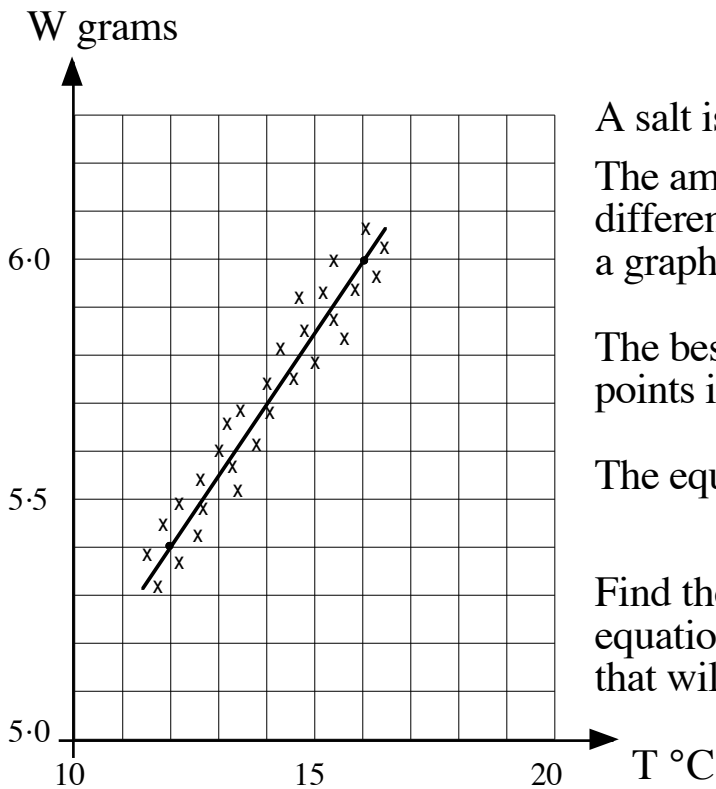


rate of change £1.50 per mile , $m = 1.5$
 no initial charge , $C = 0$
 equation $y = 1.5x$

rate of change £0.50 per mile , $m = 0.5$
 initial charge £4 , $C = 4$
 equation $y = 0.5x + 4$

same cost for 4 miles, but
 under 4 miles CITY cheaper
 over 4 miles RENTAL cheaper

SCATTER DIAGRAM: LINE OF BEST FIT



A salt is dissolved in a litre of solvent. The amount of salt that dissolves at different temperatures is recorded and a graph plotted.

The best-fitting straight line through the points is drawn.

The equation of the graph is of the form

$$W = mT + C.$$

Find the equation of the line and use the equation to calculate the mass of salt that will dissolve at 30 °C.

using two well-separated points on the line (16 , 6.0) (12 , 5.4)

$$m = \frac{6.0 - 5.4}{16 - 12} = \frac{0.6}{4} = 0.15$$

substituting for one point on the line $\begin{matrix} a & b \\ (16 , 6.0) \end{matrix}$

$$y - b = m(x - a)$$

$$y - 6.0 = 0.15(x - 16)$$

$$y - 6.0 = 0.15x - 2.4$$

$$y = 0.15x + 3.6$$

$$\underline{\underline{W = 0.15T + 3.6}}$$

$$T = 30$$

$$W = 0.15 \times 30 + 3.6$$

$$= 4.5 + 3.6$$

$$= 8.1$$

$$\underline{\underline{8.1 \text{ grams}}}$$

CHAPTER 14: SIMULTANEOUS EQUATIONS

The equation of a line is a rule connecting the x and y coordinates of any point on the line.

For example,
$$\begin{array}{r} x \quad y \\ (-2, 12) \end{array} \quad \begin{array}{l} y + 2x = 8 \\ 12 + 2 \times (-2) = 8 \end{array}$$

To sketch a line, find the points where the line meets the axes.

SOLVE SIMULTANEOUS EQUATIONS: GRAPHICAL METHOD

Sketch the two lines and the point of intersection is the solution.

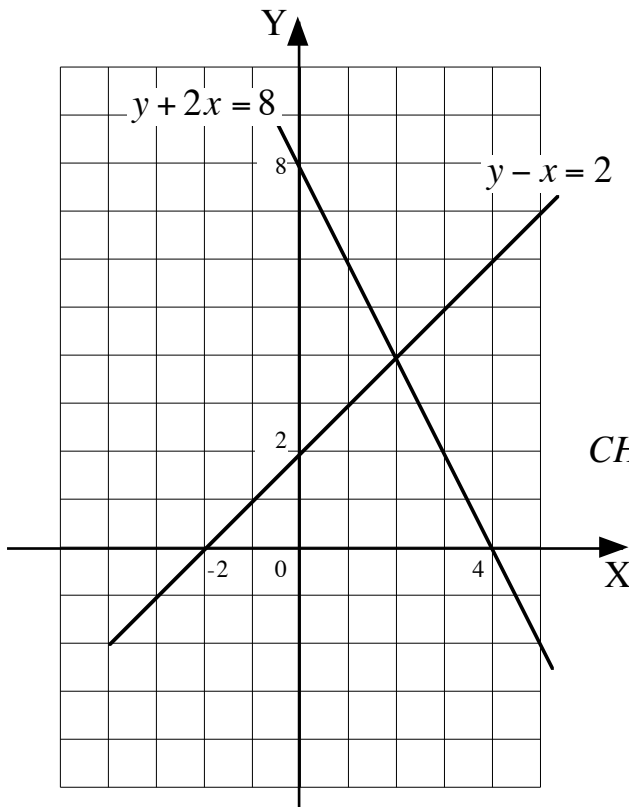
Solve **graphically** the system of equations:
$$\begin{array}{l} y + 2x = 8 \\ y - x = 2 \end{array}$$

$$\begin{array}{l} y + 2x = 8 \quad (1) \\ y + 2 \times 0 = 8 \quad \text{substituted for } x = 0 \\ y = 8 \quad \text{plot } (0, 8) \end{array}$$

$$\begin{array}{l} y - x = 2 \quad (2) \\ y - 0 = 2 \quad \text{substituted for } x = 0 \\ y = 2 \quad \text{plot } (0, 2) \end{array}$$

$$\begin{array}{l} y + 2x = 8 \quad (1) \\ 0 + 2x = 8 \quad \text{substituted for } y = 0 \\ x = 4 \quad \text{plot } (4, 0) \end{array}$$

$$\begin{array}{l} y - x = 2 \quad (2) \\ 0 - x = 2 \quad \text{substituted for } y = 0 \\ x = -2 \quad \text{plot } (-2, 0) \end{array}$$



point of intersection (2,4)

SOLUTION: $x = 2$ and $y = 4$

CHECK: $x = 2$ and $y = 4$
substituted in both equations

$$\begin{array}{l} y + 2x = 8 \quad (1) \\ 4 + 2 \times 2 = 8 \\ 8 = 8 \end{array} \quad \begin{array}{l} y - x = 2 \quad (2) \\ 4 - 2 = 2 \\ 2 = 2 \end{array}$$

SOLVE SIMULTANEOUS EQUATIONS: SUBSTITUTION METHOD

Rearrange one equation to $y =$ and substitute for y in the other equations
(or $x =$)

Solve **algebraically** the system of equations: $y + 2x = 8$

$$3y - x = 10$$

$$y + 2x = 8 \quad (1)$$

$$y = 8 - 2x$$

can choose to rearrange to $y =$ or $x =$

choosing $y =$ avoids fractions as $x = 4 - \frac{1}{2}y$

$$3y - x = 10 \quad (2)$$

$$3(8 - 2x) - x = 10$$

$$24 - 6x - x = 10$$

$$-7x = -14$$

$$x = 2$$

replace y by $8 - 2x$

solve

$$y = 8 - 2x \quad (1)$$

$$= 8 - 2 \times 2$$

$$y = 4$$

can choose either equation (1) or (2)

substituted for $x = 2$

SOLUTION: $x = 2$ and $y = 4$

CHECK:

$$3y - x = 10 \quad (2)$$

$$3 \times 4 - 2 = 10$$

$$10 = 10$$

using the other equation

substituted for $x = 2$ and $y = 4$

SOLVE SIMULTANEOUS EQUATIONS: ELIMINATION METHOD

Can add or subtract multiples of the equations to eliminate either the x or y term.

Solve **algebraically** the system of equations: $4x + 3y = 5$
 $5x - 2y = 12$

$$\begin{array}{lll} 4x + 3y = 5 & (1) \times 2 & \text{can choose to eliminate } x \text{ or } y \text{ term} \\ 5x - 2y = 12 & (2) \times 3 & \text{choosing } y \text{ term, LCM } (3y, 2y) = 6y \\ \\ 8x + 6y = 10 & (3) & \text{multiplied each term of (1) by 2 for } +6y \\ 15x - 6y = 36 & (4) & \text{multiplied each term of (2) by 3 for } -6y \\ \\ 23x & = 46 & (3) + (4) \quad \text{added equations, adding "like" terms} \\ x = 2 & & +6y \text{ and } -6y \text{ added to 0 (ie eliminate)} \end{array}$$

$$\begin{array}{ll} 4x + 3y = 5 & (1) \\ 4 \times 2 + 3y = 5 & \text{can choose either equation (1) or (2)} \\ 8 + 3y = 5 & \text{substituted for } x = 2 \\ 3y = -3 & \\ y = -1 & \end{array}$$

SOLUTION: $x = 2$ and $y = -1$

CHECK:

$$\begin{array}{ll} 5x - 2y = 12 & (2) \\ 5 \times 2 - 2 \times (-1) = 12 & \text{using the other equation} \\ 10 - (-2) = 12 & \text{substituted for } x = 2 \text{ and } y = -1 \\ 12 = 12 & \end{array}$$

CHAPTER 15: QUADRATIC EQUATIONS

A function pairs one number with another, its IMAGE. It can be defined by a formula.

