2012 Paper 1

1. $u_1 = 3\times1+4 = 7$

$u_2 = 3\times7+4 = 25$

C

2. $\frac{dy}{dx} = 3x^2 - 6$

$m = 3(-2)^2 - 6$

$= 12 - 6$

$= 6$

D

3. $(x-3)^2 - 9 + 14$

$= (x-3)^2 + 5$

B

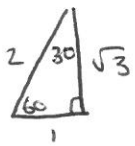
4. $m = \tan\theta$

$= \tan 150^\circ$

$= -\tan 30^\circ$

$= -\frac{1}{\sqrt{3}}$

B



5. $\cos 2a = 2\cos^2 a - 1$

$= 2\times\left(\frac{4}{5}\right)^2 - 1$

$= 2\times\frac{16}{25} - 1$

$= \frac{32}{25} - 1$

$= \frac{7}{25}$

A

6. $\frac{dy}{dx} = -6x^{-3} + \frac{6}{2}x^{\frac{1}{2}}$

$= -6x^{-3} + 3x^{\frac{1}{2}}$

C

7. $\underset{\sim}{u} \cdot \underset{\sim}{v} = 0$

$(-3 \times 1) + (1 \times t) + (2t \times (-1)) = 0$

$-3 + t - 2t = 0$

$-t = 3$

$t = -3$ A

8. $V = \frac{4}{3}\pi r^3$

$V'(r) = \frac{12}{3}\pi r^2$

$= 4\pi r^2$

$V'(2) = 4\pi \times 2^2$

$= 16\pi$ C

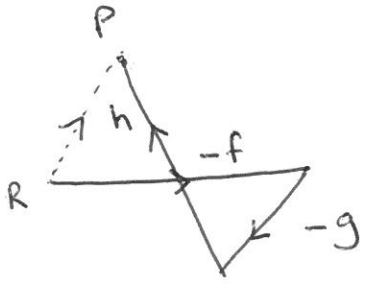
9. $y = \cos(x - \frac{\pi}{6}) - 1$

↓ moved right by $\frac{\pi}{6}$ ↓ moved down by 1

A

10. B

$-\underset{\sim}{f} - \underset{\sim}{g} + \underset{\sim}{h} = \overrightarrow{RP}$



11. $\int \frac{1}{6} x^{-2} \, dx$

$= \frac{\frac{1}{6} x^{-1}}{-1} + C$

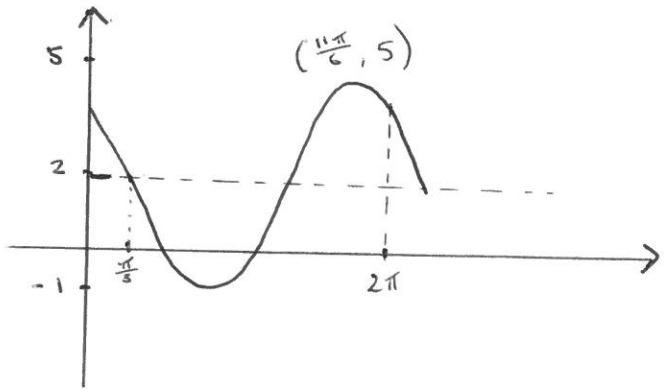
$= -\frac{1}{6} x^{-1} + C$

D

12. $2 - 3\sin\left(x - \frac{\pi}{3}\right)$

$-3\sin\left(x - \frac{\pi}{3}\right) + 2$

reflect over x	Amplitude = 3	move right $\frac{\pi}{3}$	move up 2



max value = 5

$x = 270° + 60° = 330° = \frac{11\pi}{6}$ B

13. $y = k(x+2)(x+1)$

$6 = k(0+2)(0+1)$

$6 = k \times 2 \times 1$

$6 = 2k$

$k = 3$

$y = 3(x+2)(x+1)$

D

14. $\int (2x-1)^{\frac{1}{2}}$

$= \frac{(2x-1)^{\frac{3}{2}}}{\frac{3}{2} \times 2} + C$

$= \frac{(2x-1)^{\frac{3}{2}}}{3} + C$

$= \frac{1}{3}(2x-1)^{\frac{3}{2}} + C$

A

15. $|\underset{\sim}{u}| = \sqrt{3^2 + (-1)^2 + 0^2}$

$= \sqrt{9+1}$

$= \sqrt{10}$

$\underset{\sim}{u} = \frac{1}{\sqrt{10}} \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix}$

