Math Games For Tutoring

— INTRODUCTION —

These EFM activities were selected to help teachers and tutors provide directed practice on specific topics. While limited use of flashcards and worksheets has its place, these EFM activities seek to provide practice to improve and reinforce skills in a playful and enjoyable way. We selected these activities from the larger collection of EFM activities based on the quickness of introduction, the amount of skill practice supplied, and the minimal materials needed (no more than a deck of cards). You may enjoy searching through the full collection of EFM activities looking for ones that you like that didn't quite make the cut.

— TOPICS AND ACTIVITIES —

The following is a quick list of the activities for each math topic. The activities are given with the chapter number and page number (chapter / page). In addition to the games and puzzles, a few overview pages are included with useful progressions of techniques for mastering a particular skill.

- **Counting and Numbers** — (2/5) Memory Challenge; (2/5) Go Fish!; (2/5) Bingo with Cards
- **Number Line** — (2/2) Island Hopping - Counting; (2/2) String Number Line; (2/9) Number Line Battleship
- **Comparing Single-Digit Numbers** — (2/8) War - Comparing; (2/9) I'm Thinking of a Number; (2/13) The In-Between Game
- **Comparing Double-Digit Numbers 1-2** — (3/7) Going Up; (3/7) Going Up Some More; (3/7) Creating Order; (4/10) Get Closest; (4/10) War - Double-Digit Comparison; (4/10) Math Blackjack; (4/10) Fill In the Blanks Comparison
- **Adding 1-2** — (2/6) Nim With One or Two; (2/12 Bag Game - Add and Subtract; (3/2) Mystery Change; (3/10) Hit the Target; (4/8) Pairing Down; (4/10) Math Blackjack
- **Adding - Subtracting 1-3** — (4/2) Finger Adding-subtracting Page; (4/5) Mental Adding-Subtracting Page; (4/4) War - Addition and Subtraction; (4/8) Memory Challenge Revisited; (4/14) Sum Difference
- **Number Bonds** — (3/6) Go Fish with Sums; (4/4) Target Gin Rummy; (4/6) Part-Whole Triangles
- **Multiplying - Factoring 1-4** — (5/2) Mental Multiplying Page; (4/7) Short Steps to a Target; (5/5) Cover Factors and Multiples; (5//4) Revealing Products; (5/7) War - Multiplication; (5/7) Beep; (5/7) Turning the Tables; (5/15) Grabbing Factors; (5/9) Practice Prime Factorizations; (5/13) The Product Game
- **Mixed Operations** — (5/6) Secret Ops; (5/6) Parentheses Puzzles; (5/14) Counting Neighbors; (5/14) Limited Calculators
- **Adding - Subtracting Double Digit** — (5/3) Missing Numbers; (5/11) Letter Substitution; (5/11) 5-Card Draw to a Target
- **Rectangle Area** — (5/8) Number Shapes Revisited; (5/8) The Paddock Game; (5/8) Divide Up the Box

Counting and Numerals

— MEMORY CHALLENGE —
GAME

Select two or four sets of playing cards and remove numbers above your child's comfort level. Place the cards face down in a 3 by 3 grid, with the remaining cards put in a draw pile. Take turns flipping two cards face-up. If the cards match, the player gets to keep the cards, replaces the two cards from the draw pile, and continues their turn. If the cards don't match, the player flips the cards back over and ends their turn. The game ends when the last pair of cards is taken. The player with the most cards wins.
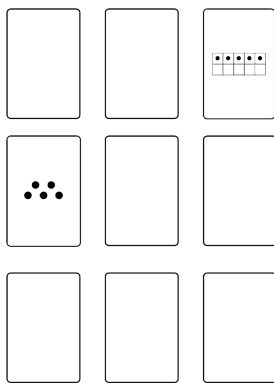
One variation allows at most one match per turn. Another option is to have a target sum, larger than the largest card used, and then two cards match when their sum is the target.

— GO FISH —
GAME

Remove cards above the highest number your child is comfortable with. If there are more than two players and you have restricted the numbers significantly, you may need to use several decks.

Start by dealing 5 cards to each player and put the remaining cards in a common draw pile. During a turn, a player "fishes" by asking any player if they have a card matching one of their cards – if they do, they hand over the card, and if they don't, they say "Go FIsh!" and the player must pick a card from the draw pile.
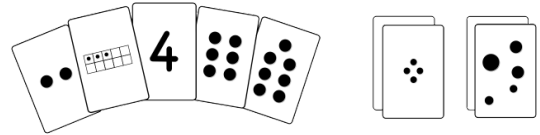
When a player has a pair of matching cards, those cards are placed in a "book" in front of them. The game is over after all the cards have been put into books. The player with the most books wins.

— BINGO WITH CARDS —
GAME

Play this with Number Cards, or if there are many players, with numbered regular playing cards. Reserve two suits worth of cards for a draw pile, and divide the remaining cards among the players. Each player will randomly select 16 cards to place in a 4 by 4 grid face up in front of them.
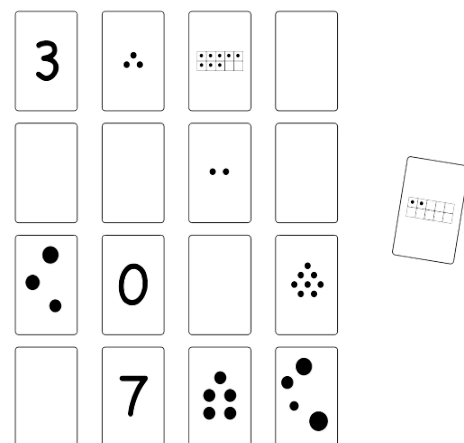
Cards are then drawn from the draw pile and the number is called out. Each player may turn over one card from their grid that matches the number drawn. If a player has more than one card that matches, the player must choose which one to turn over. The first player to get four cards flipped over in a row horizontally, vertically, or diagonally wins the game and calls out "Bingo!"

