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Exponentials & Logs

Higher Maths Exam Questions

Source: 2019 P1 Q14 Higher Maths	
(1)	(a) Evaluate $\log_{10} 4 + 2\log_{10} 5$. (b) Solve $\log_2(7x-2) - \log_2 3 = 5, \quad x \geq 1$.
Answers: $(a)\ 2$ $(b)\ 14$	

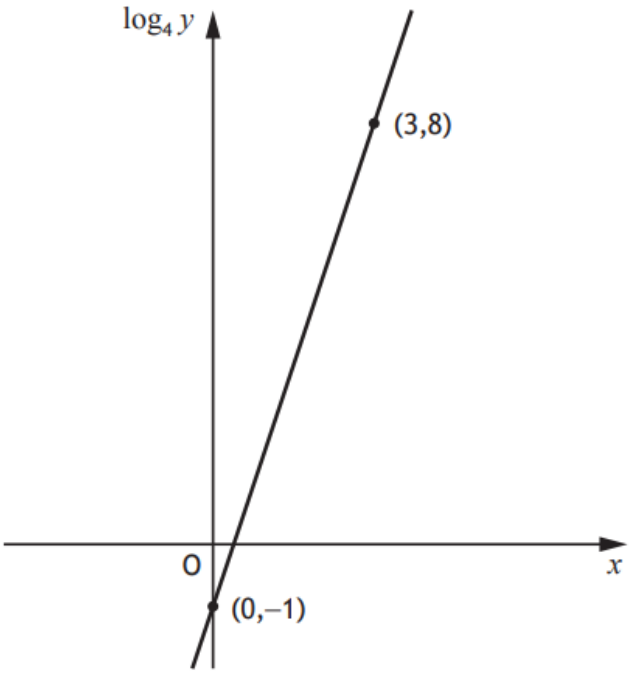
Source: 2019 P2 Q9 Higher Maths	
(2)	Electricity on a spacecraft can be produced by a type of nuclear generator. The electrical power produced by this generator can be modelled by $$P_t = 120e^{-0.0079t}$$ where P_t is the electrical power produced, in watts, after t years. (a) Determine the electrical power initially produced by the generator. (b) Calculate how long it takes for the electrical power produced by the generator to reduce by 15%.
Answers: $(a)\ 120\ watts$ $(b)\ 20.572\ldots years$	

Source: 2019 P2 Q12 Higher Maths

(3)

Two variables, x and y, are connected by the equation $y = ab^x$.

The graph of $\log_4 y$ against x is a straight line as shown.



Find the values of a and b.

Answers: $a = \frac{1}{4}$ $b = 64$

Source: 2018 P1 Q6 Higher Maths

(4)

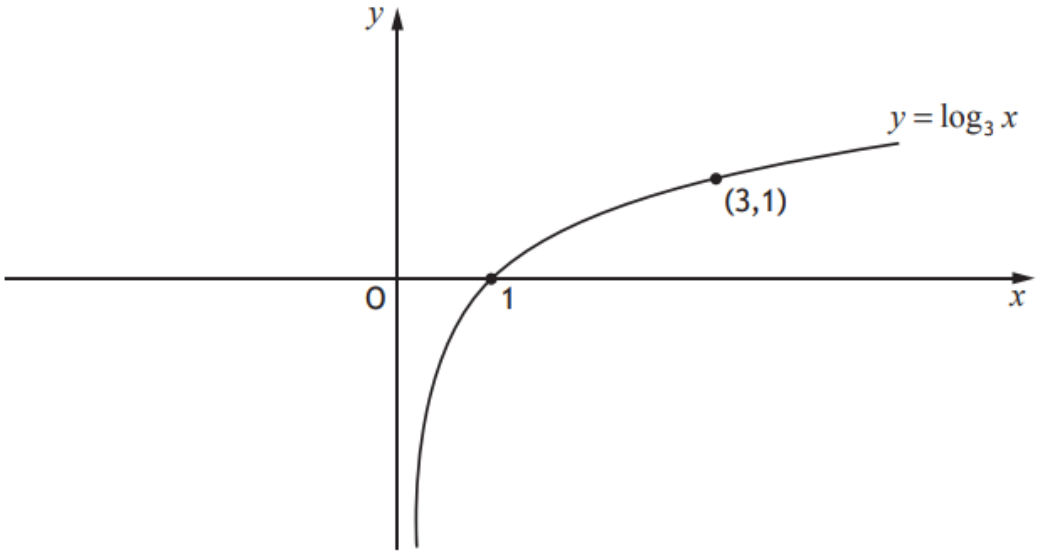
Find the value of $\log_5 250 - \frac{1}{3}\log_5 8$.

Answer: $Exact\ value = 3$

Source: 2018 P1 Q11 Higher Maths

(5)

The diagram shows the curve with equation $y = \log_3 x$.



(a) On the diagram in your answer booklet, sketch the curve with equation $y = 1 - \log_3 x$.

(b) Determine the exact value of the x-coordinate of the point of intersection of the two curves.