D

16. $y = 3\cos^4 x$

$= 3(\cos x)^4$

$\frac{dy}{dx} = 12(\cos x)^3 \times (-\sin x)$

$= -12\cos^3 x \sin x$

C

17. $\underset{\sim}{a} \cdot (\underset{\sim}{a} + \underset{\sim}{b}) = 7$

$\underset{\sim}{a} \cdot \underset{\sim}{a} + \underset{\sim}{a} \cdot \underset{\sim}{b} = 7$

$(3\times3) + (4\times4) + (0\times0) + \underset{\sim}{a} \cdot \underset{\sim}{b} = 7$

$9 + 16 + 0 + \underset{\sim}{a} \cdot \underset{\sim}{b} = 7$

$\underset{\sim}{a} \cdot \underset{\sim}{b} = -18$

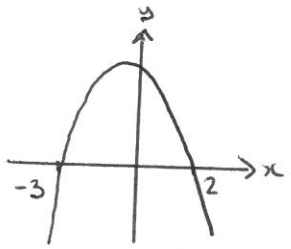
D

18. D

19. $6 - x - x^2 < 0$

$(3 + x)(2 - x) < 0$

$x = -3 \qquad x = 2$



$6 - x - x^2 < 0$

when $x < -3$, $x > 2$ B

20. $\frac{\log_b 9a^2}{\log_b 3a}$

$= \frac{\log_b (3a)^2}{\log_b 3a}$

$= \frac{2 \log_b 3a}{\log_b 3a}$

$= 2$

A

21. (a)(i)

4	1	-5	2	8
		4	-4	-8
	1	-1	-2	0

since the remainder = 0, $(x-4)$ is a factor.

(ii) $(x-4)(x^2 - x - 2) = 0$

$(x-4)(x-2)(x+1) = 0$

(iii) $x = 4$, $x = 2$, $x = -1$

(b) $\int_0^2 x^3 - 5x^2 + 2x + 8$

$= \left[\frac{x^4}{4} - \frac{5x^3}{3} + x^2 + 8x \right]_0^2$

$= \left(\frac{2^4}{4} - \frac{5(2)^3}{3} + 2^2 + 8(2) \right) - (0)$

$= \frac{16}{4} - \frac{40}{3} + 4 + 16$

$= 4 - 13\frac{1}{3} + 20$

$= 24 - 13\frac{1}{3}$

$= 10\frac{2}{3}$

22. (a) $\cos x - \sqrt{3}\sin x = k\cos(x+a)$

$= k\cos a\cos x - k\sin a\sin x$

$k\cos a = 1$

$k\sin a = \sqrt{3}$

$k = \sqrt{1^2 + (\sqrt{3})^2}$

$= \sqrt{4}$

$= 2$

$\tan a = \frac{\sqrt{3}}{1}$

$= 60$

$a = \frac{\pi}{3}$

$2\cos\left(x + \frac{\pi}{3}\right)$

(b) $y = \cos x - \sqrt{3}\sin x$

$= 2\cos(x + \frac{\pi}{3})$

Crosses x-axis when $y = 0$:

$2\cos(x + \frac{\pi}{3}) = 0$

$\cos(x + \frac{\pi}{3}) = 0$

$x + 60^\circ = 90^\circ, 270^\circ$

$x = 30^\circ, 210^\circ$

$x = \frac{\pi}{6}, \frac{7\pi}{6}$

$(\frac{\pi}{6}, 0)$ $(\frac{7\pi}{6}, 0)$

Crosses y-axis when $x = 0$:

