Vectors

Higher Maths Exam Questions

Source: 2019 P1 Q9 Higher Maths	

(1) Vectors **u** and **v** have components $\begin{pmatrix} p \\ -2 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 2p+16 \\ -3 \\ 6 \end{pmatrix}$, $p \in \mathbb{R}$.

(a) (i) Find an expression for **u.v**.

(ii) Determine the values of p for which **u** and **v** are perpendicular.

(b) Determine the value of p for which **u** and **v** are parallel.

Answers: $(a)(i)\ 2p^2 + 16p + 30$ $(ii)\ p = -5\ \&\ -3$ $(b)\ p = -32$

Source: 2019 P2 Q3 Higher Maths	

(2) E,ABCD is a rectangular based pyramid.
$\overrightarrow{AB} = $ **p**, $\overrightarrow{AD} = $ **q** and $\overrightarrow{AE} = $ **r**.
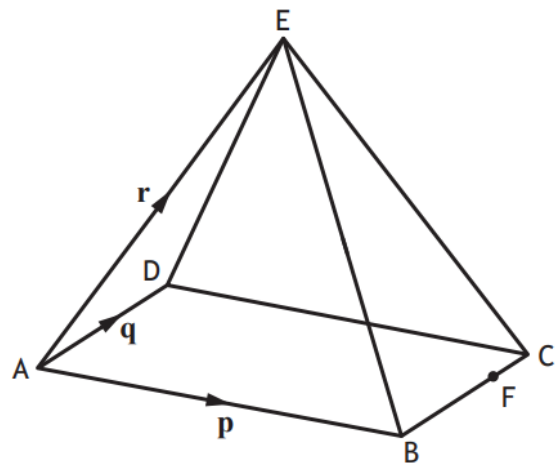
(a) Express $\overrightarrow{BE}$ in terms of **p** and **r**.

Point F divides BC in the ratio 3:1.

(b) Express vector $\overrightarrow{EF}$ in terms of **p**, **q** and **r**.

Answers: $(a)\ \overrightarrow{BE} = -p + r$ $(b)\ \overrightarrow{EF} = p - r + \frac{3}{4}q\ \ or\ \ equivalent$

(3)   The vectors **u** and **v** are such that

- $|\mathbf{u}| = 4$

- $|\mathbf{v}| = 5$

- $\mathbf{u}.(\mathbf{u} + \mathbf{v}) = 21$

Determine the size of the angle between the vectors **u** and **v**.

Answer: $Angle = 75.5° \ or \ 1.31 \ radians$

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(4)   A $(-3, 4, -7)$, B $(5, t, 5)$ and C $(7, 9, 8)$ are collinear.

(a) State the ratio in which B divides AC.

(b) State the value of t.

Answers: $(a) \ Ratio \ 4 : 1$ $(b) \ t = 8$

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(5)   Vectors **a** and **b** are such that $\mathbf{a} = 4\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ and $\mathbf{b} = -2\mathbf{i} + \mathbf{j} + p\mathbf{k}$.

(a) Express $2\mathbf{a} + \mathbf{b}$ in component form.

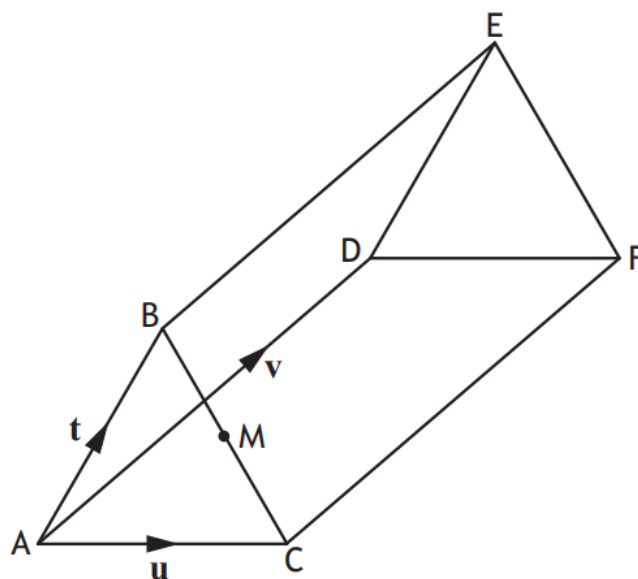(b) Hence find the values of p for which $|2\mathbf{a} + \mathbf{b}| = 7$.

Answers: $(a) \begin{pmatrix} 6 \\ -3 \\ 4 + p \end{pmatrix}$ $(b) \ p = -2, \ -6$

(6)

The diagram shows a triangular prism ABC,DEF.

