

ADVANCED HIGHER MATHEMATICS

PROOF BY INDUCTION

Prove by induction that, for all positive integers n :

1.
$$\sum_{k=1}^n (2k + 3) = n(n + 4)$$

2.
$$\sum_{k=1}^n (4k + 1) = n(2n + 3)$$

3.
$$\sum_{k=1}^n k = \frac{1}{2}n(n + 1)$$

4.
$$\sum_{k=1}^n k^3 = \frac{1}{4}n^2(n + 1)^2$$

5.
$$\sum_{k=1}^n k(k + 1) = \frac{1}{3}n(n + 1)(n + 2)$$

6.
$$\sum_{k=1}^n k(k + 3) = \frac{1}{3}n(n + 1)(n + 5)$$

7.
$$\sum_{k=1}^n (k^2 - 1) = \frac{1}{6}n(n - 1)(2n + 5)$$

8.
$$\sum_{k=1}^n k(k + 2) = \frac{1}{6}n(n + 1)(2n + 7)$$

9.
$$\sum_{k=1}^n k(k + 1)(k + 2) = \frac{1}{4}n(n + 1)(n + 2)(n + 3)$$

10.
$$\sum_{k=1}^n k(k - 1) = \frac{1}{3}n(n - 1)(n + 1)$$

11. $5^n + 3$ is divisible by 4

12. $6^n + 4$ is divisible by 10