Higher 2007 Paper 1

1. $3x - y + 2 = 0$

$-y = -3x - 2$

$y = 3x + 2$

$m = 3 \qquad (-1, 4)$

$y - 4 = 3(x + 1)$

$y = 3x + 7$

2. $\vec{BC} = 2\vec{AB}$

$\underset{\sim}{c} - \underset{\sim}{b} = 2(\underset{\sim}{b} - \underset{\sim}{a})$

$\underset{\sim}{c} - \underset{\sim}{b} = 2\underset{\sim}{b} - 2\underset{\sim}{a}$

$\underset{\sim}{c} = 3\underset{\sim}{b} - 2\underset{\sim}{a}$

$= 3\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} - 2\begin{pmatrix} -2 \\ 1 \\ -1 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ 9 \\ 6 \end{pmatrix} - \begin{pmatrix} -4 \\ 2 \\ -2 \end{pmatrix}$

$= \begin{pmatrix} 7 \\ 7 \\ 8 \end{pmatrix}$

$C(7, 7, 8)$

3. (a) $g(f(x)) = 1 - 2(x^2 + 1)$

$= -2x^2 - 1$

(b) $g(g(x)) = 1 - 2(1 - 2x)$

$= 4x - 1$

4. $b^2 - 4ac < 0$ if there are no real roots

$$(-1)^2 - 4 \times k \times (-1) < 0$$

$$1 + 4k < 0$$

$$4k < -1$$

$$k < -\frac{1}{4}$$

5. Large Circle

centre $(7, 8)$ radius $= \sqrt{(-7)^2 + (-8)^2 - 77}$

$= \sqrt{36}$

$= 6$

Small Circles

B$(7, 8)$ radius $= 2$

C$(11, 8)$

D$(15, 8)$

Equation of D

$$(x - 15)^2 + (y - 8)^2 = 4$$

6. $\sin 2x = 6\cos x$

$2\sin x \cos x - 6\cos x = 0$

$2\cos x(\sin x - 3) = 0$

$2\cos x = 0$

$\cos x = 0$

$x = 90°, 270°$

$\sin x - 3 = 0$

$\sin x = 3$

No solution

(sin only ranges from 1 to -1)

7. (a) $u_1 = \frac{1}{4} \times 0 + 16 = 16$

$u_2 = \frac{1}{4} \times 16 + 16 = 20$

$u_3 = \frac{1}{4} \times 20 + 16 = 21$

(b) (i) since $-1 < \frac{1}{4} < 1$

(ii) $L = \frac{1}{4}L + 16$

$\frac{3}{4}L = 16$

$3L = 64$

$L = \frac{64}{3}$

8. (a) Cuts x-axis when $y=0$

$$x^3 - 4x^2 + x + 6 = 0$$

3	1	-4	1	6
		3	-3	-6
	1	-1	-2	0

since remainder $=0$, $(x-3)$ is a root. Therefore $(3,0)$ is a point on the graph.

(b)