QUADRATIC FUNCTIONS

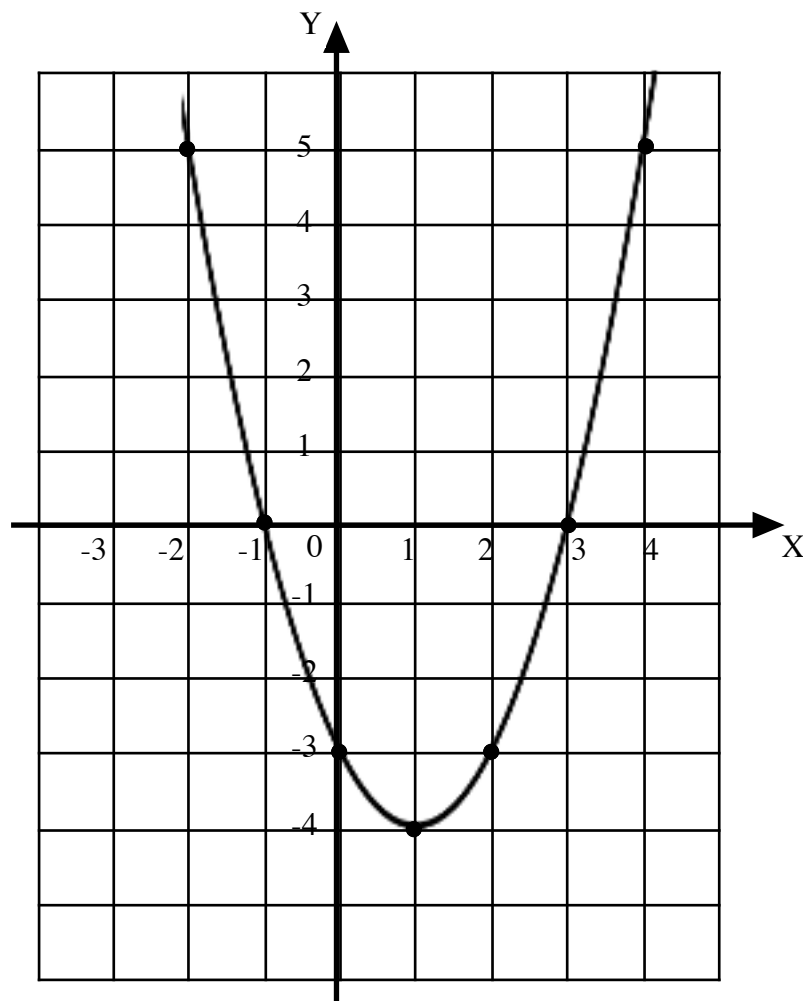
$f(x) = ax^2 + bx + c$, $a \neq 0$, where a , b and c are constants.

$$f(x) = x^2 - 2x - 3$$

$f(-2) = (-2)^2 - 2 \times (-2) - 3 = 4 + 4 - 3 = 5$, the image of -2 is 5.

x	-2	-1	0	1	2	3	4
$f(x)$	5	0	-3	-4	-3	0	5
points	(-2,5)	(-1,0)	(0,-3)	(1,-4)	(2,-3)	(3,0)	(4,5)

If all possible values of x are considered, a graph will show the images (the RANGE). The graph is a symmetrical curve called a PARABOLA.



The graph meets the x-axis where $x^2 - 2x - 3 = 0$. The **zeros** of the graph are -1 and 3 which are the **roots** of the equation.

QUADRATIC EQUATIONS

An equation of the form $ax^2 + bx + c = 0$, $a \neq 0$, where a , b and c are constants.

The values of x that satisfy the equation are the **roots** of the equation.

The quadratic formula can be used to solve the equation.

QUADRATIC FORMULA

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, a \neq 0$$

Find the **roots** of the equation $3x^2 - 5x - 1 = 0$, correct to two decimal places.

$$3x^2 - 5x - 1 = 0$$

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = 3, b = -5, c = -1$$

$$x = \frac{5 \pm \sqrt{37}}{6}$$

$$b^2 - 4ac = (-5)^2 - 4 \times 3 \times (-1) = 37$$

$$-b = -(-5) = +5$$

$$2a = 2 \times 3 = 6$$

$$= \frac{5 - \sqrt{37}}{6} \quad \text{or} \quad \frac{5 + \sqrt{37}}{6}$$

$$= \frac{-1.0827\dots}{6} \quad \text{or} \quad \frac{11.0827\dots}{6}$$

$$x = -0.1804\dots \quad \text{or} \quad 1.8471\dots$$

roots are -0.18 and 1.85

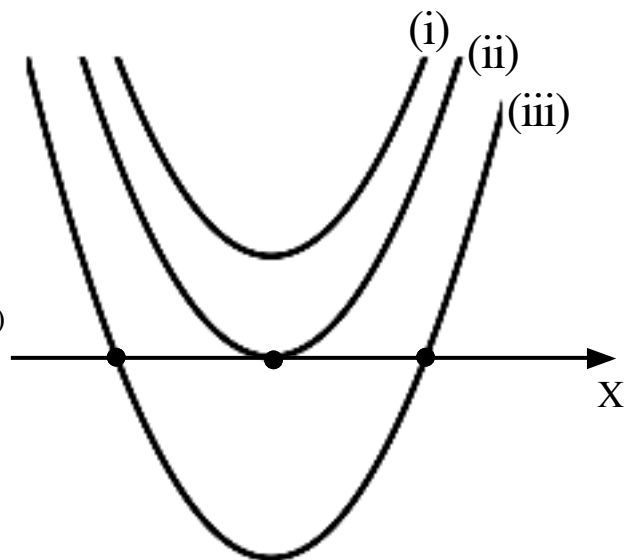
DISCRIMINANT

$b^2 - 4ac$ gives the nature of the roots

(i) $b^2 - 4ac < 0$ NO REAL ROOTS
ie. negative

(ii) $b^2 - 4ac = 0$ ONE REAL ROOT (repeated)
(two EQUAL roots)

(iii) $b^2 - 4ac > 0$ TWO REAL ROOTS
(and DISTINCT)



Find k so that $2x^2 - 4x - k = 0$ has **real** roots.

$$\begin{array}{ll}
 2x^2 - 4x - k = 0 & \text{for real roots, } b^2 - 4ac \geq 0 \\
 ax^2 + bx + c = 0 & 16 + 8k \geq 0 \\
 a = 2, b = -4, c = -k & 8k \geq -16 \\
 b^2 - 4ac = (-4)^2 - 4 \times 2 \times (-k) & k \geq -2 \\
 = 16 + 8k &
 \end{array}$$

FACTORISATION

RATIONAL ROOTS (non-surds) $b^2 - 4ac =$ a square number ie. 0, 1, 4, 9, 16, ...
the quadratic can be factorised to solve the equation.

IRRATIONAL ROOTS (surds) $b^2 - 4ac \neq$ a square number
solve the equation by formula.

Solve:

$$(1) \quad 4n - 2n^2 = 0$$

$$2n(2 - n) = 0$$

$$2n = 0 \quad \text{or} \quad 2 - n = 0$$

$$\underline{\underline{n = 0 \quad \text{or} \quad n = 2}}$$

$$(2) \quad 2t^2 + t - 6 = 0$$

$$(2t - 3)(t + 2) = 0$$

$$2t - 3 = 0 \quad \text{or} \quad t + 2 = 0$$

$$2t = 3$$

$$\underline{\underline{t = \frac{3}{2} \quad \text{or} \quad t = -2}}$$

The equation may need to be rearranged:

$$(3) \quad (w + 1)^2 = 2(w + 5)$$

$$w^2 + 2w + 1 = 2w + 10$$

$$w^2 - 9 = 0$$

$$(w + 3)(w - 3) = 0$$

$$w + 3 = 0 \quad \text{or} \quad w - 3 = 0$$

$$\underline{\underline{w = -3 \quad \text{or} \quad w = 3}}$$

$$(4) \quad x + 2 = \frac{15}{x}, \quad x \neq 0$$

$$x(x + 2) = 15$$

$$x^2 + 2x = 15$$

$$x^2 + 2x - 15 = 0$$

$$(x + 5)(x - 3) = 0$$

$$x + 5 = 0 \quad \text{or} \quad x - 3 = 0$$

$$\underline{\underline{x = -5 \quad \text{or} \quad x = 3}}$$

GRAPHS

A sketch of the graph of a quadratic function should show where the parabola meets the axes and the maximum or minimum turning point.

Sketch the graph $y = x^2 - 2x - 3$.

(i) meets the Y-axis where $x = 0$

$$y = 0^2 - 2 \times 0 - 3 = -3 \text{ point } (0, -3)$$

(ii) meets the X-axis where $y = 0$

$$x^2 - 2x - 3 = 0$$

$$(x + 1)(x - 3) = 0$$

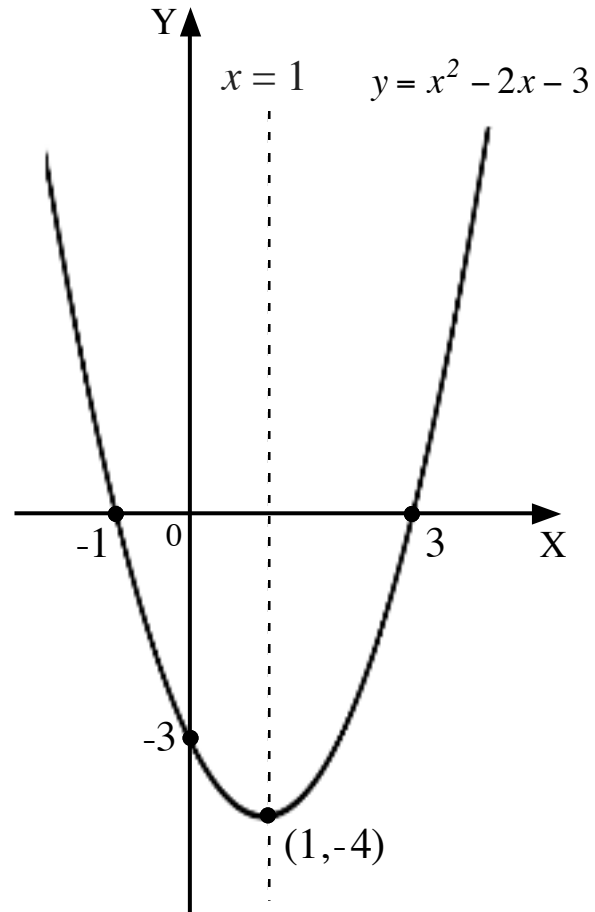
$$x = -1 \text{ or } x = 3$$

points $(-1, 0)$ and $(3, 0)$

(iii) axis of symmetry

vertical line half-way between the zeros

$$x = \frac{-1 + 3}{2} = \frac{2}{2} = 1, \text{ equation } x = 1.$$



(iv) turning point

lies on the axis of symmetry $x = 1$

$$y = 1^2 - 2 \times 1 - 3 = -4 \text{ point } (1, -4)$$

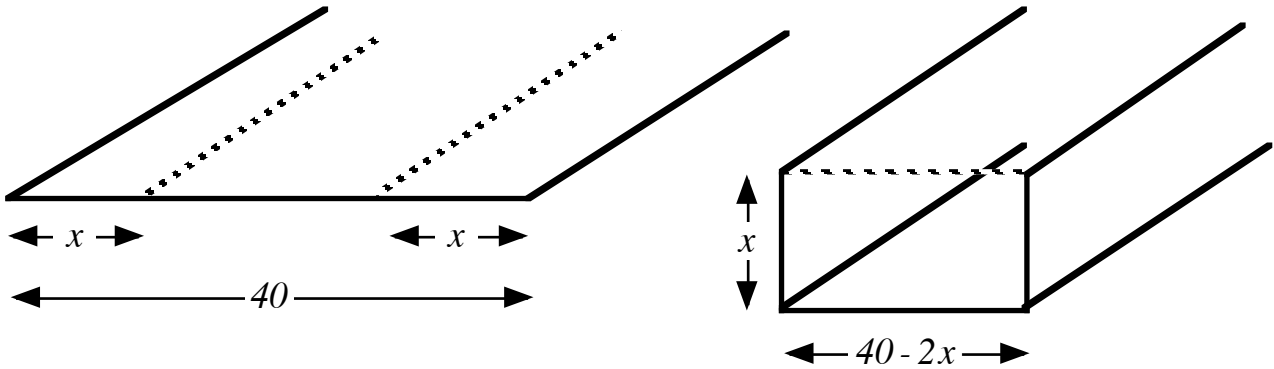
Note: (1) from the graph, $(1, -4)$ is a **minimum turning point**.