$y = 2\cos(0 + \frac{\pi}{3})$

$= 2\cos\frac{\pi}{3}$

$= 2 \times \cos 60^\circ$

$= 2 \times \frac{1}{2}$

$= 1$

$(0, 1)$

23. (a) $m_{PQ} = \frac{9 - (-3)}{-1 - 3}$

$= \frac{12}{-4}$

$= -3$

$m_\perp = \frac{1}{3}$

$M_{PQ} = \left(\frac{3 + (-1)}{2}, \frac{(-3) + 9}{2}\right)$

$= (1, 3)$

$y - 3 = \frac{1}{3}(x - 1)$

$3y - 9 = x - 1$

$3y - x = 8$

(b) $m = -3$

$y - (-2) = -3(x - 1)$

$y + 2 = -3x + 3$

$y + 3x = 1$

(c) $3y - x = 8$

$y + 3x = 1$

$9y - 3x = 24$

$y + 3x = 1$

$10y = 25$

$y = 2.5$

$y + 3x = 1$

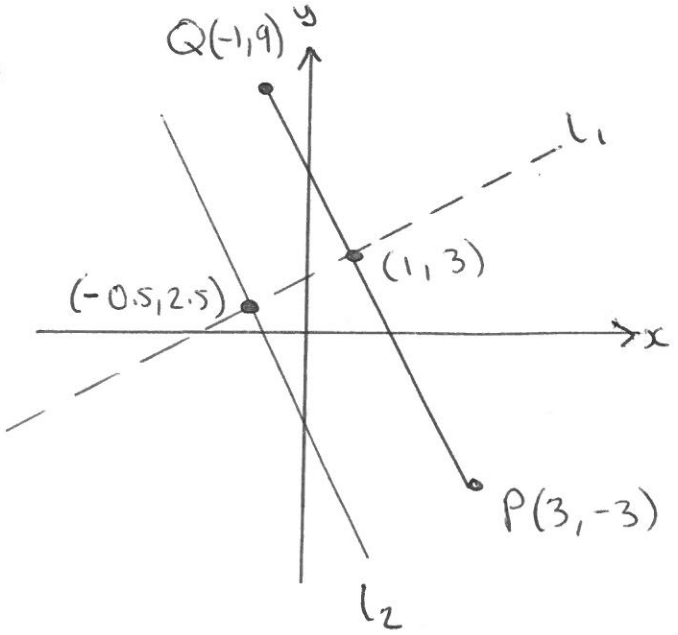
$2.5 + 3x = 1$

$3x = -1.5$

$x = -0.5$

$(-0.5, 2.5)$

(d)



$$d = \sqrt{(1-(-0.5))^2 + (3-2.5)^2}$$

$$= \sqrt{(1.5)^2 + (0.5)^2}$$

$$= \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$$

$$= \sqrt{\frac{9}{4} + \frac{1}{4}}$$

$$= \sqrt{\frac{10}{4}}$$

$$= \frac{\sqrt{10}}{\sqrt{4}}$$

$$= \frac{\sqrt{10}}{2}$$

Higher 2012 Paper 2

1. (a) $f(g(x)) = (x+4)^2 + 3$

$g(f(x)) = x^2 + 3 + 4$

$= x^2 + 7$

(b) $(x+4)^2 + 3 + x^2 + 7 = 0$

$x^2 + 8x + 16 + 10 + x^2 = 0$

$2x^2 + 8x + 26 = 0$

$b^2 - 4ac$

$= 8^2 - 4 \times 2 \times 26$

$= 64 - 208$

$= -144$

Since $b^2 - 4ac < 0$,

there are no real roots.

2. (a)

$y = 2x + 5$

$x^2 + (2x+5)^2 - 6x - 2(2x+5) - 30 = 0$

$x^2 + 4x^2 + 20x + 25 - 6x - 4x - 10 - 30 = 0$

$5x^2 + 10x - 15 = 0$

$5(x^2 + 2x - 3) = 0$

$5(x+3)(x-1) = 0$

$x = -3 \qquad x = 1$

$y = -6 + 5$

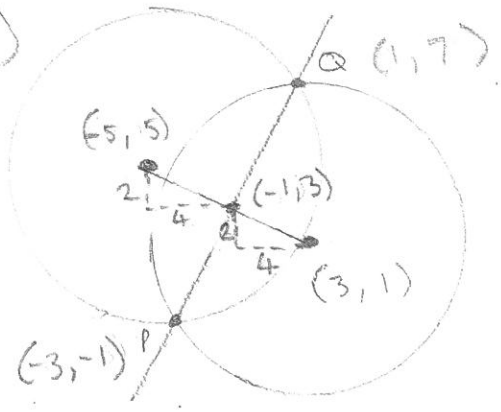
$= -1$

$y = 2 + 5$

$= 7$

$P(-3, -1) \qquad Q(1, 7)$

(b)