Number Line

— ISLAND HOPPING – COUNTING —
PUZZLE

These puzzles have numbered islands (circles) connected by bridges (lines) drawn on paper. The challenge is to find a path that connects the islands in order. The easiest versions have numbers that go from 1 to the number of islands.


1 - 7   0 - 6
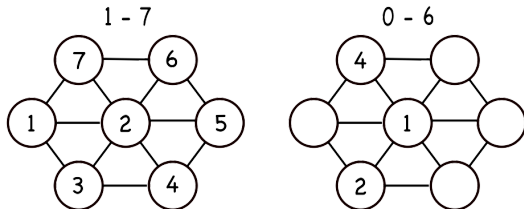
To add challenge, leave out some of the numbers so your child figures out what is missing and where it belongs. Also, instead of starting at 1, design the puzzles to start at 0 or other numbers (usually noting this at the top of the puzzle).

Make this into a physical puzzle by placing pieces of paper with consecutive numbers on the floor in some twisty path. To add challenge, replace some of the numbered pieces with blank ones. Also, challenge your child to start at the largest number and go downward.

— STRING NUMBER LINE —
INVESTIGATION

In addition to putting a number line on a piece of paper on a wall, create a number line using a string. Tie or clip a section of string between two objects. Use paper clips to attach sliding Number Cards from 0 to 10 along the string in order.
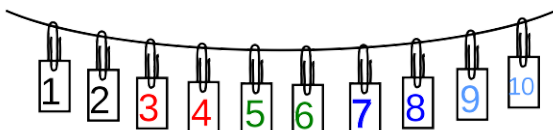
Here are some things you can do with this. You and your child will probably discover many more.

- Switch two numbers and have your child find the mistake.
- Leave out a number and have your child find which one is missing.

— NUMBER LINE BATTLESHIP —
GAME

Each player has two number lines, one for their battleships and one for their guesses. These number lines are drawn on paper and go from 0 to 12 (or higher if the children can count higher). Out of sight, on their battleship number line, each player places tokens on two numbers that will be their battleships.

After the setup, the players take turns guessing numbers. When a player makes a guess, the other player says how close the guess was to the nearest target - the guesser then marks down that information on their second number line. The first person to get all the targets wins.
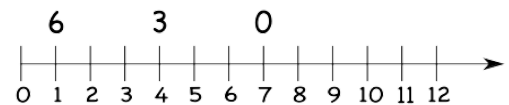

My guesses: the number is the distance, 0 is a hit

My battleships and opponent's guesses

There are several variations to make this more challenging. A wider range of numbers can be used. The response to a guess can be a range of distances rather than an exact amount - for example: "the nearest ship is 1 or 2 away." Another variation is to have ships that are two or three spaces in length.

Comparing Single-Digit Numbers

— WAR – SINGLE-DIGIT COMPARING —
GAME

Remove the cards from two, four, or six Number Decks outside your child's comfort range. Split the cards evenly in two piles face down. Flip over the top cards and the player with the larger card keeps both cards. If the cards match, flip the next two cards and the winner gets all four cards. The player with the most cards after one or more runs through all the cards is the winner.
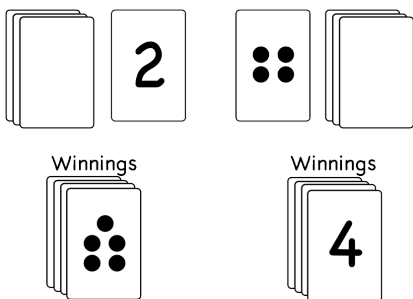
To change things up, sometimes play that the smaller of the two cards wins.

— THE IN-BETWEEN GAME —
GAME

Use one set of cards from 0 to 10. Use either Number Cards or playing cards with Queen as 0 and Ace as 1. Each player also gets 20 tokens.
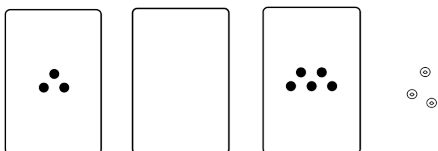

For the player whose turn it is, deal two cards face up and a third card face down between them. The player decides to bet 0 to 3 tokens that the third card is between the two cards. If the player is right, the player gets that many tokens from the other player. If the player is wrong, that many tokens go to the other player.

You can play five rounds or until one player runs out of tokens.

— I'M THINKING OF A NUMBER —
GAME

This is an introductory, warmup version of a fun game we will revisit again soon. There are two people involved. The Puzzler thinks of a number and the Questioner discovers the number. The Puzzler declares, "I am thinking of a number from 0 to 8" (or whatever upper limit they like). The Questioner then asks questions of the form "How does your number compare to 3?" The Puzzler answers that the number is smaller, equal to, or larger than that number.

Here is an example of a game with a target of 5:
Puzzler: I'm thinking of a number from 0 to 8.
Question: How does your number compare to 3?
Puzzler: My number is larger than 3.
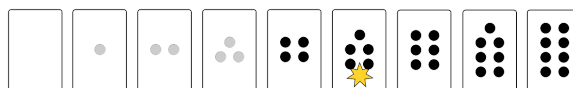
Question: How does your number compare to 6?
Puzzler: My number is smaller than 6.
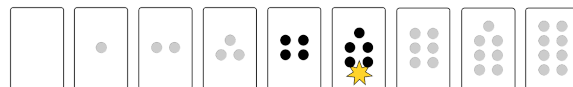
Question: How does your number compare to 5?
Puzzler: Well done! My number is equal to 5.

With younger children, you can play this game with counting cards from 0 to 8 (or whatever the limits are) placed face up in order on the table. Under one of the cards you hide a star or whatever is fun. After each guess, the child (Questioner) turns over all the cards that have been eliminated until the star is discovered.

