

Differentiation

Higher Maths Exam Questions

Source: 2019 P1 Q1 Higher Maths

- (1) Find the x -coordinates of the stationary points on the curve with equation $y = \frac{1}{2}x^4 - 2x^3 + 6$.

Answers: $x = 0$ and 3

Source: 2019 P2 Q7b Higher Maths

- (2) (a) Express $-6x^2 + 24x - 25$ in the form $p(x+q)^2 + r$.
- (b) Given that $f(x) = -2x^3 + 12x^2 - 25x + 9$, show that $f(x)$ is strictly decreasing for all $x \in \mathbb{R}$.

Answers: (a) $-6(x-2)^2 - 1$

(b)

Method 1

- ⁴ differentiate
- ⁵ link with (a) and identify sign of $(x-2)^2$
- ⁶ communicate reason

Method 2

- ⁴ differentiate
- ⁵ identify maximum value of $f'(x)$
- ⁶ communicate reason

Method 1

- ⁴ $-6x^2 + 24x - 25$
- ⁵ $f'(x) = -6(x-2)^2 - 1$ and $(x-2)^2 \geq 0 \forall x$
- ⁶ eg $\therefore -6(x-2)^2 - 1 < 0 \forall x$
 \Rightarrow always strictly decreasing

Method 2

- ⁴ $-6x^2 + 24x - 25$
- ⁵ 'maximum value is -1 ' or annotated sketch including x -axis
- ⁶ $-1 < 0$ or 'graph lies below x -axis'
 $\therefore f'(x) < 0 \forall x$
 \Rightarrow always strictly decreasing

Source: 2018 P2 Q3 Higher Maths

(3)

A function, f , is defined on the set of real numbers by $f(x) = x^3 - 7x - 6$.
Determine whether f is increasing or decreasing when $x = 2$.

Answer:

•¹ differentiate

•² evaluate derivative at $x = 2$

•³ interpret result

•¹ $3x^2 - 7$

•² 5

•³ (f is) increasing

Source: 2018 P2 Q9 Higher Maths

(4)

A sector with a particular fixed area has radius x cm.

The perimeter, P cm, of the sector is given by

$$P = 2x + \frac{128}{x}.$$

Find the minimum value of P .

Answer: *Minimum value of $P = 32$*

Source: 2017 P1 Q8 Higher Maths

(5)

Calculate the rate of change of $d(t) = \frac{1}{2t}$, $t \neq 0$, when $t = 5$.

Answer: $x = -\frac{1}{50}$

Source: 2017 P2 Q4 Higher Maths

- (6)
- (a) Express $3x^2 + 24x + 50$ in the form $a(x+b)^2 + c$.
- (b) Given that $f(x) = x^3 + 12x^2 + 50x - 11$, find $f'(x)$.
- (c) Hence, or otherwise, explain why the curve with equation $y = f(x)$ is strictly increasing for all values of x .

Answers: (a) $3(x+4)^2 + 2$ (b) $3x^2 + 24x + 50$

- (c)
- | Method 1 | Method 1 |
|---|---|
| <ul style="list-style-type: none"> •⁶ link with (a) and identify sign of $(x+4)^2$ •⁷ communicate reason | <ul style="list-style-type: none"> •⁶ $f'(x) = 3(x+4)^2 + 2$ and $(x+4)^2 \geq 0 \forall x$ •⁷ $\therefore 3(x+4)^2 + 2 > 0 \Rightarrow$ always strictly increasing |
| Method 2 | Method 2 |
| <ul style="list-style-type: none"> •⁶ identify minimum value of $f'(x)$ •⁷ communicate reason | <ul style="list-style-type: none"> •⁶ eg minimum value = 2 or annotated sketch •⁷ $2 > 0 \therefore (f'(x) > 0) \Rightarrow$ always strictly increasing |

Source: 2017 P2 Q7 Higher Maths

- (7)
- (a) Find the x -coordinate of the stationary point on the curve with equation $y = 6x - 2\sqrt{x^3}$.
- (b) Hence, determine the greatest and least values of y in the interval $1 \leq x \leq 9$.

Answers: (a) $x = 4$ (b) Greatest = 8, Least = 0

Source: 2016 P1 Q2 Higher Maths

- (8)
- Given that $y = 12x^3 + 8\sqrt{x}$, where $x > 0$, find $\frac{dy}{dx}$.