$M_{PQ} = \left(\frac{-3+1}{2}, \frac{-1+7}{2}\right)$

$= (-1, 3)$

$C(-5, 5)$

$r = \sqrt{-3^2 + (-1)^2 - (-30)}$

$= \sqrt{9+1+30}$

$= \sqrt{40}$

$(x+5)^2 + (y-5)^2 = 40$

3. For S.P's $f'(x) = 0$

$3x^2 - 4x - 4 = 0$

$(3x + 2)(x - 2) = 0$

$3x + 2 = 0$ $\quad$ $x - 2 = 0$

$3x = -2$ $\quad$ $x = 2$

$x = -\frac{2}{3}$

x	-1	$-\frac{2}{3}$	0	2	3
$f'(x)$	$+$	0	$-$	0	$+$
	/	—	\	—	/

$y = \left(-\frac{2}{3}\right)^3 - 2\left(-\frac{2}{3}\right)^2 - 4\left(-\frac{2}{3}\right) + 6$

$= -\frac{8}{27} - 2\left(\frac{4}{9}\right) + \frac{8}{3} + 6$

$= -\frac{8}{27} - \frac{8}{9} + \frac{8}{3} + 6$

$= -\frac{8}{27} - \frac{24}{27} + \frac{72}{27} + 6$

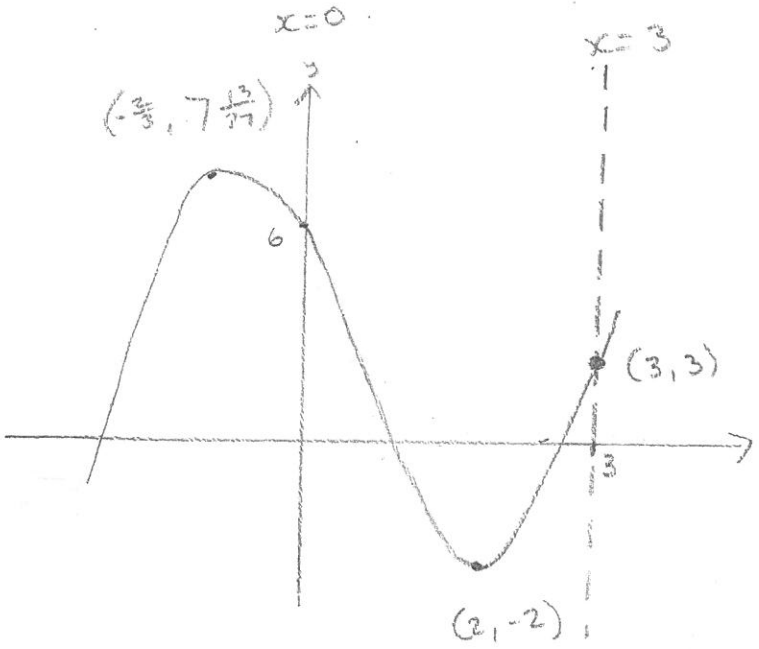
$= \frac{40}{27} + 6$

$= 7\frac{13}{27}$ $\quad$ $\left(-\frac{2}{3}, 7\frac{13}{27}\right)$

$y = 8 - 8 - 8 + 6$

$= -2$

$(2, -2)$



When $x=0$, $y = 0^3 - 2(0)^2 - 4(0) + 6$

$= 6$

When $x = 3$, $y = 3^3 - 2(3)^2 - 4(3) + 6$

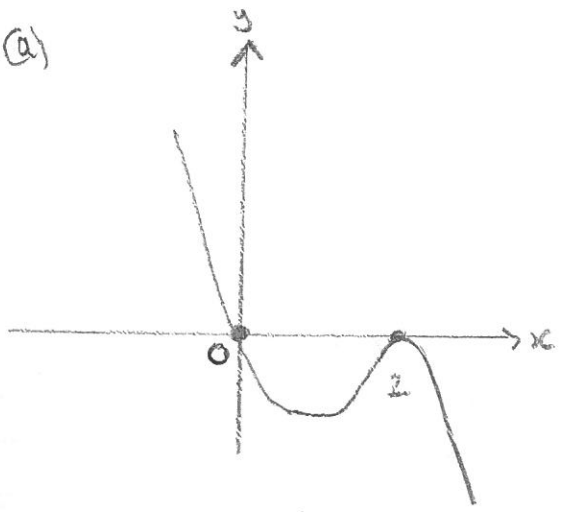
$= 27 - 18 - 12 + 6$

$= 3$

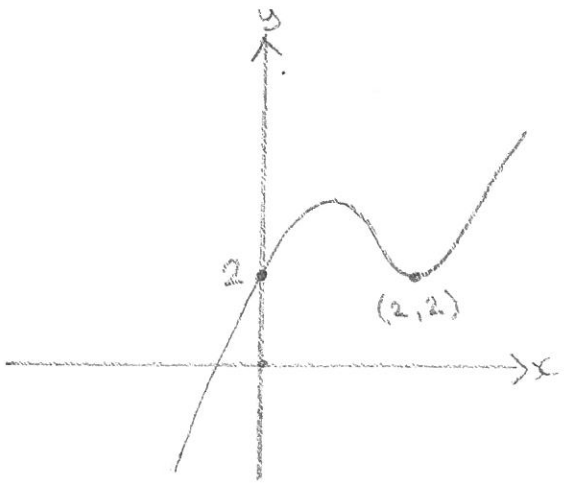
Max value $= 6$

Min value $= -2$

4. (a)



(b)



5. (a) $\vec{BA} = \underset{\sim}{a} - \underset{\sim}{b}$

$= \begin{pmatrix} 3 \\ -3 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

(b) $\vec{BC} = \underset{\sim}{c} - \underset{\sim}{b}$

$= \begin{pmatrix} 4 \\ k \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ k+3 \\ -1 \end{pmatrix}$

(i) $\cos A\hat{B}C = \dfrac{\vec{BA} \cdot \vec{BC}}{|\vec{BA}|\,|\vec{BC}|}$

$|\vec{BA}| = \sqrt{2}$

$|\vec{BC}| = \sqrt{4 + k^2 + 6k + 9 + 1}$

