

ADVANCED HIGHER MATHEMATICS

THE BINOMIAL THEOREM I

1. Expand:
- (a) $(a+h)^3$ (b) $(p+q)^4$ (c) $(m+n)^5$ (d) $(x-y)^4$
 - (e) $(c-d)^5$ (f) $(u-v)^5$
2. Expand:
- (a) $(x+2)^3$ (b) $(a+1)^4$ (c) $(y+5)^3$ (d) $(p+2)^4$
 - (e) $(1+x)^5$ (f) $(m+3)^4$ (g) $(c+6)^3$ (h) $(x+2)^5$
 - (i) $(a+4)^3$ (j) $(2+k)^4$ (k) $(x+3)^3$ (l) $(4+x)^5$
 - (m) $(x-2)^3$ (n) $(d-1)^4$ (o) $(2-x)^4$ (p) $(a-8)^3$
 - (q) $(3-k)^5$ (r) $(y-6)^4$ (s) $(1-x)^3$ (t) $(3-x)^4$
3. Expand:
- (a) $(2x+1)^3$ (b) $(2a+3)^4$ (c) $(1+2x)^5$ (d) $(3m+2)^3$
 - (e) $(3x+2)^4$ (f) $(2x+5)^3$ (g) $(2x+y)^4$ (h) $(3a+2b)^3$
 - (i) $(2x-1)^4$ (j) $(4a-3)^3$ (k) $(3x-2)^5$ (l) $(2x-y)^3$
 - (m) $(x-3y)^4$ (n) $(2x-3y)^5$ (o) $(x-2y)^5$ (p) $(3x-y)^3$
 - (q) $(2x+7)^3$ (r) $(2x+3y)^4$ (s) $(1-3p)^3$ (t) $(3+2a)^5$
 - (u) $(2x-4y)^3$ (v) $(1+3x)^4$ (w) $(3x-4y)^5$ (x) $(5-3x)^4$

- ① (a) $a^3 + 3a^2b + 3ab^2 + b^3$
- (b) $p^4 + 4p^3q + 6p^2q^2 + 4pq^3 + q^4$
- (c) $m^5 + 5m^4n + 10m^3n^2 + 10m^2n^3 + 5mn^4 + n^5$
- (d) $x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4$
- (e) $c^3 - 3c^2d + 3cd^2 - d^3$
- (f) $u^5 - 5u^4v + 10u^3v^2 - 10u^2v^3 + 5uv^4 - v^5$
- ② (a) $x^3 + 6x^2 + 12x + 8$
- (b) $a^4 + 4a^3 + 6a^2 + 4a + 1$
- (c) $y^3 + 15y^2 + 75y + 125$
- (d) $p^4 + 8p^3 + 24p^2 + 32p + 16$
- (e) $1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5$
- (f) $m^4 + 12m^3 + 54m^2 + 108m + 81$
- (g) $c^3 + 18c^2 + 108c + 216$
- (h) $x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32$
- (i) $a^3 + 12a^2 + 48a + 64$
- (j) $16 + 32k + 24k^2 + 8k^3 + k^4$
- (k) $x^3 + 9x^2 + 27x + 27$
- (l) $1024 + 1280x + 640x^2 + 160x^3 + 20x^4 + x^5$
- (m) $x^3 - 6x^2 + 12x - 8$
- (n) $d^4 - 4d^3 + 6d^2 - 4d + 1$
- (o) $16 - 32x + 24x^2 - 8x^3 + x^4$
- (p) $a^3 - 24a^2 + 192a - 512$

- (1) $243 - 405k + 270k^2 - 90k^3 + 15k^4 - k^5$
- (1) $y^4 - 24y^3 + 216y^2 - 1296y + 1296$
- (2) $1 - 3x + 3x^2 - x^3$
- (4) $81 - 108x + 54x^2 - 12x^3 + x^4$
- 3) (a) $8x^3 + 12x^2 + 6x + 1$
- (b) $16a^4 + 96a^3 + 216a^2 + 216a + 81$
- (c) $1 + 10x + 40x^2 + 80x^3 + 80x^4 + 32x^5$
- (d) $27m^3 + 54m^2 + 36m + 8$
- (e) $81x^4 + 216x^3 + 216x^2 + 96x + 16$
- (f) $8x^3 + 60x^2 + 150x + 125$
- (g) $16x^4 + 32x^3y + 24x^2y^2 + 8xy^3 + y^4$
- (4) $27a^3 + 54a^2b + 36ab^2 + 8b^3$
- (i) $16x^4 - 32x^3 + 24x^2 - 8x + 1$
- (j) $64a^3 - 144a^2 + 108a - 27$
- (k) $243x^5 - 810x^4 + 1080x^3 - 720x^2 + 240x - 32$
- (l) $8x^3 - 12x^2y + 6xy^2 - y^3$
- (m) $x^4 - 12x^3y + 54x^2y^2 - 108xy^3 + 81y^4$
- (n) $8x^3 - 36x^2y + 54xy^2 - 27y^3$
- (o) $x^5 - 10x^4y + 40x^3y^2 - 80x^2y^3 + 80xy^4 - 32y^5$
- (p) $27x^3 - 27x^2y + 9xy^2 - y^3$
- (4) $8x^3 + 84x^2 + 294x + 343$
- (7) $16x^4 + 96x^3y + 216x^2y^2 + 216xy^3 + 81y^4$