$$(x-3)(x^2 - x - 2) = 0$$

$$(x-3)(x-2)(x+1) = 0$$

$x=3$ $x=2$ $x=-1$

$(3,0)$ $(2,0)$ $(-1,0)$ $A(2,0)$

(c) $$\int_0^2 x^3 - 4x^2 + x + 6$$

$$= \left[\frac{x^4}{4} - \frac{4x^3}{3} + \frac{x^2}{2} + 6x \right]_0^2$$

$$= \left(\frac{16}{4} - \frac{32}{3} + \frac{4}{2} + 12 \right) - (0)$$

$$= \left(4 - 10\frac{2}{3} + 2 + 12 \right) - (0)$$

$$= 7\frac{1}{3}$$

9. (a) Crosses x-axis when $y=0$,

$$3x - x^3 = 0$$

$$x(3 - x^2) = 0$$

$x = 0$ $\quad 3 - x^2 = 0$

$$x^2 = 3$$

$$x = \pm\sqrt{3}$$

$(0,0)$, $(-\sqrt{3},0)$, $(\sqrt{3},0)$

Crosses y-axis when $x = 0$,

$$y = 3(0) - (0)^3$$

$$= 0$$

$(0,0)$

(b) For S.P's $f'(x) = 0$

$$f'(x) = 3 - 3x^2$$

$$3 - 3x^2 = 0$$

$$3(1 - x^2) = 0$$

$$3(1+x)(1-x) = 0$$

$x = -1$ $\quad x = 1$

x	-2	-1	0	1	2
$f'(x)$	$-$	0	$+$	0	$-$
slope	\	_	/	—	\

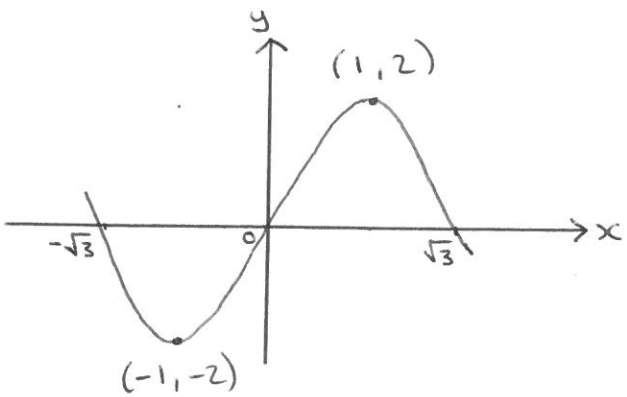
min t.p. at $(-1,-2)$ max t.p. at $(1,2)$

When $x = -1$,

$$y = 3(-1) - (-1)^3$$

$$= -2$$

(c)



When $x = 1$

$$y = 3(1) - (1)^3$$

$$= 2$$

10. $y = \sqrt{3x^2+2} = (3x^2+2)^{\frac{1}{2}}$

$$\frac{dy}{dx} = \frac{1}{2}(3x^2+2)^{-\frac{1}{2}} \times 6x$$

$$= 3x(3x^2+2)^{-\frac{1}{2}}$$

11. (a) $f(x) = \sqrt{3}\cos x + \sin x = k\cos(x-a)$

$= k(\cos x \cos a + \sin x \sin a)$

$= k\cos a \cos x + k \sin a \sin x$

$k\cos a = \sqrt{3}$

$k \sin a = 1$

S ✓	A ✓
T	C ✓

$k = \sqrt{(\sqrt{3})^2 + 1^2}$

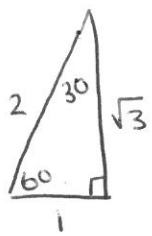
$= \sqrt{4}$

$= 2$

$\tan a = \frac{1}{\sqrt{3}}$

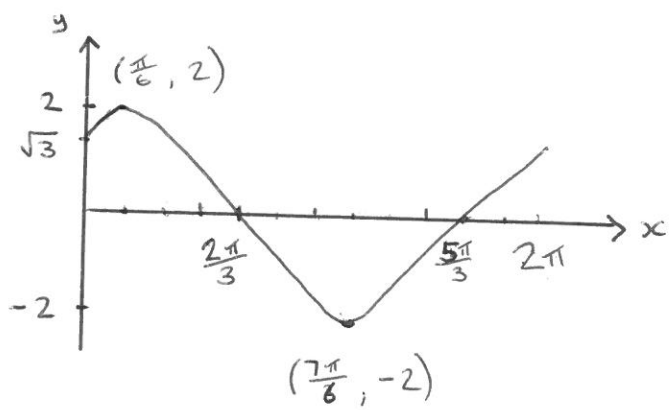
$a = 30°$

$a = \frac{\pi}{6}$



$f(x) = 2\cos(x - \frac{\pi}{6})$

(b)



When $x = 0$,

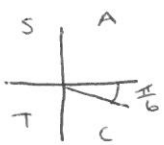
$y = 2\cos(0 - \frac{\pi}{6})$

$= 2\cos(-\frac{\pi}{6})$

$= 2 \times \frac{\sqrt{3}}{2}$

$= \sqrt{3}$

$(0, \sqrt{3})$



Higher 2007 Paper 2

1. (a) $Q(0, 2, 2)$ (b) $\underset{\sim}{p} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ $\underset{\sim}{q} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

(c) $\cos POQ = \dfrac{\underset{\sim}{p} \cdot \underset{\sim}{q}}{|\underset{\sim}{p}||\underset{\sim}{q}|}$

$|\underset{\sim}{p}| = \sqrt{0^2 + 1^2 + 1^2}$
$= \sqrt{2}$

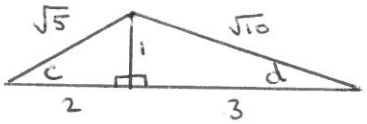
$|\underset{\sim}{q}| = \sqrt{1^2 + 2^2 + 1^2}$
$= \sqrt{6}$

