2011 Paper 1

1. $2\begin{pmatrix}2\\5\\-7\end{pmatrix} - \begin{pmatrix}1\\0\\-1\end{pmatrix} - \frac{1}{2}\begin{pmatrix}-4\\2\\0\end{pmatrix}$

$= \begin{pmatrix}4\\10\\-14\end{pmatrix} - \begin{pmatrix}1\\0\\-1\end{pmatrix} - \begin{pmatrix}-2\\1\\0\end{pmatrix}$

$= \begin{pmatrix}5\\9\\-13\end{pmatrix}$ C

2. $3y + 2x = 6$

$3y = -2x + 6$

$y = -\frac{2}{3}x + 2$

Parallel $m = -\frac{2}{3}$

B

3. Move 2 left, 1 down

D

4. $\frac{dy}{dx} = 3x^2 - 2$

$m = 3\times 2^2 - 2$

$= 12 - 2$

$= 10$ D

5. $x^2 - 8x + 7$

$[(x-4)^2 - 16] + 7$

$(x-4)^2 - 9$ A

6. $m_\perp = \frac{1}{2}$

$y + 3 = \frac{1}{2}(x - 2)$

C

7.

	1	−1	1	3
1		1	0	1
	1	0	1	4

D

8. $m = \tan 30°$

$= \frac{1}{\sqrt{3}}$ A

9. B

10. $2\cos x = \sqrt{3}$

$\cos x = \frac{\sqrt{3}}{2}$

$x = 30°, 330°$

$= \frac{\pi}{6}, \frac{11\pi}{6}$

D

11. $\int 4x^{\frac{1}{2}} + x^{-3}$

$= \frac{4x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{-2}}{-2} + C$

$= \frac{8x^{\frac{3}{2}}}{3} - \frac{x^{-2}}{2} + C$

D

12. $\sin(p+q)$

$= \sin p\cos q + \cos p \sin q$

$= \left(\frac{2}{\sqrt{5}} \times \frac{\sqrt{5}}{3}\right) + \left(\frac{1}{\sqrt{5}} \times \frac{2}{3}\right)$

$= \frac{2\sqrt{5}}{3\sqrt{5}} + \frac{2}{3\sqrt{5}}$

$= \frac{2}{3} + \frac{2}{3\sqrt{5}}$ C

13. $f(x) = 4\sin 3x$

$f'(x) = 3 \times 4\cos 3x$

$= 12\cos 3x$

$f'(0) = 12 \times \cos 3x0$

$= 12 \times \cos 0$

$= 12 \times 1$

$= 12$ C

14. $\underset{\sim}{P} \cdot \underset{\sim}{q} = |\underset{\sim}{p}||\underset{\sim}{q}|\cos\theta$

$= 3 \times 3 \times \cos 60^\circ$

$= 9 \times \frac{1}{2}$

$= \frac{9}{2}$

B

15. $\overrightarrow{ST} = t - s$ $\overrightarrow{TU} = u - t$

$= \begin{pmatrix} -12 \\ -9 \\ 15 \end{pmatrix}$ $= \begin{pmatrix} -8 \\ -6 \\ 10 \end{pmatrix}$

$\overrightarrow{ST} = \frac{3}{2}\overrightarrow{TU}$

T divides SU in the ratio

3 : 2

B

16. $\int \frac{1}{3}x^{-4}$

$= \frac{\frac{1}{3}x^{-3}}{-3} + C$

$= -\frac{1}{9}x^{-3} + C$

$= -\frac{1}{9x^3} + C$ A

17. $y = kx(x+1)(x-2)$

$x \quad y$

$(1, 2)$ $\quad 2 = k \times 1 (1+1)(1-2)$

$2 = k \times 2 \times (-1)$

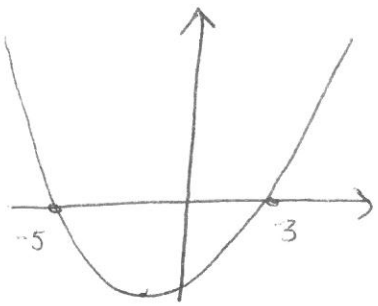
$2 = -2k$

$-2k = 2$

$k = -1$

$y = -x(x+1)(x-2)$ A

18.



