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Straight Lines

Higher Maths Exam Questions

Source: 2019 P1 Q5 Higher Maths	
(1)	(a) Show that the points A(1,5,–3), B(4,–1,0) and C(8,–9,4) are collinear. (b) State the ratio in which B divides AC.

Answers:

$(a)\ \overrightarrow{AB} = \begin{pmatrix} 3 \\ -6 \\ 3 \end{pmatrix}\ \overrightarrow{BC} = \begin{pmatrix} 4 \\ -8 \\ 4 \end{pmatrix} \quad \overrightarrow{AB} = \frac{3}{4}\overrightarrow{BC}$.

$AB\ is\ parallel\ to\ BC\ (common\ diection)\ and\ B\ is\ a\ common\ point.$

$Therefore\ A, B\ and\ C\ are\ collinear.$

$(b)\ Ratio\ 3:4$

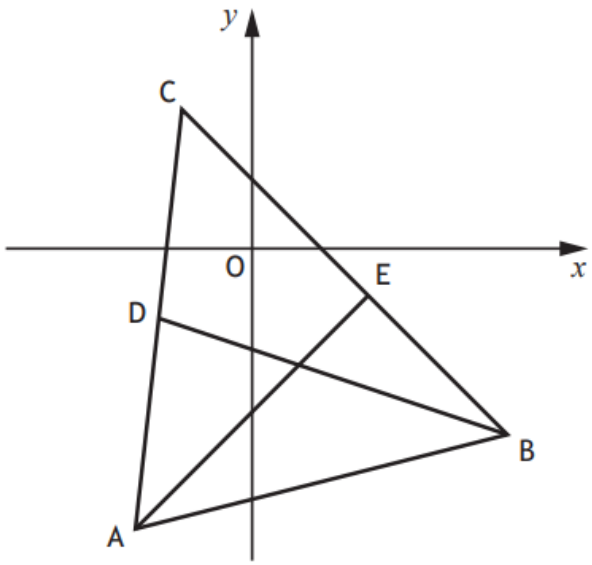
Source: 2019 P1 Q7 Higher Maths	
(2)	The line, L, makes an angle of 30° with the positive direction of the x-axis. Find the equation of the line perpendicular to L, passing through (0,–4).

Answer: $y = -\sqrt{3}x - 4$

Source: 2019 P2 Q1 Higher Maths

(3)

Triangle ABC has vertices A(–5,–12), B(11,–8) and C(–3,6).



(a) Find the equation of the median BD.

(b) Find the equation of the altitude AE.

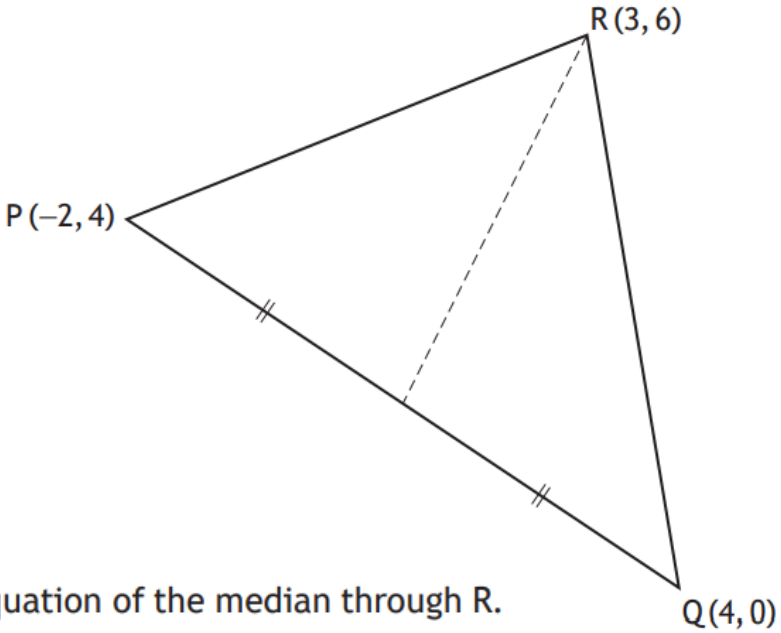
(c) Find the coordinates of the point of intersection of BD and AE.