- (5) $1 - 9p + 27p^2 - 27p^3$
- (x) $243 + 810a + 1080a^2 + 720a^3 + 240a^4 + 32a^5$
- (u) $8x^3 - 48x^2y + 96xy^2 - 64y^3$
- (v) $1 + 12x + 54x^2 + 108x^3 + 81x^4$
- (w) $243x^5 - 1620x^4y + 4320x^3y^2 - 5760x^2y^3 + 3840xy^4 - 1024y^5$
- (x) $625 - 1500x + 1350x^2 - 540x^3 + 81x^4$

THE BINOMIAL THEOREM 2

1. Expand:

- (a) $(x^2 + 1)^3$ (b) $(x^2 + 2)^4$ (c) $(x^2 + 4)^3$
- (d) $(a^2 + 2)^3$ (e) $(p^2 + 1)^4$ (f) $(x^2 + 2)^5$
- (g) $(m^2 - 5)^3$ (h) $(x^2 - 3)^4$ (i) $(x^4 + 1)^3$
- (j) $(x^3 + 2)^4$ (k) $(y^3 - 2)^5$ (l) $(2 - x^3)^3$
- (m) $\left(x + \frac{1}{x}\right)^3$ (n) $\left(x + \frac{1}{x}\right)^4$ (o) $\left(\frac{2}{x + \frac{1}{x}}\right)^3$
- (p) $\left(x - \frac{1}{x}\right)^5$ (q) $\left(x - \frac{2}{x}\right)^4$ (r) $\left(2x + \frac{1}{x}\right)^5$
- (s) $\left(2x + \frac{3}{x}\right)^4$ (t) $\left(x^2 + \frac{1}{x}\right)^3$ (u) $\left(x^2 + \frac{2}{x}\right)^4$
- (v) $\left(\frac{x}{y} + \frac{y}{x}\right)^5$ (w) $\left(x^3 + \frac{1}{x}\right)^4$ (x) $\left(x^2 - \frac{1}{x^2}\right)^3$
- (y) $\left(x^2 - \frac{2}{x}\right)^5$ (z) $\left(\frac{x^2}{y} - \frac{y}{x}\right)^4$

2. Expand:

- (a) $(2x^2 + 1)^3$ (b) $(2x^2 + 3)^4$ (c) $(3x^2 + 2)^3$
- (d) $(4x^2 + 3)^3$ (e) $(2x^2 - 1)^5$ (f) $(2x^2 - 5)^3$
- (g) $(2x^2 + y)^4$ (h) $(x^2 + 2y^2)^3$ (i) $(2x^2 + 3y^2)^3$
- (j) $(2x^3 + 1)^4$ (k) $\left(2x^2 + \frac{1}{x}\right)^3$ (l) $\left(3x^2 - \frac{2}{x}\right)^4$

3. (a) Expand $(x+1)^3$.

(b) Hence find the coefficient of x^2 in the expansion of $(x+2)(x+1)^3$.

4. (a) Expand $(x+2)^3$.

(b) Hence find the coefficient of x^3 in the expansion of $(x+1)(x+2)^3$.

5. (a) Expand $(x+1)^4$.

(b) Hence find the coefficient of x^2 in the expansion of $(x+3)(x+1)^4$.

6. (a) Expand $(x+2)^5$.

(b) Hence find the coefficient of x^3 in the expansion of $(x-4)(x+2)^5$.

7. (a) Expand $(x-2)^4$.
 (b) Hence find the coefficient of x^3 in the expansion of $(x+2)(x-2)^4$.

8. (a) Expand $(x+1)^4$.
 (b) Hence find the coefficient of x^2 in the expansion of $(2x+1)(x+1)^4$.

9. (a) Expand $(x-3)^3$.
 (b) Hence find the coefficient of x^3 in the expansion of $(x^2+2)(x-3)^3$.

10. (a) Expand $(2x+1)^4$.
 (b) Hence find the coefficient of x^2 in the expansion of $(x-2)(2x+1)^4$.