Answers:

(a)

- •1 curve reflected in x-axis and translated 1 unit vertically
- •2 accurate sketch

- •1 a generally decreasing curve above the x-axis for $1 < x < 3$
- •2 asymptote at $x = 0$ **and** passing through $(3,0)$ **and** continuing to decrease for $x \geq 3$

(b) $\sqrt{3}$ *or* $3^{\frac{1}{2}}$

Source: 2018 P2 Q11 Higher Maths

(6)

A supermarket has been investigating how long customers have to wait at the checkout.

During any half hour period, the percentage, $P\%$, of customers who wait for less than t minutes, can be modelled by

$$P = 100\left(1 - e^{kt}\right), \text{ where } k \text{ is a constant.}$$

(a) If 50% of customers wait for less than 3 minutes, determine the value of k.

(b) Calculate the percentage of customers who wait for 5 minutes or longer.

Answers: $(a)\ k = -0.231$ $(b)\ 31.5\%$

Source: 2017 P1 Q12 Higher Maths

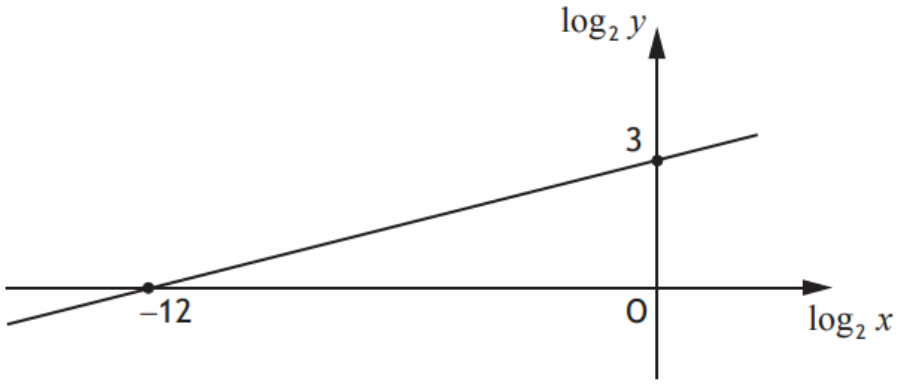
(7) Given that $\log_a 36 - \log_a 4 = \frac{1}{2}$, find the value of a.

Answer: $a = 81$

Source: 2017 P2 Q9 Higher Maths

(8) Two variables, x and y, are connected by the equation $y = kx^n$.

The graph of $\log_2 y$ against $\log_2 x$ is a straight line as shown.



Find the values of k and n.

Answer: $k = 8$ $n = \frac{1}{4}$

Source: 2016 P1 Q14 Higher Maths

(9) (a) Evaluate $\log_5 25$.

(b) Hence solve $\log_4 x + \log_4 (x-6) = \log_5 25$, where $x > 6$.

Answers: (a) 2 (b) 8

Source: 2016 P2 Q6 Higher Maths

(10) Scientists are studying the growth of a strain of bacteria. The number of bacteria present is given by the formula

$$B(t) = 200e^{0\cdot107t},$$

where t represents the number of hours since the study began.

(a) State the number of bacteria present at the start of the study.

(b) Calculate the time taken for the number of bacteria to double.

Answers: $(a)\ 200 \quad (b)\ t = 6.428$

Source: 2015 P1 Q6 Higher Maths

(11) Evaluate $\log_6 12 + \frac{1}{3}\log_6 27$.

Answer: 2

Source: Specimen P2 Q4 Higher Maths

(12) (a) Express $y = \log_4 2x$ in the form $y = \log_4 x + k$, clearly stating the value of k.

(b) Hence, or otherwise, describe the relationship between the graphs of $y = \log_4 2x$ and $y = \log_4 x$.

(c) Determine the coordinates of the point where the graph of $y = \log_4 2x$ intersects the x-axis.

(d) Sketch and annotate the graph of $y = f^{-1}(x)$, where $f(x) = \log_4 2x$.

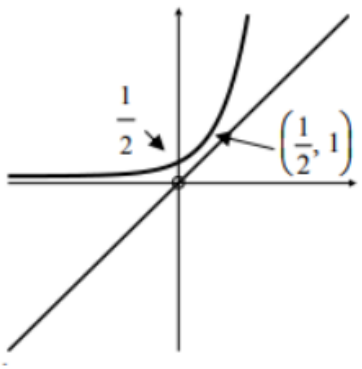
Answers:

(a) $y = log_4 x + \frac{1}{2}$

(b) Ans: Graph of $y = \log_4 x$ moved up by $\frac{1}{2}$ or graph of $y = \log_4 x$ compressed horizontally by a factor of 2.

(c) $x = \frac{1}{2}$

(d)



Source: Specimen P2 Q7 Higher Maths

(13) Given that $P(t) = 30e^{t-2}$ decide whether each of the statements below is true or false. Justify your answers.

Statement A $P(0) = 30$.

Statement B When $P(t) = 15$, the only possible value of t is 1·3 to one decimal place.