(2) the **minimum value** of the function is -4 .

APPLICATIONS

Problems involving maxima or minima which can be modelled by a quadratic equation.

A sheet of metal 40 cm. wide is folded x cm from each end to form a gutter. To maximise water flow the rectangular cross-section should be as large as possible.



Find the maximum cross-sectional area possible.

$$\begin{aligned}
 A &= lb \\
 &= x(40 - 2x) \quad \text{sketch the graph } A = 40x - 2x^2 \\
 &= 40x - 2x^2
 \end{aligned}$$

Zeros:

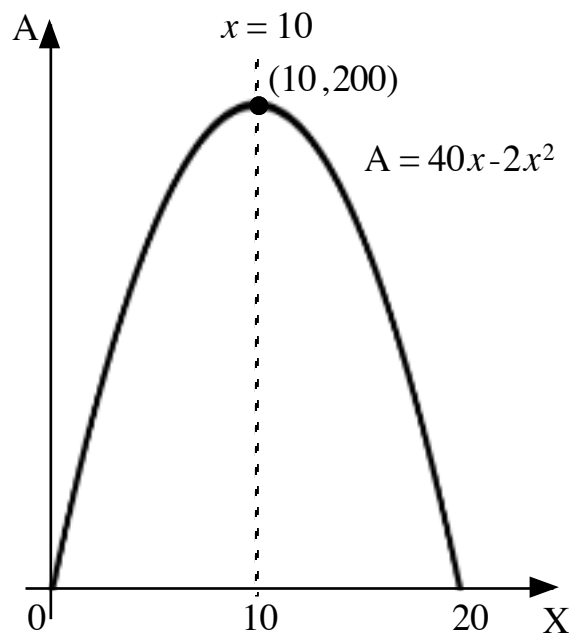
$$\begin{aligned}
 40x - 2x^2 &= 0 \\
 2x(20 - x) &= 0 \\
 2x = 0 \quad \text{or} \quad 20 - x = 0 \\
 x = 0 \quad \text{or} \quad x = 20
 \end{aligned}$$

Turning Point:

$$\begin{aligned}
 (0 + 20) \div 2 &= 10 \\
 \text{axis of symmetry } x &= 10
 \end{aligned}$$

$$\begin{aligned}
 y &= 40x - 2x^2 \\
 y &= 40 \times 10 - 2 \times 10^2 = 200
 \end{aligned}$$

maximum turning point $(10, 200)$



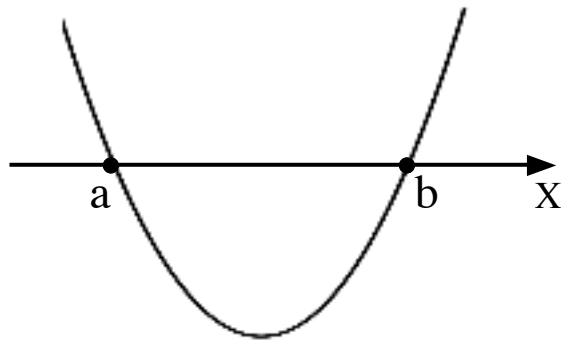
Maximum area 200 square centimetres.

EQUATION OF A GRAPH

$$y = k(x - a)(x - b)$$

a and b are the zeros of the graph

k is a constant.



Write the equation of the graph.

$$y = k(x - a)(x - b)$$

zeros -1 and 3

$$y = k(x + 1)(x - 3)$$

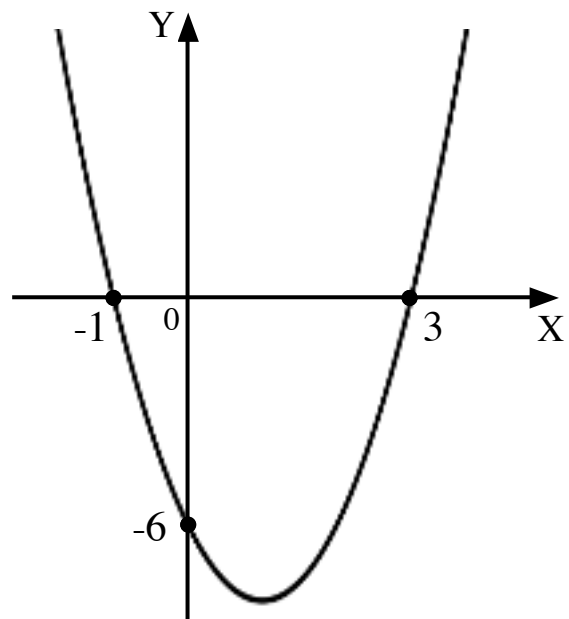
for $\begin{matrix} x & y \\ (0, & -6) \end{matrix}$, $-6 = k(0 + 1)(0 - 3)$

$$-6 = k \times 1 \times (-3)$$

$$-6 = -3k$$

$$k = 2$$

$$y = 2(x + 1)(x - 3)$$



Write the equation of the graph.

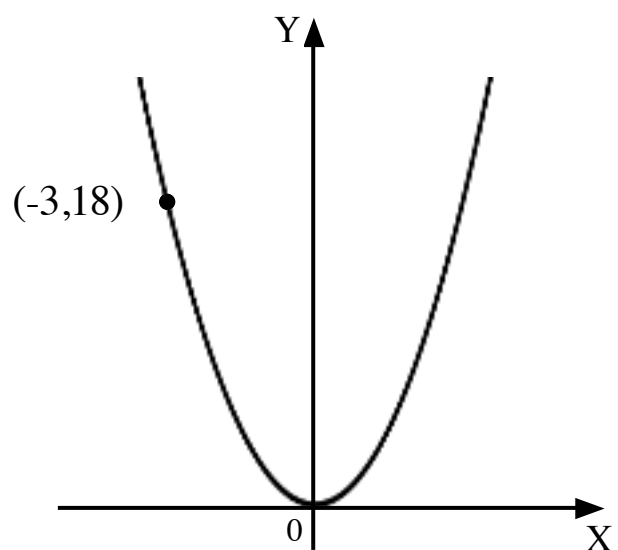
$$y = kx^2$$

for $\begin{matrix} x & y \\ (-3, & 18) \end{matrix}$, $18 = k \times (-3)^2$

$$18 = 9k$$

$$k = 2$$

$$y = 2x^2$$



COMPLETED SQUARE:

Quadratic functions written in the form $y = \pm 1(x - a)^2 + b$, a and b are constants.

axis of symmetry $x = a$

turning point (a, b) , minimum for $+1$, maximum for -1

$y = x^2 - 8x + 21$ can be written as $y = (x - 4)^2 + 5$

meets the y-axis where $x = 0$

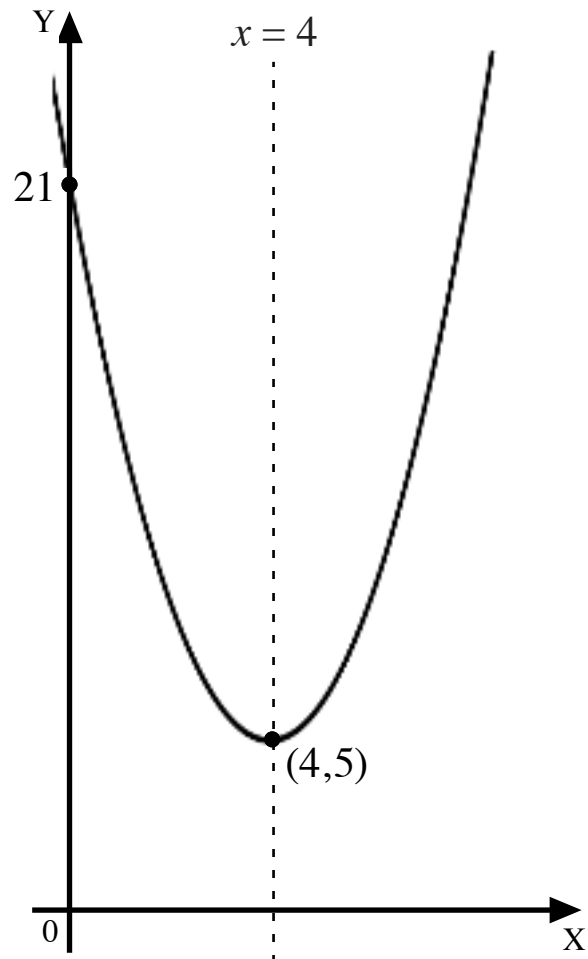
$y = (0 - 4)^2 + 5 = 16 + 5 = 21$

point $(0, 21)$

turning point: $y = +1(x - 4)^2 + 5$

minimum turning point $(4, 5)$

axis of symmetry $x = 4$



COMPLETING THE SQUARE:

$x^2 - 8x + 21$

$x^2 - 8x$ $+ 21$ *coefficient of x , -8 , is halved and squared, $(-4)^2$ to 16*

$= x^2 - 8x + 16 - 16 + 21$ *add and subtract 16*

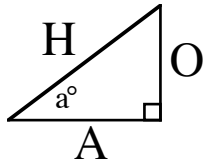
$= (x - 4)^2 - 16 + 21$ *complete square around trinomial*

$= (x - 4)^2 + 5$

CHAPTER 16: TRIGONOMETRY: RIGHT-ANGLED TRIANGLES

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The sides of a right-angled triangle are labelled:



Opposite: opposite the angle a° .

