Higher 2008 Paper 1

1. $U_{11} = 0.3 \times 10 + 6$

$= 9$

$U_{12} = 0.3 \times 9 + 6$

$= 8.7$ C

2. radius $= 6$

$(x+7)^2 + (y-6)^2 = 36$

D

3. $\underset{\sim}{u} \cdot \underset{\sim}{v} = 0$

$(k \times 0) + (-1 \times 4) + (1 \times k) = 0$

$-4 + k = 0$

$k = 4$

C

4. $L = 0.4L - 240$

$0.6L = -240$

$6L = -2400$

$L = -400$

B

5. $m_{radius} = \frac{9-5}{7-2}$

$= \frac{4}{5}$

$m_{tangent} = -\frac{5}{4}$

$y - 9 = -\frac{5}{4}(x-7)$

A

6. $2\sin x - \sqrt{3} = 0$

$\sin x = \frac{\sqrt{3}}{2}$

$x = \cancel{60}, 120^\circ$

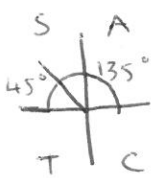
$x = \cancel{\frac{\pi}{3}}, \frac{2\pi}{3}$

B

7. $m = \tan 135^\circ$

$= -\tan 45^\circ$

$= -1$

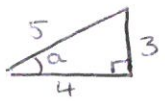


C

8. D

- Reflect over x
- Move 2 to the right.

9. $\sin(x+a)$



$= \sin x\cos a + \cos x \sin a$

$= \frac{4}{5}\sin x + \frac{3}{5}\cos x$

B

10. $b^2 - 4ac$

$= 1^2 - 4 \times 1 \times 1$

$= 1 - 4$

$= -3$ A

No real roots.

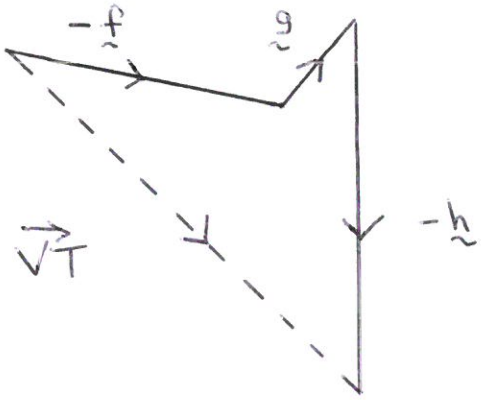
11. $\vec{EP} = \underset{\sim}{p} - \underset{\sim}{e}$ $\vec{PF} = \underset{\sim}{f} - \underset{\sim}{p}$

$= \begin{pmatrix}1\\5\\7\end{pmatrix} - \begin{pmatrix}-2\\-1\\4\end{pmatrix}$ $= \begin{pmatrix}7\\17\\13\end{pmatrix} - \begin{pmatrix}1\\5\\7\end{pmatrix}$

$= \begin{pmatrix}3\\6\\3\end{pmatrix}$ $= \begin{pmatrix}6\\12\\6\end{pmatrix}$

B 1 : 2

12.



C

13. $y = k(x-1)(x-4)$

$(0,12)$ $12 = k(0-1)(0-4)$

$12 = 4k$

$k = 3$ A

14. $\int 4\sin(2x+3)\,dx$

$= -\frac{1}{2} \times 4\cos(2x+3) + C$

$= -2\cos(2x+3) + C$ B

15. $f(x) = (x^3+4)^2$

$f'(x) = 2(x^3+4) \times 3x^2$

$= 6x^2(x^3+4)$

C

16. $2[x^2+2x]+7$

$= 2[(x+1)^2 - 1] + 7$

$= 2(x+1)^2 - 2 + 7$

$= 2(x+1)^2 + 5$ A

17. $f(x) = \sqrt{9 - x^2}$

For $9 - x^2 \geq 0$,

$-3 \leq x \leq 3$ C

18. $\underset{\sim}{q} \cdot (\underset{\sim}{p} + \underset{\sim}{q})$

$= \underset{\sim}{q} \cdot \underset{\sim}{p} + \underset{\sim}{q} \cdot \underset{\sim}{q}$

