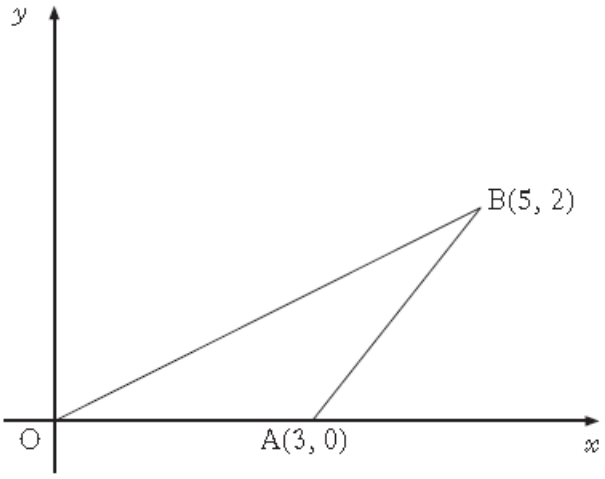
November revision questions (Higher)

1. A curve has equation $y = 3x^2 - x^3$.

(*a*) Find the coordinates of the stationary points on this curve and determine their nature. **6**

(*b*) State the coordinates of the points where the curve meets the coordinate axes and sketch the curve. **2**

2. A(3, 0), B(5, 2) and the origin are the vertices of a triangle as shown in the diagram.



(*a*) Obtain the equation of the perpendicular bisector of AB. **4**

(*b*) The median from A has equation $y + 2x = 6$.

Find T, the point of intersection of this median and the perpendicular bisector of AB. **2**

(*c*) Calculate the angle that AT makes with the positive direction of the x-axis. **2**

3. A curve has equation $y = x^4 - 2x^3 + 5$.

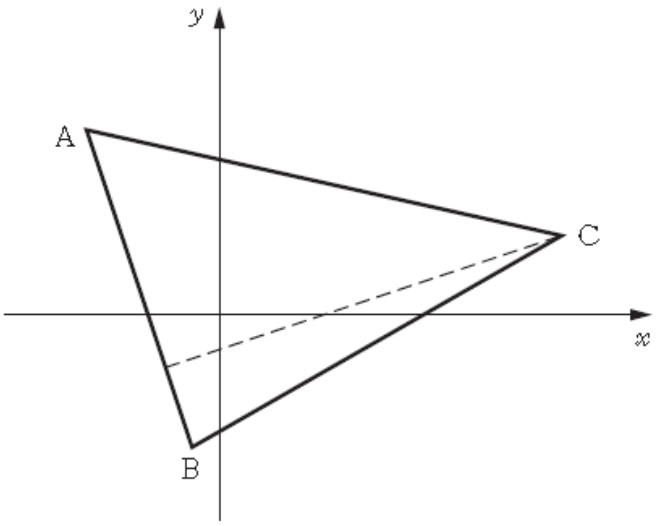
Find the equation of the tangent to this curve at the point where $x = 2$. **4**

4. The function $f(x) = \frac{4}{x^2} + x$ is defined on the domain $x > 0$, $x \in \mathbb{R}$, the set of real numbers.

Find the maximum and minimum values of $f(x)$ on the closed interval $1 \leq x \leq 4$. **6**

5. The vertices of triangle ABC are A(–5, 7), B(–1, –5) and C(13, 3) as shown in the diagram.

The broken line represents the altitude from C.



(*a*) Show that the equation of the altitude from C is $x - 3y = 4$. **4**

(*b*) Find the equation of the median from B. **3**

(*c*) Find the coordinates of the point of intersection of the altitude from C and the median from B. **2**

6. Functions f and g are defined on suitable domains by

$$f(x) = 10 + x \text{ and } g(x) = (1 + x)(3 - x) + 2.$$

(*a*) Find an expression for $f(g(x))$. **2**

(b) Another function h is given by $h(x) = \dfrac{1}{f(g(x))}$.

What values of x cannot be in the domain of h? **2**

7. **Find the equation of the line passing through the point $(-2, 3)$ which is parallel to the line with equation $y + 4x = 7$.** **2**

8. **Given that $y = 12x^3 + 8\sqrt{x}$, where $x > 0$, find $\dfrac{dy}{dx}$.** **3**

9. A version of the following problem first appeared in print in the 16th Century.

A frog and a toad fall to the bottom of a well that is 50 feet deep.

Each day, the frog climbs 32 feet and then rests overnight. During the night, it slides down $\frac{2}{3}$ of its height above the floor of the well.

The toad climbs 13 feet each day before resting.

Overnight, it slides down $\frac{1}{4}$ of its height above the floor of the well.

Their progress can be modelled by the recurrence relations:

- $f_{n+1} = \frac{1}{3} f_n + 32, \quad f_1 = 32$
- $t_{n+1} = \frac{3}{4} t_n + 13, \quad t_1 = 13$

where f_n and t_n are the heights reached by the frog and the toad at the end of the nth day after falling in.

(a) Calculate t_2, the height of the toad at the end of the second day. 1

(b) Determine whether or not either of them will eventually escape from the well. 5

10. A sequence is defined by the recurrence relation $u_{n+1} = \frac{1}{3} u_n + 10$ with $u_3 = 6$.

(a) Find the value of u_4. 1

(b) Explain why this sequence approaches a limit as $n \to \infty$. 1

(c) Calculate this limit. 2

11 Functions f and g are defined on $\mathbb{R}$, the set of real numbers.
The inverse functions f^{-1} and g^{-1} both exist.

(a) Given $f(x) = 3x + 5$, find $f^{-1}(x)$. 3

(b) If $g(2) = 7$, write down the value of $g^{-1}(7)$. 1

12. (a) Find the x-coordinates of the stationary points on the graph with equation $y = f(x)$, where $f(x) = x^3 + 3x^2 - 24x$.

4

(b) Hence determine the range of values of x for which the function f is strictly increasing. 2

Answers

1. a. Min TP at (0, 0) and Max TP at (2, 4)
   b. (3, 0) and (0, 0). Sketch.

2. a. $y = -x + 5$

   b. $x = 1, y = 4$

   c. 116.6^{o}

3. $y = 8x - 11$

4. Max is 5, Min is 3

5. a. $3y = x - 4$

   b. $y = 2x - 3$

   c. $x = 1, y = -1$

6. a. $10 + (1 + x)(3 - x) + 2$

   b. 5 and -3

7. $y = -4x - 5$

8. $36x^2 + 4x^{-\frac{1}{2}}$

9. a. 22.75 or equivalent

   b. Limit (frog) = 48, Limit (toad) = 52. Since 52>50, toad will escape

10. a. $u_4 = 12$

    b. A limit exists since $-1 < \frac{1}{3} < 1$

    c. 15

11. a. $f^{-1}(x) = \frac{x-5}{3}$

    b. 2

12. a. -4 and 2

    b. x<-4 and x>2