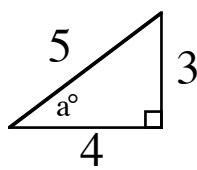
Adjacent: next to the angle a° .

Hypotenuse: opposite the right angle.

The ratios of sides $\frac{O}{H}$, $\frac{A}{H}$ and $\frac{O}{A}$ have values which depend on the size of angle a° .

These are called the sine, cosine and tangents of a° , written $\sin a^\circ$, $\cos a^\circ$ and $\tan a^\circ$.

For example,



$$S = \frac{O}{H}$$

$$C = \frac{A}{H}$$

$$T = \frac{O}{A}$$

$$\sin a^\circ = \frac{3}{5}$$

$$\cos a^\circ = \frac{4}{5}$$

$$\tan a^\circ = \frac{3}{4}$$

The trig. function acts on an angle to produce the value of the ratio.

The inverse trig. function acts on the value of a ratio to produce the angle.

For example,

$$\sin 30^\circ = 0.5$$

$$\sin^{-1} 0.5 = 30^\circ$$

ACCURACY

Rounding the angle or the value in a calculation can result in significant errors.

For example,

$$100 \times \tan 69.5^\circ = 267.462... \approx 267$$

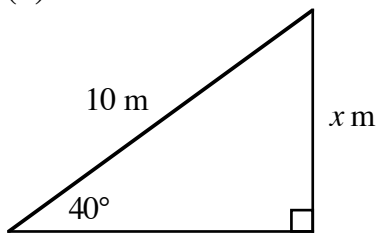
$$\tan^{-1} 2.747 = 69.996... \approx 70.0$$

$$100 \times \tan 70^\circ = 274.747... \approx 275$$

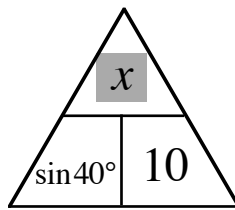
$$\tan^{-1} 2.7 = 69.676... \approx 69.7$$

FINDING AN UNKNOWN SIDE

(1) Find x .



rearrange for x

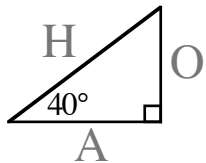


$$\sin 40^\circ = \frac{x}{10}$$

$$x = 10 \times \sin 40^\circ \quad \text{ensure calculator set to **DEGREES**}$$

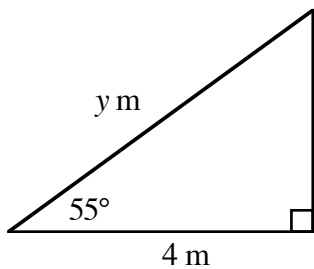
$$= 6.427\dots$$

$$x = 6.4$$

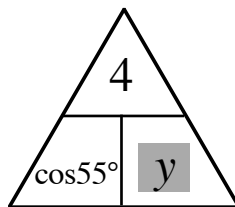


know H , find O
 $\downarrow\downarrow \quad \downarrow\downarrow$
SOH-CAH-TOA

(2) Find y .



rearrange for y

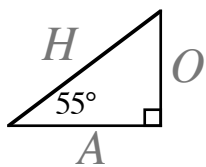


$$\cos 55^\circ = \frac{4}{y}$$

$$y = \frac{4}{\cos 55^\circ} \quad \text{4} \div \cos 55^\circ, \text{ calculator set to **DEGREES**}$$

$$= 6.973\dots$$

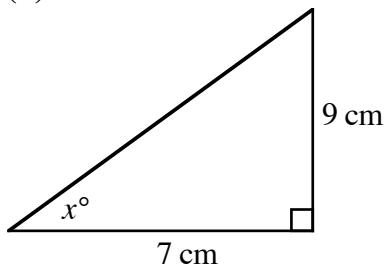
$$y = 7.0$$



know A , find H
 $\downarrow \quad \downarrow\downarrow \quad \downarrow\downarrow$
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FINDING AN UNKNOWN ANGLE

(3) Find x .

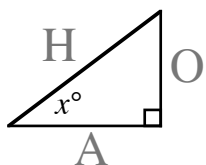


$$\tan x^\circ = \frac{9}{7}$$

$$x = \tan^{-1}\left(\frac{9}{7}\right) \quad \text{use brackets for (9} \div \text{7), calculator set to **DEGREES**}$$

$$= 52.125\dots$$

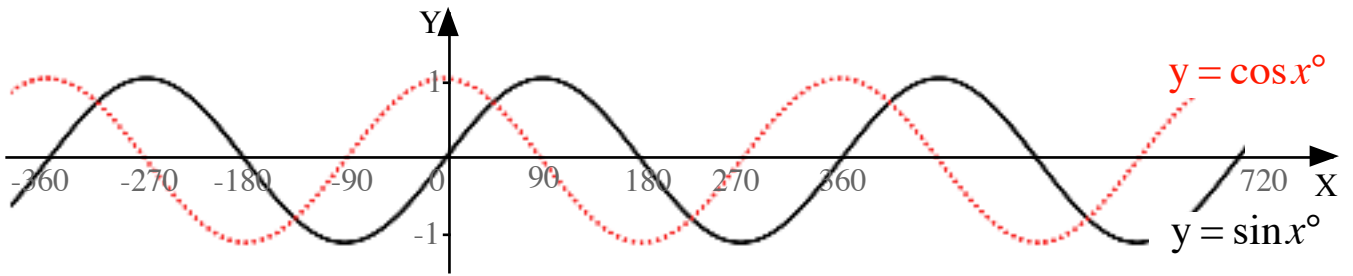
$$x = 52.1$$



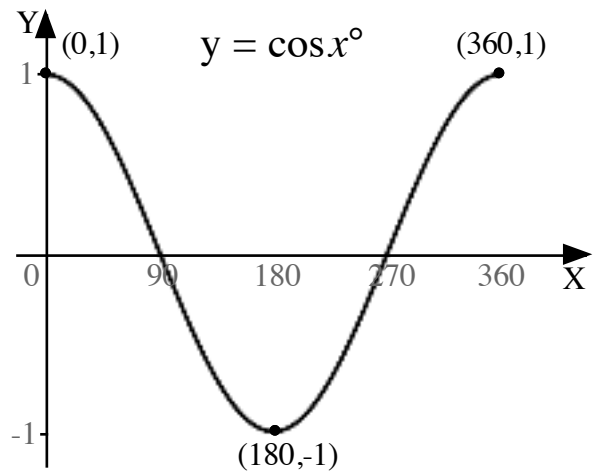
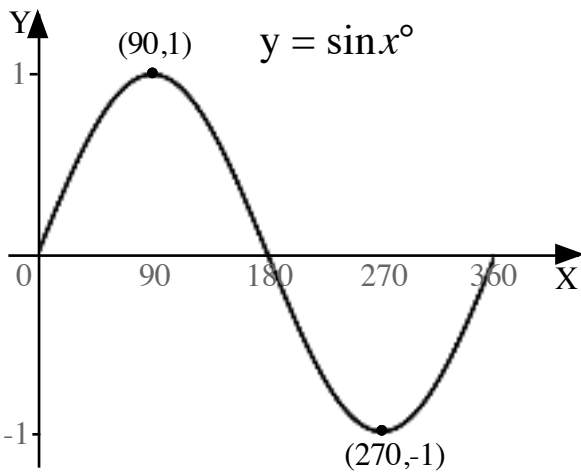
know O , know A
 $\downarrow \quad \downarrow \quad \downarrow\downarrow \quad \downarrow\downarrow$
SOH-CAH-TOA

CHAPTER 17: TRIGONOMETRY: GRAPHS & EQUATIONS

The cosine graph is the sine graph shifted 90° to the left.

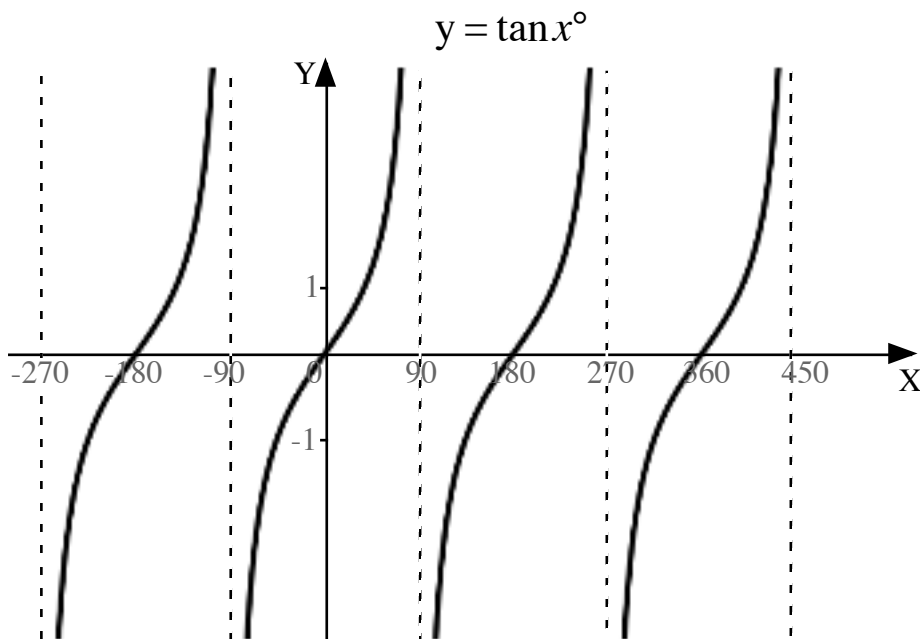


The graphs have a PERIOD of 360° (repeat every 360°).



