

St. Maurice's High School



# Numeracy Across The Curriculum

A staff guide to how the Maths department teach certain topics in Numeracy.



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# 1. Basic Calculations

Vocabulary:

<u>Addition (+)</u>	<u>Subtraction (-)</u>	<u>Multiplication (×)</u>	<u>Division (÷)</u>	<u>Equals (=)</u>
Sum of	Less than	Multiply	Divide	Is equal to
More than	Take away	Times	Share	Same as
Add	Minus	Product	Split	Makes
Total	Subtract	Lots of	Groups of	Will be
And	Difference	Sets of		
Plus	Between			

## Mental Agility

Mental Agility is when pupils are required to work out the answer in their head without doing any written working. This is practiced regularly within the BGE often as starter questions and pupils undertake a Mental Agility assessment once per term.

Pupils are encouraged to use various strategies for this type of questions, for example:

### Partitioning

$$25 + 47 = 25 + 40 = 65 + 7 = 72 \quad \text{Tens or ones first}$$

$$276 - 153 = 276 - 100 = 176 - 50 = 126 - 3 = 123$$

### Re-ordering

$$43 + 18 + 17 = 43 + 17 + 18 = 60 + 18 = 78$$

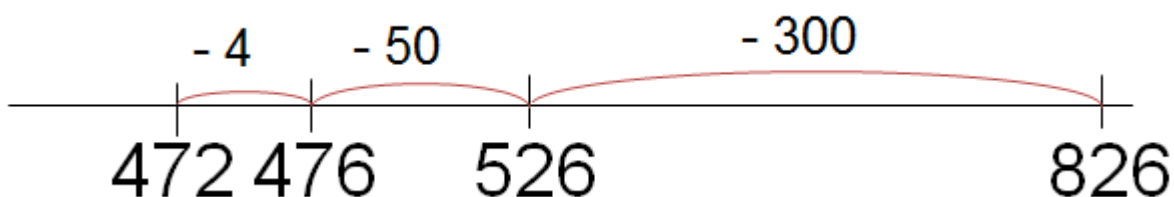
### Compensation

$$37 + 19 = 37 + 20 - 1 = 56$$

$$184 - 28 = 184 - 30 + 2 = 156$$

### Number Line

$$826 - 354 = \quad \text{Good strategy for visual learners}$$



# Addition

For written **ADDITION** calculations carried out without a calculator, the method below is used throughout the maths department.

Example 1

Add  $487 + 729$

$$\begin{array}{r} 487 \\ + 729 \\ \hline 1216 \end{array}$$

The calculation begins with the UNITS column on the far right, and any tens are carried to the next column. This process is repeated for the hundreds column (and any other columns) until the calculation has been completed.

Instead of showing a sum like the one above, pupils can work out  $487 + 700 + 20 + 9$  by showing jottings (rough steps). We would not expect this calculation to be worked out mentally.

Example 2

Add  $56.82 + 39.6$

$$\begin{array}{r} 56.82 \\ + 39.60 \\ \hline 96.42 \end{array}$$

The calculation begins with the HUNDREDTHS column on the far right, and any tenths are carried to the next column. This process is repeated for the tenths, units and tens (and any other columns) until the calculation has been completed. Pupils would be encouraged to keep the decimal point in a separate column to ensure the digits were in the correct place. Depending on the ability of the class, some pupils may be encouraged to add the zero after the 6 to help with this as well.

Other methods are possible and pupils are encouraged to use the method which they feel most confident with.



# Subtraction

For written **SUBTRACTION** calculations carried out without a calculator, the method below – called **DECOMPOSITION** – is used throughout the maths department.

Example 1

Subtract 257 from 821

$$\begin{array}{r} 7 \quad 11 \quad 1 \\ \cancel{8} \quad \cancel{2} \quad 1 \\ - 2 \quad 5 \quad 7 \\ \hline 5 \quad 6 \quad 4 \end{array}$$

Example 2

Subtract 23.7 from 76.19

$$\begin{array}{r} 7 \quad 5 \quad \cancel{6} \quad \cdot \quad 1 \quad 1 \quad 9 \\ - 2 \quad 3 \quad \cdot \quad 7 \quad 0 \\ \hline 5 \quad 2 \quad \cdot \quad 4 \quad 9 \end{array}$$

Starting from the right hand column, the pupil carries out the calculation “top minus bottom”. If the calculation would not result in a digit which is positive, then the calculation “cannot be done” without borrowing a TEN from the next column. This process is repeated until all the possible calculations have been carried out. This process is also true for decimal calculations. Pupils would be encouraged to keep the decimal point in a separate column to ensure the digits were in the correct place. Depending on the ability of the class, some pupils may be encouraged to add a zero at the end of the second number to help with this.

When Is This Taught?

All pupils are given regular opportunities to practise this basic skill at all levels of study. Naturally, there is considerable prior knowledge of this when pupils begin S1. As with addition, some pupils – usually but not always those of lower ability – refer to these types of calculations as “chimney sums”.



# Multiplication

Times tables - Pupils are required to learn these, and practise regularly, for all mathematics courses.

For written **MULTIPLICATION** calculations carried out without a calculator, the methods below are used throughout the maths department to help pupils develop a variety of approaches to non-calculator multiplication.