Above x-axis when

$x < -5$, $x > 3$

C

19. $\log_3 y = x$

$y = 3^x$

when $x = 0$, $y = 3^0$

$= 1 \quad (0, 1)$

C

20. $x \geq 2$ as the $\sqrt{x-2}$ cannot be negative, e.g. if $x = 0$ or 1

All values of sin range from -1 to 1, since Sin is squared the values can only range from 0 to 1

$0 \leq g(x) \leq 1$

D

21. (a) $m_{BD} = \frac{-3-12}{2-7}$

$= -\frac{15}{-5}$

$= 3$

$y - 12 = 3(x-7)$

$y - 12 = 3x - 21$

$y = 3x - 9$

(b) $-3x + y = -9$

$x + 3y = 23$

$-3x + y = -9$

$3x + 9y = 69$

$10y = 60$

$y = 6$

$x + 18 = 23$

$x = 5$

E(5, 6)

(c) $M_{AB} = \left(\frac{-1+7}{2}, \frac{8+12}{2}\right)$

$= (3, 10)$

$m_{AB} = \frac{12-8}{7-(-1)}$

$= \frac{4}{8}$

$= \frac{1}{2}$

$m_{\perp} = -2$

$y - 10 = -2(x-3)$

$y - 10 = -2x + 6$

$y = -2x + 16$

(d) $(5, 6)$ $6 = -2 \times 5 + 16$

$6 = -10 + 16$

$6 = 6$

since LHS = RHS the point (5,6) lies on the line.

22. (a) (i) $(x-2)(x^2+1) = 0$

$x = 2$ $x^2 = -1$

$(2, 0)$

(ii) $y = (0-2)(0^2+1)$

$= (-2) \times (1)$

$= -2$

$(0, -2)$

(b) $f(x) = (x-2)(x^2+1)$

$= x^3 + x - 2x^2 - 2$

$= x^3 - 2x^2 + x - 2$

For S.P's $f'(x) = 0$

$3x^2 - 4x + 1 = 0$

$(3x-1)(x-1) = 0$

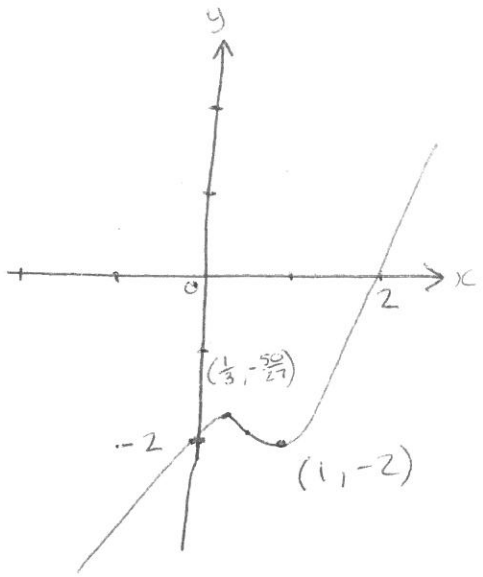
$x = \frac{1}{3}$ $x = 1$

x	0	$\frac{1}{3}$	$\frac{1}{2}$	1	2
$f'(x)$	+	0	−	0	+
slope	/	—	\	—	/

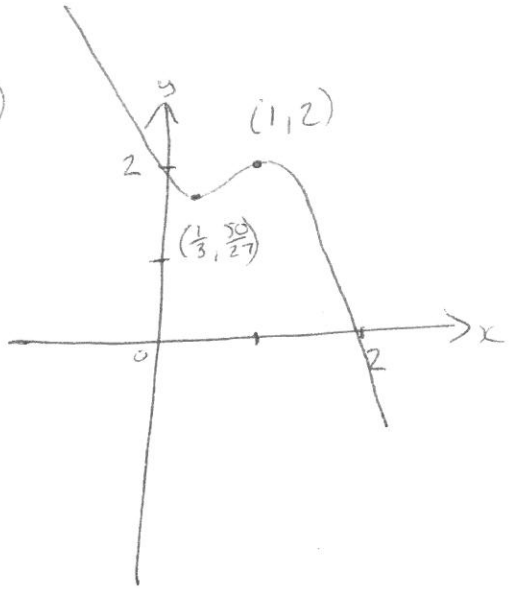
max tp $\left(\frac{1}{3}, -\frac{50}{27}\right)$ min tp $(1, -2)$