Turning points:
maximum $(90,1)$, minimum $(270,-1)$

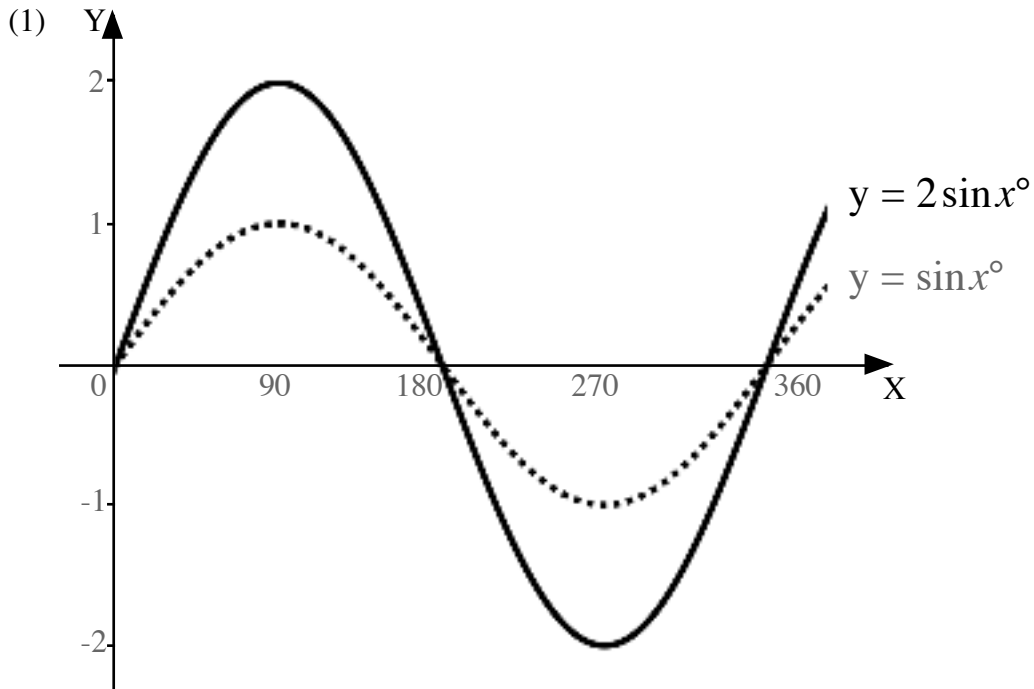
maximum $(0,1)$, minimum $(180,-1)$



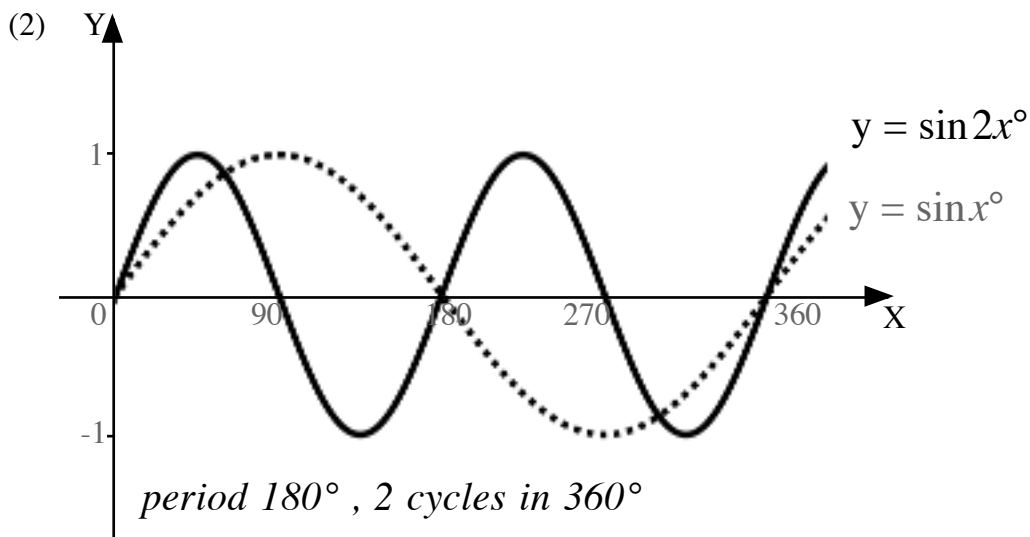
The tangent graph has a PERIOD of 180° .

TRANSFORMATIONS Same rules for $y = \sin x^\circ$ and $y = \cos x^\circ$.

Y-STRETCH $y = n \sin x^\circ$ y -coordinates multiplied by n .
amplitude n units
maximum value $+n$, minimum value $-n$



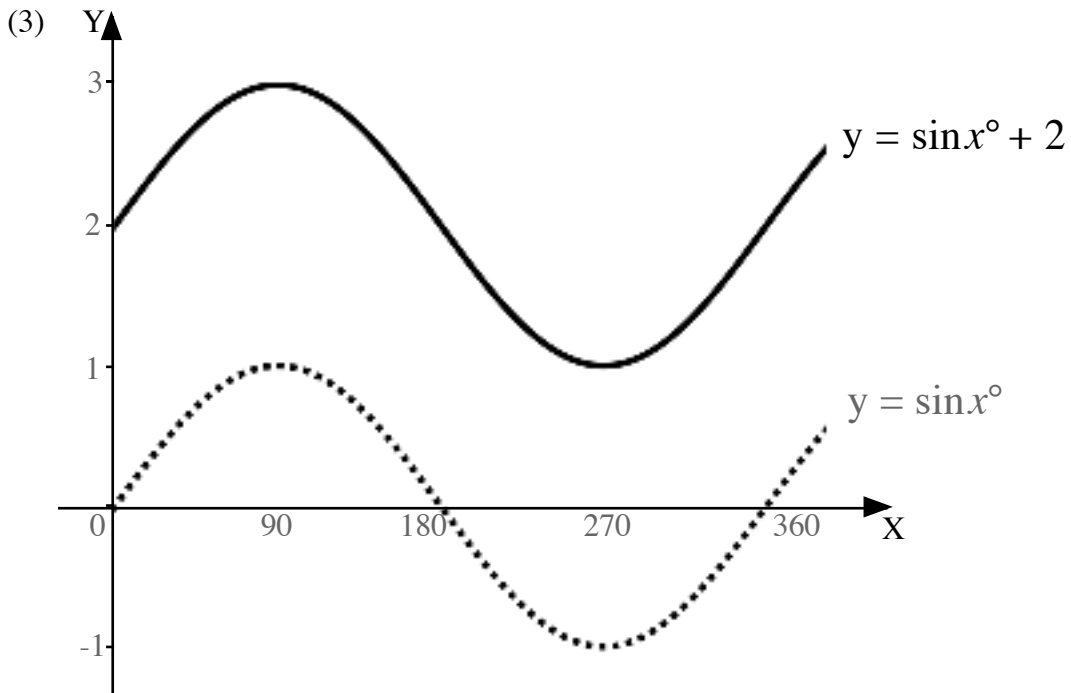
X-STRETCH $y = \sin nx^\circ$ x -coordinates divided by n .
period $360^\circ \div n$. There are n cycles in 360° .



Y-SHIFT

$$y = \sin x^\circ + n$$

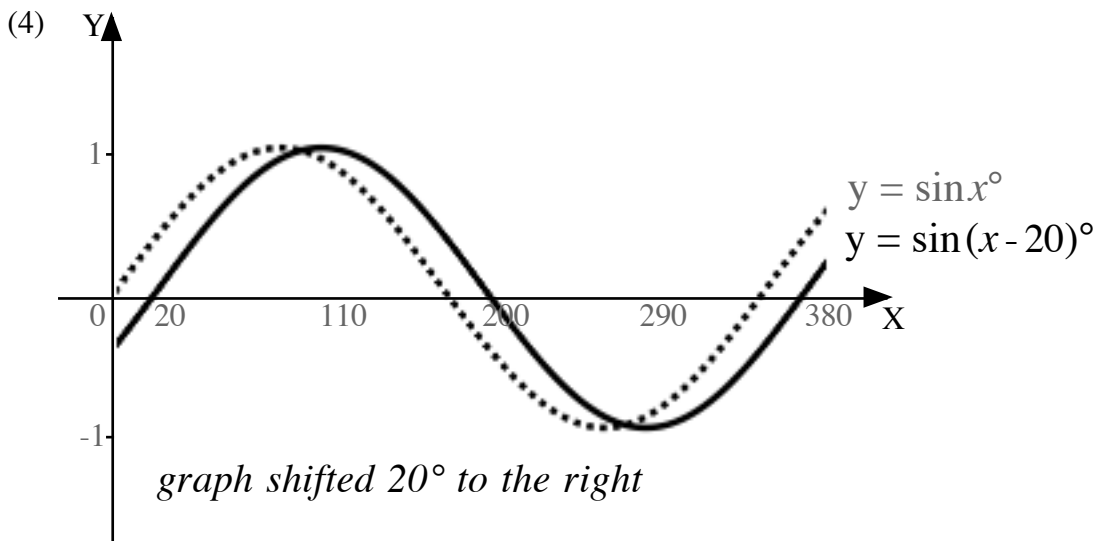
add **n** units to y-coordinates
graph shifted **n** units vertically.



X-SHIFT

$$y = \sin(x + n)^\circ$$

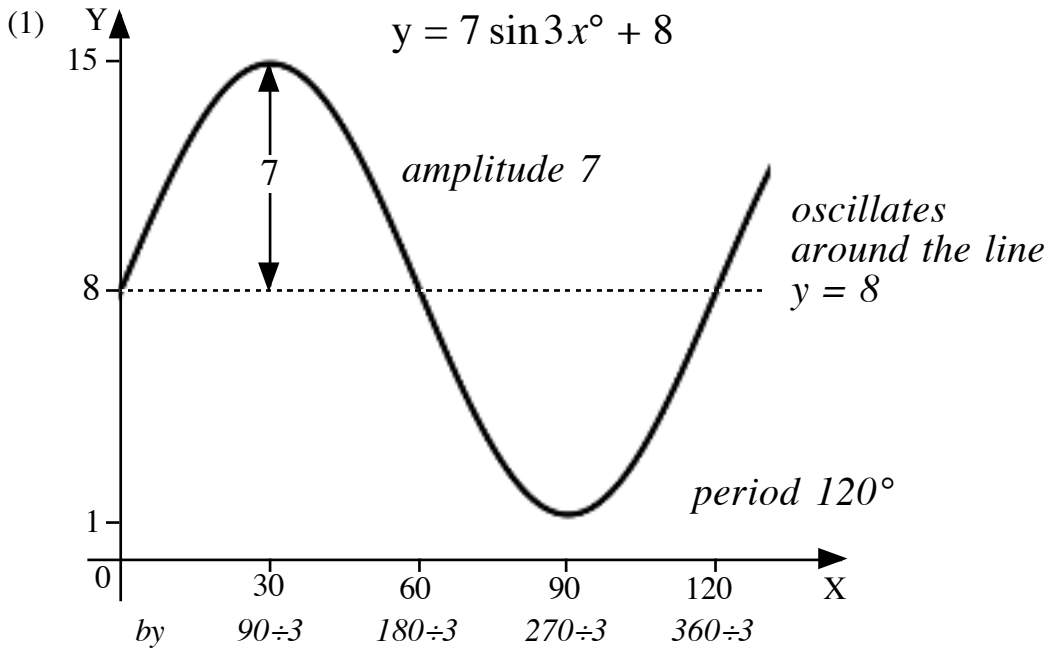
subtract **n** units from the x-coordinates
graph shifted $-n^\circ$ horizontally.