— GOING UP —
GAME

Use one deck (two if there are several players) of cards that go from 0 to 20. Each player is dealt four cards face up in the order they are dealt, and the remaining cards form a draw pile.

You can either play that a player must select one of their existing cards in advance or, the friendlier version, that the card can be selected after seeing the new card. Either way, one of their cards is replaced by the top card of the pile and the replaced card is put on the draw pile bottom.

The first player to get their cards in order wins. Make this easier by allowing duplicate cards. Make this harder by using more cards in a hand.

— GOING UP SOME MORE —
GAME

Each player draws a staircase with anywhere from 4 to 10 steps (more steps for older players).
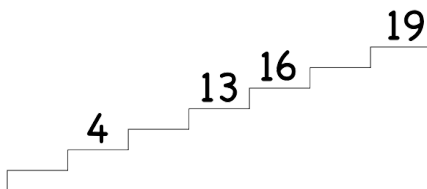
Create a number using Number Cards numbered from 0 to 9. The first selection will be from the cards 0 to 1, and this is the tens digit. Then select a ones digit from all the cards. The two digits together form the number. If possible, the player must place this number on their steps. The first player to fill their steps in increasing order from bottom to top wins. Make it easier by allowing neighboring steps to have the same value.

— CREATING ORDER —
GAME

Start with a deck of Number Cards that go from 0 to 40 (higher with more players). Deal ten cards to each player (fewer cards for younger players). The dealt cards are placed in front of each player in the order they are dealt. The remaining cards are placed face down in a draw pile. The top card is placed facing up as the first card in a discard pile. The goal of the game is to get the cards in ascending order from left to right.
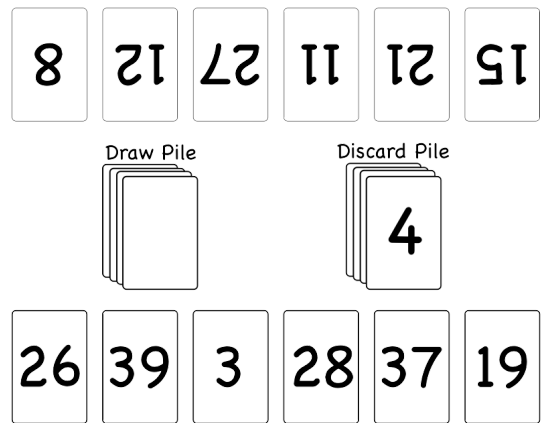
During a turn, a player may select either the top card of the discard pile or the draw pile - this card must be used to replace one of the cards in front of the player, and the replaced card is put face up on the top of the discard pile.

You can play that the first player to get their cards in order wins, and that's all there is to it. Or, you can use a point system for each round of play. In this system, the winner receives 15 points. The other players receive one point for every card they have in order starting with their lowest card.

Comparing Double-Digit Numbers - 2

— GET CLOSEST —

GAME

Write the numbers 5, 10, 25, and 50 vertically on a piece of paper. Put a single blank space on each side of the 5, and two blank spaces on each side of the other numbers. One player fills in the blanks on the left side and the other fills the other side. Each player also has one extra blank to use once with a number to ignore.

Play with a deck of Number Cards from 0 to 9. Randomly pick a card from the deck, and put it back in after it is used. Both players must use that number somewhere in the spaces that haven't been filled in yet. Once all the spaces are filled, the player's values are compared to each of the target numbers. Whichever player gets closest to each target number gets a point, with both players getting a point if they are equally close. Whoever has the most points wins.

Vary this game by having a different set of target numbers. You can also choose to score the game by summing up all the errors for each player. For this, the player with the smaller score wins.

— WAR – DOUBLE-DIGIT COMPARISON —

GAME

Shuffle a deck of playing cards with the face cards and tens removed, and split it evenly between two players. Each player turns over two cards and puts them side by side to form a two-digit number. The player with the larger number keeps all four cards. If there is a tie, each player turns over two more cards with the winner getting all eight cards. After one or more passes through the cards, the player with the most cards wins.

— MATH BLACKJACK —

GAME

Traditionally, the target number is 21, but for a young child use a smaller number such as 12. Adjust the contents of the playing cards for your child. For example, for a very young child this might be the cards 1 to 4 in the four suits.

Two cards are dealt to each player - one is face up and one is face down (the receiving player is the only one to look at the face down card). During a turn, the player has the option of asking for one more card until the player decides to stop. After every player has had a turn, the players compare the sum of their cards. The player with the sum closest to the target without going over wins.

— FILL IN THE BLANKS COMPARISON —

GAME

Shuffle a deck of playing cards with the numbers 1 to 9. Deal two cards to each player face down. Then, each player turns over one card and decides whether that card will be the tens or ones card. After deciding, each player's remaining card is turned over and is used to fill the remaining place. The player with the larger number wins. You can also play that the smaller number wins. You can decide whether it is more dramatic to show the cards as they are turned over, or wait until all the decisions are made and the final numbers are formed.

To practice a bit of addition, as well as making the decisions trickier, draw three cards to turn over one at a time to form a two-digit number and a single-digit number. The goal is to create the largest sum of the two numbers.

Adding - 1

— NIM WITH ONE AND TWO —
GAME

A target number, say 10, is chosen. Let your child choose whether to go first or second. The total starts at 0. During a turn, a person chooses to add 1 or 2 to the current total. The first person to reach the target wins. Once children learn to play this verbally, it is a great travel game.