$= 10 + (|\underset{\sim}{q}||\underset{\sim}{q}|\cos\theta)$

$= 10 + (4 \times 4 \times \cos 0^\circ)$

$= 10 + (16 \times 1)$

$= 26$ C

19. $y = 2m^x$

$54 = 2m^3$

$m^3 = 27$

$m = 3$ B

20. $(q, 2)$
$x \quad y$

$2 = \log_3(q - 4)$

$\log_3(q - 4) = 2$

$q - 4 = 3^2$ D

$q - 4 = 9$

$q = 13, \quad q = 13$

Section B

21. (a) For S.P's $f'(x) = 0$

$f'(x) = 3x^2 - 3 = 0$

$3(x^2 - 1) = 0$

$3(x + 1)(x - 1) = 0$

$x = -1 \quad x = 1$

x	-2	-1	0	1	2
$f'(x)$	+	0	−	0	+
Slope	/	—	\	—	/

max t.p. at $(-1, 4)$ min t.p. at $(1, 0)$

$y = (-1)^3 - 3(-1) + 2$

$= 4$

$y = (1)^3 - 3(1) + 2$

$= 0$

(b) (i)

1	1	0	−3	2
		1	1	−2
	1	1	−2	0

Since the remainder = 0, $(x-1)$ is a factor.

(ii)

