

# TRINITY HIGH SCHOOL

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Mathematics

CFE Higher

NATIONAL

QUALIFICATIONS

Higher Practice Examination January 2019

Paper 2 (Calculator allowed)

Time allowed - 1 hour 30 minutes

## **Read carefully**

- 1. Calculators may be used in this paper.**
2. Full credit will be given only where the solution contains appropriate working.
3. State the units for your answers where appropriate.
4. Answers obtained from readings from scale drawings will not receive any credit.

## FORMULAE LIST

### Circle:

The equation  $x^2 + y^2 + 2gx + 2fy + c = 0$  represents a circle centre  $(-g, -f)$  and radius  $\sqrt{g^2 + f^2 - c}$ .

The equation  $(x - a)^2 + (y - b)^2 = r^2$  represents a circle centre  $(a, b)$  and radius  $r$ .

**Scalar Product:**  $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos \theta$ , where  $\theta$  is the angle between  $\mathbf{a}$  and  $\mathbf{b}$

or  $\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$  where  $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ .

**Trigonometric formulae:**  $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

**Table of standard derivatives:**

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

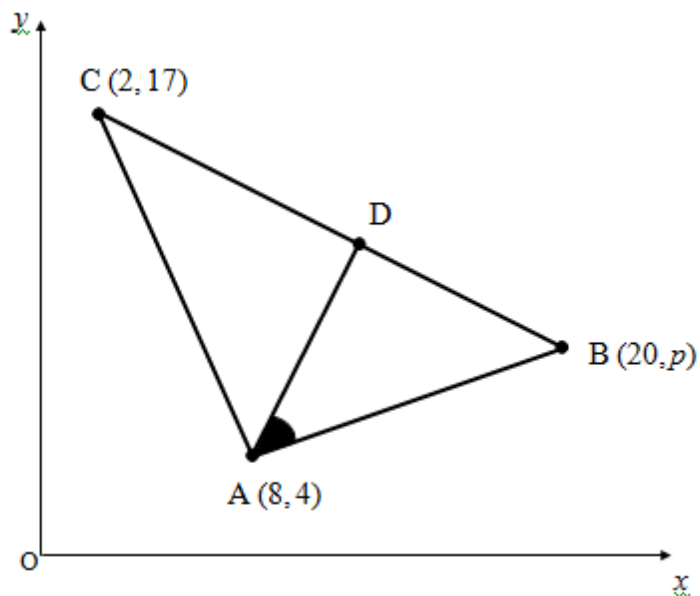
**Table of standard integrals:**

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + c$
$\cos ax$	$\frac{1}{a} \sin ax + c$

## Marks

1. In the diagram below triangle ABC has vertices A(8, 4), B(20, p) and C(2, 17) as shown.

AD is perpendicular to BC [Diagram not drawn to scale].



- (a) Given that the gradient of BC is  $-\frac{1}{2}$ , find the value of p.  
**3**
- (b) Find the equation of the altitude AD in the form  $Ax + By + C = 0$   
**3**
- (c) By considering the gradients of the side AB and the altitude AD, calculate the size of the shaded angle DAB.  
**3**

2. A function  $f$  is defined by the formula  $f(x) = 4x^2(x-3)$  where  $x \in \mathbb{R}$ .

(a) Write down the coordinates of the points where the curve with equation  $y = f(x)$  meets the  $x$ - and  $y$ -axes. **2**

(b) Find the stationary points of  $y = f(x)$  and determine the nature of each. **6**

(c) Sketch the curve  $y = f(x)$ . **2**

3. The circles with equations  $(x-3)^2 + (y-4)^2 = 25$  and  $x^2 + y^2 - kx - 8y - 2k = 0$  have the same centre.

Determine the radius of the larger circle. **5**

4. Two sequences are defined by the recurrence relations

$$U_{n+1} = aU_n + 3 \text{ and } V_{n+1} = a^2V_n + 5.$$

It is known that both sequences tend towards the same limit as  $n \rightarrow \infty$ .

