AH National Qualifications SPECIMEN ONLY

S847/77/11

Mathematics
Paper 1 (Non-calculator)

Date — Not applicable

Duration — 1 hour

Total marks — 35

Attempt ALL questions.

You may NOT use a calculator.

To earn full marks you must show your working in your answers.

State the units for your answer where appropriate.

You will not earn marks for answers obtained by readings from scale drawings.

Write your answers clearly in the spaces provided in the answer booklet. The size of the space provided for an answer is not an indication of how much to write. You do not need to use all the space.

Additional space for answers is provided at the end of the answer booklet. If you use this space you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.

S847711

FORMULAE LIST

Standard derivatives

$f(x)$	$f'(x)$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
$\tan x$	$\sec^2 x$
$\cot x$	$-\text{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\text{cosec}\, x$	$-\text{cosec}\, x \cot x$
$\ln x$	$\frac{1}{x}$
e^x	e^x

Standard integrals

$f(x)$	$\int f(x)\,dx$
$\sec^2(ax)$	$\frac{1}{a}\tan(ax)+c$
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(\frac{x}{a}\right)+c$
$\frac{1}{a^2+x^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c$
$\frac{1}{x}$	$\ln\lvert x\rvert+c$
e^{ax}	$\frac{1}{a}e^{ax}+c$

Summations

(Arithmetic series) $$S_n = \frac{1}{2}n\left[2a+(n-1)d\right]$$

(Geometric series) $$S_n = \frac{a\left(1-r^n\right)}{1-r},\ r \neq 1$$

$$\sum_{r=1}^{n} r = \frac{n(n+1)}{2},\quad \sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6},\quad \sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$$

Binomial theorem

$$(a+b)^n = \sum_{r=0}^{n}\binom{n}{r}a^{n-r}b^r \quad \text{where} \quad \binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

Maclaurin expansion

$$f(x) = f(0)+f'(0)x+\frac{f''(0)x^2}{2!}+\frac{f'''(0)x^3}{3!}+\frac{f^{iv}(0)x^4}{4!}+\ldots$$

De Moivre's theorem

$$\left[r(\cos\theta + i\sin\theta)\right]^n = r^n\left(\cos n\theta + i\sin n\theta\right)$$

Vector product

$$\mathbf{a}\times\mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta\,\hat{\mathbf{n}}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \mathbf{i}\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - \mathbf{j}\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + \mathbf{k}\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

Matrix transformation

Anti-clockwise rotation through an angle, θ, about the origin, $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

[Turn over

MARKS

Total marks — 35

Attempt ALL questions

1. Matrix A is defined by $A = \begin{pmatrix} 5 & 6 \\ -1 & -2 \end{pmatrix}$

(a) Find A^{-1} **2**

(b) State A'. **1**

2. Given $f(x) = 2x\tan x$, where $0 < x < \frac{\pi}{2}$, obtain $f'\left(\frac{\pi}{4}\right)$. **3**

3. Use Gaussian elimination to solve the following system of equations. **4**

$$\begin{aligned} x + y + 3z &= 2 \\ 2x + y + z &= 2 \\ 3x + 2y + 5z &= 5 \end{aligned}$$

4. The velocity, v, of a particle P at time t is given by

$$v = e^{3t} + 2e^{t}.$$

Find the distance covered by P between $t = 0$ and $t = \ln 3$. **3**

5. The complex number $z = 2 + i$ is a root of the polynomial equation $z^4 - 6z^3 + 16z^2 - 22z + 15 = 0$.

Find the remaining roots. **6**

6. Use the substitution $t = x^4$ to obtain $\int \frac{x^3}{1 + x^8}\,dx$. **3**

MARKS

7. Prove by induction that, for all positive integers n,

$$\sum_{r=1}^{n} 3^{r-1} = \frac{1}{2}\left(3^n - 1\right).$$ **5**

8. A function is defined on a suitable domain by $f(x) = \dfrac{3x^2 + 2}{x^2 - 2}$.

(a) Obtain equations for the asymptotes of the graph of $y = f(x)$. **3**

(b) Determine whether the graph of $y = f(x)$ has any points of inflection.
Justify your answer. **5**

[END OF SPECIMEN QUESTION PAPER]

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Marking Instructions

These marking instructions have been provided to show how SQA would mark this specimen question paper.