22. (c)

(i)



(ii)



23. (a) $\cos 2x - 3\cos x + 2 = 0$

$2\cos^2 x - 1 - 3\cos x + 2 = 0$

$2\cos^2 x - 3\cos x + 1 = 0$

$(2\cos x - 1)(\cos x - 1) = 0$

S	A ✓
T	C ✓

$\cos x = \frac{1}{2}$ $\quad$ $\cos x = 1$

$x = 60°, 300°$ $\quad$ $x = 0°, 360°$

(b)

If, $\cos 2x - 3\cos x + 2 = 0$ then, $\cos 4x - 3\cos 2x + 2 = 0$

$x = 60, 300°$ $\quad$ $x = 0°$

$2x = 60, 300°$ $\quad$ $2x = 0°$

$x = 30°, 150°$ $\quad$ $x = 0$

(Need to add 360 to answers for $2x$ as range is $0 \leq x < 360$)

$2x = 60, 300, 420, 660$ $\quad$ $2x = 0, 360$

$x = 30, 150, 210, 330$ $\quad$ $x = 0, 180$

2011 Paper 2

1. (a) $B(4, 4, 0)$

(b) $\overrightarrow{DB} = b - d$

$= \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ 6 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ 2 \\ -6 \end{pmatrix}$

$\overrightarrow{DM} = m - d$

$= \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ 6 \end{pmatrix}$

$= \begin{pmatrix} 0 \\ -2 \\ -6 \end{pmatrix}$

(c) $\cos\theta = \dfrac{\underline{a} \cdot \underline{b}}{|\underline{a}||\underline{b}|}$

$= \dfrac{2\times 0 + 2\times(-2) + (-6)\times(-6)}{\sqrt{44} \times \sqrt{40}}$

$= \dfrac{32}{\sqrt{44}\sqrt{40}}$

$= 0.762\ldots$

$\theta = 40.3^\circ$

2. (a) $g(f(x)) = 3(x^3 - 1) + 1$

$= 3x^3 - 3 + 1$

$= 3x^3 - 2$

(b) $g(f(x)) + xh(x)$

$= 3x^3 - 2 + 4x^2 - 5x$

$= 3x^3 + 4x^2 - 5x - 2$

(c) (i)

1	3	4	-5	-2
		3	7	2
	3	7	2	0

since the remainder = 0, $(x-1)$ is a factor.