(a) Find the value of  $a$ .

**5**

(b) Hence, or otherwise, evaluate the limit of the sequences as  $n \rightarrow \infty$ .

**1**

5. The value  $V$  (in £ million) of a cruise ship  $t$  years after launch is given by the formula

$$V = 252e^{-0.06335t}$$

(a) What was its value when launched?

**1**

(b) The owners decide to sell the ship when its value falls below £20 million.  
After how many years will it be sold?

**4**

- 6 (a) Relative to a suitable set of coordinate axes, Diagram 1 shows the line  $2x - y + 5 = 0$  intersecting the circle  $x^2 + y^2 - 6x - 2y - 30 = 0$  at the points P and Q.

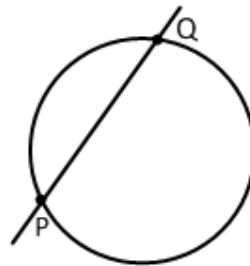


Diagram 1

Find the coordinates of P and Q.

6

- (b) Diagram 2 shows the circle from (a) and a second congruent circle, which also passes through P and Q.

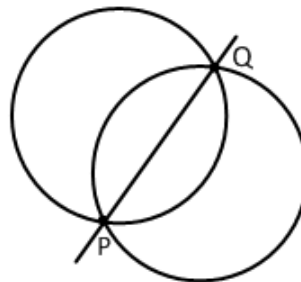


Diagram 2

Determine the equation of this second circle.

6

7. A function is defined on a suitable domain as  $h(x) = 2\sin 2x - \sqrt{3}\cos^2 x$ .

Calculate the rate of change of this function at the point where  $x = \frac{\pi}{3}$ .

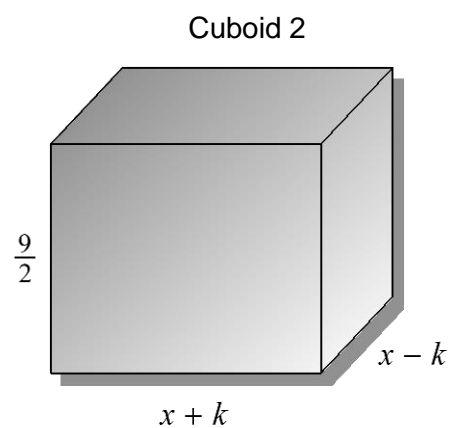
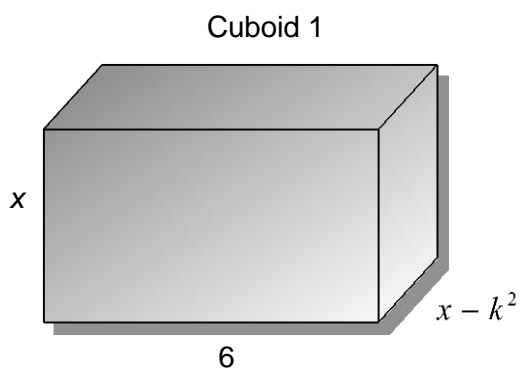
5

8. Solve the equation

$$\log_5(3 - 2x) + \log_5(2 + x) = 1, \text{ where } x \text{ is a real number.}$$

4

9. The two cuboids below have the **same volume**.



- (a) By writing down expressions for the two volumes,  $V_1$  and  $V_2$ , show that the following equation can be constructed

$$3(x^2 - 4k^2x + 3k^2) = 0$$

4

- (b) Given that  $k > 0$ , find the **exact value** of  $k$  for which the equation  $3(x^2 - 4k^2x + 3k^2) = 0$  has **equal** roots. **5**

10. The tangent to the curve  $y = x + \frac{p}{\sqrt{x}}$ , at the point where  $x = 4$ , is parallel to the line with equation  $x + y = 10$ .

Find the value of  $p$ .

**5**

End of question paper



Additional space for working