Answers:

Ans: A False and B True

6

- $\bullet^1$ valid reason for statement A
- $\bullet^2$ selecting true or false for statement A with valid reason
- $\bullet^3$ setting $P(t) = 15$
- $\bullet^4$ taking log to base e
- $\bullet^5$ completing valid reason
- $\bullet^6$ selecting true or false for statement B with valid reason

- $\bullet^1$ $P(0) = 30e^{-2} = 4 \cdot 06$
- $\bullet^2$ false, since $P(0) \neq 30$
  (do not award without valid reason)
- $\bullet^3$ $15 = 30e^{t-2}$
- $\bullet^4$ $\ln e^{t-2} = \ln 0 \cdot 5$
- $\bullet^5$ $t - 2 = \ln 0 \cdot 5$
  $t = \ln 0 \cdot 5 + 2$ (1·3)
- $\bullet^6$ true, since $t = 1 \cdot 3$ to one decimal place and there is only one solution
  (do not award without valid reason)

Source: 2013 P2 Q5 Higher Maths

(14) Solve the equation

$\log_5(3-2x) + \log_5(2+x) = 1$, where x is a real number.

Answers: $x = \frac{1}{2}$ $x = -1$

Source: Exemplar P2 Q7 Higher Maths

(15)

The concentration of the pesticide, *Xpesto*, in soil can be modelled by the equation.

$$P_t = P_0 e^{-kt}$$

where:

- P_0 is the initial concentration;
- P_t is the concentration at time t;
- t is the time, in days, after the application of the pesticide.

Once in the soil, the half-life of a pesticide is the time taken for its concentration to be reduced to one half of its initial value.

(a) If the half-life of *Xpesto* is 25 days, find the value of k to 2 significant figures.

On all *Xpesto* packaging, the manufacturer states that 80 days after application the concentration of *Xpesto* in the soil will have decreased by over 90%.

(b) Is this statement correct? Justify your answer.

Answers:

(a) $k = 0.028$

(b) No, the concentration will not have decreased by over 90%. 89% decrease.

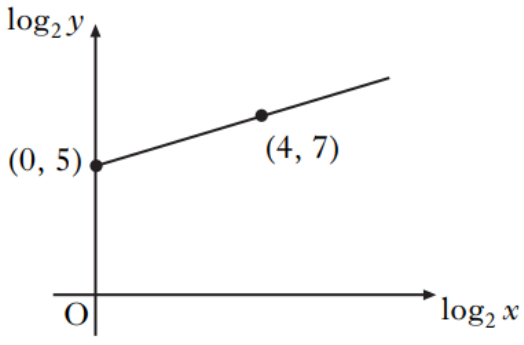
Source: 2011 P2 Q5 Higher Maths

(16)

Variables x and y are related by the equation $y = kx^n$.

The graph of $\log_2 y$ against $\log_2 x$ is a straight line through the points (0, 5) and (4, 7), as shown in the diagram.

Find the values of k and n.



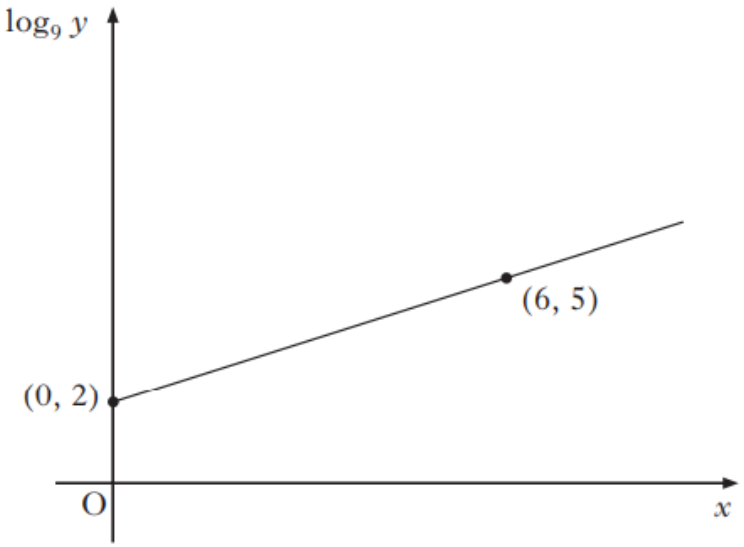
Answers: $k = 32, \quad n = \frac{1}{2}$

Source: 2014 P1 Q24 Higher Maths

(17) Two variables, x and y, are related by the equation

$$y = ka^x.$$

When $\log_9 y$ is plotted against x, a straight line passing through the points (0, 2) and (6, 5) is obtained, as shown in the diagram.



Find the values of k and a.

Answers: $k = 81$ $a = 3$

Source: 2010 P2 Q7 Higher Maths

(18) (*a*) Given that $\log_4 x = \mathrm{P}$, show that $\log_{16} x = \frac{1}{2}\mathrm{P}$.

(*b*) Solve $\log_3 x + \log_9 x = 12$.

Answers:

(*a*)

•1	ss	convert from log to exponential form	•1	$x = 4^{\mathrm{P}}$
•2	ss	know to and convert back to log form	•2	$\log_{16} x = \log_{16} 4^{\mathrm{P}}$
•3	pd	process and complete	•3	$\log_{16} x = \mathrm{P} \times \log_{16} 4$ and complete

(*b*) $x = 6561$