$= \dfrac{(0 \times 1) + (1 \times 2) + (1 \times 1)}{\sqrt{2} \times \sqrt{6}}$

$= \dfrac{3}{\sqrt{12}}$

$POQ = 30^\circ$

2. (a) $\sin(c + d) = \sin c \cos d + \cos c \sin d$



$= \left(\dfrac{1}{\sqrt{5}} \times \dfrac{3}{\sqrt{10}}\right) + \left(\dfrac{2}{\sqrt{5}} \times \dfrac{1}{\sqrt{10}}\right)$

$= \dfrac{3}{\sqrt{50}} + \dfrac{2}{\sqrt{50}}$

$= \dfrac{5}{\sqrt{50}}$

$= \dfrac{5}{\sqrt{25}\sqrt{2}}$

$= \dfrac{5}{5\sqrt{2}}$

$= \dfrac{1}{\sqrt{2}}$

2. (b) (i) $\sin 2c = 2\sin c \cos c$

$= 2 \times \left(\frac{1}{\sqrt{5}}\right) \times \left(\frac{2}{\sqrt{5}}\right)$

$= \frac{4}{5}$

(ii) $\cos 2d = \cos^2 d - \sin^2 d$

$= \left(\frac{3}{\sqrt{10}}\right)^2 - \left(\frac{1}{\sqrt{10}}\right)^2$

$= \frac{9}{10} - \frac{1}{10}$

$= \frac{8}{10} = \frac{4}{5}$

3. $x^2 + (6-2x)^2 + 6x - 4(6-2x) - 7 = 0$

$x^2 + 36 - 24x + 4x^2 + 6x - 24 + 8x - 7 = 0$

$5x^2 - 10x + 5 = 0$

$5(x^2 - 2x + 1) = 0$

$5(x-1)(x-1) = 0$

Equal roots, therefore line is a tangent.

$x - 1 = 0$

$x = 1$

$y = 6 - 2 \times 1$

$= 6 - 2$

$= 4$

Point of contact is $(1, 4)$

4. (a) $y = a\sin(bx) + c$

$a = 2 \qquad b = 3 \qquad c = -1$

(b) Crosses x-axis when $y = 0$,

$$2\sin 3x - 1 = 0$$
$$2\sin 3x = 1$$
$$\sin 3x = \tfrac{1}{2}$$
$$3x = 30^\circ, 150^\circ, 390^\circ$$
$$x = 10^\circ, 50^\circ, \cancel{130^\circ}$$

S	A
T	C

x-co-ordinate of P is 50°.

5. (a) $\frac{dy}{dx} = x - 8 = m$

$$x - 8 = 4$$
$$x = 12$$

$y = \frac{1}{2}(12)^2 - 8(12) + 34$

$= 72 - 96 + 34$

$= 10$

Q(12, 10)

(b) $x - 8 = -4$

$x = 4$

$y = \frac{1}{2}(4)^2 - 8(4) + 34$

$= 8 - 32 + 34$

$= 10$

P(4, 10)

5. (c) $m_{CQ} = -\frac{1}{4}$ $\quad Q(12, 10)$

$$y - 10 = -\frac{1}{4}(x - 12)$$

$$4y - 40 = -x + 12$$

$$4y = -x + 52 \quad \leftarrow \text{Equation of } CQ$$

x-coordinate of C is halfway between P and Q

$$(4 + 12) \div 2 = 8$$

$$4y = -8 + 52$$

$$4y = 44$$

$$y = 11 \qquad C(8, 11)$$

6. (a) (i) $ST^2 = 10^2 + 10^2$

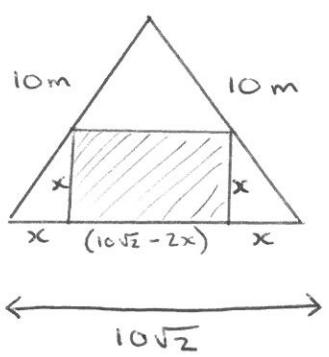
$= 100 + 100$

$ST = \sqrt{200}$

$= \sqrt{100}\sqrt{2}$

$= 10\sqrt{2}$

(ii)



$A = L \times b$

$= x(10\sqrt{2} - 2x)$

$= 10\sqrt{2}x - 2x^2$

6. (b) $A(x) = 10\sqrt{2}x - 2x^2$

For S.P's $A'(x) = 0$

$$10\sqrt{2} - 4x = 0$$

$$-4x = -10\sqrt{2}$$

$$x = \frac{10\sqrt{2}}{4}$$

$$x = \frac{5\sqrt{2}}{2}$$

x	0	$\frac{5\sqrt{2}}{2}$	4
$A'(x)$	$+$	0	$-$
	/	—	\

Maximum area when $x = \frac{5\sqrt{2}}{2}$

Dimensions

Length $= 10\sqrt{2} - 2\left(\frac{5\sqrt{2}}{2}\right) = 10\sqrt{2} - 5\sqrt{2} = 5\sqrt{2}$