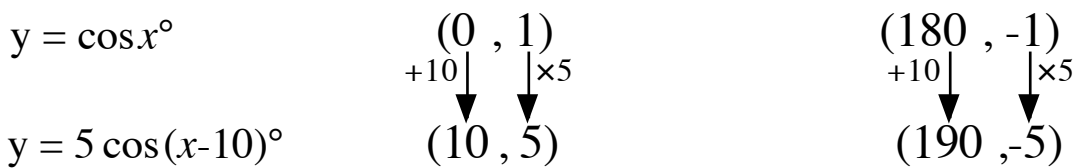
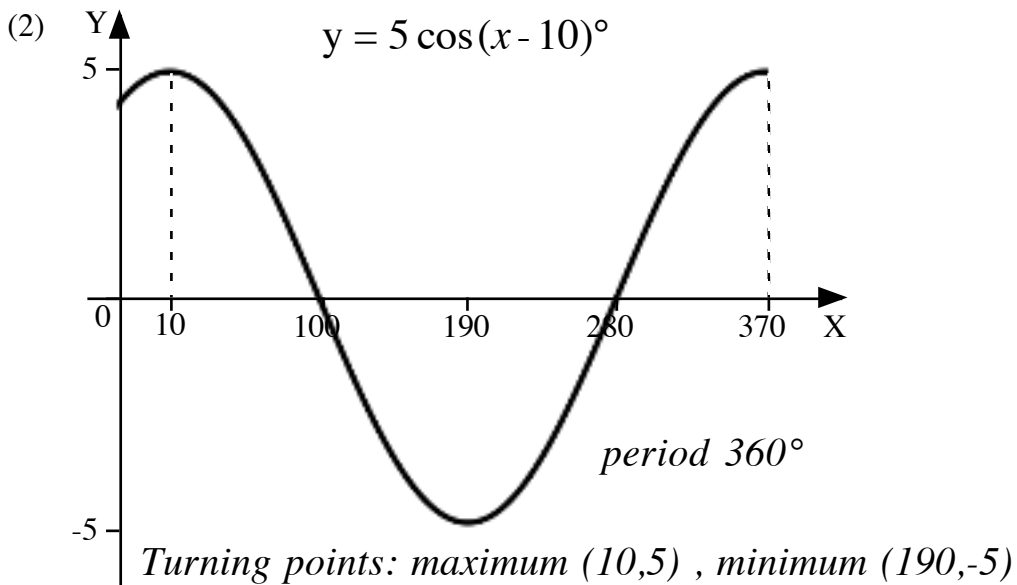
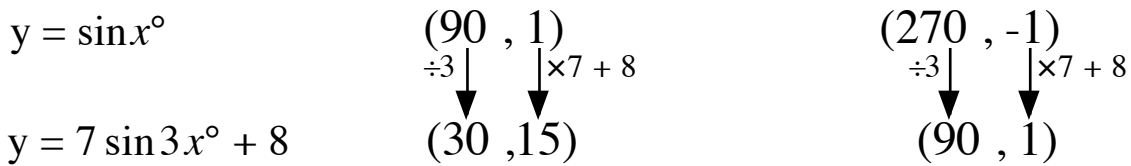
graph shifted 20° to the right

NOTE: for $y = \sin(x + 20)^\circ$ the graph $y = \sin x^\circ$ would be shifted 20° to the left.

COMBINING TRANSFORMATIONS

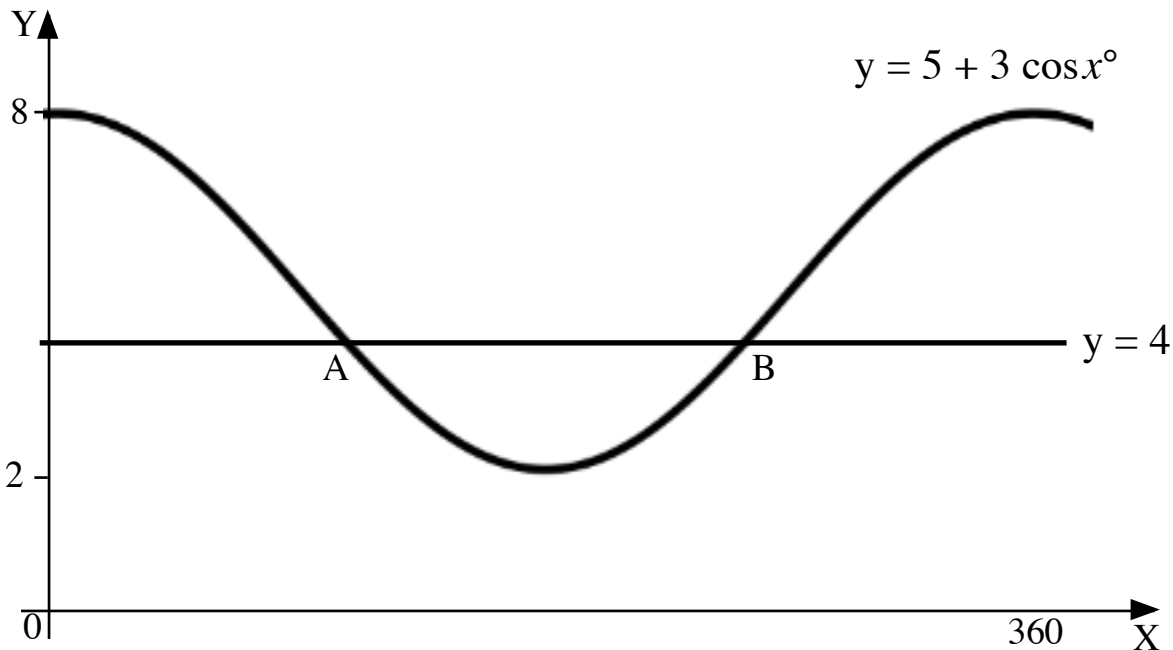


Turning points: maximum $(30, 15)$, minimum $(90, 1)$



EQUATIONS

The graphs with equations $y = 5 + 3 \cos x^\circ$ and $y = 4$ are shown.
Find the x coordinates of the points of intersection A and B.



$$5 + 3 \cos x^\circ = 4$$

$$3 \cos x^\circ = -1$$

$$\cos x^\circ = -\frac{1}{3}$$

$$\underline{\underline{x = 109 \cdot 5 \text{ or } 250 \cdot 5}}$$

* **A, S, T, C** is where functions are **positive**:

S	A
<i>cosine</i> <i>negative</i> ✓	<i>cosine</i> <i>positive</i> ✗
$180 - a = 109 \cdot 5$	$a = \cos^{-1} 1/3 = 70 \cdot 528 \dots$
$180 + a = 250 \cdot 5$	$360 - a = 289 \cdot 5$
<i>cosine</i> <i>negative</i> ✓	<i>cosine</i> <i>positive</i> ✗
T	C

- * A all functions are positive
 S sine function only is positive
 T tangent function only is positive
 C cosine function only is positive

IDENTITIES

$$\sin^2 x^\circ + \cos^2 x^\circ = 1$$

$$\tan x^\circ = \frac{\sin x^\circ}{\cos x^\circ}$$

Simplify $\frac{1 - \cos^2 x^\circ}{\sin x^\circ \cos x^\circ}$.

$$= \frac{\sin^2 x^\circ}{\sin x^\circ \cos x^\circ}$$

$$= \frac{\sin x^\circ \sin x^\circ}{\sin x^\circ \cos x^\circ}$$

$$= \frac{\sin x^\circ}{\cos x^\circ}$$

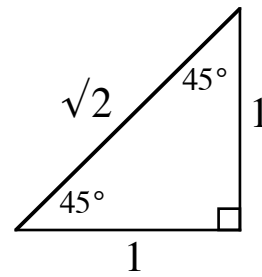
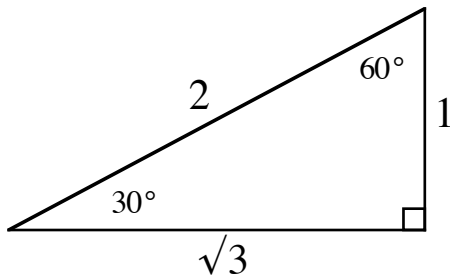
$$= \tan x^\circ$$

since $\sin^2 x^\circ + \cos^2 x^\circ = 1$

$$\sin^2 x^\circ = 1 - \cos^2 x^\circ$$

"cancel" $\sin x^\circ$

EXACT VALUES



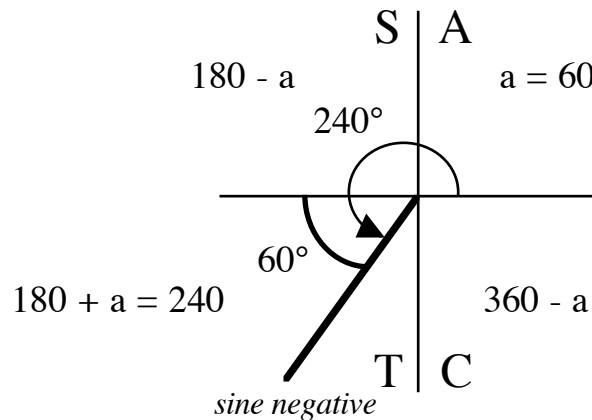
For example,

$$\sin 60^\circ = \frac{\sqrt{3}}{2}, \quad \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}}, \quad \tan 45^\circ = \frac{1}{1} = 1$$

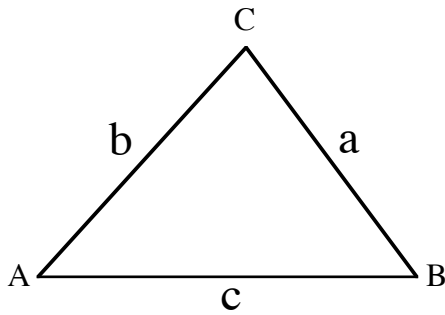
ANGLES > 90°

$$\begin{aligned} & \sin 240^\circ \\ &= \sin(180^\circ + 60^\circ) \\ &= -\sin 60^\circ \\ &= -\frac{\sqrt{3}}{2} \end{aligned}$$



CHAPTER 18: TRIGONOMETRY: TRIANGLE FORMULAE

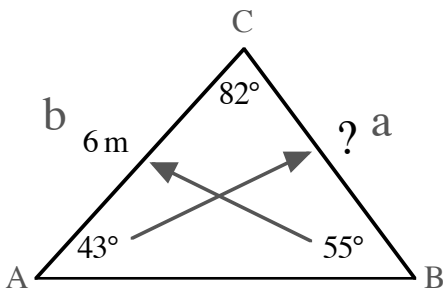
SINE RULE



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

NOTE: requires at least one side and its opposite angle to be known.