$$(x-1)(x^2+x-2)=0$$

$$(x-1)(x-1)(x+2)=0$$

(c) Crosses x-axis when $y=0$,

$$x^3-3x+2=0$$

$$(x-1)(x-1)(x+2)=0$$

$x=1$ $x=1$ $x=-2$

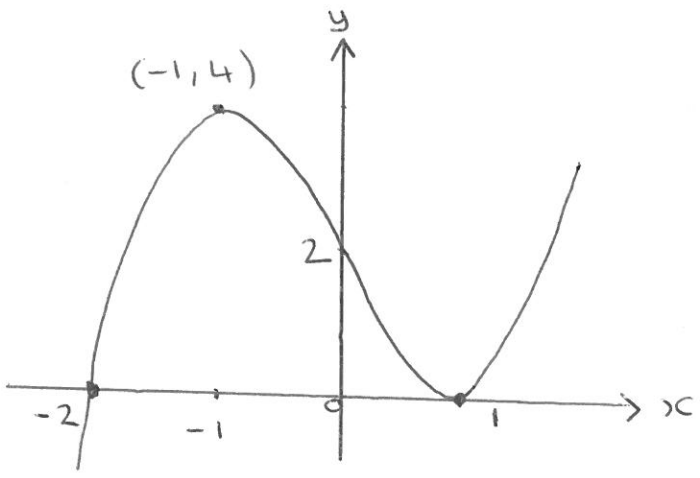
$(1,0)$ $(-2,0)$

Crosses y-axis when $x=0$,

$$y=0^3-3(0)+2$$

$$=2$$

$(0,2)$



22. (a) $\frac{dy}{dx} = 3x^2 - 12x + 8 = -1$

$3x^2 - 12x + 9 = 0$

$3(x^2 - 4x + 3) = 0$

$3(x - 1)(x - 3) = 0$

$x = 1 \qquad x = 3$

$y = 1^3 - 6(1)^2 + 8(1)$

$= 3$

$(1, 3)$

$y = 3^3 - 6(3)^2 + 8(3)$

$= -3$

$(3, -3)$

(b) $x^3 - 6x^2 + 8x = 4 - x$

$x^3 - 6x^2 + 9x - 4 = 0$

$(x - 1)(x^2 - 5x + 4) = 0$

$(x - 1)(x - 1)(x - 4) = 0$

↓

Equal roots when $x - 1 = 0$

Tangent occurs when $x = 1$

1	1	-6	9	-4
		1	-5	4
	1	-5	4	0

$y = 4 - 1$

$= 3 \qquad A(1, 3)$

23. (a) $h(f(x)) = \log_2(x^2 - x + 10)$

$h(g(x)) = \log_2(5 - x)$

(b) $h(f(x)) - h(g(x)) = 3$

$\log_2(x^2 - x + 10) - \log_2(5 - x) = 3$

$\log_2\left(\frac{x^2 - x + 10}{5 - x}\right) = 3$

$\frac{x^2 - x + 10}{5 - x} = 2^3$

$x^2 - x + 10 = 8(5 - x)$

$x^2 - x + 10 = 40 - 8x$

$x^2 + 7x - 30 = 0$

$(x - 3)(x + 10) = 0$

$x = 3 \qquad x = -10$

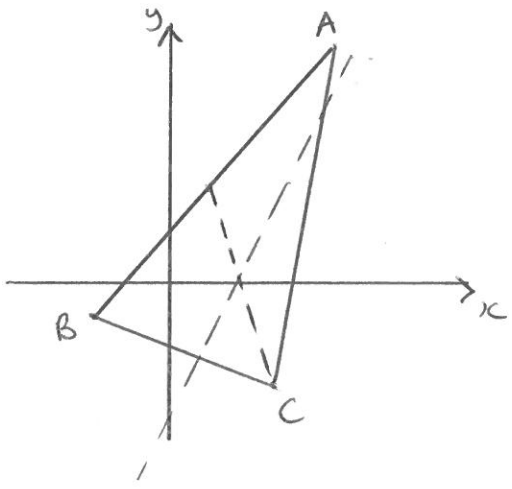
Paper 2

1. (a) $m_{BC} = \frac{-5 - (-1)}{5 - (-3)} = \frac{-4}{8} = -\frac{1}{2}$

$m_{\perp} = 2$

$M_{BC} = \left(\frac{-3 + 5}{2}, \frac{-1 + (-5)}{2}\right)$

$= (1, -3)$



$y - (-3) = 2(x - 1)$

$y + 3 = 2x - 2$

$y = 2x - 5$

(b) $M_{AB} = \left(\frac{7+(-3)}{2}, \frac{9+(-1)}{2}\right) = (2, 4)$

$m_{CM} = \frac{4-(-5)}{2-5} = \frac{9}{-3} = -3$

$y - 4 = -3(x-2)$

$y - 4 = -3x + 6$

$y = -3x + 10$

(c) $y - 2x = -5$

$y + 3x = 10$

$-5x = -15$

$x = 3$

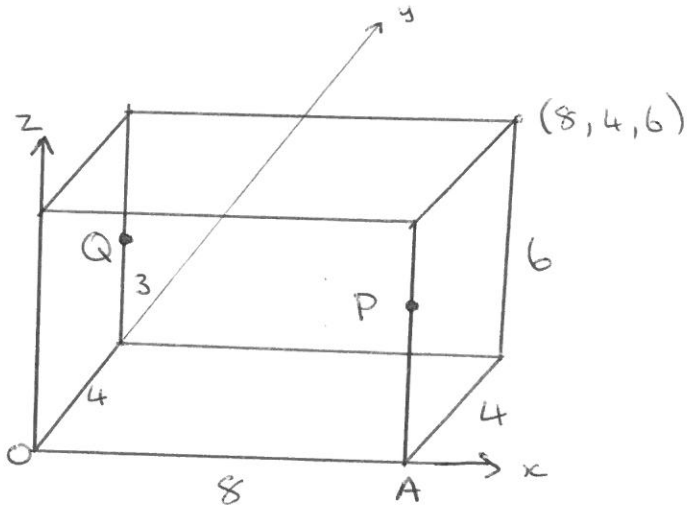
$y - 2(3) = -5$

$y - 6 = -5$

$y = 1 \qquad (3, 1)$

2. (a) $P(8, 0, 4)$

$Q(0, 4, 3)$



(b) $A(8, 0, 0)$

$\vec{PQ} = \underset{\sim}{q} - \underset{\sim}{p}$

$= \begin{pmatrix} 0 \\ 4 \\ 3 \end{pmatrix} - \begin{pmatrix} 8 \\ 0 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} -8 \\ 4 \\ -1 \end{pmatrix}$

$|\vec{PQ}| = \sqrt{(-8)^2 + 4^2 + (-1)^2}$

$= \sqrt{81} = 9$

$\vec{PA} = \underset{\sim}{a} - \underset{\sim}{p}$

$= \begin{pmatrix} 8 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 8 \\ 0 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} 0 \\ 0 \\ -4 \end{pmatrix}$