For younger children, use an actual pile of objects to play with instead of writing anything down. Similarly, a number line to the target number could be used and a marker could be advanced along the line one or two spaces during each move.
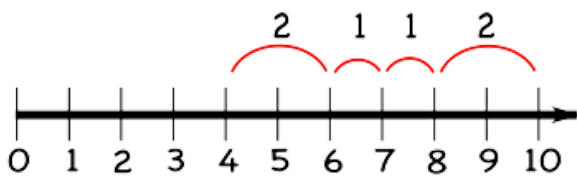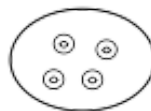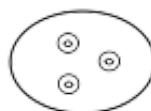

This game can also be played with subtraction. In this version, the starting total starts at the target, which in this example is 10. On a given turn the player chooses whether to subtract 1 or 2. The first person to reach 0 wins.

There are many variations to this popular game. You can use larger target numbers as your child's skills improve. One variation is that instead of winning, the player forced to hit or go beyond the target number loses. You can also experiment with what happens if you allow a player to add (or subtract) 1, 2, or 3 for each turn.

— BAG GAME – ADD AND SUBTRACT —
ACTIVITY

You and your child each pretend to have a bag with some number of things. One person creates a story such as: "Your bag has 3 raisins and mine has one more. How many do I have?" After your child gets comfortable with this, let your child come up with the question some of the time - this is often great fun for them, especially if you make the occasional "mistake."

These stories can get more elaborate with experience. For example, the story could be "I have two fewer cookies than you do, and together we have six cookies. How many cookies do you have?" Another example, "You have twice as many candies as I do, and together we have nine pieces. How many pieces do you have?"

— MYSTERY CHANGE —
ACTIVITY

Have your child count some small number of objects. While they look away, change the number of objects. When they look back, ask what change you made. They can test their theory by reenacting what they think happened.

Once this becomes easy, you can have them be more creative with their answers. For example, if 4 became 6, the answer might be that you doubled the 4 and then took 2 away.

Adding - 2

— HIT THE TARGET —
GAME

From a deck of playing cards, remove the face cards and any numbers that are larger than what the children are comfortable with. Shuffle the cards and turn over five cards to use and a sixth card that is the target. Leave the remaining cards as a draw pile to fill in cards as they get removed.

During a turn, if a player can use the sum or difference of two of the five cards to equal the sixth, the player gets all three cards and they are replaced from the draw pile. If the player fails, then a new sixth card is turned over and the turn moves to the next player. The player with the most cards at the end of the game wins.

— MATH BLACKJACK —
GAME

Traditionally, the target number is 21, but for a young child use a smaller number such as 12. Adjust the contents of the playing cards for your child. For example, for a very young child this might be the cards 1 to 4 in the four suits.

Two cards are dealt to each player - one is face up and one is face down (the receiving player is the only one to look at the face down card). During a turn, the player has the option of asking for one more card until the player decides to stop. After every player has had a turn, the players compare the sum of their cards. The player with the sum closest to the target without going over wins.

— PAIRING DOWN —
GAME

Start with a number line that goes from 1 to some number, say 20. During a turn, choose two numbers and a result, none of which have been crossed out, and write down an addition or subtraction equation that involves those numbers. The two numbers in the equation are crossed off, and the result is circled.

The next player must use the result as one of the two numbers. If it is played competitively, the winner is the last player with a legal move. It can also be played cooperatively to see how few numbers are left untouched.

$$9 + 11 = 20 \qquad 20 - 19 = 1$$
$$1 + 12 = 13 \qquad 13 + 5 = 18$$
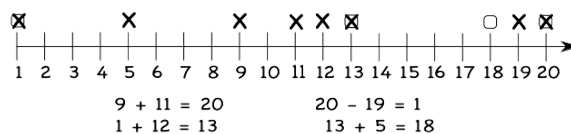
Adding - Subtracting - 1

— FINGER ADDITION W/ COMPENSATION —

Use compensation for sums between 11 and 18 to make them much easier. Suppose you're adding 7 + 8. One person puts up 7 fingers and the other person puts up 8 fingers. Then, one person gives away as many fingers as are needed to put up 10 fingers on the other person's hands. In this example, 7 + 8 could turn into 5 + 10 (giving away 2) or 10 + 5 (giving away 3).

Be dramatic and give away the fingers by having one person's hand bang into the other person's hand and having the fingers transfer "magically."

— FINGER ADDITION THE EASY WAY —

Use "counting on" to make addition easy. Suppose you're adding 8 + 3. For ease, pick the larger number being added to start the counting on. Have your child make a closed fist and say "8." Then, lift one more finger each time as your child counts out loud "9, 10, 11." When 3 fingers are raised, the counting stops. At that point, you have that 8 plus 3 is 11.
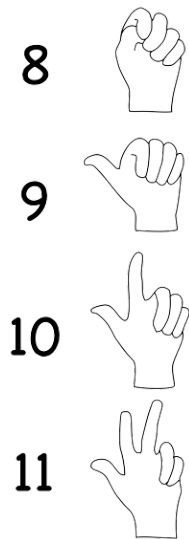
With practice and further ideas, your child will memorize these math facts. However, there is no hurry for memorization and it can wait until more experience with the quantities and relationships between the numbers has been gained.

— FINGER SUBTRACTION THE EASY WAY —

Subtraction can be thought of as "take away" or "difference," and both models are essential to a complete understanding. Have your child practice both ways of thinking of subtraction using these finger subtraction methods.
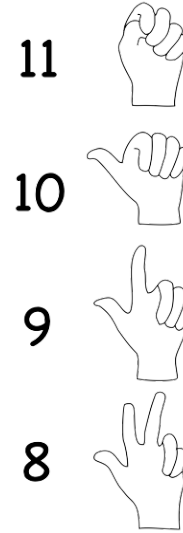
We'll look at 11 - 3.

Take Away: Start with a closed fist and say "11." Then, lifting one more finger each time, count "10, 9, 8." When your child sees 3 fingers raised, the counting should stop. At that point you have that taking 3 away from 11 leaves 8.