Example 1                     $39 \times 6$

Method 1

$$\begin{array}{r}
 39 \\
 \times 6 \\
 \hline
 5 \\
 234
 \end{array}$$

Method 2

$$\begin{aligned}
 30 \times 6 &= 180 \\
 9 \times 6 &= 54 \\
 \text{Therefore } 39 \times 6 &= 234 \text{ (180 + 54)}
 \end{aligned}$$

Method 3 (mental strategy)

$$\begin{aligned}
 40 \times 6 &= 240 \\
 40 \text{ is } 1 \text{ too many so take } 6 \times 1 &\text{ away} \\
 \text{Therefore } 39 \times 6 &= 234 \text{ (240 - 6)}
 \end{aligned}$$

Example 2

Long multiplication is a more complex skill. It has been taught at Primary school and is reinforced during Secondary stages.

$$\begin{array}{r}
 \phantom{+} \phantom{1} \phantom{8} \phantom{3} \phantom{0} \\
 \phantom{+} \phantom{1} \phantom{8} \phantom{3} \phantom{0} \\
 \phantom{+} \phantom{1} \phantom{8} \phantom{3} \phantom{0} \\
 \phantom{+} \phantom{1} \phantom{8} \phantom{3} \phantom{0} \\
 \hline
 + \phantom{1} \phantom{8} \phantom{3} \phantom{0} \\
 \phantom{+} \phantom{1} \phantom{8} \phantom{3} \phantom{0} \\
 \hline
 2 \phantom{3} \phantom{7} \phantom{9}
 \end{array}
 \begin{array}{l}
 6 \phantom{1} \\
 \times 3 \phantom{9} \\
 \hline
 5 \phantom{4} \phantom{9} \leftarrow \times 9 \\
 \phantom{5} \phantom{4} \phantom{9} \phantom{0} \leftarrow \times 30 \\
 \hline
 \phantom{5} \phantom{4} \phantom{9} \phantom{0} \phantom{9} \leftarrow \text{add answers}
 \end{array}$$

## Multiplying by multiples of 10 and 100.

Pupils are taught that to multiply by 10 every **digit** is moved one place to the left so the point moves one place to the right.

To multiply by 100 every **digit** is moved two places to the left so the point moves two places to the right.

It can be easier for pupils to remember that the point moves however to know that it is mathematically correct to move the digits.

### Example 1

a) Multiply 354 by 10


Th	H	T	U
	3	5	4
3	5	4	0

Every digit moves up one place and a zero is added if necessary.

b) Multiply 50.6 by 100

Th	H	T	U	.	t
		5	0	.	6
5	0	6	0	.	0

All digits are moved 2 places to the left hence the point moves 2 places to the right.

$$50.6 \times 100 = 5060$$


### Example 2

Pupils will also be familiar with this method:

a)  $2.36 \times 20$   
 $2.36 \times 2 = 4.72$   
 $4.72 \times 10 = 47.2$

So,  $2.36 \times 20 = 47.2$

b)  $38.4 \times 50$   
 $38.4 \times 5 = 192$   
 $192 \times 10 = 1920$

So,  $38.4 \times 50 = 1920$



# Division

For written **DIVISION** calculations carried out without a calculator, the methods below are used throughout the maths department.

## Example 1

There are 192 pupils in first year, shared equally between 8 classes.  
How many pupils are in each class?

$$\begin{array}{r} 24 \\ 8 \overline{) 192} \\ \underline{16} \phantom{0} \\ 32 \\ \underline{32} \\ 0 \end{array}$$

There are 24 pupils in each class

## Example 2

Divide 4.74 by 3

$$\begin{array}{r} 1.58 \\ 3 \overline{) 4.74} \\ \underline{3} \phantom{0} \phantom{0} \\ 17 \phantom{0} \\ \underline{15} \phantom{0} \\ 24 \\ \underline{24} \\ 0 \end{array}$$

When dividing a decimal fraction by a whole number, the decimal point must stay in line.

## Example 3

A jug contains 2.2 litres of juice. If it is poured evenly into 8 glasses, how much juice is in each glass?

$$\begin{array}{r} 0.275 \\ 8 \overline{) 2.275} \\ \underline{16} \phantom{0} \phantom{0} \\ 67 \phantom{0} \\ \underline{64} \phantom{0} \\ 35 \\ \underline{32} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

If you have a remainder at the end of a calculation, add a zero onto the end of the decimal and continue with the calculation.

## Long Division

This is *not* part of the Numeracy experiences and outcomes.

Pupils would estimate the answer and then use a calculator to get the exact answer.





# Order of Operations

Pupils will be shown the order in which a series of calculations should be worked out. We use BIDMAS as a memory aid:

**B**rackets

**I**ndices

\*note Indices are powers eg.  $5^2 (= 5 \times 5) = 25$

**D**ivide

**M**ultiply

**A**dd

**S**ubtract

## Examples

1)  $10 + 2 \times 7$     **not  $12 \times 7$**   
=  $10 + 14$   
= 24

2)  $12 - 10 \div 2$     **not  $2 \div 2$**   
=  $12 - 5$   
= 7

3)  $(5 + 4) \times 3$   
=  $9 \times 3$   
= 27

4)  $20 - (1 + 3)^2$   
=  $20 - 4^2$   
=  $20 - 16$   
= 4

## NOTE:

- Any calculations within brackets must be done first
- Any indices are done after brackets
- Multiplication and division have equal priority
- Addition and subtraction have equal priority

## When Is This Taught?

“Order of Operations” is introduced in Primary School then taught within S1 course and reinforced throughout stages. The understanding of BIDMAS is vital for study on algebra.



## 2. Fractions

### Calculating a Fraction of a Quantity

#### METHOD 1 - all pupils are shown this method

Example 1 Calculate  $\frac{1}{3}$  of £18

So  
 $\frac{1}{3}$  of £18  
= £18 ÷ 3  
= £6

Example 2 Calculate  $\frac{2}{3}$  of £18

So  
 $\frac{2}{3}$  of £18  
= 18 ÷ 3 × 2  
= £12

For calculations of this type we may teach the rule

“Divide by the bottom, multiply by the top”

Fractions, Decimals and Percentages are all closely related to each other. Pupils should have the following common conversions memorised:

Percentage	1%	10%	20%	25%	$33\frac{1}{3}\%$	50%	$66\frac{2}{3}\%$	75%	100%
Fraction	$\frac{1}{100}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	1
Decimal	0.01	0.1	0.2	0.25	0.33...	0.5	0.66...	0.75	1



Adding/Subtracting Fractions

Pupils must ensure that fractions have the **same denominator**. Once this has been done, pupils add the numerators and the denominator stays the same.

