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Recurrence Relations

Higher Maths Exam Questions

Source: 2019 P1 Q4 Higher Maths

(1)

A sequence is generated by the recurrence relation

$$u_{n+1} = mu_n + c,$$

where the first three terms of the sequence are 6, 9 and 11.

(a) Find the values of m and c.

(b) Hence, calculate the fourth term of the sequence.

Answers: $(a)\;\; m = \frac{2}{3}$ $\quad (b)\;\; 4th\ term = \frac{37}{3} = 12\frac{1}{3}$

Source: 2019 P2 Q4 Higher Maths

(2)

In a forest, the population of a species of mouse is falling by 2·7% each year.

To increase the population scientists plan to release 30 mice into the forest at the end of March each year.

(a) u_n is the estimated population of mice at the start of April, n years after the population was first estimated.

It is known that u_n and u_{n+1} satisfy the recurrence relation $u_{n+1} = au_n + b$.

State the values of a and b.

The scientists continue to release this species of mouse each year.

(b) (i) Explain why the estimated population of mice will stabilise in the long term.

(ii) Calculate the long term population to the nearest hundred.

Answers: $(a)\;\; a = 0.973 \quad b = 30$

$(b)\;\; (i)\;\; A\ limit\ exists\ as\ the\ recurrence\ relation\ is\ linear\ and -1 < 0.973 < 1$

$(ii)\;\; L = 1100$

Source: 2017 P1 Q9 Higher Maths

(3)

A sequence is generated by the recurrence relation $u_{n+1} = m u_n + 6$ where m is a constant.

(a) Given $u_1 = 28$ and $u_2 = 13$, find the value of m.

(b) (i) Explain why this sequence approaches a limit as $n \to \infty$.

(ii) Calculate this limit.

Answer: $(a)\ m = \frac{1}{4}$

$(b)\ (i)\ A\ limit\ exists\ as\ the\ recurrence\ relation\ is\ linear\ and\ -1 < \frac{1}{4} < 1$

$(ii)\ L = 8$

Source: 2017 P2 Q8 Higher Maths

(4)

Sequences may be generated by recurrence relations of the form $u_{n+1} = k u_n - 20$, $u_0 = 5$ where $k \in \mathbb{R}$.

(a) Show that $u_2 = 5k^2 - 20k - 20$.

(b) Determine the range of values of k for which $u_2 < u_0$.

Answers: $(a)\ Proof$ $(b)\ -1 < k < 5$

Source: 2016 P1 Q3 Higher Maths

(5)

A sequence is defined by the recurrence relation $u_{n+1} = \frac{1}{3}u_n + 10$ with $u_3 = 6$.

(a) Find the value of u_4.

(b) Explain why this sequence approaches a limit as $n \to \infty$.

(c) Calculate this limit.

Answers: $(a)\ u_4 = 12$

$(b)\ A\ limit\ exists\ as\ the\ recurrence\ relation\ is\ linear\ and\ -1 < \frac{1}{3} < 1$

$(c)\ L = 15$

Source: 2015 P2 Q3 Higher Maths

(6)

A version of the following problem first appeared in print in the 16th Century.

A frog and a toad fall to the bottom of a well that is 50 feet deep.

Each day, the frog climbs 32 feet and then rests overnight. During the night, it slides down $\frac{2}{3}$ of its height above the floor of the well.

The toad climbs 13 feet each day before resting.

Overnight, it slides down $\frac{1}{4}$ of its height above the floor of the well.

Their progress can be modelled by the recurrence relations:

- $f_{n+1} = \frac{1}{3} f_n + 32, \qquad f_1 = 32$
- $t_{n+1} = \frac{3}{4} t_n + 13, \qquad t_1 = 13$

where f_n and t_n are the heights reached by the frog and the toad at the end of the nth day after falling in.

(a) Calculate t_2, the height of the toad at the end of the second day.

(b) Determine whether or not either of them will eventually escape from the well.

Answers: $(a)\ 22.75 \quad (b)\ 52 > 50\ therefore\ the\ toad\ will\ escape$

Source: Specimen P2 Q2 Higher Maths

(7)

A wildlife reserve has introduced conservation measures to build up the population of an endangered mammal. Initially the reserve population of the mammal was 2000. By the end of the first year there were 2500 and by the end of the second year there were 2980.

It is believed that the population can be modelled by the recurrence relation:

$u_{n+1} = au_n + b$,

where a and b are constants and n is the number of years since the reserve was set up.

(a) Use the information above to find the values of a and b.

(b) Conservation measures will end if the population stabilises at over 13 000. Will this happen? Justify your answer.

Answers: $(a)\ \ a = 0.96\ \ b = 580 \qquad (b)\ \ Yes,\ \ stabilises\ at\ 14{,}500$

Source: Exemplar P2 Q1 Higher Maths

(8)

A sequence is defined by $u_{n+1} = -\frac{1}{2}u_n$ with $u_0 = -16$.

(a) Determine the values of u_1 and u_2.

(b) A second sequence is given by 4, 5, 7, 11, ...

It is generated by the recurrence relation $v_{n+1} = pv_n + q$ with $v_1 = 4$.

Find the values of p and q.

(c) Either the sequence in (a) or the sequence in (b) has a limit.

(i) Calculate this limit.

(ii) Why does this other sequence not have a limit?

Answers: $(a)\ \ u_1 = 8,\ u_2 = -4 \quad (b)\ p = 2,\ \ q = 3$

$(c)\ (i)\ limit = 0 \qquad (ii)\ Outside\ interval\ \ -1 < p < 1$

Source: 2018 P2 Q7 Higher Maths

(9)

(a) (i) Show that $(x-2)$ is a factor of $2x^3-3x^2-3x+2$.

(ii) Hence, factorise $2x^3-3x^2-3x+2$ fully.

The fifth term, u_5, of a sequence is $u_5=2a-3$.

The terms of the sequence satisfy the recurrence relation $u_{n+1}=au_n-1$.

(b) Show that $u_7=2a^3-3a^2-a-1$.

For this sequence, it is known that

- $u_7=u_5$
- a limit exists.

(c) (i) Determine the value of a.

(ii) Calculate the limit.

Answers: (a) (i) *Use synthetic division to show remainder equals zero*

(ii) $(x-2)(2x-1)(x+1)$

(b) $2a^3-3a^2-a-1$

(c) (i) $a=\frac{1}{2}$ (ii) $L=-2$