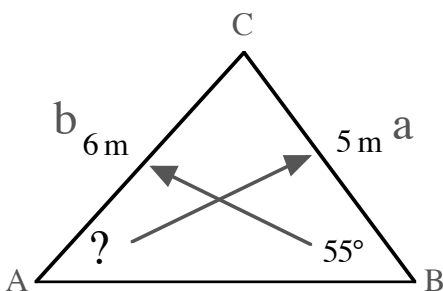
FINDING AN UNKNOWN SIDE



Find the length of side BC

$$\begin{aligned} \frac{a}{\sin A} &= \frac{b}{\sin B} \\ \frac{a}{\sin 43^\circ} &= \frac{6}{\sin 55^\circ} \\ a &= \frac{6}{\sin 55^\circ} \times \sin 43^\circ \\ &= 4.995\dots \\ BC &\approx 5.0 \text{ m} \end{aligned}$$

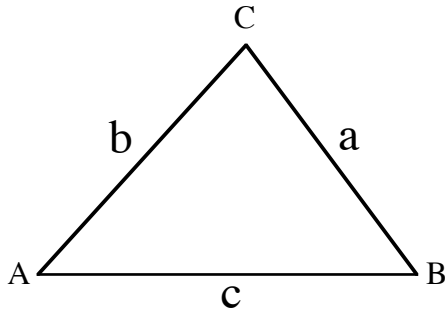
FINDING AN UNKNOWN ANGLE



Find the size of angle BAC.

$$\begin{aligned} \frac{\sin A}{a} &= \frac{\sin B}{b} \\ \frac{\sin A}{5} &= \frac{\sin 55^\circ}{6} \\ \sin A &= \frac{\sin 55^\circ}{6} \times 5 \\ &= 0.682\dots \\ A &= \sin^{-1} 0.682\dots \\ &= 43.049\dots \\ \angle BAC &\approx 43.0^\circ \end{aligned}$$

COSINE RULE

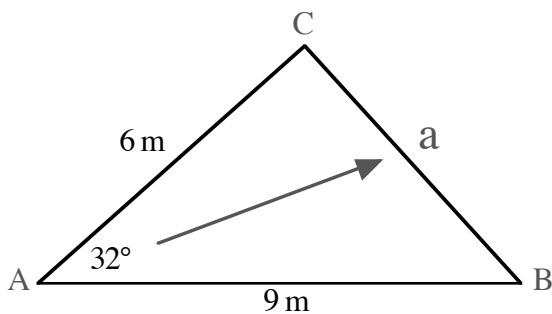


$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

FINDING AN UNKNOWN SIDE

NOTE: requires knowing 2 sides and the angle between them.



$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$= 6^2 + 9^2 - 2 \times 6 \times 9 \times \cos 32^\circ$$

$$a^2 = 25.410\dots$$

$$a = \sqrt{25.410\dots}$$

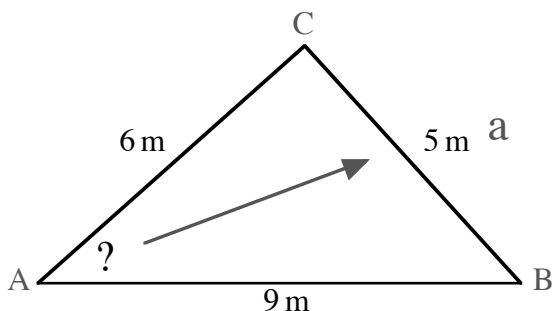
$$= 5.040\dots$$

$$BC \approx 5.0 \text{ m}$$

Find the length of side BC.

FINDING AN UNKNOWN ANGLE

NOTE: requires knowing all 3 sides.



$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$= \frac{6^2 + 9^2 - 5^2}{2 \times 6 \times 9}$$

$$\cos A = 0.85185\dots$$

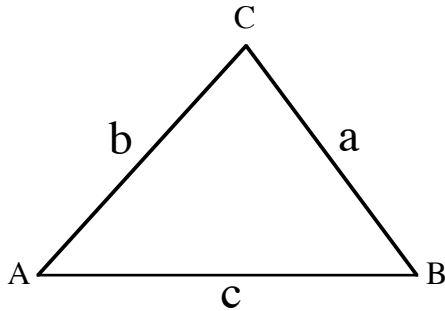
$$A = \cos^{-1}(0.85185\dots)$$

$$= 31.586\dots$$

$$\angle BAC \approx 31.6^\circ$$

Find the size of angle BAC.

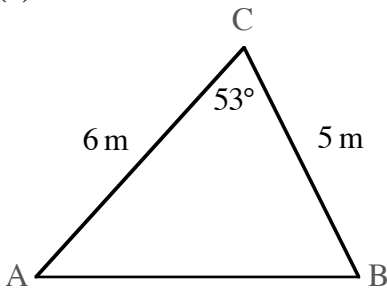
AREA FORMULA



$$\text{Area } \Delta ABC = \frac{1}{2}ab \sin C$$

NOTE: requires knowing 2 sides and the angle between them.

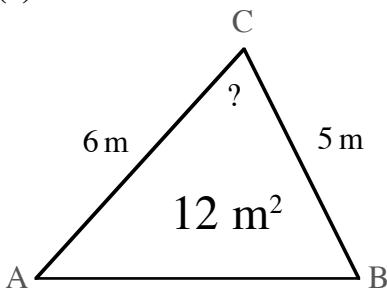
(1)



$$\begin{aligned} \text{Area } \Delta ABC &= \frac{1}{2}ab \sin C \\ &= \frac{1}{2} \times 5 \times 6 \times \sin 53^\circ \\ &= 11.979\dots \\ \text{Area} &\approx 12.0 \text{ m}^2 \end{aligned}$$

Find the area of the triangle.

(2)



$$\begin{aligned} \text{Area } \Delta ABC &= \frac{1}{2}ab \sin C \\ 12 &= \frac{1}{2} \times 5 \times 6 \times \sin C && \text{double both sides} \\ 24 &= 30 \times \sin C \\ \sin C &= 24 \div 30 = 0.8 \end{aligned}$$

Find angle ACB.

$$\begin{aligned} C &= \sin^{-1}(0.8) \\ &= 53.130\dots^\circ \quad \text{or} \quad 126.869\dots^\circ \\ &\quad \text{(from } 180^\circ - 53.130\dots^\circ \\ &\quad \text{as angle could be obtuse)} \end{aligned}$$

$$\angle ACB \approx 53.1^\circ \quad \text{from diagram, angle acute}$$

CHAPTER 19: VECTORS

SCALAR quantities have size(magnitude).

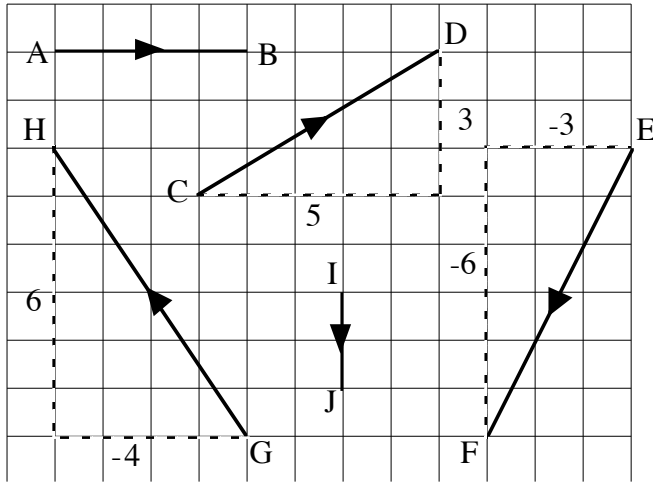
eg. time, speed, volume

VECTOR quantities have **size** and **direction**.

eg. force, velocity

A **directed line segment** represents a vector.

Vectors can be written in component form as **column vectors**



$$\vec{AB} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \quad \vec{CD} = \begin{pmatrix} 5 \\ 3 \end{pmatrix} \quad \vec{EF} = \begin{pmatrix} -3 \\ -6 \end{pmatrix}$$

$$\vec{GH} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \quad \vec{IJ} = \begin{pmatrix} 0 \\ -2 \end{pmatrix}$$

negative vector, opposite direction

$$\vec{HG} = \begin{pmatrix} 4 \\ -6 \end{pmatrix}$$

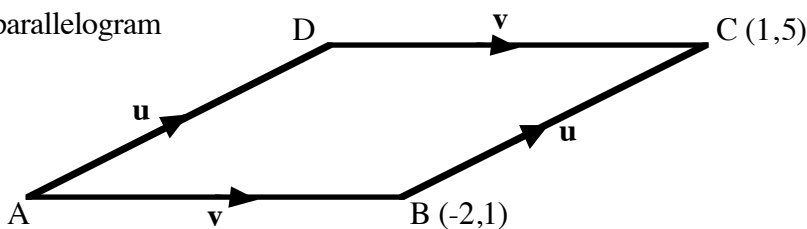
SIZE follows from Pyth. Thm

$$\mathbf{u} = \begin{pmatrix} a \\ b \end{pmatrix}$$

$$|\mathbf{u}| = \sqrt{a^2 + b^2}$$

$$|\vec{GH}| = \sqrt{(-4)^2 + 6^2} = \sqrt{52} = 2\sqrt{13} \text{ units}$$

parallelogram



$$\vec{AD} = \vec{BC} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

same size and direction
same vector **u**
same component form

ADD/SUBTRACT

by column vectors

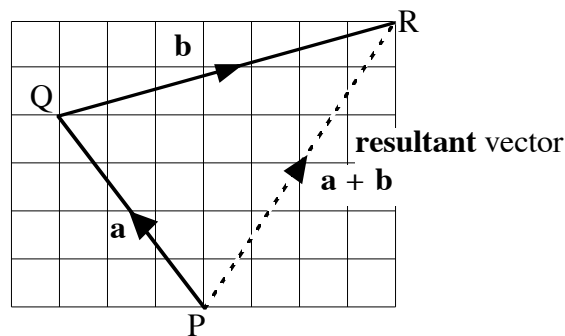
add or subtract components.

$$\begin{pmatrix} -3 \\ 4 \end{pmatrix} + \begin{pmatrix} 7 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$$

by diagram

“head-to-tail” addition

$$\vec{PQ} + \vec{QR} = \vec{PR}$$



MULTIPLY by a number

$$\mathbf{u} = \begin{pmatrix} a \\ b \end{pmatrix} \quad k\mathbf{u} = \begin{pmatrix} ka \\ kb \end{pmatrix}$$

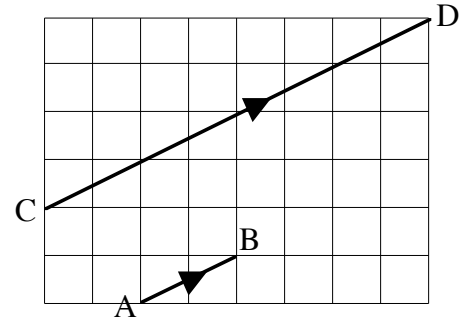
$$\vec{AB} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad \vec{CD} = \begin{pmatrix} 8 \\ 4 \end{pmatrix}$$

if $\mathbf{v} = k\mathbf{u}$

$$\vec{CD} = 4\vec{AB}$$

then \mathbf{u} and \mathbf{v} are parallel

CD is parallel to AB

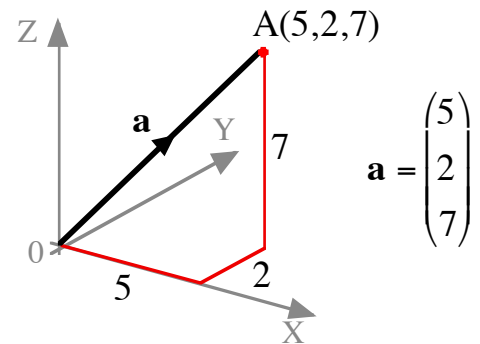


3D

Vectors in 3D operate in the same way as vectors in 2D.