Example 1

$$\frac{2}{7} + \frac{3}{7}$$

$$= \frac{5}{7}$$

Example 2

$$\frac{1}{3} + \frac{1}{4}$$

$$\begin{matrix} \times 4 & & \times 3 \\ \curvearrowright & & \curvearrowleft \end{matrix}$$

$$= \frac{4}{12} + \frac{3}{12}$$

$$= \frac{7}{12}$$

Example 3

$$\frac{1}{3} + \frac{1}{4}$$

$$\times$$

$$= \frac{4+3}{12}$$

$$= \frac{7}{12}$$

Multiplying Fractions

Multiply the top two numbers and the bottom two numbers to make a new fraction.

$$\frac{3}{2} \times \frac{2}{5}$$

$$= \frac{6}{10}$$

$$\curvearrowright \div 2$$

$$= \frac{3}{5}$$

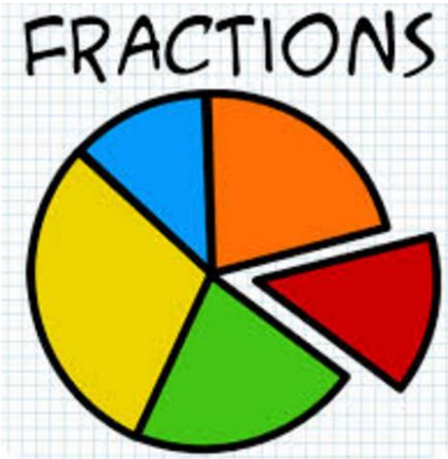
Always leave the fraction in its simplest form

OR you can **cross cancel** before you multiply

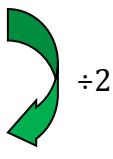
$$\frac{3}{\cancel{2}^1} \times \frac{\cancel{2}^1}{5}$$

$$= \frac{3}{1} \times \frac{1}{5}$$

$$= \frac{3}{5}$$



## Dividing Fractions

$$\begin{aligned} & \frac{2}{3} \div \frac{4}{5} \\ &= \frac{2}{3} \times \frac{5}{4} \\ &= \frac{10}{12} \\ &= \frac{5}{6} \end{aligned}$$


When dividing fractions, we turn the second fraction upside down and multiply

NOTE: only more able pupils will encounter division of fractions

## 3. Percentages

Calculating the Percentage of a Quantity using a Calculator

**METHOD 1 - all pupils are shown this method**

Example 1 Calculate 18% of £860

So

$$\begin{aligned} & 18\% \text{ of } \text{£}860 \\ &= (18 \div 100) \times 860 \\ &= \underline{\underline{\text{£}154.80}} \end{aligned}$$

For calculations of this type we may teach the rule

**“Percentage divided by 100, then multiply by the amount”**

Example 2 Calculate 3.2% of £1850

So

$$\begin{aligned} & 3.2\% \text{ of } \text{£}1850 \\ &= (3.2 \div 100) \times 1850 \\ &= \underline{\underline{\text{£}59.20}} \end{aligned}$$

Using the same rule as above still makes it relatively straightforward to deal with the decimal percentage.

## METHOD 2 – pupils would be made aware of these methods

Example 1 Calculate 22% of £550

$$\begin{aligned} \text{So} & \\ & 22\% \text{ of } \pounds 550 \\ & = 0.22 \times 550 \\ & = \underline{\underline{\pounds 121}} \end{aligned}$$

For calculations of this type we may teach the rule

“convert the percentage to its decimal equivalent, then multiply by the quantity”

Example 2 Calculate 1.7% of £300

$$\begin{aligned} \text{So} & \\ & 1.7\% \text{ of } \pounds 300 \\ & = 0.017 \times 300 \\ & = \underline{\underline{\pounds 5.10}} \end{aligned}$$

Using the same rule as above makes the calculation more difficult for many pupils, although it is similar to the previous method.

When Is This Taught?

<b>S1-3</b>	All pupils are expected to be able to carry out simple percentages and, where appropriate, pupils are extended to decimal percentages.
<b>S4-6</b>	Pupils following N4 and N5 are expected to be able to carry out these calculations. N3 pupils are not required to calculate decimal percentages.

### Calculating the “Standard” Percentages

50% is the same as finding a half $\left(\frac{1}{2}\right)$	⇒ <b>DIVIDE BY 2</b>
25% is the same as finding one quarter $\left(\frac{1}{4}\right)$	⇒ <b>DIVIDE BY 4</b>
10% is the same as finding one tenth $\left(\frac{1}{10}\right)$	⇒ <b>DIVIDE BY 10</b>
5% is to find $\frac{1}{2}$ of 10%	⇒ <b>DIVIDE BY 10 THEN DIVIDE BY 2</b>
1% is the same as finding one hundredth $\left(\frac{1}{100}\right)$	⇒ <b>DIVIDE BY 100</b>
$33\frac{1}{3}\%$ is the same as finding one third $\left(\frac{1}{3}\right)$	⇒ <b>DIVIDE BY 3</b>

## Using the "Standard" Percentages to find other percentages

Example 1 Calculate 30% of £4000

$$\begin{aligned} & 10\% \text{ of } £4000 = £400 \\ \Rightarrow & 30\% \text{ of } £4000 = 400 \times 3 = \underline{£1200} \end{aligned}$$

Pupils are taught to **divide by 10** (to get 10%) and **multiply by 3** (to get 30%)

Example 2 Calculate 75% of £332

There are different possibilities here, but the quickest way is to **divide by 4** (to get 25%) then **multiply by 3**.

$$\begin{aligned} & 25\% \text{ of } £332 = 332 \div 4 = £83 \\ \Rightarrow & 75\% = 83 \times 3 = \underline{£249} \end{aligned}$$