Answers: $(a)\ 3y = -x - 13$ $(b)\ y = x - 7$ $(c)\ (2, -5)$

Source: 2018 P1 Q1 Higher Maths

(4)

PQR is a triangle with vertices P (–2, 4), Q (4, 0) and R (3, 6).



Find the equation of the median through R.

Answer: $y = 2x$

Source: 2018 P1 Q8 Higher Maths

(5) A line has equation $y - \sqrt{3}x + 5 = 0$.

Determine the angle this line makes with the positive direction of the x-axis.

Answer: $60°$ *or* $\frac{\pi}{3}$

Source: 2017 P1 Q7 Higher Maths

(6) A $(-3, 5)$, B $(7, 9)$ and C $(2, 11)$ are the vertices of a triangle.

Find the equation of the median through C.

Answer: $x = 2$

Source: 2017 P1 Q11 Higher Maths

(7) A and B are the points $(-7, 2)$ and $(5, a)$.

AB is parallel to the line with equation $3y - 2x = 4$.

Determine the value of a.

Answer: $a = 10$

Source: 2015 P1 Q9 Higher Maths

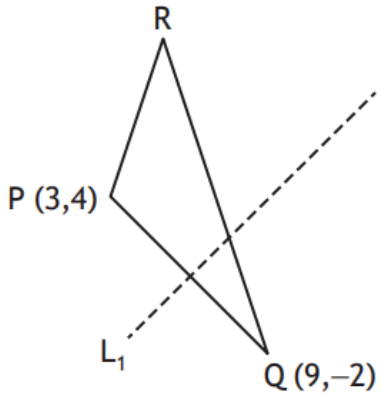
(8) A, B and C are points such that AB is parallel to the line with equation $y + \sqrt{3}x = 0$ and BC makes an angle of $150°$ with the positive direction of the x-axis.

Are the points A, B and C collinear?

Answer: $m_{ab} = -\sqrt{3}$ *and* $m_{bc} = -\frac{1}{\sqrt{3}}$ *thereore points are not collinear*

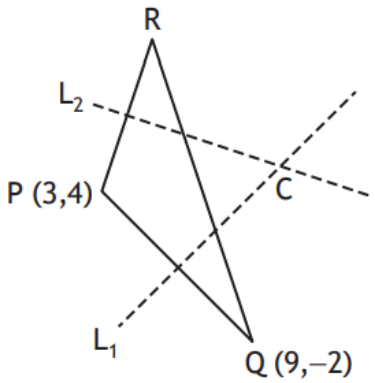
Source: 2018 P2 Q5a,b Higher Maths

(9) PQR is a triangle with P(3,4) and Q(9,–2).



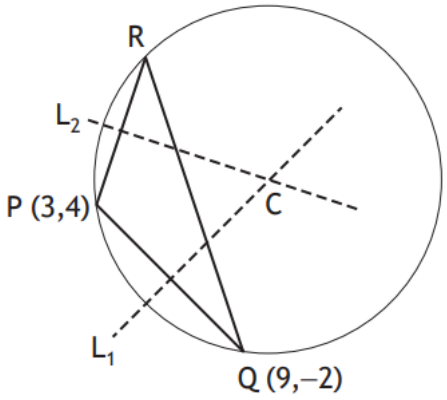
(a) Find the equation of L_1, the perpendicular bisector of PQ.

The equation of L_2, the perpendicular bisector of PR is $3y + x = 25$.



(b) Calculate the coordinates of C, the point of intersection of L_1 and L_2.

C is the centre of the circle which passes through the vertices of triangle PQR.



(c) Determine the equation of this circle.