$\overrightarrow{AB} = t$, $\overrightarrow{AC} = u$ and $\overrightarrow{AD} = v$.

(a) Express $\overrightarrow{BC}$ in terms of **u** and **t**.

M is the midpoint of BC.

(b) Express $\overrightarrow{MD}$ in terms of **t, u** and **v**.

Answers: $(a) \overrightarrow{BC} = -t + u$ $(b) \overrightarrow{MD} = -\dfrac{1}{2}t - \dfrac{1}{2}u + v$

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(7)

Vectors **u** and **v** are defined by $u = \begin{pmatrix} -1 \\ 4 \\ -3 \end{pmatrix}$ and $v = \begin{pmatrix} -7 \\ 8 \\ 5 \end{pmatrix}$.
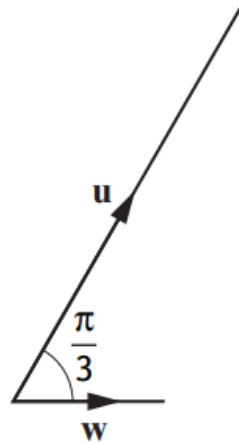
(a) Find **u.v**.

(b) Calculate the acute angle between **u** and **v**.

Answers: $(a)\, u.v = 24$ $(b)\, 66.38°\ or\ 1.16\ radians$

(8)

Vectors **u** and **v** are $\begin{pmatrix} 5 \\ 1 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -8 \\ 6 \end{pmatrix}$ respectively.

(a) Evaluate **u.v**.

(b)

Vector **w** makes an angle of $\dfrac{\pi}{3}$ with **u** and $|\mathbf{w}| = \sqrt{3}$.

Calculate **u.w**.

Answers: $(a)\ u.v = 1$ $(b)\ u.w = 4.5$

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(9)

Three vectors can be expressed as follows:

$\overrightarrow{FG} = -2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}$

$\overrightarrow{GH} = 3\mathbf{i} + 9\mathbf{j} - 7\mathbf{k}$

$\overrightarrow{EH} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$
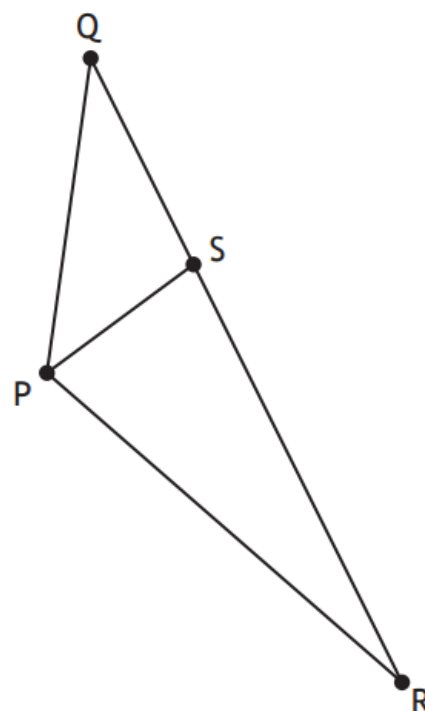
(a) Find $\overrightarrow{FH}$.

(b) Hence, or otherwise, find $\overrightarrow{FE}$.

Answers: $(a)\ u.v = 24$ $(b)\ 66.38°\ or\ 1.16\ radians$

(10) In the diagram, $\overrightarrow{PR} = 9\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$ and $\overrightarrow{RQ} = -12\mathbf{i} - 9\mathbf{j} + 3\mathbf{k}$.

Q

S

(a) Express $\overrightarrow{PQ}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$.

P

The point S divides QR in the ratio 1:2.

(b) Show that $\overrightarrow{PS} = \mathbf{i} - \mathbf{j} + 4\mathbf{k}$.

(c) Hence, find the size of angle QPS.

R

Answers: $(a) -3i - 4j + 5k$ $(b) Proof$ (c) 45.6° or 0.795 $radians$

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(11) (a) A and C are the points $(1, 3, -2)$ and $(4, -3, 4)$ respectively.

Point B divides AC in the ratio $1 : 2$.

Find the coordinates of B.
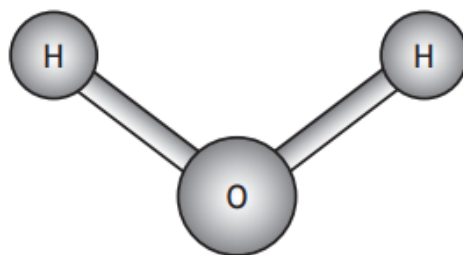
(b) $k\overrightarrow{AC}$ is a vector of magnitude 1, where $k > 0$.

Determine the value of k.

