2010 Paper 1

1. $2x - 3y - 6 = 0$

$$-3y = -2x + 6$$

$$y = \frac{2}{3}x - 2$$

$$m_{\perp} = -\frac{3}{2}$$ A

2. $U_1 = 2 \times 1 + 3 = 5$

$$U_2 = 2 \times 5 + 3 = 13$$

C

3. $3\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} - 2\begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix}$

$$= \begin{pmatrix} 6 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} -2 \\ 4 \\ 8 \end{pmatrix}$$

$$= \begin{pmatrix} 8 \\ -4 \\ -5 \end{pmatrix}$$ D

4. A

5. $x^2 + 8x + 3$

$$= (x+4)^2 - 16 + 3$$

$$= (x+4)^2 - 13$$ B

6. For equal roots

$$b^2 - 4ac = 0$$

$$(-3)^2 - 4 \times k \times 2 = 0$$

$$9 - 8k = 0$$

$$-8k = -9$$

$$k = \frac{9}{8}$$ D

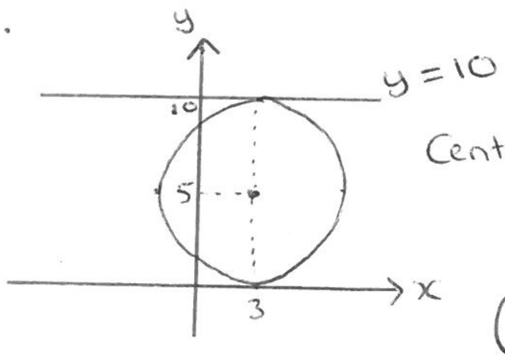
7. $L = \frac{1}{4}L + 7$

$$\frac{3}{4}L = 7$$

$$3L = 28$$

$$L = \frac{28}{3}$$ C

8.



Centre (3, 5)

B

9. $\int (2x^{-4} + \cos 5x)\, dx$

$= \frac{2x^{-3}}{-3} + \frac{1}{5}\sin 5x + C$

$= -\frac{2}{3}x^{-3} + \frac{1}{5}\sin 5x + C$

(C)

10. If perpindicular then

$\underset{\sim}{a} \cdot \underset{\sim}{b} = 0$

$\begin{pmatrix} x \\ 5 \\ 7 \end{pmatrix} \quad \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}$

$(x \times (-3)) + (5 \times 2) + (7 \times (-1)) = 0$

$-3x + 10 - 7 = 0$

$-3x + 3 = 0$

$-3x = -3$

$x = 1$

(B)

11. $g(\frac{\pi}{6}) = \frac{\pi}{6} + \frac{\pi}{6}$

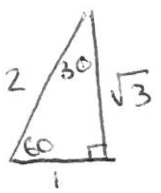
$= \frac{2\pi}{6}$

$= \frac{\pi}{3}$

$f(g(\frac{\pi}{6})) = \cos\frac{\pi}{3}$

$= \cos 60°$

$= \frac{1}{2}$



(D)

12. $f(x) = \frac{1}{x^{\frac{1}{5}}} = x^{-\frac{1}{5}}$

$f'(x) = -\frac{1}{5}x^{-\frac{6}{5}}$

(A)

13. If $a > 0$, then $\vee$

If $b^2 - 4ac > 0$, then unequal roots.

(B)

14. $\int_{-2}^{2} (14 - x^2) - (2x^2 + 2)\, dx$

$\int_{-2}^{2} (12 - 3x^2)\, dx$

(C)

