(*a*) (i) Show that $x = 1$ is a root of $x^3 + 8x^2 + 11x - 20 = 0$.

(ii) Hence factorise $x^3 + 8x^2 + 11x - 20$ fully. **4**

(*b*) Solve $\log_2(x + 3) + \log_2(x^2 + 5x - 4) = 3$. **5**

The size of the human population, N, can be modelled using the equation $N = N_0e^{rt}$ where N_0 is the population in 2006, t is the time in years since 2006, and r is the annual rate of increase in the population.

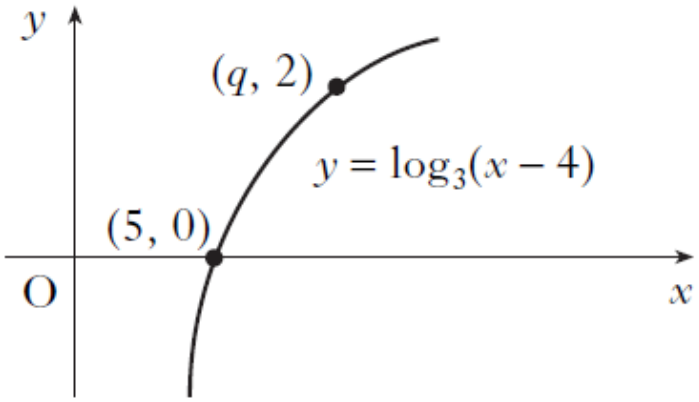
(*a*) In 2006 the population of the United Kingdom was approximately 61 million, with an annual rate of increase of 1·6%. Assuming this growth rate remains constant, what would be the population in 2020? **2**

(*b*) In 2006 the population of Scotland was approximately 5·1 million, with an annual rate of increase of 0·43%.

Assuming this growth rate remains constant, how long would it take for Scotland's population to double in size? **3**

The diagram shows part of the graph of $y = \log_3(x - 4)$.

The point $(q, 2)$ lies on the graph.



What is the value of q?

A 6

B 7

C 8

D 13

Functions f, g and h are defined on suitable domains by

$$f(x) = x^2 - x + 10,\ g(x) = 5 - x \text{ and } h(x) = \log_2 x.$$

(*a*) Find expressions for $h(f(x))$ and $h(g(x))$. **3**

(*b*) Hence solve $h(f(x)) - h(g(x)) = 3$. **5**

The curve with equation $y = \log_3(x - 1) - 2{\cdot}2$, where $x > 1$, cuts the x-axis at the point $(a, 0)$.

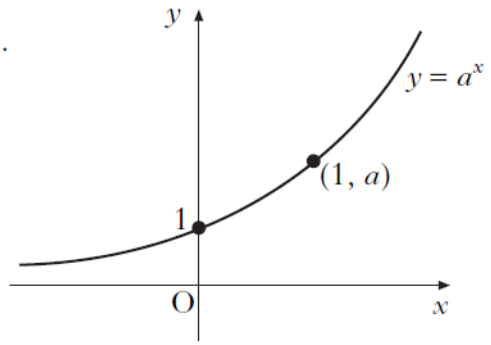
Find the value of a. **4**

The diagram shows the graph of $y = a^x$, $a > 1$.

On separate diagrams, sketch the graphs of:

(*a*) $y = a^{-x}$; **2**

(*b*) $y = a^{1-x}$. **2**



Two variables x and y satisfy the equation $y = 3 \times 4^x$.

(*a*) Find the value of a if $(a, 6)$ lies on the graph with equation $y = 3 \times 4^x$. **1**

(*b*) If $(-\frac{1}{2}, b)$ also lies on the graph, find b. **1**

(*c*) A graph is drawn of $\log_{10} y$ against x. Show that its equation will be of the form $\log_{10} y = Px + Q$ and state the gradient of this line. **4**