Example 3 Calculate 6% of £140

The method taught here is to **divide by 100** (to get 1%) then to **multiply by 6**.

$$\begin{aligned} & 1\% \text{ of } £140 = £1.40 \\ \Rightarrow & 6\% = 1.40 \times 6 = \underline{£8.40} \end{aligned}$$

### Types of questions

Mental Agility	50% of £80 - Pupils should be able to work out the answer in their head
Non-calculator	20% of £135 - Would expect pupils to show some working for this
Calculator	17% of £294 - Pupils would use a calculator for this

# 4. Information Handling

## Bar Graphs

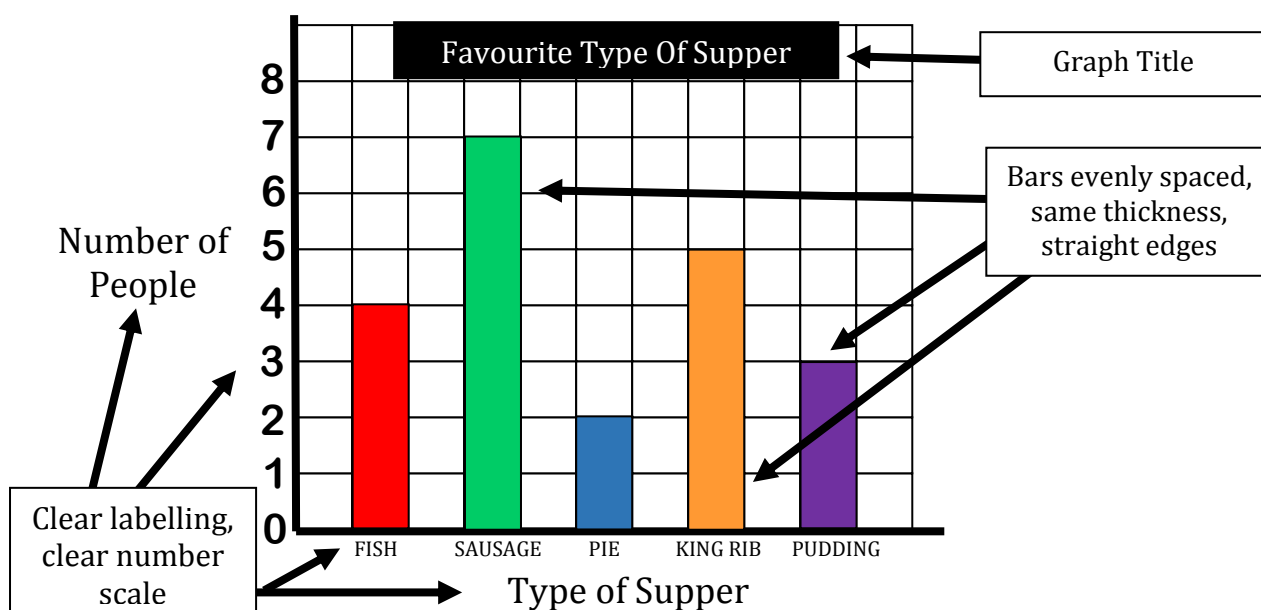
When drawing bar graphs in maths, all pupils are taught to adhere to the following rules:

1. There should be a vertical number scale clearly labelled.
2. The data being surveyed should be clearly labelled along the bottom of the bar graph.
3. The bars should be of uniform thickness, and drawn carefully using a ruler.
4. Where DISCRETE data is involved (i.e. if the survey is asking for a response from a group of choices), the bars should be evenly spaced apart and should not be touching each other.
5. The bars should only be touching if the data is CONTINUOUS (e.g. measuring rainfall, heights, weights etc)
6. The completed graph should be given a title.

### Example 1 – Bar Graph for Discrete Data

A class of 30 pupils were asked to name their favourite chip shop supper. The results are shown below. Draw a neat bar graph to illustrate this information.

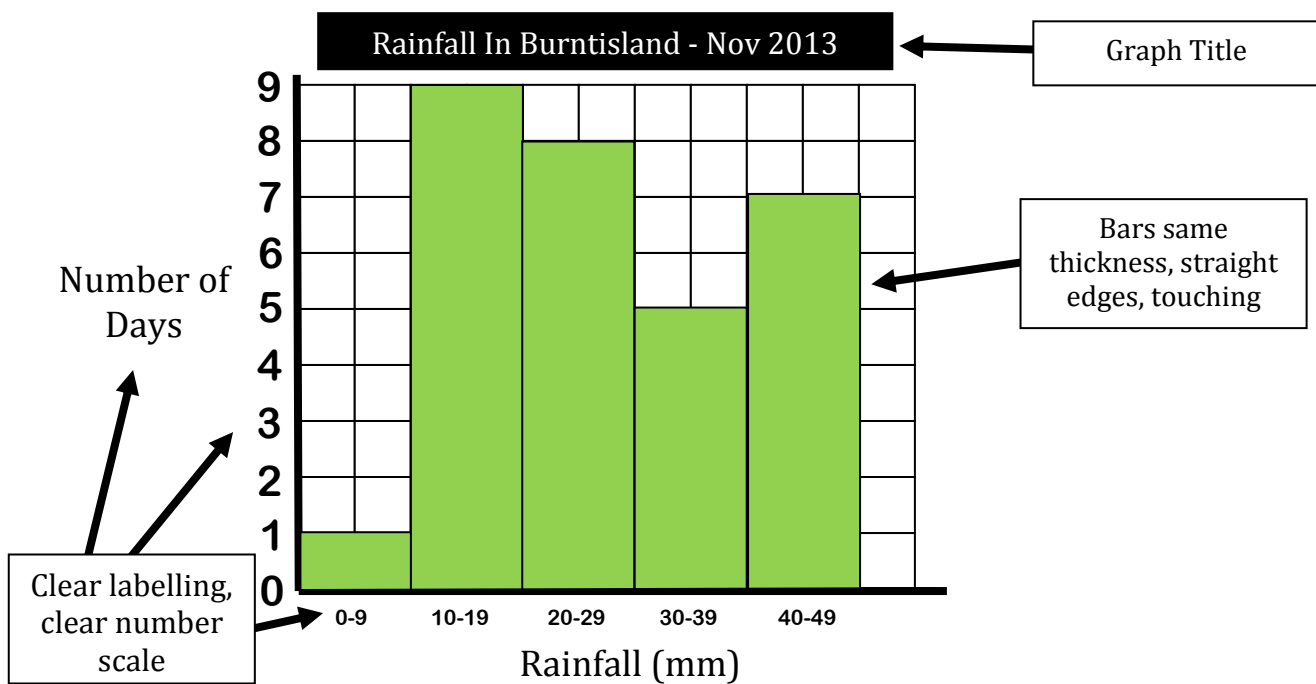
Favourite Supper	Number Of People
Fish	4
Sausage	7
Pie	2
King Rib	5
Pudding	3