15. $f'(1) = 1^2 - 9$

$= -8$

Graph decreasing when $x = 1$

$f'(-3) = (-3)^2 - 9$

$= 0$

Graph stationary when $x = -3$

C

16. Roots when $x=1$ and $x=5$

$y = k(x-1)^2(x-5)$

$\begin{matrix} x & y \\ (0, & 10) \end{matrix}$

$10 = k(0-1)^2(0-5)$

$10 = k \times 1 \times (-5)$

$10 = -5k$

$k = -2$ A

17. $S'(t) = 2t - 5$

$S'(3) = 2 \times 3 - 5$

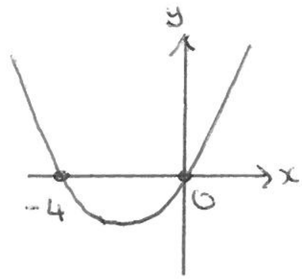
$= 1$ B

18. Sketch graph to see when x^2+4x is above 0.

$x^2+4x=0$

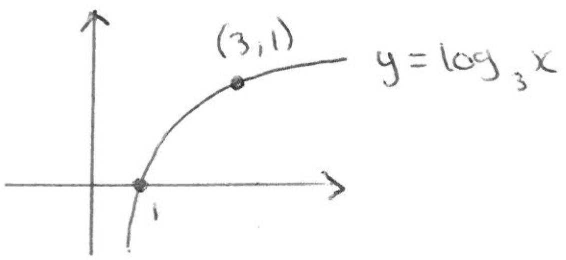
$x(x+4)=0$

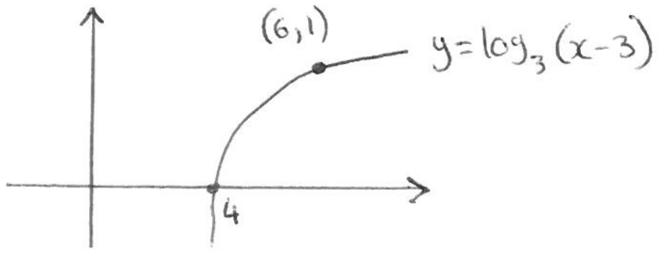
$x=0 \quad x=-4$



Above 0 when $x<-4$, $x>0$ B

19.



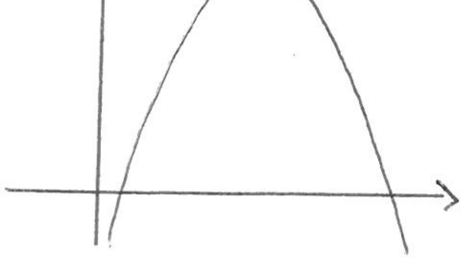


C

20. $y = f(2x) - 3$

Squashed horizontally by a factor of 2

Moved 3 places down.



A

Section B

21. (a) $M_{AC} = \left(\frac{4+18}{2}, \frac{0+20}{2}\right)$

$= (11, 10)$

$m_{BQ} = \frac{10-16}{11-(-4)}$

$= \frac{-6}{15}$

$= -\frac{2}{5}$

$y - 10 = -\frac{2}{5}(x-11)$

$5y - 50 = -2(x-11)$

$5y - 50 = -2x + 22$

$5y + 2x = 72$

(b) When $x = 6$,

$5y + 12 = 72$

$5y = 60$

$y = 12$

Therefore $(6,12)$ lies on the line BQ.

(c) $\vec{BT} = \underset{\sim}{t} - \underset{\sim}{b}$

$= \begin{pmatrix} 6 \\ 12 \end{pmatrix} - \begin{pmatrix} -4 \\ 16 \end{pmatrix}$

$= \begin{pmatrix} 10 \\ -4 \end{pmatrix}$

$\vec{TQ} = \underset{\sim}{q} - \underset{\sim}{t}$

$= \begin{pmatrix} 11 \\ 10 \end{pmatrix} - \begin{pmatrix} 6 \\ 12 \end{pmatrix}$

$= \begin{pmatrix} 5 \\ -2 \end{pmatrix}$

$\vec{BT} = 2\vec{TQ}$, T divides BQ in the ratio $2:1$

22. (a) (i)

$$\begin{array}{r|rrrr} 1 & 2 & 1 & -8 & 5 \\ & & 2 & 3 & -5 \\ \hline & 2 & 3 & -5 & \boxed{0} \end{array}$$

Since the remainder $= 0$, $(x-1)$ is a factor of $f(x)$.

(ii) $(x-1)(2x^2+3x-5) = 0$

$(x-1)(2x+5)(x-1) = 0$

(b) $x = 1 \quad x = -\frac{5}{2} \quad x = 1$

(c) $2x^3 + x^2 - 6x + 2 = 2x - 3$

$2x^3 + x^2 - 8x + 5 = 0$

$(x-1)(2x+5)(x-1) = 0$

Tangent occurs where we have equal roots,

$x - 1 = 0$ $\quad$ $y = 2\times 1 - 3$

$x = 1$ $\quad$ $= -1$ $\quad$ $G(1, -1)$

(d) $2x + 5 = 0$ $\quad$ $y = 2\times(-\frac{5}{2}) - 3$

$2x = -5$ $\quad$ $= -\frac{10}{2} - 3$

$x = -\frac{5}{2}$ $\quad$ $H(-\frac{5}{2}, -8)$

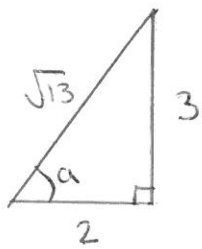
$= -8$

23. (a)(i) $3x - 2y = 0$ $\quad$ $\tan\theta = m$

$-2y = -3x$ $\quad$ $\tan a = \frac{3}{2}$

$y = \frac{3}{2}x$

(ii)