Difference: This uses counting on, much as we did for Easy Way Finger Addition. What we are doing is finding out which number we need to add to 3 to get 11. Have your child make a closed fist and say "3." Then, lifting one more finger each time, count "4, 5, 6, 7, 8, 9, 10, 11." When your child says 11, there are 8 fingers raised - the difference between 3 and 11 is 8!
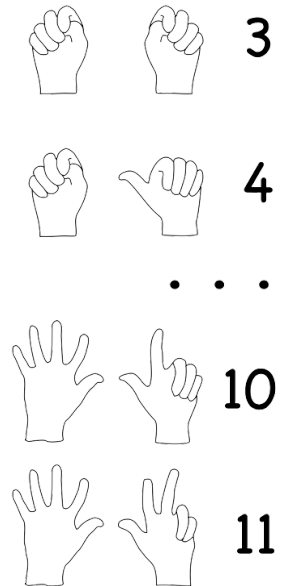
— FLASH CARDS —

The ease of practicing math facts using flash cards makes them tempting. However, they are often abused by well-meaning helpers and can contribute to math hatred. Beyond the psychological damage that occurs with overly-enthusiastic drill, using flash cards misses out on learning important relationships between numbers. Feel free to use flash cards to give focused practice for a small handful of facts, but please keep the practice very relaxed.

This page has methods that practice structural insights until the facts become automatic.

— REVIEWING ADDITION SKILLS —

To do the activities on this page your child should know how to mentally do the following:

- add and subtract 0, 1, 2 (and perhaps 3)
- do adding twins and near twins
- know the number bonds for 10
- add 10 to single-digit numbers.

If your child is weak with any of these skills, this is the time to practice those skills some more.

— ADDITION COMPENSATION —

Compensation is a powerful technique for making mental math easier. When adding two numbers, you can get the same sum by shifting over part of one number to the other. Adding 8 or 9 is easy using compensation. For example, add 6 + 9 by shifting 1 from the 6 to the 9, which gives 5 + 10. Similarly, 4 + 8 becomes 2 + 10.

Use compensation from twins and near twins for what remains: 3 + 5, 3 + 6, 4 + 7, and 5 + 7. For example, 5 + 7 is the same as 6 + 6.

Some math facts can be done several ways. Challenge your child to find more than one way to do a problem. For example, 5 + 7 can become 6 + 6, but it can also become 2 + 10. This kind of math play will lead to lasting insights.

— REVIEWING SUBTRACTION SKILLS —

Before starting these subtraction activities, practice any of the following skills that happen to be weak for your child:

- add and subtract 0, 1, 2 (and perhaps 3)
- subtract numbers 1 or 2 apart
- know the number bonds for 10 and how they make subtracting from 10 easy
- subtract 10 from numbers from 11 to 19.

— USING 10 AS AN INTERMEDIATE STOP —

For problems with numbers larger than 10, such as 13 - 8, break them into two differences. The distance from 13 to 8 is the distance from 13 to 10 plus the distance from 10 to 8. So, 13 - 8 becomes (13 - 10) + (10 - 8) = 3 + 2 = 5.

— SUBTRACTION COMPENSATION —

Compensation for subtraction means adding or subtracting the same amount for both numbers to maintain their distance. Use compensation on 13 - 8 by adding 2 to both numbers to turn the problem into 15 - 10. Single-digit problems can also be done this way. For example, 3 can be added to both numbers in 7 - 3 to make it 10 - 6.

Adding - Subtracting - 3

— WAR – ADDITION AND SUBTRACTION —

GAME

Evenly split a shuffled deck of playing cards with the face cards removed. If you like, use dominoes instead. Both players turn over their top two cards and add them. The player with the larger sum wins all four cards. If the sums are equal, the next two pairs of cards are added and the winner gets all eight cards. Play this with a single pass through the deck or multiple passes. Either way, the winner is the player who has the most cards.

For variety, play this using the difference of the two cards. Or, you can add three cards at a time. Another option is to assign one person to be Odd and the other Even. For this, each player turns over a card and the evenness or oddness of the sum determines who gets the cards.

— SUM DIFFERENCE —

ACTIVITY

One person gives two numbers, one a sum and the other a difference, and the other person is challenged to find the two numbers that have that sum and difference. For example, if one person says the sum is 12 and the difference is 6, the other person says the numbers are 3 and 9.

Because of how easy it is to create these questions, this is a good activity to let your child be the questioner. Not all combinations of numbers for the sum and difference will produce reasonable answers. If you start with two numbers and then say their sum and difference, that will guarantee that there is an answer.

Challenge an older child with the question of why some sums and differences have reasonable answers and others do not.

— MEMORY CHALLENGE REVISITED —

GAME

There are many versions of this game. The idea is always the same: deal a grid of cards face down, say 4 by 4, and the players take turns turning over two cards. If the cards match, the player keeps the cards, two more cards are dealt into the empty spaces, and the player gets another turn. If the cards don't match, the cards are turned back over and the player's turn ends. The player with the most cards wins.

Here are other ideas for how cards can match:

- Use a target sum - two cards match if their sum is the target.
- Use a target difference - two cards match if their difference is the target.
- Use cards with addition or subtraction problems together with cards that have the answers - cards match if the problem matches the answer.

Number Bonds

— GO FISH WITH SUMS —
GAME

Using a target sum that your child is comfortable with, remove cards at or above that target from the Number Card deck. If there are more than two players and you have restricted the numbers significantly, you may need to use several decks.

Play starts by dealing 5 cards to each player. Put the remaining cards in a common draw pile. Players take turns "fishing" for cards whose number will sum to the target sum with cards they already have.

For example, a player might ask a player, "Do you have any 4's?" If that player has some 4's, they are handed over and the original player gets another turn. However, if that player does not have any 4's, then the player says "Go Fish!" and a card is drawn from the draw pile. If the drawn card matches with a card they have, the player may have another turn; otherwise, the turn is over and play continues to the left.