## Example 2 – Bar Graph for Continuous Data

The rainfall, in millimetres, was recorded each day in November 2013 for the town of Burntisland. The results are shown in the table below. Draw a bar graph to illustrate this information.

Rainfall(mm)	Number Of Days
0-9	1
10-19	9
20-29	8
30-39	5
40-49	7



### When Is This Taught?

<b>S1</b>	Bar graphs are taught at Primary School and are revised in S1.
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# Line Graphs

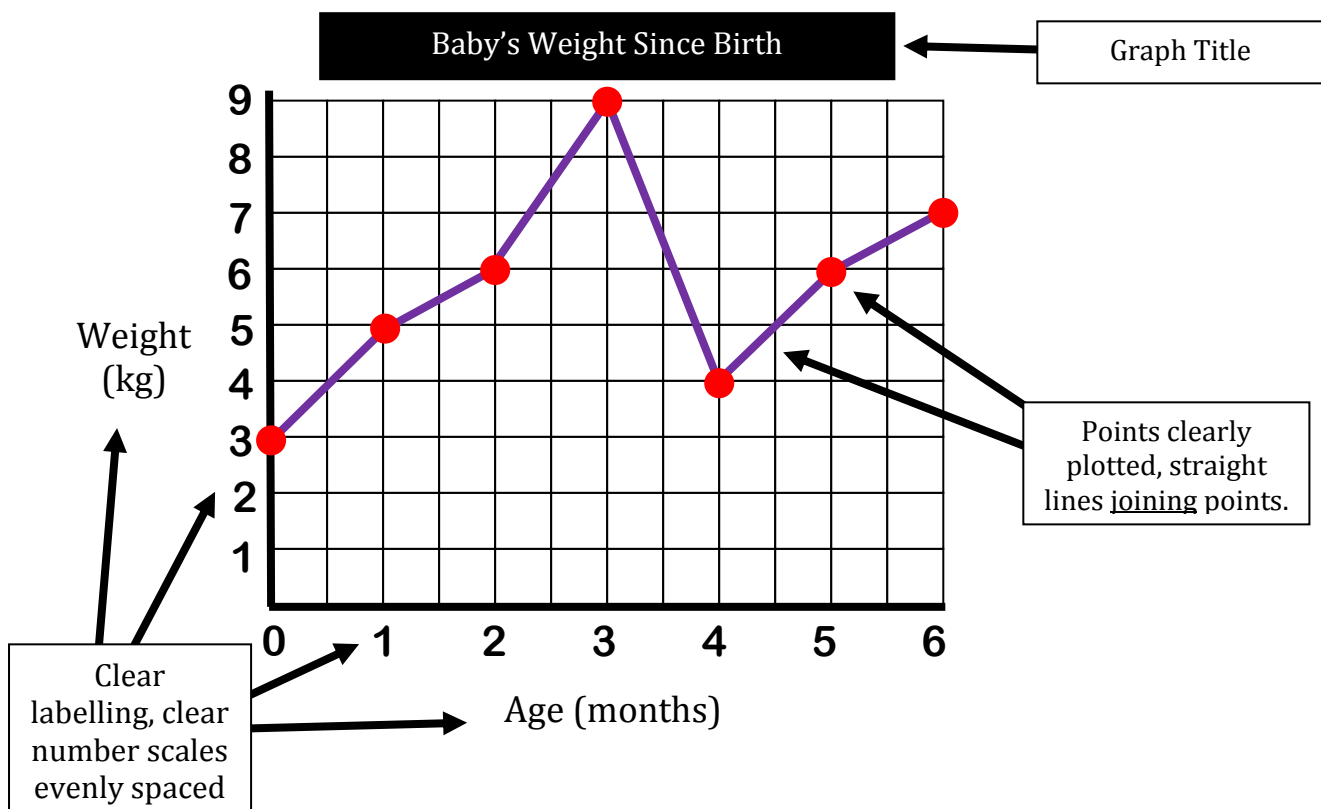
When drawing line graphs in maths, all pupils are taught to adhere to the following rules:

1. Both scales should be clearly labelled.
2. The graph should have a title.
3. Points should be plotted clearly and accurately.
4. Lines should be drawn with a ruler.
5. Line graphs are used mainly to track the progress of a quantity over a period of time. More generally, they are used for data which has been **MEASURED** rather than for data which has been **COUNTED**.

## Example – Line Graph

The weight, in kilograms, of a baby is recorded for the first 6 months after it is born. The results are shown in the table below. Draw a line graph to illustrate this information.

