Higher 2009 Paper 1

1. $U_2 = 3 \times 2 + 4$

$= 10$

$U_3 = 3 \times 10 + 4$

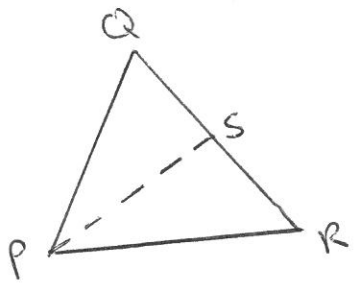
$= 34$ **A**

2. $r = \sqrt{4^2 + 3^2 + 75}$

$= \sqrt{100}$

$= 10$ **B**

3.



$M_{QR} = (1, 5)$

$m_{PS} = \frac{5 - (-2)}{1 - (-3)}$

$= \frac{7}{4}$

D

4. $\frac{dy}{dx} = 15x^2 - 12$

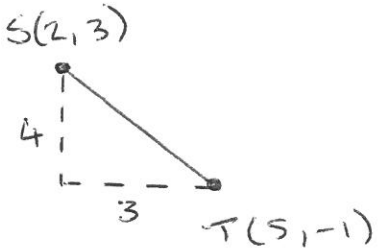
$m = 15(1)^2 - 12$

$= 15 - 12$

$= 3$

C

5.



$ST = \sqrt{4^2 + 3^2}$

$= \sqrt{25}$

$= 5$

$m = \frac{-1 - 3}{5 - 2}$

$= -\frac{4}{3}$ **B**

6. $L = 0.7L + 10$

$0.3L = 10$

$3L = 100$

$L = \frac{100}{3}$

A

7. $\cos 2x$

$= 2\cos^2 x - 1$

$= 2\left(\frac{1}{\sqrt{5}}\right)^2 - 1$

$= 2\left(\frac{1}{5}\right) - 1$

$= \frac{2}{5} - 1$

$= -\frac{3}{5}$ A

8. $f(x) = \frac{1}{4x^3}$

$= \frac{1}{4}x^{-3}$

$f'(x) = -\frac{3}{4}x^{-4}$

$= -\frac{3}{4x^4}$ D

9. $x^2 + (2x)^2 = 5$

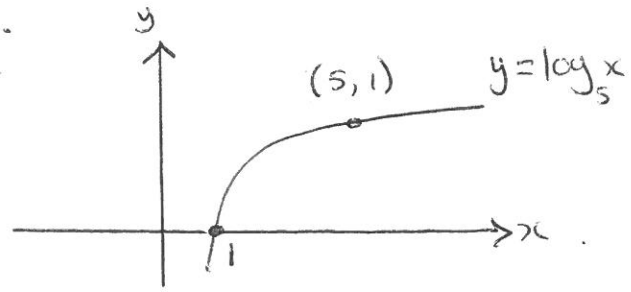
$x^2 + 4x^2 = 5$

$5x^2 = 5$

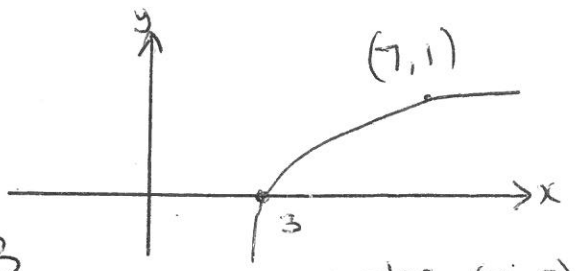
$x^2 = 1$

$x = \pm 1$ A

10.