$x^2 = 3^2 + 2^2$

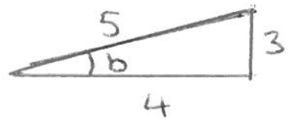
$= 9 + 4$

$x = \sqrt{13}$ $\quad$ $\sin a = \frac{3}{\sqrt{13}}$

(b) $3x - 4y = 0$ $\quad$ $\tan\theta = m$

$-4y = -3x$ $\quad$ $\tan b = \frac{3}{4}$

$y = \frac{3}{4}x$



$\sin b = \frac{3}{5}$

$\cos b = \frac{4}{5}$

(c) (i)

$$\sin(a-b) = \sin a\cos b - \cos a\sin b$$

$$= \left(\frac{3}{\sqrt{13}} \times \frac{4}{5}\right) - \left(\frac{2}{\sqrt{13}} \times \frac{3}{5}\right)$$

$$= \frac{12}{5\sqrt{13}} - \frac{6}{5\sqrt{13}}$$

$$= \frac{6}{5\sqrt{13}}$$

(ii) $$\sin(b-a) = \sin b\cos a - \cos b\sin a$$

$$= \left(\frac{3}{5} \times \frac{2}{\sqrt{13}}\right) - \left(\frac{4}{5} \times \frac{3}{\sqrt{13}}\right)$$

$$= \frac{6}{5\sqrt{13}} - \frac{12}{5\sqrt{13}}$$

$$= -\frac{6}{5\sqrt{13}}$$

2010 Paper 2

1\. (a) $M(0,1,0)$ $\quad N(4,2,2)$

(b) $\vec{VM} = \underset{\sim}{m} - \underset{\sim}{v}$ $\qquad \vec{VN} = \underset{\sim}{n} - \underset{\sim}{v}$

$= \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}$ $\qquad = \begin{pmatrix} 4 \\ 2 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 0 \\ -1 \\ -3 \end{pmatrix}$ $\qquad = \begin{pmatrix} 4 \\ 0 \\ -1 \end{pmatrix}$

(c) $\cos\theta = \dfrac{\vec{VM} \cdot \vec{VN}}{|\vec{VM}|\,|\vec{VN}|}$

$|\vec{VM}| = \sqrt{0^2 + (-1)^2 + (-3)^2}$

$= \sqrt{10}$

$= \dfrac{(0 \times 4) + (-1 \times 0) + (-3 \times (-1))}{\sqrt{10} \times \sqrt{17}}$

$|\vec{VN}| = \sqrt{4^2 + 0^2 + (-1)^2}$

$= \sqrt{17}$

$= \dfrac{3}{\sqrt{10}\sqrt{17}}$

$= 0.23\ldots$

$\cos^{-1}\theta = 76.7^\circ$

2\. (a) $12\cos x - 5\sin x = k\cos(x+a)$

$= k(\cos x \cos a - \sin x \sin a)$

$= k\cos a \cos x - k\sin a \sin x$

$k\cos a = 12$

$k\sin a = 5$

$k = \sqrt{12^2 + 5^2}$

$= \sqrt{169}$

$= 13$

S | A
T | C

$\tan a = \frac{5}{12}$

$a = 22.6^\circ$

2. (b) (i) $12\cos x - 5\sin x = 13\cos(x + 22.6)^\circ$

max value $= 13$ min value $= -13$

(ii) max when $x = 0 - 22.6 = \cancel{-22.6^\circ}$

$x = 360 - 22.6 = 337.4^\circ$

min when $x = 180 - 22.6 = 157.4^\circ$

3. (a) (i) $x^2 + (3-x)^2 + 14x + 4(3-x) - 19 = 0$

$x^2 + 9 - 6x + x^2 + 14x + 12 - 4x - 19 = 0$

$2x^2 + 4x + 2 = 0$

$2(x^2 + 2x + 1) = 0$

$2(x+1)(x+1) = 0$

Equal roots so line is a tangent.

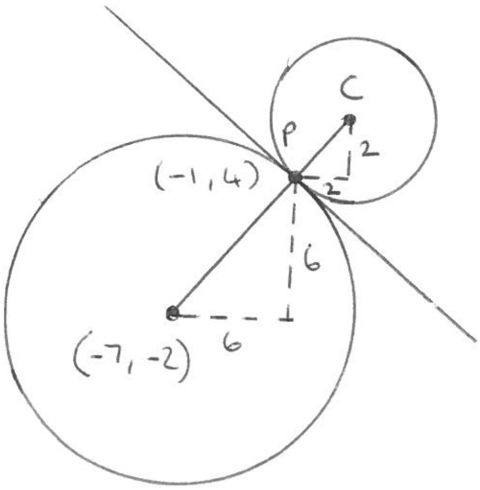
(ii) $x + 1 = 0$

$x = -1$ $y = 3 - (-1)$

$= 4$

$P(-1, 4)$

3. (b)