Months since birth	0	1	2	3	4	5	6
Weight (kg)	3	5	6	9	4	6	7



# Scatter Graphs

Scatter graphs can also be called scatter diagrams, scatter plots or scatter charts. They are used to show the relationship between two variables in a set of data. This is different from a line graph because time is not usually one of the variables. We still follow the same rules:

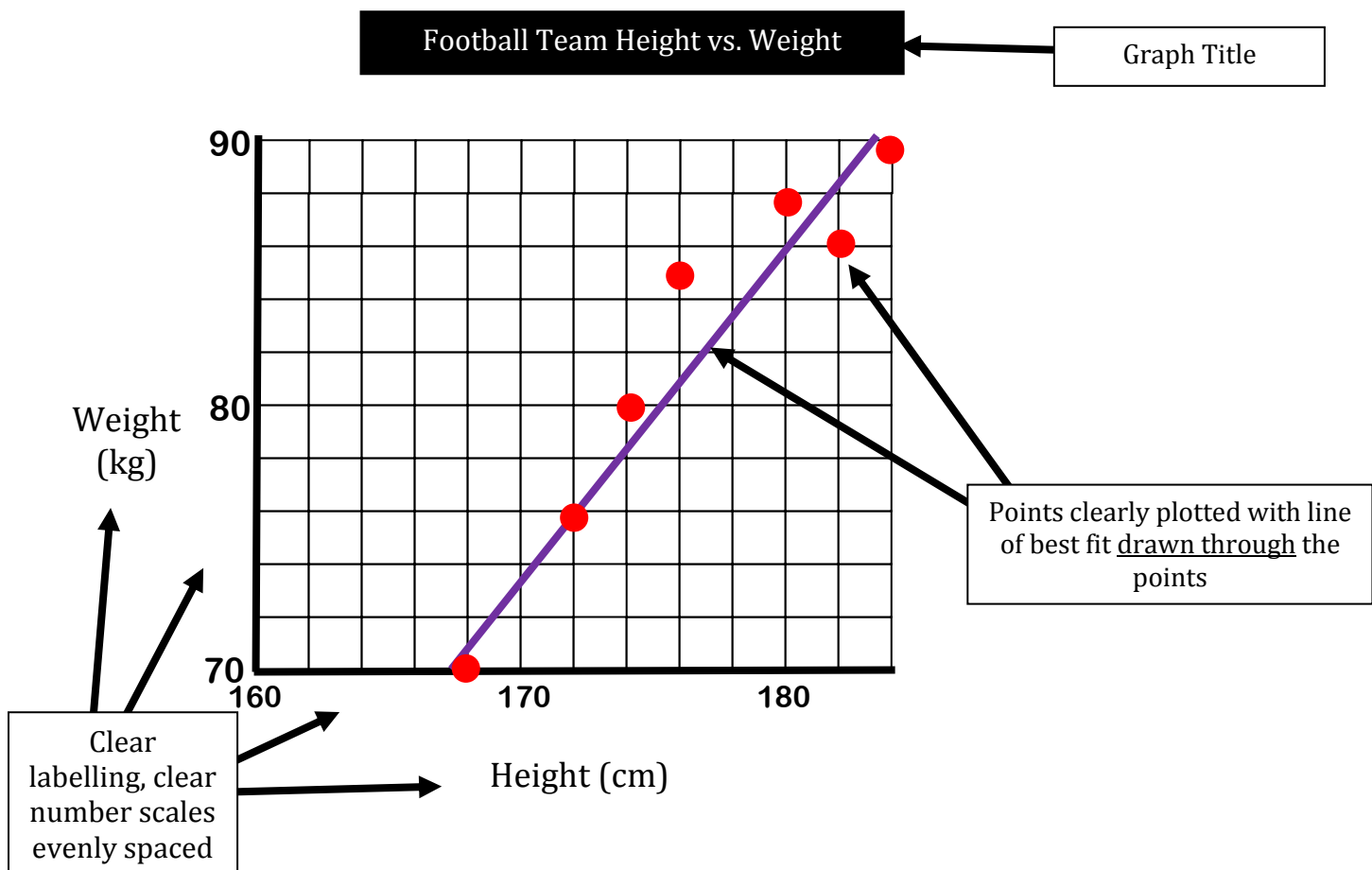
1. Both scales should be clearly labelled.
2. The graph should have a title.
3. Points should be plotted clearly and accurately.

However, instead of joining the points up, we draw one straight line through the data which we call a **line of best fit**.

## Example – Scatter Graph

The height and weight of a football team are recorded below:

	Joe	Bob	Tom	Jack	Pete	Paul	James
Height (cm)	168	176	174	184	180	172	182
Weight (kg)	70	85	80	90	88	76	86