When a player has a pair of cards that sum to the total, the player puts that pair on the table in front of them. The game is over when all cards are in pairs. The player with the most pairs wins.

To create some variety, allow players to use more than two cards to create a group of cards that sum to the target sum. Another possibility is to say that two cards match when their difference is a specified target difference.

— TARGET GIN RUMMY —
GAME

Agree on a target sum, say 10. Remove the face cards and deal seven cards to each player. The remaining cards become a draw pile, and its top card is flipped over to start the discard pile. The goal is to hold seven cards that are broken into separate groups of one or more cards that add up to the target. During a turn, the player has the choice of picking up the top card of the discard pile or the unseen card at the top of the draw pile. That player then discards a card. When a player successfully fills their whole hand, the player lays down the hand and says "Gin!"

Taking the difference with pairs of cards can be used instead of addition. In that case, deal an even number of cards to each player.

— PART-WHOLE TRIANGLES —
GAME

Use number cards 1 to 13. To start, each player is dealt 6 cards face up. There is a draw pile with one card turned over to start a discard pile.

During a turn, a player takes the top discard card or the unknown card at the top of the draw pile. The chosen card must replace a card they already have, and the replaced card is discarded. The goal for each player is to create a 6-card pyramid where each card is the sum of the two below it. The first player to achieve this wins.

Experiment with smaller or larger ranges of cards to accommodate the skills of younger or older players.

Multiplying - Factoring - 1

— INTRODUCTION —

These teaching methods provide structured strategies for learning single-digit multiplication. As your child practices these strategies they will be learning important numerical relationships, and they will also eventually memorize these facts. Your child should already be good with doubling and skip counting by any number.

— 3 x 4 = 4 x 3 —

By now, your child is so familiar with addition that it's no surprise that 2 + 3 is the same as 3 + 2.
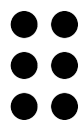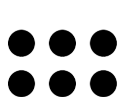
Although not as obvious, the same is true for multiplication. This illustration makes it easy to see that two rows of three is the same as three rows of two – you just change your point of view! It doesn't matter which order you multiply two numbers – you get the same answer either way!

In addition to being cool in a nerdy way, it also means that your child needs to master only about half as many multiplication facts – once your child knows 3 x 4, they also know 4 x 3.

— SKIP COUNTING IS MULTIPLICATION —

Skip counting is great for getting better at addition and subtraction. It also is a big help in getting started with multiplication.

Although skip counting is not the fastest way to find a result, it is reliable and your child should use it as long as they need to. Suppose your child needs to find 7 x 3. Skip count by 3's seven times or skip count by 7's three times to get 21.

— MULTIPLYING BY 5 AND 10 —

These facts are quickly learned, provide a backbone for other multiplication facts, and they help with understanding two-digit place value.

— SQUARES —

Just as addition twins are favorite math facts for addition, squares are often favorites for multiplication. Learning these provides another foundation for learning other multiplication facts.

— DOUBLING —

Use this strategy for multiplying by even numbers. For example, the result of 6 x 7 is the double of 3 x 7. So, 6 x 7 is the double of 21, which is 42.

— 1 MORE OR 1 LESS —

This strategy is effective for the remaining multiplication facts.

For example, 9 x 7 is one 7 less than 10 x 7 = 70. So it is 70 - 7 = 63. This can be done for all the 9's.

Similarly, 3 x 7 is one more 7 than doubling 7, so it is 7 + 14 = 21. This can be done for all the 3's.

— MULTIPLYING BY 9 —

Although multiplying by 9 is covered by the last strategy, they are fun to learn in their own right. If you write out the multiples of 9 in order, you'll see that the tens digit is always one less than the number you are multiplying by and the ones digit plus the tens digit always adds up to 9!

Multiplying - Factoring - 2

— SHORT STEPS TO A TARGET —
GAME

Create a random 2-digit starting number using two playing cards from 1 to 9 - the first card will give you the tens place and the other the ones place. Starting at that number, you are allowed to use a total of 5 jumps each of which is either by 1 or 10. The goal is to get as close to 50 as possible, and the score is the difference from 50. The lowest total score after several rounds wins.

Young players benefit from referring to a 100-chart. Using that chart will also emphasize place value as they go up or down by 10. You can vary this game by allowing steps of 1, 2 or 10, or steps of 1, 2, 5, or 10. Also, use target numbers other than 50 sometimes.

— COVER FACTORS AND MULTIPLES —
GAME

Have a board of numbers from 1 to 30. There are two kinds of tokens – a single token reserved for "the last move," and a pile of other tokens.

The first player gets to pick any number and cover it with the last move token. After that, a player replaces the last move token with the other type of token and moves the last move token to any number that is a factor or multiple of the number from the last move. The losing player is the one forced to cover the number 1.

As children get better at this game, they will discover rules to follow governing reasonable first moves. The most basic rule is that the first move cannot be on a prime number in the upper half of the numbers. Adjust the range of numbers for the skill level of the players - you can use 1 to 24, 1 to 48, or even 1 to 60.

— REVEALING PRODUCTS —
PUZZLE

Start with a blank multiplication table that has 4 product rows and 4 product columns. There are also groups of four missing numbers at the top and left sides – these will have some of the numbers from 2 to 9, and these numbers can be duplicated.

Fill in the table out of sight of your child, and then flip over or cover all the numbers. Your child can ask to reveal, one at a time, up to 10 of the 16 product entries. The goal is to figure out the entries for the top and left sides before running out of turns.