$|\vec{PA}| = \sqrt{0^2 + 0^2 + (-4)^2}$

$= \sqrt{16} = 4$

(c) $\cos\theta = \frac{\vec{PQ} \cdot \vec{PA}}{|\vec{PQ}|\,|\vec{PA}|}$

$= \frac{(-8 \times 0) + (4 \times 0) + (-1 \times (-4))}{9 \times 4}$

$= \frac{4}{36}$

$\theta = 83.6^\circ$

3. (a) (i) $f(x) = \sqrt{7}\cos x$ (ii) $g(x) = -3\sin x$

$p = \sqrt{7}$ $q = -3$

(b) $\sqrt{7}\cos x - 3\sin x = k\cos(x+a)$

$= k(\cos x \cos a - \sin x \sin a)$

$= k\cos a \cos x - k\sin a \sin x$

$k\cos a = \sqrt{7}$

$k\sin a = 3$

$k = \sqrt{(\sqrt{7})^2 + 3^2}$

$= \sqrt{16}$

$= 4$

S | A
T | C

$\tan a = \frac{3}{\sqrt{7}}$

$a = 0.85$

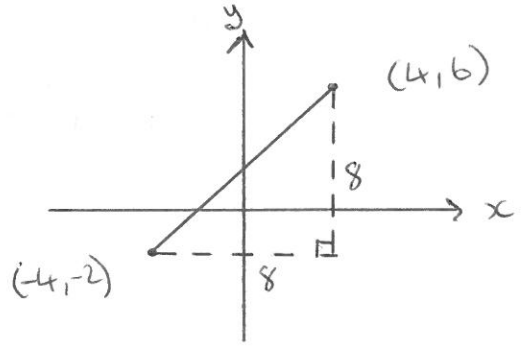
$4\cos(x + 0.85)$

(c) $f'(x) + g'(x) = -4\sin(x + 0.85)$

4. (a) Centre $(-4, -2)$ $r = \sqrt{4^2 + 2^2 - (-38)}$

$= \sqrt{58}$

(b) Centre $(4, 6)$



Distance $= \sqrt{8^2 + 8^2}$

$= \sqrt{128}$

$r = \sqrt{26}$

$\sqrt{26} + \sqrt{58} = 12.7$ $\sqrt{128} = 11.3$

Since the distance between the centres is less than the two radii combined, the circles must intersect.

(c) $y = 4 - x$

$$x^2 + (4-x)^2 + 8x + 4(4-x) - 38 = 0$$

$$x^2 + 16 - 8x + x^2 + 8x + 16 - 4x - 38 = 0$$

$$2x^2 - 4x - 6 = 0$$

$$2(x^2 - 2x - 3) = 0$$

$$2(x+1)(x-3) = 0$$

$x = -1$ $\qquad$ $x = 3$

$y = 4 - (-1)$ $\qquad$ $y = 4 - 3$

$= 5$ $\qquad$ $= 1$

$(-1, 5)$ $\qquad$ $(3, 1)$

5. $\cos 2x + 2\sin x = \sin^2 x$

$$1 - 2\sin^2 x + 2\sin x - \sin^2 x = 0$$

$$-3\sin^2 x + 2\sin x + 1 = 0$$

$$3\sin^2 x - 2\sin x - 1 = 0$$

$$(3\sin x + 1)(\sin x - 1) = 0$$

$3\sin x = -1$ $\qquad$ $\sin x = 1$

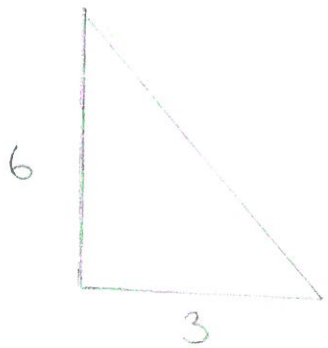
$\sin x = -\frac{1}{3}$ $\qquad$ $x = 90^\circ$