# Pictographs

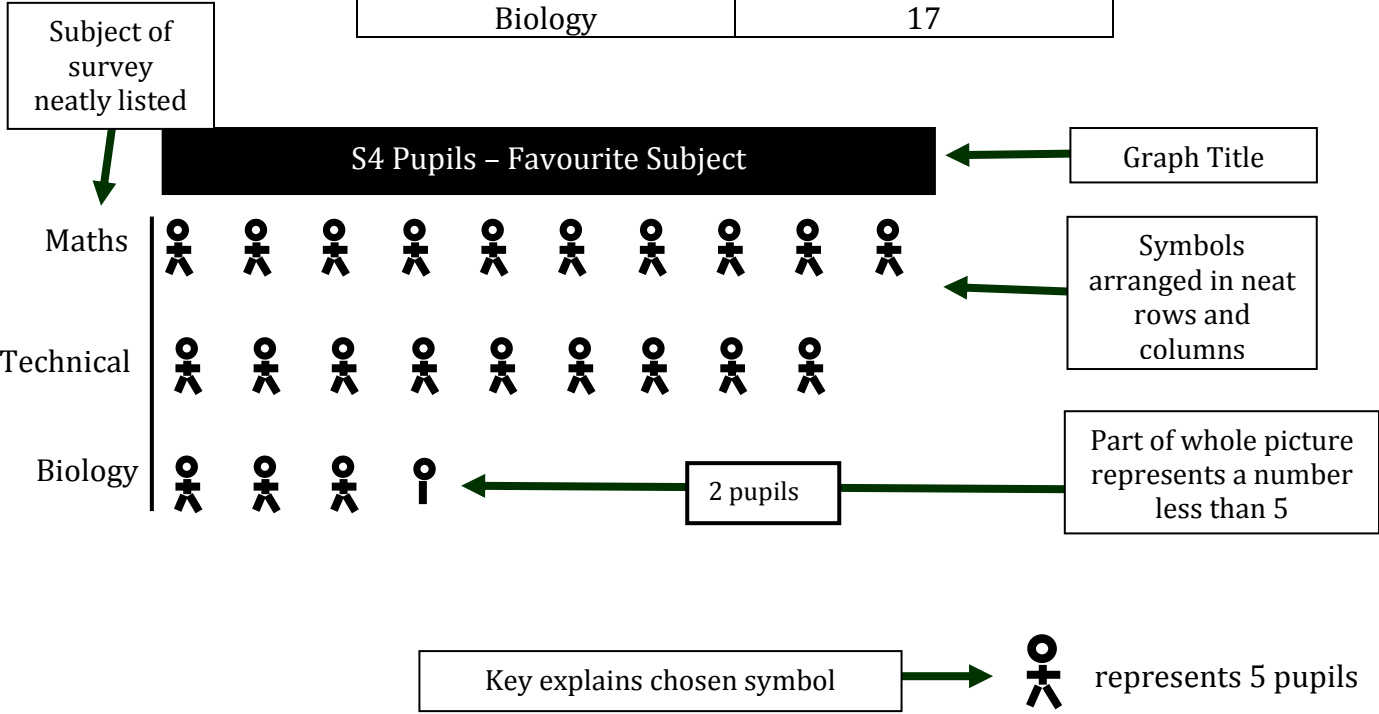
Pictographs are useful for representing discrete data if the numbers being recorded are slightly larger. When drawing pictographs, the following rules are observed:

1. A clear explanation of what quantity the chosen symbol represents must be included.
2. It can be useful to ensure that the chosen symbol can be divided up into parts to represent different numbers (see example below)
3. Pictographs are usually drawn from left to right, and the chosen symbols are also aligned in columns.

### Example – Pictographs

At St. Andrew’s High School , all fourth year pupils were asked to name their favourite subject. The results are shown in the table below. Draw a pictograph to illustrate the information.

Favourite Subject	Number of Pupils
Maths	50
Technical	45
Biology	17



## Pie Charts

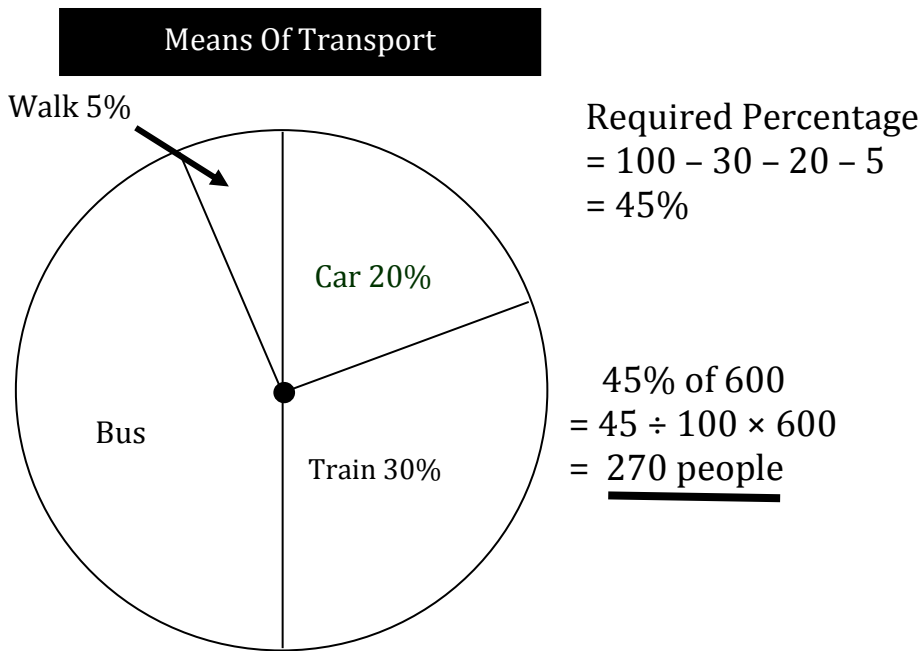
When drawing pie charts in mathematics, the following rules are observed:

1. Pie charts can be used to display information which has been given as a percentage. Pupils may be asked to interpret the graph to find the answer to a particular question. (Example 1)
2. Pie charts can also be constructed by using the fact that one whole circle represents an angle of 360°, and calculating fractions of this whole. (Example 2)

### Example 1 – Using a Pie Chart to calculate an answer

600 people were surveyed to find out how they travel to work. The results are shown in the pie chart below.

How many people travel to work by bus?