11. (a) Expand $(2x-1)^3$.
 (b) Hence find the coefficient of x^2 in the expansion of $(2x+3)(2x-1)^3$.

12. (a) Expand $(3x+2)^3$.
 (b) Hence find the coefficient of x^2 in the expansion of $(2x-1)(3x+2)^3$.

13. (a) Expand $(x+1)^2$ and $(x+2)^3$.
 (b) Hence find the coefficient of x^3 in the expansion of $(x+1)^2(x+2)^3$.

14. (a) Expand $(x-1)^2$ and $(x+2)^4$.
 (b) Hence find the coefficient of x^3 in the expansion of $(x-1)^2(x+2)^4$.

15. (a) Expand $(x+1)^3$ and $(x-2)^5$.
 (b) Hence find the coefficient of x^3 in the expansion of $(x+1)^3(x-2)^5$.

16. Expand fully:

- (a) $(x+2)(x+1)^3$ (b) $(x+1)(x+2)^3$ (c) $(x+3)(x-1)^3$
- (d) $(x-2)(x+3)^3$ (e) $(2x+1)(x-2)^3$ (f) $(x+2)(x+1)^4$
- (g) $(x-1)(x+2)^4$ (h) $(x+1)^2(x+2)^3$ (i) $(x-2)^2(x+1)^3$

ANSWERS

- ① (a) $x^6 + 3x^4 + 3x^2 + 1$
 (b) $x^8 + 8x^6 + 24x^4 + 32x^2 + 16$
 (c) $x^6 + 12x^4 + 48x^2 + 64$
 (d) $a^6 + 6a^4 + 12a^2 + 8$
 (e) $p^8 + 4p^6 + 6p^4 + 4p^2 + 1$
 (f) $x^{10} + 10x^8 + 40x^6 + 80x^4 + 80x^2 + 32$
 (g) $w^6 - 15w^4 + 75w^2 - 125$
 (h) $x^8 - 12x^6 + 54x^4 - 108x^2 + 81$
 (i) $x^{12} + 3x^8 + 3x^4 + 1$
 (j) $x^{12} + 8x^9 + 24x^6 + 32x^3 + 16$
 (k) $y^{15} - 10y^{12} + 40y^9 - 80y^6 + 80y^3 - 32$
 (l) $8 - 12x^3 + 6x^6 - x^9$
 (m) $x^3 + 3x + \frac{3}{x} + \frac{1}{x^3}$
 (n) $x^4 + 4x^2 + 6 + \frac{4}{x^2} + \frac{1}{x^4}$
 (o) $x^3 + 6x + \frac{12}{x} + \frac{8}{x^3}$
 (p) $x^5 - 5x^3 + 10x - \frac{10}{x} + \frac{5}{x^3} - \frac{1}{x^5}$
 (q) $x^4 - 8x^2 + 24 - \frac{32}{x^2} + \frac{16}{x^4}$
 (r) $32x^5 + 80x^3 + 80x + \frac{40}{x} + \frac{10}{x^3} + \frac{1}{x^5}$

- (s) $16x^4 + 96x^2 + 216 + \frac{216}{x^2} + \frac{81}{x^4}$
 (t) $x^6 + 3x^3 + 3 + \frac{1}{x^3}$
 (u) $x^8 + 8x^5 + 24x^2 + \frac{32}{x} + \frac{16}{x^4}$
 (v) $\frac{x^5}{y^5} + \frac{5x^3}{y^3} + \frac{10x}{y} + \frac{10y}{x} + \frac{5y^3}{x^3} + \frac{y^5}{x^5}$
 (w) $x^{12} + 4x^8 + 6x^4 + 4 + \frac{1}{x^4}$
 (x) $x^6 - 3x^2 + \frac{3}{x^2} - \frac{1}{x^6}$
 (y) $x^{10} - 10x^7 + 40x^4 - 80x + \frac{80}{x^2} - \frac{32}{x^5}$
 (z) $\frac{x^8}{y^4} - \frac{4x^5}{y^2} + 6x^2 - \frac{4y^2}{x} + \frac{y^4}{x^4}$
- ② (a) $8x^6 + 12x^4 + 6x^2 + 1$
 (b) $16x^8 + 96x^6 + 216x^4 + 216x^2 + 81$
 (c) $27x^6 + 54x^4 + 36x^2 + 8$
 (d) $64x^6 + 144x^4 + 108x^2 + 27$
 (e) $32x^{10} - 80x^8 + 80x^6 - 40x^4 + 10x^2 - 1$
 (f) $8x^6 - 60x^4 + 150x^2 - 125$
 (g) $16x^8 + 32x^6y + 24x^4y^2 + 8x^2y^3 + y^4$
 (h) $x^6 + 6x^4y^2 + 12x^2y^4 + 8y^6$