B



11. $(4\sin x - \sqrt{5})(\sin x + 1) = 0$

$4\sin x - \sqrt{5} = 0$

$\sin x = \frac{\sqrt{5}}{4}$

$x = *, *$

2 solutions

$\sin x + 1 = 0$

$\sin x = -1$

$x = 180$

1 solution

Total = 3 solutions B

12. $b^2 - 4ac$

$= (-1)^2 - 4 \times 2 \times (-9)$

$= 1 + 72$

$= 73$

C

13. $k = \sqrt{1^2 + (\sqrt{3})^2}$

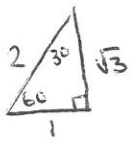
$= \sqrt{1+3}$

$= \sqrt{4}$

$= 2$

$\tan a = \frac{1}{\sqrt{3}}$

$a = 30^\circ$



B

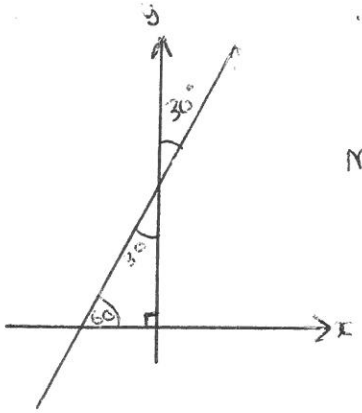
14. $f(x) = 2\sin(3x - \frac{\pi}{2}) + 5$

- Amplitude of 2
- 3 waves
- moved $\frac{\pi}{2}$ to the right
- moved up 5

max value = 7 min = -3

C

15.



$m = \tan 60^\circ$

$= \frac{\sqrt{3}}{1}$

A

16. $\int_0^1 4x^3 - 9x^2 \, dx$

$= -\left[x^4 - 3x^3\right]_0^1$

B

17. $|A| = \sqrt{\left(-\frac{3}{5}\right)^2 + 0^2 + \left(\frac{4}{5}\right)^2}$

$= \sqrt{\frac{9}{25} + 0 + \frac{16}{25}}$

$= \sqrt{\frac{25}{25}}$

$= \sqrt{1}$

$= 1$ ← unit vector

$\begin{pmatrix} -\frac{3}{5} \\ 0 \\ \frac{4}{5} \end{pmatrix} = \frac{1}{5}\begin{pmatrix} -3 \\ 0 \\ 4 \end{pmatrix}$

Parallel

A

18. $f(x) = (4 - 3x^2)^{-\frac{1}{2}}$

$f'(x) = -\frac{1}{2}(4 - 3x^2)^{-\frac{3}{2}} \times (-6x)$

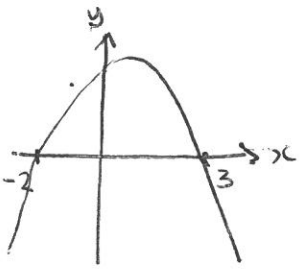
$= 3x(4 - 3x^2)^{-\frac{3}{2}}$

D

19. $6 + x - x^2 < 0$

$(3 - x)(2 + x)$

$x = 3 \qquad x = -2$



Less than zero when,

$x < -2$, $x > 3$

C

20. $A(r) = 2\pi r^2 + 6\pi r$

$A'(r) = 4\pi r + 6\pi$

$A'(2) = 8\pi + 6\pi$

$= 14\pi$

C

Section B

21. (a) when $y = 0$,

$6x - 7(0) + 18 = 0$

$6x + 18 = 0$

$6x = -18$

$x = -3$

$P(-3, 0)$

(b) $m_{QR} = \frac{-2 - 6}{8 - 4} = \frac{-8}{4}$

$= -2$

$m_{PT} = \frac{1}{2}$

$y - 0 = \frac{1}{2}(x + 3)$

$2y = x + 3$

(c) $y - 6 = -2(x - 4)$

$y - 6 = -2x + 8$

$y + 2x = 14$

$2y - x = 3$

$y + 2x = 14$

$4y - 2x = 6$

$y + 2x = 14$

$5y = 20$

$y = 4$

$4 + 2x = 14$

$2x = 10$

$x = 5$

$T(5, 4)$

22. (a) (i) $\overrightarrow{DE} = \underset{\sim}{e} - \underset{\sim}{d}$ $\overrightarrow{EF} = \underset{\sim}{f} - \underset{\sim}{e}$

$= \begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix} - \begin{pmatrix} 10 \\ -8 \\ -15 \end{pmatrix}$ $= \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix}$

$= \begin{pmatrix} -9 \\ 6 \\ 12 \end{pmatrix}$ $= \begin{pmatrix} -3 \\ 2 \\ 4 \end{pmatrix}$.