General marking principles for Advanced Higher Mathematics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

The marking instructions for each question are generally in two sections:

- *generic scheme — this indicates why each mark is awarded*
- *illustrative scheme — this covers methods which are commonly seen throughout the marking*

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

(c) One mark is available for each •. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example 6 × 6 = 12, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

$$x^2+5x+7=9x+4$$
$$x-4x+3=0$$
$$x=1$$

This is a transcription error and so the mark is not awarded. [$x-4x+3=0$]

This is no longer a solution of a quadratic equation, so the mark is not awarded. [$x=1$]

The following example is an exception to the above

$$x^2+5x+7=9x+4$$
$$x-4x+3=0$$
$$(x-3)(x-1)=0$$
$$x=1 \text{ or } 3$$

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded. [$x-4x+3=0$]

(i) **Horizontal/vertical marking**
If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	•5	•6
•5	$x = 2$	$x = -4$
•6	$y = 5$	$y = -7$

Horizontal: •5 $x = 2$ and $x = -4$
•6 $y = 5$ and $y = -7$

Vertical: •5 $x = 2$ and $y = 5$
•6 $x = -4$ and $y = -7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$

$\frac{43}{1}$ must be simplified to 43

$\frac{15}{0{\cdot}3}$ must be simplified to 50

$\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 100 must be known.

(k) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example

  $(x^3 + 2x^2 + 3x + 2)(2x + 1)$ written as

  $(x^3 + 2x^2 + 3x + 2) \times 2x + 1$

  $= 2x^4 + 5x^3 + 8x^2 + 7x + 2$

  gains full credit

- repeated error within a question, but not between questions or papers

(l) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(m) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(n) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(o) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Marking instructions for each question

Question			Generic scheme	Illustrative scheme	Max mark
1.	(a)		•¹ find determinant •² find inverse	•¹ -4 •² $-\frac{1}{4}\begin{pmatrix}-2 & -6\\ 1 & 5\end{pmatrix}$	2
	(b)		•³ find transpose	•³ $\begin{pmatrix}5 & -1\\ 6 & -2\end{pmatrix}$	1
2.			•¹ evidence of use of product rule •² complete differentiation •³ substitute and evaluate	•¹ $(\ldots)\tan x + 2x(\ldots)$ •² $2\tan x + 2x\sec^2 x$ •³ $2+\pi$	3
3.			•¹ set up augmented matrix •² obtain two zeros •³ complete row operations •⁴ obtain solution	•¹ $\left[\begin{array}{ccc\|c}1 & 1 & 3 & 2\\ 2 & 1 & 1 & 2\\ 3 & 2 & 5 & 5\end{array}\right]$ •² $\left[\begin{array}{ccc\|c}1 & 1 & 3 & 2\\ 0 & -1 & -5 & -2\\ 0 & -1 & -4 & -1\end{array}\right]$ •³ $\left[\begin{array}{ccc\|c}1 & 1 & 3 & 2\\ 0 & -1 & -5 & -2\\ 0 & 0 & 1 & 1\end{array}\right]$ •⁴ $x=2,\ y=-3,\ z=1$	4
4.			•¹ form integral with correct limits •² integrate •³ find distance	•¹ $\int_0^{\ln 3}\left(e^{3t}+2e^t\right)dt$ •² $\left[\frac{1}{3}e^{3t}+2e^t\right]_0^{\ln 3}$ •³ $12\frac{2}{3}$	3