- (2) $8x^6 + 36x^4y^2 + 54x^2y^4 + 27y^6$
 (b) $16x^{12} + 32x^9 + 24x^6 + 8x^3 + 1$
 (b) $8x^6 + 12x^3 + 6 + \frac{1}{x^3}$
 (b) $81x^8 - 216x^5 + 216x^2 - \frac{96}{x} + \frac{16}{x^4}$

- (3) (a) $x^3 + 3x^2 + 3x + 1$ (b) 9
 (4) (a) $x^3 + 6x^2 + 12x + 8$ (b) 7
 (5) (a) $x^4 + 4x^3 + 6x^2 + 4x + 1$ (b) 22
 (6) (a) $x^5 + 10x^4 + 40x^3 + 80x^2 + 160x + 32$ (b) -80
 (7) (a) $x^4 - 8x^3 + 24x^2 - 32x + 16$ (b) 8
 (8) (a) $x^4 + 4x^3 + 6x^2 + 4x + 1$ (b) 14
 (9) (a) $x^3 - 9x^2 + 27x - 27$ (b) 29
 (10) (a) $16x^4 + 32x^3 + 24x^2 + 8x + 1$ (b) -40
 (11) (a) $8x^3 - 12x^2 + 6x - 1$ (b) -24
 (12) (a) $27x^3 + 54x^2 + 36x + 8$ (b) 18
 (13) (a) $(x+1)^2 = x^2 + 2x + 1$;
 $(x+2)^3 = x^3 + 6x^2 + 12x + 8$
 (b) 25

- (14) (a) $(x-1)^2 = x^2 - 2x + 1$;
 $(x+2)^4 = x^4 + 8x^3 + 24x^2 + 32x + 16$
 (b) -8

- (15) (a) $(x+1)^3 = x^3 + 3x^2 + 3x + 1$;
 $(x-2)^3 = x^3 - 6x^2 + 12x - 8$

(b) ||

- (16) (a) $x^4 + 5x^3 + 9x^2 + 7x + 2$
 (b) $x^4 + 7x^3 + 18x^2 + 20x + 8$
 (c) $x^4 - 6x^2 + 8x - 3$
 (d) $x^4 + 7x^3 - 45x^2 - 27x - 54$
 (e) $2x^4 - 11x^3 + 18x^2 - 4x - 8$
 (f) $x^5 + 6x^4 + 14x^3 + 16x^2 + 9x + 2$
 (g) $x^5 + 7x^4 + 16x^3 + 8x^2 - 16x - 16$
 (h) $x^5 + 8x^4 + 25x^3 + 38x^2 + 28x + 8$
 (i) $x^5 - x^4 - 5x^3 + x^2 + 8x + 4$

ADVANCED HIGHER MATHEMATICS

HOMWORK ON THE BINOMIAL THEOREM

1. Expand:

- | | | |
|--------------------------------------|---------------------------------------|---|
| (a) $(x + 5)^3$ | (b) $(a + 2)^4$ | (c) $(m - 3)^5$ |
| (d) $(5 - y)^4$ | (e) $(2x + 3)^3$ | (f) $(3p + 1)^4$ |
| (g) $(2x + y)^5$ | (h) $(2k - 5)^3$ | (i) $(3x - 2y)^4$ |
| (j) $(x^2 + 2)^3$ | (k) $(y^3 - 2)^4$ | (l) $\left(x + \frac{1}{x}\right)^3$ |
| (m) $\left(x + \frac{2}{x}\right)^4$ | (n) $\left(2x - \frac{3}{x}\right)^3$ | (o) $\left(x^2 - \frac{2}{x}\right)^5$ |
| (p) $(2x^2 + 1)^4$ | (q) $(3x^2 - 2)^3$ | (r) $\left(2x^3 - \frac{2}{x}\right)^4$ |

2. (a) Expand $(x + 2)^3$.

(b) Hence find the coefficient of x^2 in the expansion of $(2x - 1)(x + 2)^3$.

3. (a) Expand $(2x - 1)^4$.

(b) Hence find the coefficient of x^3 in the expansion of $(x + 3)(2x - 1)^4$.

4. Expand fully:
- | | |
|-----|----------------------|
| (a) | $(x + 2)(x - 3)^3$ |
| (b) | $(3x - 2)(x + 1)^4$ |
| (c) | $(x - 1)^2(x + 2)^3$ |

5. Expand fully $\left(x^2 - \frac{2}{x}\right)^6$.