Since $\overrightarrow{DE} = 3\overrightarrow{EF}$, DE and EF are parallel and since E is common to both, the points D, E and F are collinear.

(ii) 3 : 1

(b) $\overrightarrow{DE} \cdot \overrightarrow{GE} = 0$ if perpindiculos.

$(-9 \times (1-k)) + (6 \times (-3)) + (12 \times (-3)) = 0$

$-9 + 9k - 18 - 36 = 0$

$9k - 63 = 0$

$9k = 63$

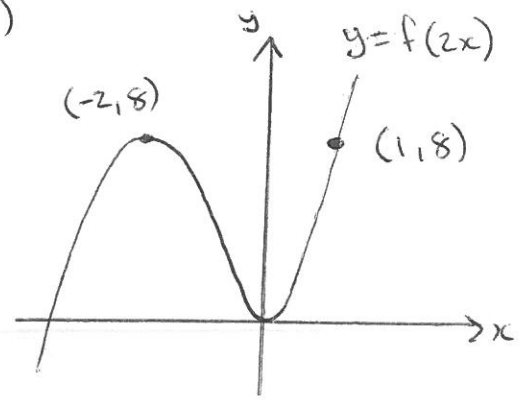
$k = 7$

$\overrightarrow{GE} = \underset{\sim}{e} - \underset{\sim}{g}$

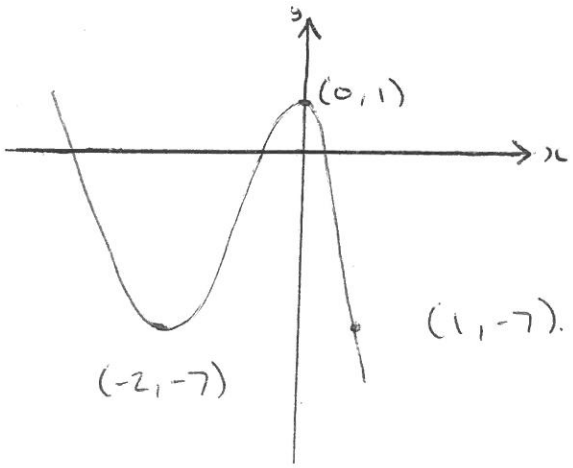
$= \begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix} - \begin{pmatrix} k \\ 1 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} 1-k \\ -3 \\ -3 \end{pmatrix}$

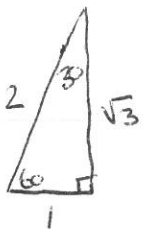
23. (a)

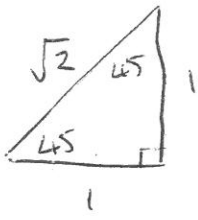


(b) $y = 1 - f(2x) \rightarrow y = -f(2x) + 1$



24. (a) $\sin\left(\frac{7\pi}{12}\right) = \sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$





$= \sin\frac{\pi}{3}\cos\frac{\pi}{4} + \cos\frac{\pi}{3}\sin\frac{\pi}{4}$

$= \left(\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}\right) + \left(\frac{1}{2} \times \frac{1}{\sqrt{2}}\right)$

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

(b) $\sin(A+B) + \sin(A-B)$

$= \sin A\cos B + \cos A\sin B + \sin A\cos B - \cos A\sin B$

$= 2\sin A\cos B$

(c) (i) $\frac{\pi}{3} - \frac{\pi}{4}$

$= \frac{4\pi}{12} - \frac{3\pi}{12}$

$= \frac{\pi}{12}$ $\qquad \frac{\pi}{12} = \left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

(ii)

$\sin\left(\frac{7\pi}{12}\right) + \sin\left(\frac{\pi}{12}\right)$

$= \sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right) + \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$= 2\sin\frac{\pi}{3}\cos\frac{\pi}{4}$ ← from 24 (b)

$= 2 \times \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}$

$= \frac{2\sqrt{3}}{2\sqrt{2}}$

$= \frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{2}\sqrt{3}}{2} = \frac{\sqrt{6}}{2}$