Breadth $= \frac{5\sqrt{2}}{2}$

7. $\int_0^2 \sin(4x+1)\,dx$

$$= \left[-\frac{1}{4}\cos(4x+1)\right]_0^2$$

$$= \left(-\frac{1}{4}\cos 9\right) - \left(-\frac{1}{4}\cos 1\right)$$

$$= 0.228 - (-0.135)$$

$$= 0.36$$

Remember to set calculator to radians.

8. $y = \log_3(x-1) - 2.2$ $\quad x \;\; y$ $\quad (a, 0)$

$\log_3(a-1) - 2.2 = 0$

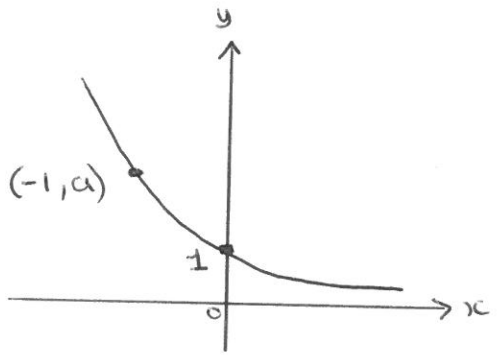
$\log_3(a-1) = 2.2$

$a - 1 = 3^{2.2}$

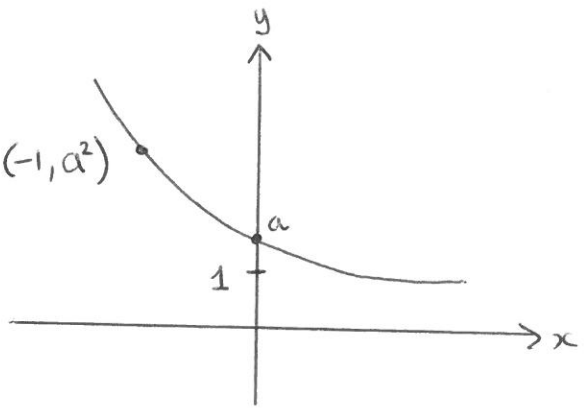
$a = 3^{2.2} + 1$

$a = 12.2$

9. (a) $y = a^{-x}$ (reflect in y-axis)



(b) $y = a^{1-x}$



10. (a) (i) $a = 2$ $b = 4$

(ii) $y = k(x-2)(x-4)$ $\quad x \quad y$ $(0, 6)$

$6 = k(0-2)(0-4)$

$6 = k \times (-2) \times (-4)$

$6 = 8k$

$k = \frac{6}{8}$

$k = \frac{3}{4}$

(b) $f'(x) = \frac{3}{4}(x-2)(x-4)$

$= \frac{3}{4}(x^2 - 6x + 8)$

$= \frac{3x^2}{4} - \frac{18x}{4} + 6$

$\int \frac{3x^2}{4} - \frac{9x}{2} + 6$

$y = \frac{3x^3}{12} - \frac{9x^2}{4} + 6x + C$

$y = \frac{x^3}{4} - \frac{9x^2}{4} + 6x + C$ $\quad x \quad y$ $(0, 6)$

$6 = \frac{0^3}{4} - \frac{9(0)^2}{4} + 6(0) + C$

$C = 6$

$f(x) = \frac{1}{4}x^3 - \frac{9}{4}x^2 + 6x + 6$

11. (a) $\overset{x}{a}, \overset{y}{6}$

$y = 3 \times 4^x$

$6 = 3 \times 4^a$

$4^a = 2$

$a = \frac{1}{2}$

(b) $\overset{x}{-\frac{1}{2}}, \overset{y}{b}$

$b = 3 \times 4^{-\frac{1}{2}}$

$b = 3 \times \frac{1}{\sqrt{4}}$

$= 3 \times \frac{1}{2} \quad = \frac{3}{2}$

(c)

$\log_{10} y = \log_{10} (3 \times 4^x)$

$\log_{10} y = \log_{10} 3 + \log_{10} 4^x$

$\log_{10} y = x \log_{10} 4 + \log_{10} 3$

$y = mx + c$

↓

gradient is the co-efficient of x

Gradient $= \log_{10} 4$