X	5	3	7	8
2	10	6	14	16
9	45	27	63	72
8	40	24	56	64
5	25	15	35	40

Imagine that all the cards were flipped over in this example. If your child chose to flip over the card that happened to have the 63 under it, then they would know it came from a 7 and a 9. Flipping over any other card in the same row or column as the 63 would indicate where the 7 and 9 are. Suppose the second card they flipped over was where the 56 is. Not only would they know that the third column was for 7, they would also know that the second row was for 9 and the third row was for 8.

Larger sizes also work. For example, a table with 5 blank rows and columns that allows for up to 12 flips will work well. Pick numbers for the top and left sides that you want your child to practice with.

Multiplying - Factoring - 3

— WAR – MULTIPLICATION —
GAME

Remove the picture cards from a deck and split it evenly between two players. To give more focused practice, remove the A's and 10's as well.

Each player turns over two cards, multiplies them, and the player with the larger product wins those four cards. If the products are equal, two more cards are turned over and the winner gets to keep all eight cards. The player with the most cards after playing for a set time is the winner.

— BEEP —
GAME

Start by identifying a group of numbers to use for the round. It can be odd numbers, or multiples of 3 together with numbers that have a 3 in them, or any group that provides good practice.

Two or more players take turns saying the numbers starting at 1. When a player has a number in the group, they must say "beep." If they fail to say beep, or say beep for a wrong number, they're out. The last player in wins!

— TURNING THE TABLES —
PUZZLE

Filling in a standard multiplication table is boring, and children quickly realize they can fill it out using addition rather than multiplication. To really practice multiplication, as well as practicing problem solving and factoring, create a mixed-up multiplication table for your child.

Make these tables by moving the rows and columns around, and leaving out most of the headings and entries in the middle. Here is an example using headings of 2 through 9:

X	5				6			
2								
		40						
				49				
	20				36			
		72						
			9					12
				48				

Solve this by starting with the distinctive entries. The 20 forces its row to be multiplying by 4, and then the 36 makes its column be 9. The 49 forces its column and row to be multiplying by 7. The 9 forces its column and row to be multiplying by 3. Now the 12 must be in the column for 4. Continue the detective work in this way and fill in the entries as the headings are discovered.

— GRABBING FACTORS —
GAME

Use a board with a 4 by 6 grid of numbers from 1 to 24. On a turn, a player chooses a number that is uncovered and has at least one factor uncovered – the player gets the selected number and the other player gets any or all of the uncovered factors (their choice as to how many). Play alternates until there are no legal numbers left. The players add up their numbers and the higher sum wins.

This can also be played as a solitaire puzzle, sometimes called Taxman. In this version, the one player selects each number and the taxman gets all the available factors. Play continues until the player no longer has a legal move - at that point, the taxman receives the remaining numbers. The goal is to have as large a sum as possible - bigger than the taxman when that's possible.

Make the range of numbers suit the ability of the players - it could be 1 to 12 or as high as 1 to 60.

— PRACTICE PRIME FACTORIZATIONS —
ACTIVITY

An excellent thing to practice when you are travelling and have time on your hands is to recite the prime factorizations for the numbers in order. This also provides practice with talking about powers. Knowing prime factorizations with ease will be helpful in many things to come, such as working with fractions. Have fun with this and don't push your child beyond their comfort level.

It goes like this: 1 is a unit, 2 is a prime, 3 is a prime, 4 is 2 squared, 5 is a prime, 6 is 2 x 3, 7 is a prime, 8 is 2 cubed, 9 is 3 squared, 10 is 2 x 5, 11 is a prime, 12 is 2 squared x 3, 13 is a prime, 14 is 2 x 7, 15 is 3 x 5, and 16 is 2 to the fourth. If your child stumbles, help them figure it out rather than simply reminding them of the answer.

— THE PRODUCT GAME —
GAME

Use a shared piece of paper filled out as follows:

1	2	3	4	5	6
7	8	9	10	12	14
15	16	18	20	21	24
25	27	28	30	32	35
36	40	42	45	48	49
54	56	63	64	72	81

1	2	3	4	5	6	7	8	9

The first player moves a token onto any number from 1 to 9 in the 1-9 squares. The second player puts another token on one of the 1-9 squares and claims the product in the 6 by 6 grid. From then on, each player chooses to move either of the two tokens and claims the product (if they can). The first player to claim 3 squares in a row wins.

Mix up the product numbers to give your child better practice identifying the products. See the Chapter 5 Bonus Material for designs of larger boards with larger ranges.

Mixed Operations

— SECRET OPS —
ACTIVITY

Near the end of Chapter 4, the Sum Difference activity had one person think of two numbers and then challenge the other person to find the numbers by telling them the sum and difference of the numbers. Secret Ops uses the same idea, only now the challenger can use any two operations, such as multiplying and subtracting.

For example, the challenger might say "Which two numbers have a product of 12 and a difference of 4?" You can extend this to three numbers, if you like - "Which three numbers have a product of 12 and a sum of 8?"

— COUNTING NEIGHBORS —
GAME

Use three dice and an 8 by 8 board of numbers from 1 to 64. A player rolls the dice and uses addition, subtraction, multiplication, and division to make any unmarked number on the board. The player marks this square and receives one point for the square plus one more point for each marked square that it touches, including diagonally. If a player cannot make a play, any other player who finds a play can claim that score. Play five or more rounds, with the largest score winning.

Some game options are to use a fourth die, and to use a smaller or larger board.

— LIMITED CALCULATORS —
PUZZLE

The premise is that you have a calculator that is badly broken and you are challenged to produce some result on the calculator. This is easy to play orally whenever you have a spare moment. Here are some examples to get you started..

Suppose you had a calculator with +, -, x, and /, but only one working number key, the 4. Could you get the result 21? If so, what is the fewest number of steps you would need? Suppose you could use 4 at most four times - which numbers could you produce? Suppose you had to use the 4 exactly four times. Play around with having other single keys and creating other results.