2009 Paper 2

1. For S.P's $\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = 3x^2 - 6x - 9 = 0$$

$$3(x^2 - 2x - 3) = 0$$

$$3(x+1)(x-3) = 0$$

$x = -1 \qquad x = 3$

x	-2	-1	0	3	4
$\frac{dy}{dx}$	+	0	−	0	+
slope	/	—	\	_	/

Max t.p at $(-1, 17)$ Min t.p. at $(3, -15)$

$y = (-1)^3 - 3(-1)^2 - 9(-1) + 12$

$= -1 - 3 + 9 + 12$

$= 17$

$y = (3)^3 - 3(3)^2 - 9(3) + 12$

$= 27 - 27 - 27 + 12$

$= -15$

2. (a) (i) $P(x) = f(g(x)) = 3(x^2 - 2) + 1 = 3x^2 - 5.$

(ii) $q(x) = g(f(x)) = (3x+1)^2 - 2 = 9x^2 + 6x - 1$

(b)

$$6x = 18x + 6$$

$$-12x = 6$$

$$x = -\frac{1}{2}$$

3. (a) (i)

1	1	8	11	−20
		1	9	20
	1	9	20	0

Since remainder = 0
$x = 1$ is a root

(ii)
$$(x-1)(x^2+9x+20) = 0$$
$$(x-1)(x+4)(x+5) = 0$$

(b) $\log_2(x+3) + \log_2(x^2+5x-4) = 3$

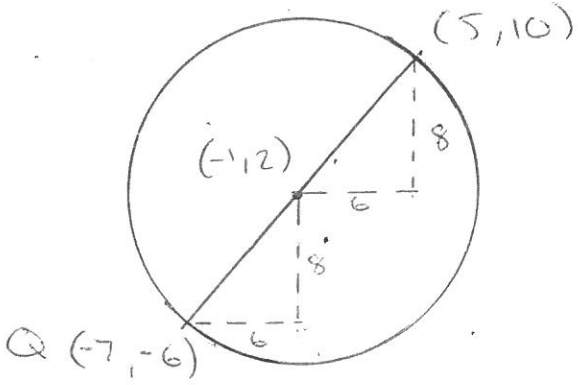
$$\log_2(x+3)(x^2+5x-4) = 3$$
$$(x+3)(x^2+5x-4) = 2^3$$
$$x^3+5x^2-4x+3x^2+15x-12 = 8$$
$$x^3+8x^2-11x-20 = 0$$
$$(x-1)(x+4)(x+5) = 0$$
$$x = 1, \cancel{x=-4}, \cancel{x=-5}$$

4. (a) When $x = 5$ and $y = 10$,

$$(x+1)^2 + (y-2)^2$$
$$= (5+1)^2 + (10-2)^2$$
$$= 6^2 + 8^2$$
$$= 100$$

Since LHS = RHS, P(5, 10) lies on the circle.

(b) Centre $(-1, 2)$



$$m_{OQ} = \frac{2-(-6)}{-1-(-7)}$$

$$= \frac{8}{6}$$

$$= \frac{4}{3}$$

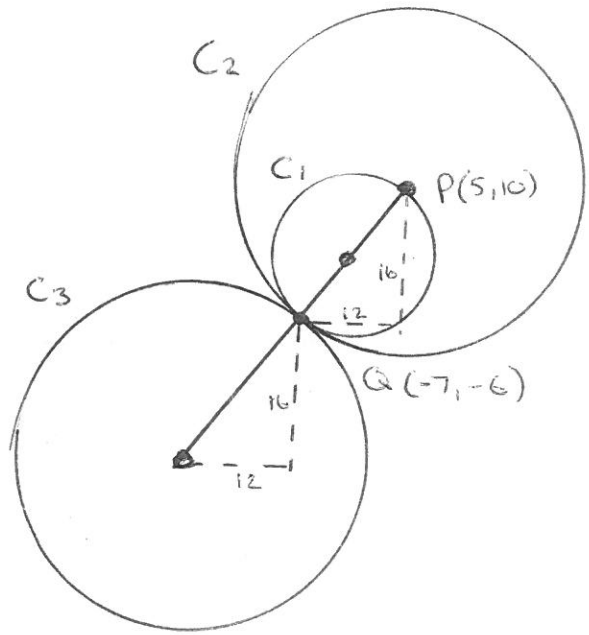
$$m_{Tangent} = -\frac{3}{4}$$

$$y-(-6) = -\frac{3}{4}(x-(-7))$$

$$4y + 24 = -3(x+7)$$

$$4y = -3x - 45$$

(c)