Question			Generic scheme	Illustrative scheme	Max mark
5.			•1 state second solution	•1 $z = 2 - i$	**6**
			•2 create two linear factors	•2 $(z-2+i)$ and $(z-2-i)$	
			•3 create quadratic factor	•3 $z^2 - 4z + 5$	
			•4 set up algebraic division or equivalent	•4 $z^2-4z+5 \overline{\smash{)}\, z^4 - 6z^3 + 16z^2 - 22z + 15}$	
			•5 complete algebraic division	•5 $\begin{array}{r} z^2 - 2z + 3 \\ z^2-4z+5 \overline{\smash{)}\, z^4 - 6z^3 + 16z^2 - 22z + 15} \\ \underline{z^4 - 4z^3 + 5z^2} \qquad\qquad \\ -2z^3 + 11z^2 - 22z + 15 \\ \underline{-2z^3 + 8z^2 - 10z} \qquad \\ 3z^2 - 12z + 15 \\ \underline{3z^2 - 12z + 15} \\ 0 \end{array}$	
			•6 obtain remaining two solutions	•6 $z = 1 \pm \sqrt{2}\, i$	
6.			•1 differentiate **AND** start to substitute	•1 $\frac{dt}{dx} = 4x^3$ **AND** evidence of substitution	**3**
			•2 rewrite integral entirely in terms of t	•2 $\frac{1}{4}\int \frac{1}{1+t^2}\, dt$	
			•3 integrate and substitute for t	•3 $\frac{1}{4}\tan^{-1} x^4 + c$	

Question			Generic scheme	Illustrative scheme	Max mark
7.			•¹ show true for $n=1$	•¹ LHS: $3^0=1$ RHS: $\frac{1}{2}(3-1)=1$ So true for $n=1$	**5**
			•² assume (statement) true for $n=k$ **AND** consider whether (statement) true for $n=k+1$	•² Suitable statement **and** $\sum_{r=1}^{k}3^{r-1}=\frac{1}{2}(3^k-1)$ **AND** $\sum_{r=1}^{k+1}3^{r-1}=\ldots$	
			•³ correct statement for sum to $(k+1)$ terms using inductive hypothesis	•³ $\ldots=\frac{1}{2}(3^k-1)+3^{(k+1)-1}$	
			•⁴ combine terms in 3^k	•⁴ $\frac{3}{2}\times3^k-\frac{1}{2}$	
			•⁵ express sum explicitly in terms of $(k+1)$ **or** achieve stated aim/goal **AND** communicate	•⁵ $\frac{1}{2}(3^{(k+1)}-1)$ If true for $n=k$ then true for $n=k+1$. Also shown true for $n=1$ therefore, by induction, true for all positive integers n.	

Question			Generic scheme	Illustrative scheme	Max mark
8.	(a)		•[1] state equations of vertical asymptotes	•[1] $x=\sqrt{2},\ x=-\sqrt{2}$	**3**
			•[2] strategy for non-vertical asymptote	•[2] $f(x)=3+\dfrac{8}{x^2-2}$ OR $f(x)=\dfrac{3+\frac{2}{x^2}}{1-\frac{2}{x^2}}$	
			•[3] equation of non-vertical asymptote	•[3] $y=3$, since $\dfrac{8}{x^2-2}\to 0$ as $x\to\pm\infty$ OR $f(x)\to\dfrac{3+0}{1-0}$ as $x\to\pm\infty$	
	(b)		•[4] first derivative	•[4] $f'(x)=\dfrac{-16x}{(x^2-2)^2}$	**5**
			•[5] start second derivative	•[5] $\dfrac{-16(x^2-2)^2-\ldots}{(x^2-2)^4}$ OR $\dfrac{\ldots-16x\times 2(x^2-2)\times 2x}{(x^2-2)^4}$	
			•[6] complete second derivative and simplify	•[6] $\dfrac{-16(x^2-2)^2+64x^2}{(x^2-2)^3}$	
			•[7] show second derivative is never zero	•[7] $32+48x^2\neq 0$	
			•[8] consider the case where $f''(x)$ is undefined and state conclusion	•[8] $f''(x)$ is undefined only when $x=\pm\sqrt{2}$. Since $f(x)$ is undefined at these values there is no point of inflection.	

[END OF SPECIMEN MARKING INSTRUCTIONS]