(ii) $(x-1)(3x^2 + 7x + 2) = 0$

$(x-1)(3x+1)(x+2) = 0$

(d) $x = 1$, $x = -\frac{1}{3}$, $x = -2$

3. (a) $u_1 = -\frac{1}{2} \times (-16) = 8$

$u_2 = -\frac{1}{2} \times 8 = -4$

(b) $V_2 = pV_1 + q \longrightarrow 5 = p \times 4 + q$

$V_3 = pV_2 + q \longrightarrow 7 = p \times 5 + q$

$4p + q = 5$

$5p + q = 7$

$p = 2$

$8 + q = 5$

$q = -3$

$p = 2$, $q = -3$

(C) (i) $u_{n+1} = -\frac{1}{2}u_n$ has a limit since $-1 < -\frac{1}{2} < 1$

$$L = -\frac{1}{2}L$$
$$\frac{3}{2}L = 0$$
$$L = 0$$

(ii) $v_{n+1} = 2v_n - 3$

No limit since $-1 \leq a \leq 1$

4. $$\int_0^3 2x + 4 - (x^3 - x^2 - 4x + 4)\,dx$$

$$= \int_0^3 -x^3 + x^2 + 6x \,.\, dx$$

$$= \left[-\frac{x^4}{4} + \frac{x^3}{3} + \frac{6x^2}{2} \right]_0^3$$

$$= \left(-\frac{81}{4} + \frac{27}{3} + \frac{54}{2} \right) - (0)$$

$$= (-20\tfrac{1}{4} + 9 + 27)$$

$$= 15\tfrac{3}{4}$$

$$\int_{-2}^{0} x^3 - x^2 - 4x + 4 - (2x + 4)\ dx$$

$$= \int_{-2}^{0} x^3 - x^2 - 6x\ dx$$

$$= \left[\frac{x^4}{4} - \frac{x^3}{3} - \frac{6x^2}{2} \right]_{-2}^{0}$$

$$= (0) - \left(\frac{16}{4} - \frac{-8}{3} - \frac{24}{2} \right)$$

$$= 0 - \left(4 + 2\frac{2}{3} - 12 \right)$$

$$= 0 - \left(-5\frac{1}{3} \right)$$

$$= 5\frac{1}{3}$$

Total Area $= 15\frac{3}{4} + 5\frac{1}{3}$

$= 15\frac{9}{12} + 5\frac{4}{12}$

$= 21\frac{1}{12}$

5. $y = kx^n$

$\log_2 y = \log_2 kx^n$

$\log_2 y = n\log_2 x + \log_2 k$

$y = mx + c$

$n = m = \frac{7-5}{4-0}$

$= \frac{2}{4}$

$= \frac{1}{2}$

$\log_2 k = 5$

$k = 2^5$

$= 32$

$y = 32x^{\frac{1}{2}}$

6. (a) $3\sin x - 5\cos x = R\sin(x-a)$

$= R(\sin x\cos a + \cos x \sin a)$

$= R\cos a \sin x + R\sin a \cos x$

$R\cos a = 3$

$R\sin a = -5$

S	A
T	C

$R = \sqrt{3^2 + (-5)^2}$

$= \sqrt{34}$

$\tan a = -\frac{5}{3}$

$a = -59°$

$a = 301°$

$= 5.3$

(b) $\int_0^t (3\cos x + 5\sin t)\,dx = 3$

$\left[3x\sin x - 5\cos x\right]_0^t = 3$

$(3\sin t - 5\cos t) - (3\sin 0 - 5\cos 0) = 3$

$(3\sin t - 5\cos t) - (0 - 5) = 3$

$3\sin t - 5\cos t + 5 = 3$

$3\sin t - 5\cos t = -2$

$\sqrt{34}\sin(t + 5.3) = -2$

$\sin(t + 5.3) = -0.3429...$

S	A
T	C

$t + 5.3 = -20.1°$

$t + 5.3 = 200.1°, 339.9°$

$t + 5.3 = 3.5, 5.9$

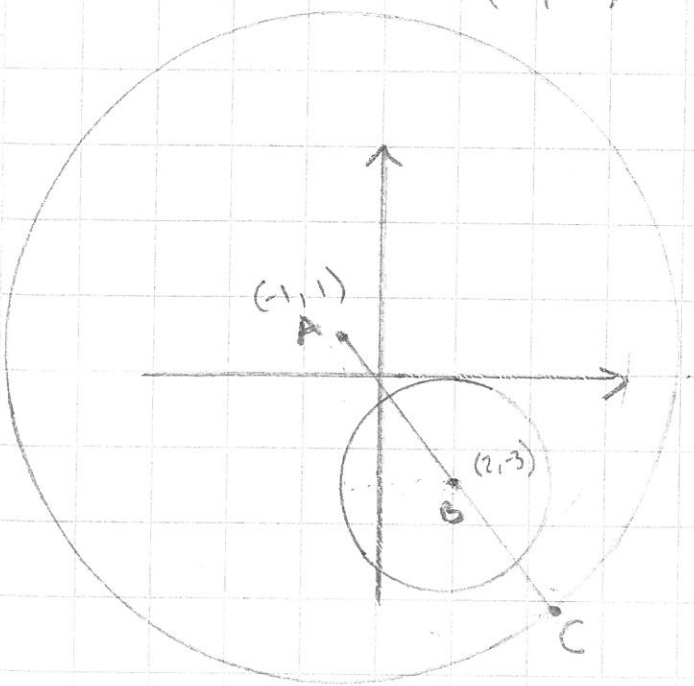
$t = -1.8, 0.6$

$t = 0.6$

7. C_1: Centre $(-1, 1)$ Radius $= \sqrt{121} = 11$

C_2: Centre $(2, -3)$ Radius $= \sqrt{-2^2 + 3^2 - p} = \sqrt{13 - p}$

$p < 13$



$AB^2 = 3^2 + 4^2 = 25$

$AB = 5$

If $AB = 5$ and the radius of $C_1 = 11$, then BC must be < 6 or the two circles will meet.

$\sqrt{13 - p} < 6$

$13 - p < 36$

$-p < 23$

$p > -23$