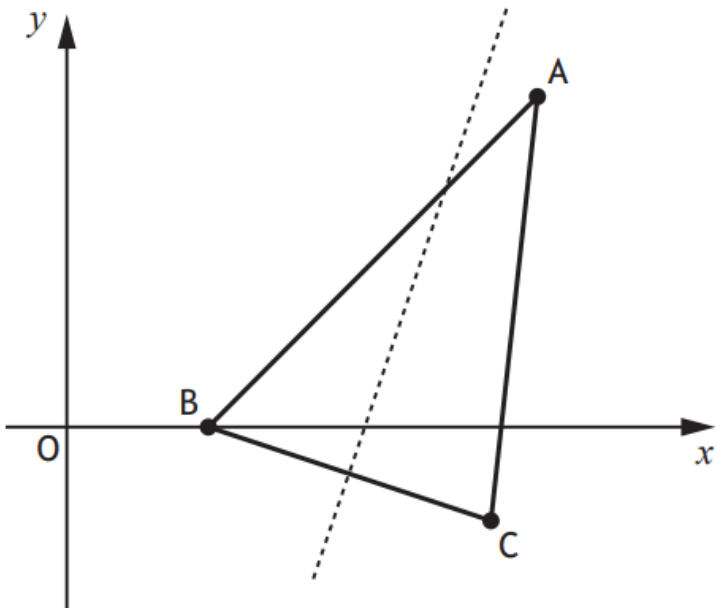
Answers: $(a)\ y = x - 5$ $(b)\ C(10,5)$ $(c)\ (x-10)^2 + (y-5)^2 = 50$

Source: 2017 P2 Q1 Higher Maths

(10) Triangle ABC is shown in the diagram below.

The coordinates of B are (3,0) and the coordinates of C are (9,–2).

The broken line is the perpendicular bisector of BC.



(a) Find the equation of the perpendicular bisector of BC.

(b) The line AB makes an angle of 45° with the positive direction of the x-axis.
Find the equation of AB.

(c) Find the coordinates of the point of intersection of AB and the perpendicular bisector of BC.

Answers: $(a)\ y = 3x - 19$ $(b)\ y = x - 3$ $(c)\ (8, 5)$

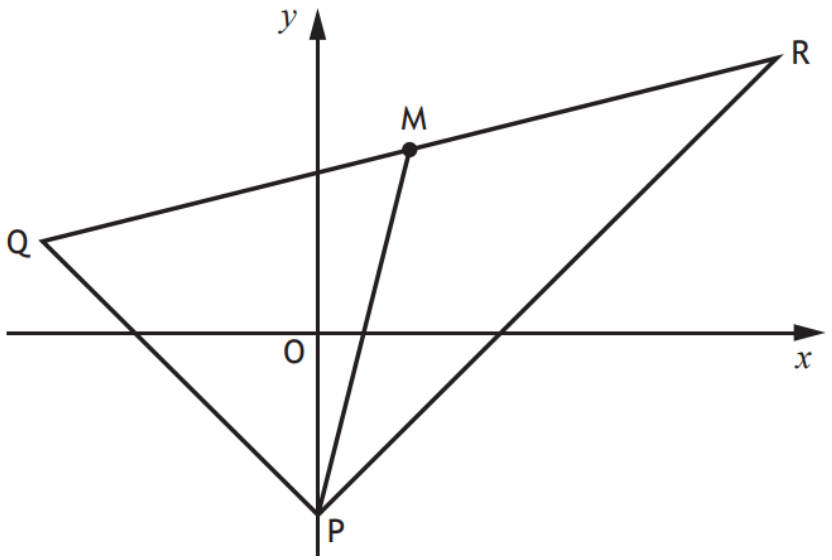
Source: 2016 P1 Q1 Higher Maths

(11) Find the equation of the line passing through the point $(-2, 3)$ which is parallel to the line with equation $y + 4x = 7$.

Answer: $y + 4x = -5$

Source: 2016 P2 Q1 Higher Maths

(12) PQR is a triangle with vertices $P(0,-4)$, $Q(-6,2)$ and $R(10,6)$.



(a) (i) State the coordinates of M, the midpoint of QR.

(ii) Hence find the equation of PM, the median through P.

(b) Find the equation of the line, *L*, passing through M and perpendicular to PR.

(c) Show that line *L* passes through the midpoint of PR.