Answer: $\frac{dy}{dx} = 36x^2 + 4x^{-\frac{1}{2}}$

Source: 2016 P1 Q9 Higher Maths

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| (9) | <p>(a) Find the x-coordinates of the stationary points on the graph with equation $y = f(x)$, where $f(x) = x^3 + 3x^2 - 24x$.</p> <p>(b) Hence determine the range of values of x for which the function f is strictly increasing.</p> |
|-----|--|

Answers: (a) $x = -4, 2$ (b) $x < -4, x > 2$

Source: 2015 P1 Q2 Higher Maths

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| (10) | Find the equation of the tangent to the curve $y = 2x^3 + 3$ at the point where $x = -2$. |
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Answer: $y = 24x + 35$

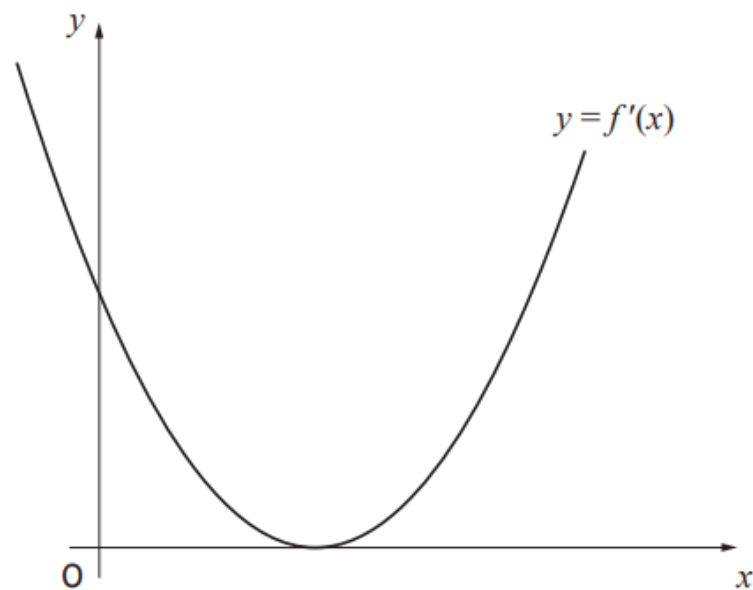
Source: 2015 P1 Q7 Higher Maths

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| (11) | <p>A function f is defined on a suitable domain by $f(x) = \sqrt{x} \left(3x - \frac{2}{x\sqrt{x}} \right)$.</p> <p>Find $f'(4)$.</p> |
|------|---|

Answer: $9\frac{1}{8}$

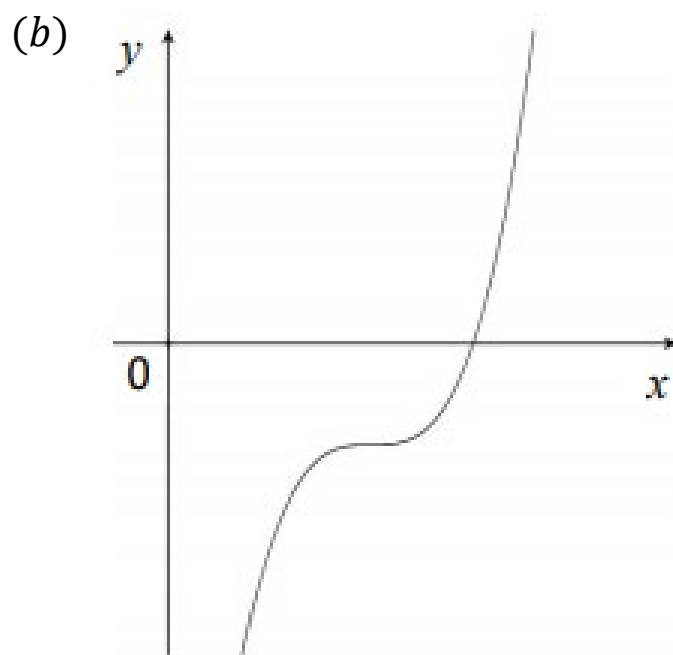
(12)

The diagram shows the graph of $y = f'(x)$. The x -axis is a tangent to this graph.



- (a) Explain why the function $f(x)$ is never decreasing.
- (b) On a graph of $y = f(x)$, the y -coordinate of the stationary point is negative. Sketch a possible graph for $y = f(x)$.

Answers: (a) $m = f'(x) \geq 0$



Source: Exemplar P1 Q1 Higher Maths

- (13) The point P (5,12) lies on the curve with equation $y = x^2 - 4x + 7$.
Find the equation of the tangent to this curve at P.

Answer: $y - 12 = 6(x - 5)$

Source: Exemplar P2 Q10 Higher Maths

- (14) Acceleration is defined as the rate of change of velocity.
An object is travelling in a straight line. The velocity, v m/s, of this object, t seconds after the start of the motion, is given by $v(t) = 8\cos(2t - \frac{\pi}{2})$.
- (a) Find a formula for $a(t)$, the acceleration of this object, t seconds after the start of the motion.
 - (b) Determine whether the velocity of the object is increasing or decreasing when $t = 10$.
 - (c) Velocity is defined as the rate of change of displacement.
Determine a formula for $s(t)$, the displacement of the object, given that $s(t) = 4$ when $t = 0$.

Answers:

- (a) $a(t) = -16\sin(2t - \frac{\pi}{2})$
- (b) $a(10) > 0$ therefore increasing
- (c) $s(t) = 4\sin(2t - \frac{\pi}{2}) + 8$

Source: 2014 P2 Q2 Higher Maths

- (15) A curve has equation $y = x^4 - 2x^3 + 5$.
Find the equation of the tangent to this curve at the point where $x = 2$.

Answer: $y - 5 = 8(x - 2)$