Points are plotted on 3 mutually perpendicular axes.

$P(x,y,z)$ has **position vector** $\mathbf{p} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$



If $\mathbf{u} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ find the value of $|\mathbf{v} - 2\mathbf{u}|$.

$$\mathbf{v} - 2\mathbf{u} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} - 2\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ -4 \\ 6 \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \\ -5 \end{pmatrix}$$

$$|\mathbf{v} - 2\mathbf{u}| = \sqrt{(-3)^2 + 4^2 + (-5)^2} = \sqrt{50} = 5\sqrt{2}$$

CHAPTER 20: STATISTICS

Studying statistical information, it is useful to consider: (1) typical result: **average**
 (2) distribution of results: **spread**

AVERAGES:

$$\text{mean} = \frac{\text{total of all results}}{\text{number of results}}$$

median = middle result of the ordered results

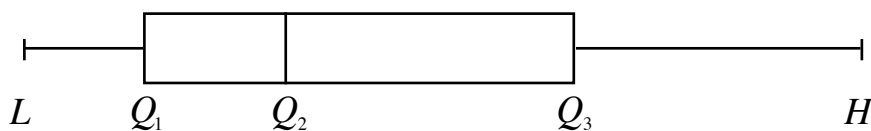
mode = most frequent result

SPREAD:

Ordered results are split into 4 equal groups so each contains 25% of the results.

The **5 figure summary** identifies: L, Q_1, Q_2, Q_3, H
 (lowest result, 1st, 2nd and 3rd quartiles, highest result)

A **Box Plot** is a statistical diagram that displays the 5 figure summary:



$$\text{range, } R = H - L$$

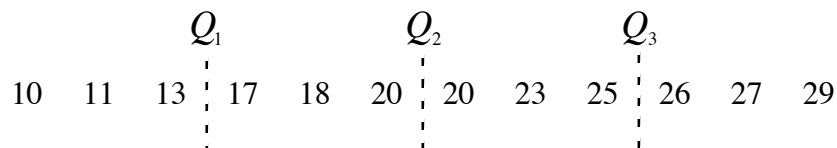
$$\text{interquartile range, } IQR = Q_3 - Q_1$$

$$\text{semi-interquartile range, } SIQR = \frac{Q_3 - Q_1}{2}$$

NOTE: If Q_1, Q_2 or Q_3 fall between two results, the mean of the two results is taken.

For example,

12 ordered results: split into 4 equal groups of 3 results



$$Q_1 = \frac{13 + 17}{2} = 15, \quad Q_2 = \frac{20 + 20}{2} = 20, \quad Q_3 = \frac{25 + 26}{2} = 25.5$$

The pulse rates of school students were recorded in Biology class.

Pulse rates: 66, 64, 71, 56, 60, 79, 77, 75, 69, 73, 75, 62, 66, 71, 66 beats per minute.

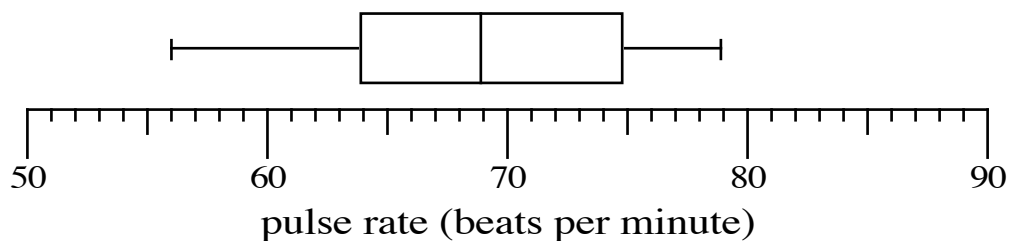
15 ordered results:

56 60 62 Q_1 64 66 66 66 Q_2 69 71 71 73 Q_3 75 75 77 79

5 Figure Summary:

$L = 56$, $Q_1 = 64$, $Q_2 = 69$, $Q_3 = 75$, $H = 79$

Box Plot:



Spread:

$$R = H - L = 79 - 56 = 23$$

$$IQR = Q_3 - Q_1 = 75 - 64 = 11$$

$$SIQR = \frac{Q_3 - Q_1}{2} = \frac{75 - 64}{2} = \frac{11}{2} = 5.5$$

Averages: ($total = 66 + 64 + 71 + \dots + 66 = 1030$)

$$MEAN = \frac{1030}{15} = 68.666\dots = 68.7$$

$$(Q_2)MEDIAN = 69$$

$$MODE = 66$$

STANDARD DEVIATION

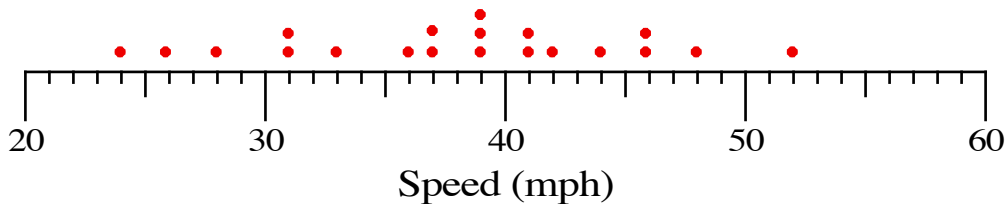
A measure of the spread of a set of data, giving a numerical value to how the data deviates from the mean. It therefore gives an indication of how good the mean is as a representative of the data set.

Formulae:

mean $\bar{x} = \frac{\sum x}{n}$ standard deviation $s = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$ or $s = \sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n - 1}}$

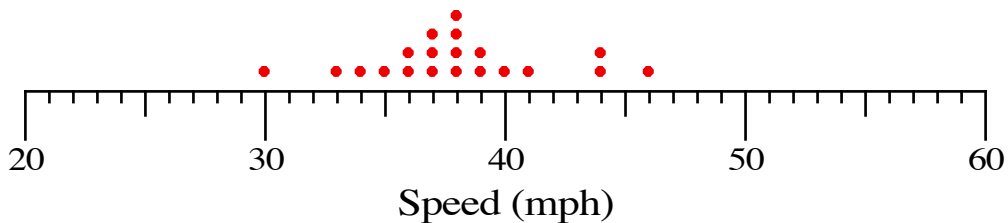
Examples,

(1) High Standard Deviation: results spread out



mean = 38 , standard deviation = 7.5

(2) Low Standard Deviation: results clustered around the mean
the results are more consistent



mean = 38 , standard deviation = 3.8

The pulse rates of 8 army recruits: 61, 64, 65, 67, 70, 72, 75, 78 beats per minute.

$$\begin{aligned}\bar{x} &= \frac{\sum x}{n} \\ &= \frac{552}{8} \\ &= 69\end{aligned}$$

x	$x - \bar{x}$	$(x - \bar{x})^2$
61	-8	64
64	-5	25
65	-4	16
67	-2	4
70	+1	1
72	+3	9
75	+6	36
78	+9	81
TOTALS	552	236

$$\begin{aligned}s &= \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}} \\ &= \sqrt{\frac{236}{7}} \\ &= 5.806\dots \\ &\approx 5.8\end{aligned}$$

Or

$$\begin{aligned}\bar{x} &= \frac{\sum x}{n} \\ &= \frac{552}{8} \\ &= 69\end{aligned}$$

x	x^2	
61	3721	
64	4096	
65	4225	
67	4489	
70	4900	
72	5184	
75	5625	
78	6084	
TOTALS	552	38324

$$\begin{aligned}s &= \sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n - 1}} \\ &= \sqrt{\frac{38324 - \frac{552^2}{8}}{7}} \\ &= \sqrt{\frac{236}{7}} \\ &= 5.806\dots \\ &\approx 5.8\end{aligned}$$

PROBABILITY

The probability of an event A occurring is $P(A) = \frac{\text{number of outcomes involving A}}{\text{total number of outcomes possible}}$

$0 \leq P \leq 1$ $P = 0$ impossible , $P = 1$ certain

$$P(\text{not A}) = 1 - P(A)$$

number of expected outcomes involving event A = number of trials \times $P(A)$

The experimental results will differ from the theoretical probability.

- (1) In an experiment a letter is chosen at random from the word ARITHMETIC and the results recorded.

letter	frequency	relative frequency
vowel	111	$111 \div 300 = 0.37$
consonant	189	$189 \div 300 = 0.63$
	total = 300	total = 1

Estimate of probability,

$$P(\text{vowel}) = 0.37$$

- (2) A letter is chosen at random from the word ARITHMETIC.

4 vowels out of 10 letters,

$$P(\text{vowel}) = \frac{4}{10} = \frac{2}{5} \quad \text{ie. } 0.4$$

for 300 trials, *number of vowels expected* = *number of trials* \times $P(\text{vowel})$
 $= 300 \times 0.4$
 $= 120$

6 consonants out of 10 letters,

$$P(\text{consonant}) = \frac{6}{10} = \frac{3}{5} \quad \text{ie. } 0.6$$

$$P(\text{not a vowel}) = 1 - 0.4 = 0.6$$

vowel **or** consonant,

$$P(\text{either}) = \frac{10}{10} = 1 \quad \text{certain}$$

vowel **and** consonant,

$$P(\text{both}) = \frac{0}{10} = 0 \quad \text{impossible}$$