Answers (a) (i) M(2, 4) (ii) $y = 4x - 4$

$(b)\ y = -x + 6$

(c)

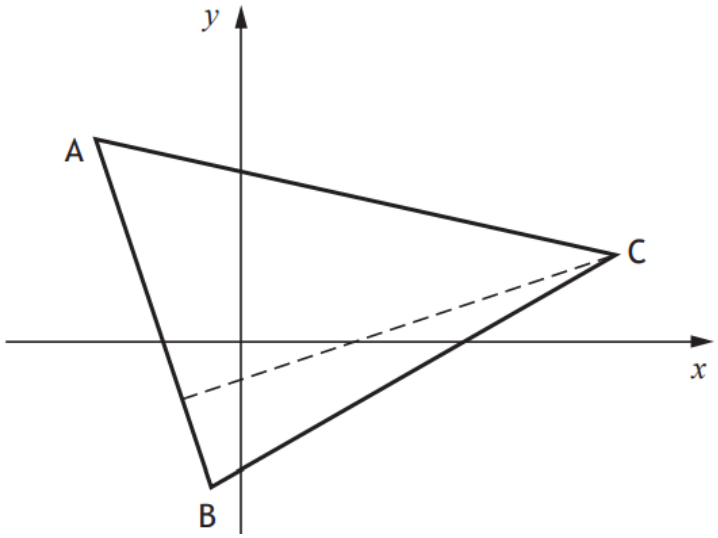
•[1] find the midpoint of PR	•[1] $(5,1)$
•[2] substitute x-coordinate into equation of L.	•[2] $y = -5 + 6\ \ (1 = -x + 6)$
•[3] verify y-coordinate and communicate conclusion	•[3] $y = 1 (x = 5)\ \therefore$ L passes through the midpoint of PR

Other methods valid – see marking scheme

Source: 2015 P2 Q1 Higher Maths

(13) The vertices of triangle ABC are A$(-5, 7)$, B$(-1, -5)$ and C$(13, 3)$ as shown in the diagram.

The broken line represents the altitude from C.



(a) Show that the equation of the altitude from C is $x - 3y = 4$.

(b) Find the equation of the median from B.

(c) Find the coordinates of the point of intersection of the altitude from C and the median from B.

Answers: $(a)\ Proof$ $(b)\ y = 2x - 3$ $(c)\ (1, -1)$

Source: Specimen P1 Q2 Higher Maths

(14) Find the coordinates of the points of intersection of the curve $y = x^3 - 2x^2 + x + 4$ and the line $y = 4x + 4$.

Answers: $(-1, 0)$ $(0, 4)$ $(3, 16)$

Source: Specimen P1 Q5 Higher Maths

(15) Line l_1 has equation $\sqrt{3}y - x = 0$.

(a) Line l_2 is perpendicular to l_1. Find the gradient of l_2.

(b) Calculate the angle l_2 makes with the positive direction of the x-axis.

Answers: $(a)\ m = -\sqrt{3}$ $(b)\ Angle = \frac{2\pi}{3}\ or\ 120°$

Source: Specimen P1 Q9 Higher Maths

(16) (a) AB is a line parallel to the line with equation $y + 3x = 25$.
A has coordinates $(-1, 10)$.
Find the equation of AB.

(b) $3y = x + 11$ is the perpendicular bisector of AB.
Determine the coordinates of B.

Answers: $(a)\ y - 10 = -3(x + 1)$ $(b)\ B\ (3, -2)$

Source: Exemplar P1 Q6 Higher Maths

(17) (a) Find the equation of l_1, the perpendicular bisector of the line joining P (3,−3) and Q (−1,9).

(b) Find the equation of l_2 which is parallel to PQ and passes through R (1,−2).

(c) Find the point of intersection of l_1 and l_2.

(d) Hence find the shortest distance between PQ and l_2.

Answers: $(a)\ y - 3 = \frac{1}{3}(x - 1)$ $(b)\ y + 2 = -3(x - 1)$

$(c)\ x = -\frac{1}{2}, \quad y = \frac{5}{2}$ $(c)\ \sqrt{\frac{5}{2}} = \frac{\sqrt{10}}{2} = \sqrt{2.5}$