$C(1, 6)$

$P(-1, 4)$

$r = \sqrt{2^2 + 2^2}$

$= \sqrt{8}$

Small circle: $(x-1)^2 + (y-6)^2 = 8$

4. $2\cos 2x - 5\cos x - 4 = 0$

$2(2\cos^2 x - 1) - 5\cos x - 4 = 0$

$4\cos^2 x - 2 - 5\cos x - 4 = 0$

$4\cos^2 x - 5\cos x - 6 = 0$

$(4\cos x + 3)(\cos x - 2) = 0$

$4\cos x + 3 = 0$ $\qquad$ $\cos x - 2 = 0$

$\cos x = -\frac{3}{4}$ $\qquad$ $\cos x = 2$

No solutions

$\begin{array}{c|c} S & A \checkmark \\ \hline T & C \checkmark \end{array}$

$360 - x$

$x = 138.6°, 221.4°$

$= 2.42, 3.86$

5. (a) (i) Find T

when $x=0$, $y = \frac{2}{5}(10 - 0^2)$

$= \frac{2}{5} \times (10)$

$= 4$ $T(0,4)$

$PQ = 10 - x^2 - 4$

$= 6 - x^2$

(ii) $A(x) = 2x(6 - x^2)$

$= 12x - 2x^3$

(b) For S.P's $A'(x) = 0$

$12 - 6x^2 = 0$

$6(2 - x^2) = 0$

$2 - x^2 = 0$

$-x^2 = -2$

$x^2 = 2$

$x = \pm\sqrt{2}$

x	-2	$-\sqrt{2}$	0	$\sqrt{2}$	2
$A'(x)$	$-$	0	$+$	0	$-$
slope	\	_	/	—	\

$A(\sqrt{2}) = 12(\sqrt{2}) - 2(\sqrt{2})^3$

$= 12\sqrt{2} - 4\sqrt{2}$

$= 8\sqrt{2}$

max area of $8\sqrt{2}$ when $x = \sqrt{2}$.

6. (a) $y = (2\times 9 - 9)^{\frac{1}{2}}$

$= 9^{\frac{1}{2}}$

$= \sqrt{9}$

$= 3$

$(9, 3)$

$\frac{dy}{dx} = \frac{1}{2}(2x-9)^{-\frac{1}{2}} \times 2$

$= (2x-9)^{-\frac{1}{2}}$

$= \dfrac{1}{(2x-9)^{\frac{1}{2}}}$

$= \dfrac{1}{\sqrt{(2x-9)}}$

$m = \dfrac{1}{\sqrt{2\times 9 - 9}}$

$= \dfrac{1}{\sqrt{9}}$

$= \frac{1}{3}$

$y - 3 = \frac{1}{3}(x-9)$

$y - 3 = \frac{1}{3}x - 3$

$y = \frac{1}{3}x$

(b) when $y = 0$

$(2x-9)^{\frac{1}{2}} = 0$

$2x - 9 = 0$

$2x = 9$

$x = \frac{9}{2}$

$A\left(\frac{9}{2}, 0\right)$

(c) Area (Triangle) $= \frac{1}{2} \times 9 \times 3$

$= 13\frac{1}{2}$

Area (curve) $= \int_{\frac{9}{2}}^{9} (2x-9)^{\frac{1}{2}}$

$= \left[\dfrac{(2x-9)^{\frac{3}{2}}}{\frac{3}{2}} \times \dfrac{1}{2} \right]_{\frac{9}{2}}^{9}$

$= \left[\dfrac{\left(\sqrt{(2x-9)}\right)^3}{3} \right]_{\frac{9}{2}}^{9}$

$= \left(\dfrac{(\sqrt{9})^3}{3} \right) - \left(\dfrac{(\sqrt{0})^3}{3} \right)$

$= 9$

Shaded Area $= 13\frac{1}{2} - 9 = 4\frac{1}{2}$

7. (a) $\log_4 x = p$

$x = 4^p$

$\log_{16} x$

$= \log_{16} 4^p$

$= p \times \log_{16} 4$

$= p \times \frac{1}{2}$

$= \frac{1}{2} p$

(b) $\log_3 x + \log_9 x = 12$

$p + \frac{1}{2} p = 12$

$\frac{3}{2} p = 12$

$3p = 24$

$p = 8$

$\log_3 x = p$

$\log_3 x = 8$

$x = 3^8$

$x = 6561$