Suppose your calculator could only add 4 or 7. Which numbers could you produce? Suppose it had 4 or 7, but now it can add and subtract. Which numbers could you produce? This is the same activity we've seen in other settings.

Suppose you only had a 1 key and could only add or double. For example, 2 x (2 x 1) + 1 is 5. What other numbers can you create?

— PARENTHESES PUZZLES —
PUZZLE

These puzzles are very easy for an adult to create. Take any equation, such as 9 = (2 + 7) x (5 - 2 x 2) and remove the parentheses. The challenge for your child is how to take 2 + 7 x 5 - 2 x 2 and add parentheses to it so that the result is 9.

Adding - Subtracting - Double Digit

— MISSING NUMBERS —
PUZZLE

Create these by taking a simple addition or subtraction equation and leaving out some of the digits. For example, the following two problems are turned into Missing Number puzzles by leaving out a couple digits.

$$
\begin{array}{r} 23 \\ +46 \\ \hline 69 \end{array} \quad
\begin{array}{r} 73 \\ -46 \\ \hline 27 \end{array} \quad \dashrightarrow \quad
\begin{array}{r} \_3 \\ +46 \\ \hline 6\_ \end{array} \quad
\begin{array}{r} 7\_ \\ -\_6 \\ \hline 27 \end{array}
$$

After getting comfortable with these, your child may enjoy doing some Letter Substitution puzzles that are described on a later Adding and Subtracting page in this chapter.

— 5-CARD DRAW TO A TARGET —
GAME

Choose a target number, say 100. Each player picks up five random cards from 0 to 9. Two 2-digit numbers are made out of these numbers, the fifth card is unused. The two numbers are added and the player closest to the target wins a point for that round. The highest number of points after a fixed number of rounds wins.

One option is to use three-digit numbers, a target number of 1000, and each player receives seven cards. Another option is to use subtraction with a smaller target number.

— LETTER SUBSTITUTION —
PUZZLE

Once your child becomes comfortable with the Missing Number puzzles from a few pages earlier in this chapter, they can start these puzzles. In these, one or more of the digits are replaced by letters. The three rules for letters are:

- A given letter is always the same digit
- The leftmost digit of a number is never 0
- Different letters must be different digits

Create these puzzles by taking an addition or subtraction problem and replacing one or more of the digits, as in the following examples:

$$
\begin{array}{r} 23 \\ +46 \\ \hline 69 \end{array} \quad \dashrightarrow \quad
\begin{array}{r} 23 \\ +4A \\ \hline A9 \end{array} \quad
\begin{array}{r} B3 \\ +4A \\ \hline A9 \end{array}
$$

The puzzles can also be created to make interesting problem-solving challenges for your child. Note that the values of the letters do not carry over from puzzle to puzzle.
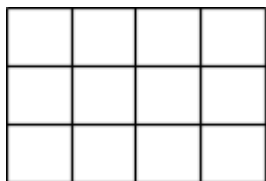
$$
\begin{array}{r} B \\ +8 \\ \hline C \end{array} \quad
\begin{array}{r} B \\ +B \\ \hline 8 \end{array} \quad
\begin{array}{r} A \\ +A \\ \hline C4 \end{array} \quad
\begin{array}{r} A \\ +2 \\ \hline BC \end{array}
$$

$$
\begin{array}{r} A \\ +B \\ \hline AC \end{array} \quad
\begin{array}{r} A \\ +BB \\ \hline A7 \end{array} \quad
\begin{array}{r} B \\ +AB \\ \hline BA \end{array} \quad
\begin{array}{r} BA \\ +BB \\ \hline CAB \end{array}
$$

Rectangle Area

— INTRODUCTION —

The area of a rectangle is its length times its width. That dry statement can be made tangible to your child in at least two ways. The first is to show a rectangle broken into a collection of squares. The second is to use number shapes to show how a quantity, such as 12, can be placed into an array - 3 by 4, 2 by 6, or 1 by 12. Playing with rectangle areas gives us a fun arena in which to play around with multiplication and factoring!

— NUMBER SHAPES REVISITED —

INVESTIGATION

Start with a large collection of small objects, such as raisins. For each number, investigate which rectangles you can make with that many objects. 1 can only be made with a 1 by 1 rectangle, and 1 is called a *unit*. The numbers, such as 5, that only have 1 by 5 and 5 by 1 rectangles, are called *primes*. Numbers that are not a unit or prime are called *composite* - they are called that because they are composed of primes being multiplied together, such as 12 = 2 x 2 x 3.

The dimensions of each rectangle are made of values that evenly divide the number and multiply together to give the number. Making rectangles is a direct way to experience divisibility. Numbers such as 16 are called *squares* because one of their rectangles is a square – one rectangle for 16 is the 4 by 4 square.

— THE PADDOCK GAME —

GAME

Each player gets a piece of graph paper. For a player's turn, use two playing cards from 1 to 10 to determine the dimensions of a rectangle. If a player's paper has room, the rectangle may be placed anywhere its interior does not overlap with an existing rectangle. Once placed, its interior is lightly shaded and its area and dimensions are written on it. If there is no room, the turn is skipped. The player with the largest total wins. For a normal piece of graph paper, this can be a long game – reduce the time by using half the paper or limiting the number of turns.

— DIVIDE UP THE BOX —

PUZZLE

A rectangle, 4 by 4 or larger, with numbers in some of its squares, is to be divided into smaller rectangles. Each number must end up in a separate rectangle whose area is that number.

Out of the sight of your child, create these puzzles by first filling in the big rectangle with smaller rectangles. Next, place the area in each rectangle. Lastly, give your child the big rectangle with only the numbers.

To solve, look first at areas that are prime numbers. Also, sometimes an area is boxed in – in this puzzle the upper "4" must relate to the upper left 2 by 2 square. Next, the upper right corner must be used in a vertical 3 by 1 rectangle. Keep going!