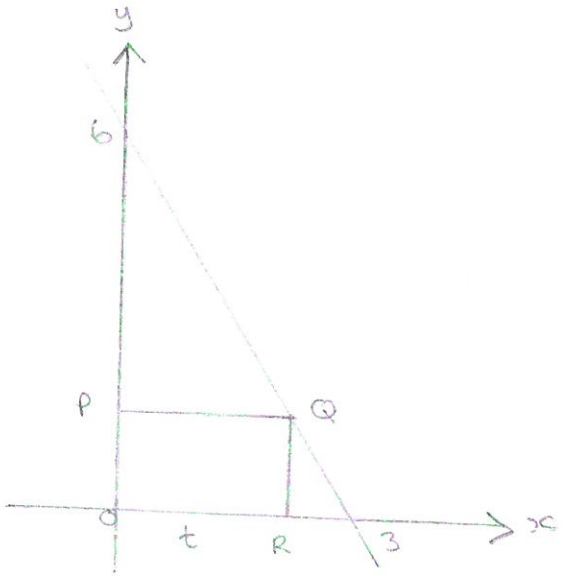
S	A
T	C

$x = -19.5^\circ$

$x = 199.5^\circ, 340.5^\circ$

6. (a) (0, 6) (3, 0)





$$m = \frac{\text{vertical}}{\text{Horizontal}} = \frac{6}{3}$$

$$m = \frac{QR}{3-t}$$

$$\frac{QR}{3-t} = \frac{6}{3}$$

$$3QR = 18 - 6t$$

$$QR = 6 - 2t$$

(b) Area = $l \times b$

$= t(6-2t)$

$= 6t - 2t^2$

For S.P's $A'(t) = 0$

$A'(t) = 6 - 4t = 0$

$4t = 6$

$t = \frac{6}{4}$

$t = \frac{3}{2}$

t	0	$\frac{3}{2}$	2
$A'(t)$	+	0	−
Slope	/	—	\

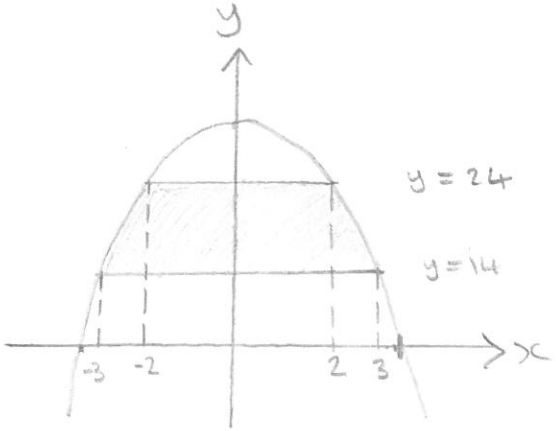
$y = -2x + 6$

$= -2 \times (\frac{3}{2}) + 6$

$= -\frac{6}{2} + 6$

$= 3$ $\quad Q(\frac{3}{2}, 3)$

7.



$32 - 2x^2 = 24$

$-2x^2 = -8$

$x^2 = 4$

$x = \pm 2$

$32 - 2x^2 = 14$

$-2x^2 = -18$

$x^2 = 9$

$x = \pm 3$

$$\int_2^3 32 - 2x^2$$

$$= \left[32x - \frac{2x^3}{3}\right]_2^3$$

$$= \left(96 - \frac{54}{3}\right) - \left(64 - \frac{16}{3}\right)$$

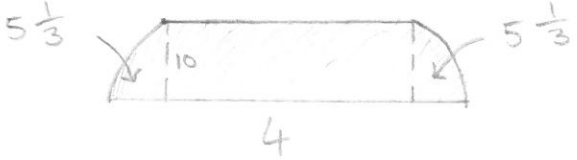
$$= (96 - 18) - \left(64 - 5\tfrac{1}{3}\right)$$

$$= 78 - 58\tfrac{2}{3}$$

$$= 19\tfrac{1}{3}$$

Shaded Area $= 19\frac{1}{3} - (14 \times 1)$

$= 5\frac{1}{3}$



$A = 4 \times 10 = 40$

Total Area $= 40 + 5\frac{1}{3} + 5\frac{1}{3}$

$= 50\frac{2}{3}$