C2

Centre $(5, 10)$

radius $= \sqrt{16^2 + 12^2}$

$= \sqrt{400}$

$= 20$

$$(x-5)^2 + (y-10)^2 = 400$$

C3

Centre $(-19, -22)$

radius $= 20$

$$(x+19)^2 + (y+22)^2 = 400$$

5. (a) $g(x) = m\cos(nx)$

$m=3 \qquad n=2$

(b) $3\cos 2x = -4\cos 2x + 3 \qquad 0 \leq x \leq \pi$

$7\cos 2x = 3$

$\cos 2x = \frac{3}{7}$

S	A ✓
T	C ✓

$2x = 1.13, 5.16$

$x = 0.565, 2.58$

$y = 3\cos(2 \times 0.565)$

$= 1.28$

$(0.6, 1.3)$

$y = 3\cos(2 \times 2.58)$

$= 1.3$

$(2.6, 1.3)$

(c) $\int_{0.6}^{2.6} -4\cos 2x + 3 - (3\cos 2x)\, dx$

$= \int_{0.6}^{2.6} -7\cos 2x + 3 \, .dx$

$= \left[\frac{1}{2} \times -7\sin 2x + 3x \right]_{0.6}^{2.6}$

$= \left[-3.5\sin 2x + 3x \right]_{0.6}^{2.6}$

$= \left(-3.5\sin(2\times 2.6) + (3\times 2.6)\right) - \left(-3.5\sin(2\times 0.6) + (3\times 0.6)\right)$

$= 10.89 - (-1.46)$

$= 12.35$

6. (a) $N = 61e^{(0.016 \times 14)}$

$= 76.3$ million

(b) $10.2 = 5.1e^{0.0043t}$

$2 = e^{0.0043t}$

$\ln 2 = \ln e^{0.0043t}$

$\ln 2 = 0.0043t \times \ln e$ $\quad (\ln e = 1)$

$\ln 2 = 0.0043t$

$t = \frac{\ln 2}{0.0043}$

$= 161.2$ years

7. (a) $\underset{\sim}{p} \cdot (\underset{\sim}{q} + \underset{\sim}{r}) = \underset{\sim}{p} \cdot \underset{\sim}{q} + \underset{\sim}{p} \cdot \underset{\sim}{r}$

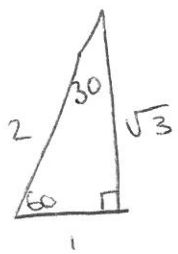
$= |\underset{\sim}{p}||\underset{\sim}{q}|\cos\theta + |\underset{\sim}{p}||\underset{\sim}{r}|\cos\theta$

$= (4 \times 3 \times \cos 30) + (4 \times |\underset{\sim}{r}| \times \cos 90^\circ)$

$= (12 \times \frac{\sqrt{3}}{2}) + (4|\underset{\sim}{r}| \times 0)$

$= \frac{12\sqrt{3}}{2} + 0$

$= 6\sqrt{3}$



$$\underset{\sim}{r} \cdot (\underset{\sim}{p} - \underset{\sim}{q}) = \underset{\sim}{r} \cdot \underset{\sim}{p} - \underset{\sim}{r} \cdot \underset{\sim}{q}$$

$$= |\underset{\sim}{r}||\underset{\sim}{p}|\cos\theta - |\underset{\sim}{r}||\underset{\sim}{q}|\cos\theta$$

$$= (|\underset{\sim}{r}| \times 4 \times \cos 90^\circ) - (|\underset{\sim}{r}| \times 3 \times \cos 120^\circ)$$

$$= 0 - (|\underset{\sim}{r}| \times 3 \times (-\cos 60))$$

$$= -(3|\underset{\sim}{r}| \times (-\tfrac{1}{2}))$$

$$= \tfrac{3}{2}|\underset{\sim}{r}|$$