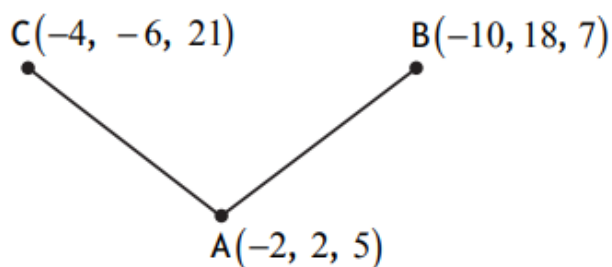Answers: $(a) B\ (2, 1, 0)$ $(b)\ k = \dfrac{1}{9}$

(12) The picture shows a model of a water molecule.

Relative to suitable coordinate axes, the oxygen atom is positioned at point $A(-2, 2, 5)$.

The two hydrogen atoms are positioned at points $B(-10, 18, 7)$ and $C(-4, -6, 21)$ as shown in the diagram below.

C(−4, −6, 21)   B(−10, 18, 7)

A(−2, 2, 5)

(a) Express $\overrightarrow{AB}$ and $\overrightarrow{AC}$ in component form.

(b) Hence, or otherwise, find the size of angle BAC.

Answers: $(a)\ \overrightarrow{AB} = \begin{pmatrix} -8 \\ 16 \\ 2 \end{pmatrix}\ \overrightarrow{BC} = \begin{pmatrix} -2 \\ -8 \\ 16 \end{pmatrix}$ $(b)\ 104.3°\ or\ 1.82\ radians$

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(13) Vectors $\mathbf{u} = 8\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{v} = -3\mathbf{i} + t\mathbf{j} - 6\mathbf{k}$ are perpendicular.

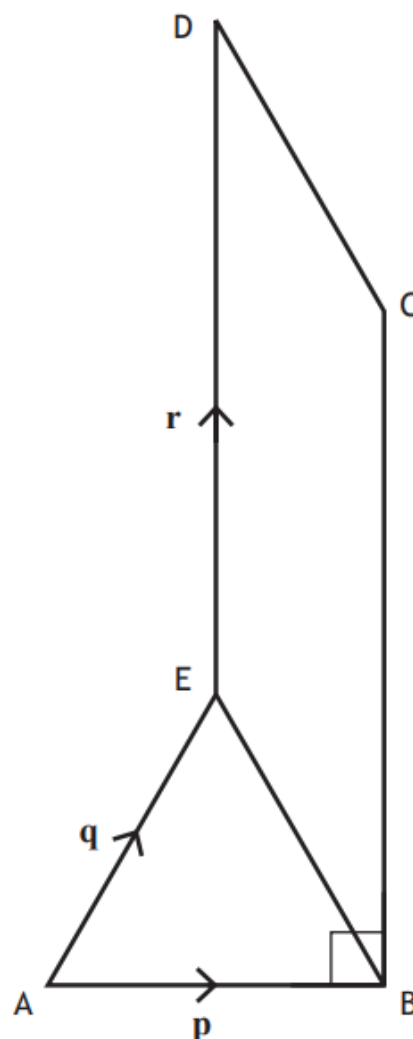Determine the value of t.

Answer: $t = 9$

(14) Vectors **p**, **q** and **r** are represented on the diagram as shown.

- BCDE is a parallelogram
- ABE is an equilateral triangle
- $|\mathbf{p}| = 3$
- Angle ABC = 90°

(a) Evaluate **p**.(**q**+**r**).

(b) Express $\overrightarrow{EC}$ in terms of **p**, **q** and **r**.

(c) Given that $\overrightarrow{AE}.\overrightarrow{EC} = 9\sqrt{3} - \dfrac{9}{2}$, find $|\mathbf{r}|$.

Answers: $(a)\ p.(q+r) = 4.5$ $(b)\ \overrightarrow{EC} = -q + p + r$ $(c)\ \dfrac{3\sqrt{3}}{cos30}$

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(15) In the diagram, P has coordinates (−6, 3, 9),

$\overrightarrow{PQ} = 6\mathbf{i} + 12\mathbf{j} - 6\mathbf{k}$ and $\overrightarrow{PQ} = 2\overrightarrow{QR} = 3\overrightarrow{RS}$.

