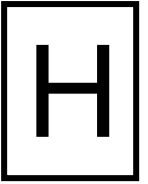Trinity High School

2019 Practice Exam

Paper 1

H

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $a.b = |a|\,|b| \cos \theta$, where θ is the angle between a and b

or $a.b = a_1b_1 + a_2b_2 + a_3b_3$ where $a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae: $\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\sin 2A = 2\sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$= 2\cos^2 A - 1$

$= 1 - 2\sin^2 A$

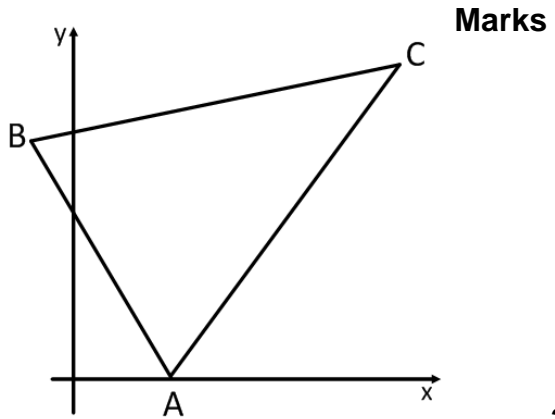**Table of standard derivatives:**

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\dfrac{1}{a}\cos ax + C$
$\cos ax$	$\dfrac{1}{a}\sin ax + C$

1. Triangle ABC has vertices A(4, 0), B(-4, 16) and C(18, 20), as shown in the diagram opposite.

Find the equation of the median fror.. ᴗ.

3

2. Express $2x^2 + 4x - 3$ in the form $a(x + b)^2 + c$

3

3. Given that $y = 4\sqrt{x} + \dfrac{2}{x^3}$, find $\dfrac{dy}{dx}$.

3

4.   The functions f and g, defined on suitable domains, are given by

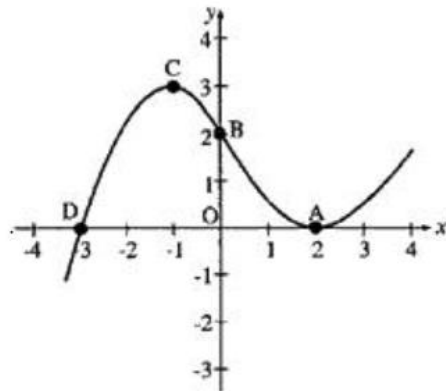$$f(x) = \frac{1}{x^2 - 4} \quad \text{and} \quad g(x) = 2x + 1$$

(a) Find an expression for $h(x)$ where $h(x) = g(f(x))$.
    Give your answer as a single fraction.

    3

(b) State a suitable domain for h.   **1**

5.   Part of the graph of $y = f(x)$ is shown in the diagram.

On separate diagrams, sketch the graphs of:

5

(i) $y = f(x - 1)$

(ii) $y = -f(x) - 2$

6.   A function, g, is defined by $g(x) = x^2 + 8$, where $x \in R$

Determine an expression for $g^{-1}(x)$

3

7.   Given that $f(x) = 5(7 - 2x)^3$, find f '(4)

3

8.   Solve $8 - 2x - x^2 > 0$ **2**

9. The points A and B have coordinates (8,4) and (2,-6) respectively.

    a.   Find the equation of the circle which has AB as its diameter.   **3**
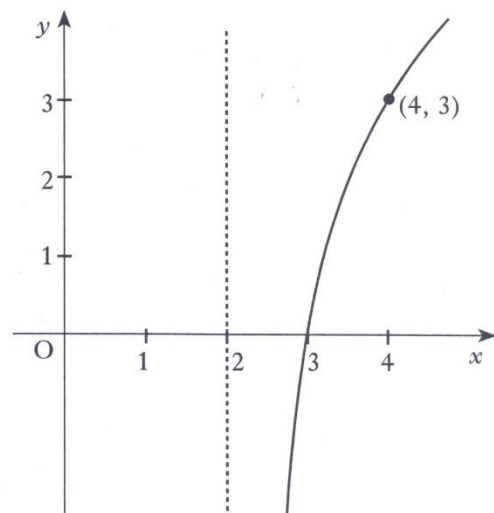
    A tangent to the circle is drawn at (10,-4).

    b.   Establish the equation of this tangent.   **3**

10.   Evaluate $log_5 2 + log_5 50 - log_5 4$ **3**

11.   The diagram shows a sketch of

       the graph of $y = f(x)$, where

       $f(x) = a\log_2(x-b)$.

       Find the values of a and b .

3

12. Find the range of values of k for which the equation $kx^2 - x - 1 = 0$ has no real roots. **3**

13. Show that the line with equation y = 4x − 2 is a tangent to the circle x² + y² − 12x − 10y + 44 = 0 and state the coordinates of the point of contact. **5**

14. A sequence is defined by the recurrence relation

$$u_{n+1} = \frac{1}{4}u_n + 16, \ u_0 = 0.$$

a. Calculate the values of $u_1, u_2 \ and \ u_3$. **2**

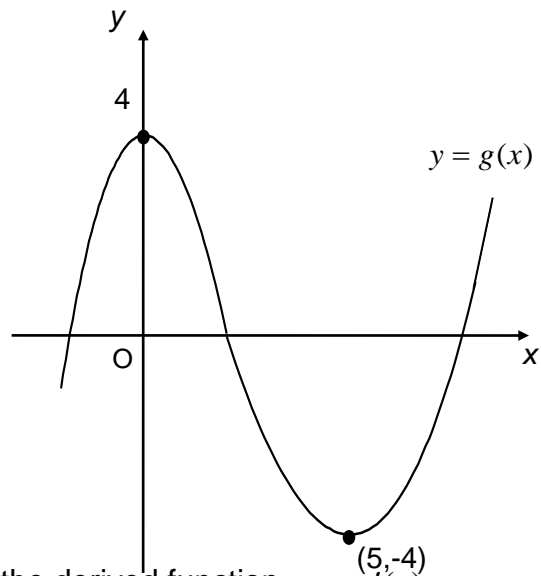b. (i) Explain why this sequence approaches a limit as n $\rightarrow \infty$ **1**

(ii) Calculate the exact value of this limit **2**

15. The diagram shows part of the graph of $y = g(x)$.

The function has stationary points at (0,4) and (5,-4).

$y = g(x)$

(5,-4)

Sketch the graph of the derived function $y = g'(x)$.

3

16. A curve has equation $y = x - \dfrac{16}{\sqrt{x}}$, $x > 0$

Find the equation of the tangent at the point where x = 4.

6

End of question paper