$= \sqrt{k^2 + 6k + 14}$

$= \dfrac{2 + 0 + 1}{\sqrt{2}\sqrt{k^2 + 6k + 14}}$

$= \dfrac{3}{\sqrt{2(k^2 + 6k + 14)}}$

(b) $\angle ABC = 30^\circ$

$\dfrac{3}{\sqrt{2(k^2 + 6k + 14)}} = \cos 30^\circ$

$\dfrac{3}{\sqrt{2(k^2 + 6k + 14)}} \rightarrow \dfrac{\sqrt{3}}{2}$

$\sqrt{3}\sqrt{2(k^2 + 6k + 14)} = 6$

$\sqrt{6(k^2 + 6k + 14)} = 6$

$6(k^2 + 6k + 14) = 36$

$k^2 + 6k + 14 = 6$

$k^2 + 6k + 8 = 0$

$(k + 4)(k + 2) = 0$

$k = -4 \quad k = -2$

6. (a) Since the range of values for $\sin x$ are from -1 to 1, the sequence has a limit.

Since $0 < x < \frac{\pi}{2}$, the interval of $\sin x$ is $0 < \sin x < 1$.

(b) $L = \sin x\, L + \cos 2x$

$\frac{1}{2} \sin x = \sin x \left(\frac{1}{2} \sin x\right) + \cos 2x$

$\frac{1}{2} \sin x = \frac{1}{2} \sin^2 x + 1 - 2\sin^2 x$

$\sin x = \sin^2 x + 2 - 4\sin^2 x$

$3\sin^2 x + \sin x - 2 = 0$

$(3\sin x - 2)(\sin x + 1) = 0$

$3\sin x = 2$ $\qquad$ $\sin x = -1$

$\sin x = \frac{2}{3}$ $\qquad$ $x = 270°$

$x = 0.73, 2.41$ $\qquad$ $x = 4.71$

7. (a) $4^x = 3^{2-x}$

$\log_a 4^x = \log_a 3^{2-x}$

$x \log_a 4 = (2-x)\log_a 3$

$x \log_a 4 = 2\log_a 3 - x\log_a 3$

$x \log_a 4 + x \log_a 3 = 2\log_a 3$

$x(\log_a 4 + \log_a 3) = 2\log_a 3$

$x \log_a 12 = 2\log_a 3$

$x \log_a 12 = \log_a 3^2$

$x \log_a 12 = \log_a 9$

$x = \dfrac{\log_a 9}{\log_a 12}$

(b) Sub $x = \dfrac{\log_a 9}{\log_a 12}$ into either graph to get y.

$y = 4^x$

$= 4^{\left(\frac{\log_a 9}{\log_a 12}\right)}$

$= 4^{0.8842}$ ← Use log button or ln button to obtain 0.8842.

$= 3.4$