Find the coordinates of S.
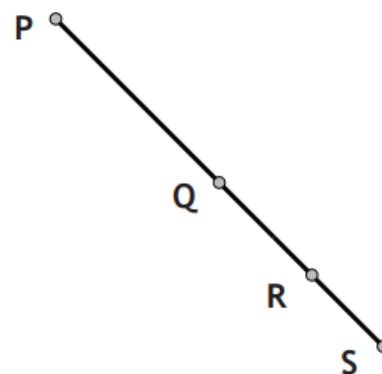
Answer: $S\ (5,\ 25,\ -2)$

(16)
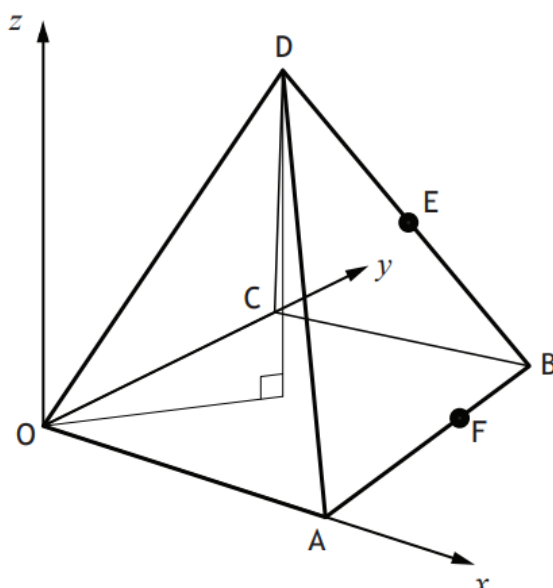
A square based right pyramid is shown in the diagram.

Square OABC has a side length of 60 units with edges OA and OC lying on the x-axis and y-axis respectively.

The coordinates of D are (30, 30, 80).

E is the midpoint of BD and F divides AB in the ratio 2:1.

(a) Find the coordinates of E and F.

(b) Calculate $\overrightarrow{ED}.\overrightarrow{EF}$.

(c) Hence, or otherwise, calculate the size of angle DEF.

Answers: (a) E $(45, 45, 40)$ F $(60, 40, 0)$ (b) $\overrightarrow{ED}.\overrightarrow{EF} = -1750$ (c) $154°$

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(17)

The points A(0, 9, 7), B(5, −1, 2), C(4, 1, 3) and D(x, −2, 2) are such that AB is perpendicular to CD.

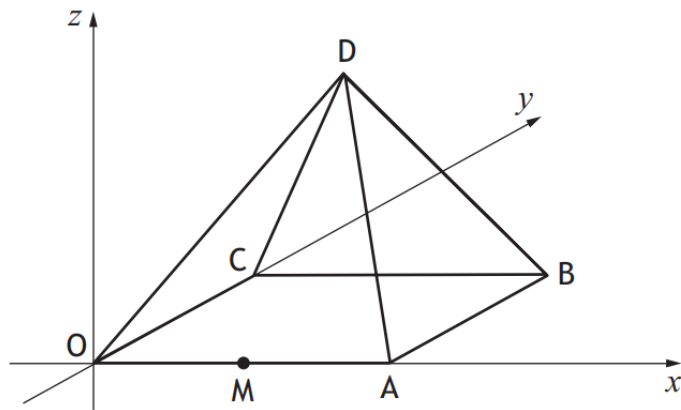Determine the value of x.

Answer: $x = -3$

(18)

D,OABC is a square-based pyramid as shown.

O is the origin and OA = 4 units.

M is the mid-point of OA.

$\overrightarrow{OD} = 2\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$

(a) Express $\overrightarrow{OB}$ in terms of $\mathbf{i}$ and $\mathbf{j}$ and $\mathbf{k}$.

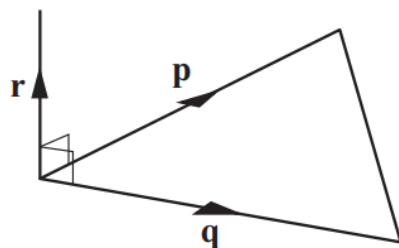(b) Express $\overrightarrow{DB}$ and $\overrightarrow{DM}$ in component form.

(c) Find the size of angle BDM.

Answers: $(a)\ \overrightarrow{OB} = 4i + 4j$ $(b)\ \overrightarrow{DB} = \begin{pmatrix} 2 \\ 2 \\ -6 \end{pmatrix}$ $\overrightarrow{DM} = \begin{pmatrix} 0 \\ -2 \\ -6 \end{pmatrix}$ $(c)\ 40.3°\ or\ 0.703\ radians$

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(19)

An equilateral triangle with sides of length 3 units is shown.

Vector $\mathbf{r}$ is 2 units long and is perpendicular to both vectors $\mathbf{p}$ and $\mathbf{q}$.

Calculate the value of the scalar product $\mathbf{p}.(\mathbf{p}+\mathbf{q}+\mathbf{r})$.

Answer: $p.(p + q + r) = \dfrac{27}{2}$