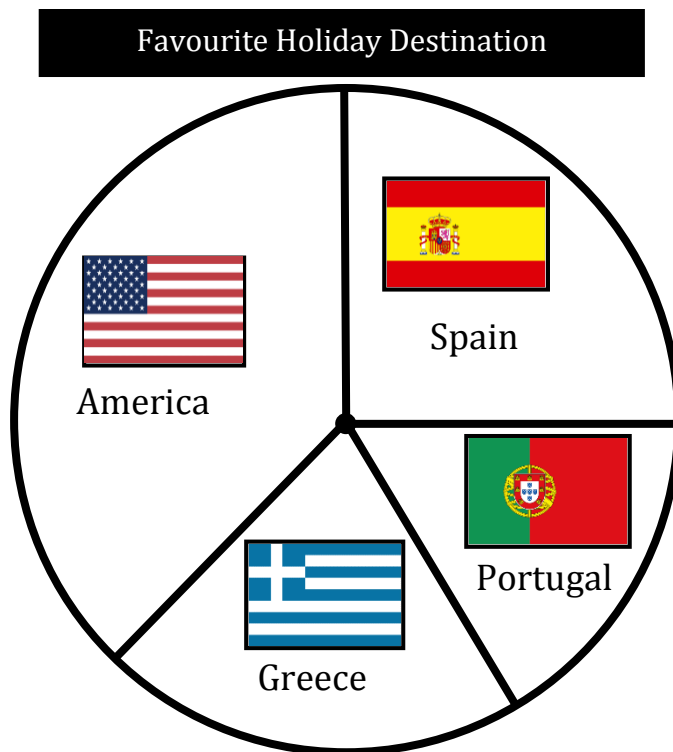
### Example 2 – Constructing A Pie Chart

A group of people were asked to name their favourite holiday destination. The results are shown in the table below. Construct a pie chart to display this information.

Favourite Destination	Number of People
Spain	60
Portugal	36
Greece	48
America	96

Total Number Of People =  $60 + 36 + 48 + 96 = 240$

$$\begin{aligned}\text{Angle for Spain} &= \frac{60}{240} \times 360^\circ = \underline{\underline{90^\circ}} \\ \text{Angle for Portugal} &= \frac{36}{240} \times 360^\circ = \underline{\underline{54^\circ}} \\ \text{Angle for Greece} &= \frac{48}{240} \times 360^\circ = \underline{\underline{72^\circ}} \\ \text{Angle for America} &= \frac{96}{240} \times 360^\circ = \underline{\underline{144^\circ}}\end{aligned}$$



Note: Each sector should be labelled with the appropriate category or a key should be shown.

## AVERAGES

Pupil are taught 3 ways of finding the average value to provide information about a set of data.

- MEAN** - The mean is found by adding all the data together and dividing by the number of values.
- MEDIAN** - The median is the middle value when all the data is written in numerical order. (If there are two middles the median is half-way between these values)
- MODE** - The mode is the value that occurs most often.

Pupils are also taught about measures of spread. For the BGE this consists of:

**RANGE** - The range of a set of data is the highest value subtract the lowest value.

Example

Class 1K scored the following marks for their homework task

Find the mean, median, mode and range for these results.

6, 9, 7, 5, 6, 6, 10, 9, 8, 4, 8, 5, 7

$$\begin{aligned} \text{Mean} &= (6+9+7+5+6+6+10+9+8+4+8+5+7)/13 \\ &= 90/13 \\ &= 6.923..... \\ &= 6.9 \text{ (to 1 decimal place)} \end{aligned}$$

Median Ordered values: 4, 5, 5, 6, 6, 6, 7, 7, 8, 8, 9, 9, 10

Median = 7

Mode = 6 (most frequent value)

Range = 10 - 4 = 6

When Is This Taught?

LINE GRAPHS, PICTOGRAPHS, PIE CHARTS AND AVERAGES

<b>Line Graphs</b>	<b>S1-3</b>	Pupils use line graphs in S1
	<b>S4-6</b>	All levels are taught to draw and interpret line graphs to varying levels of difficulty. National 4 and National 5 Maths and Lifeskills are taught how to estimate from a line of best fit.
<b>Scatter Graphs</b>	<b>S1-3</b>	Pupils use Scatter Graphs in S1
	<b>S4-6</b>	Scatter Graphs are used mainly by pupils studying National 4 and National 5 Lifeskills.
<b>Pictographs</b>	<b>S1-3</b>	Pupils use pictographs in S1.
	<b>S4-6</b>	Pictographs are used mainly by National 3 level pupils, and occasionally by National 4 level pupils.
<b>Pie Charts</b>	<b>S1-3</b>	Pupils use Pie Charts in S1, mainly with percentage calculations.
	<b>S4-6</b>	National 3 pupils are mainly required to interpret very simple pie charts. National 4 and National 5 level pupils would be expected to be able to construct a pie chart as well as interpret it.

AVERAGES

<b>S1-3</b>	Pupils are taught averages as part of the Information Handling topic in S1.
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# 5. Solving Equations

## Simple Unknowns on One Side

### MAIN METHOD 1 – all pupils are shown this method

**Example 1** Solve the equation  $a + 15 = 22$

$$\begin{array}{rcl} a + 15 & = & 22 \\ -15 & & -15 \\ \hline \Rightarrow a & = & 7 \end{array}$$

The balance method is used to isolate the letter, or unknown. Here, the pupils would be taught to do the same to both sides, that is to subtract 15.

**Example 2** Solve the equation  $b - 9 = 14$

$$\begin{array}{rcl} b - 9 & = & 14 \\ +9 & & +9 \\ \hline \Rightarrow b & = & 23 \end{array}$$

The balance method is used to isolate the letter, or unknown. Here, the pupils would be taught to do the same to both sides, that is to add on 9.

**Example 3** Solve the equation  $5p = 60$

$$\begin{array}{rcl} 5p & = & 60 \\ \div 5 & & \div 5 \\ \hline \Rightarrow p & = & 12 \end{array}$$

The balance method is again used to isolate the letter, or unknown. Here, the pupils would be taught that  $5p$  means “5 times  $p$ ”, so they would divide both sides by 5.

### OTHER METHOD 2

**Example 1** Solve the equation  $a + 15 = 22$

$$\begin{array}{rcl} a + 15 & = & 22 \\ a & = & 22 - 15 \\ \hline \Rightarrow a & = & 7 \end{array}$$

Pupils are taught the trick ‘change the side, change the operation’, so here they would move the “add 15” to the other side of the equation and it would become “subtract 15” on the other side.

**Example 2** Solve the equation  $b - 9 = 14$

$$\begin{array}{rcl} b - 9 & = & 14 \\ b & = & 14 + 9 \\ \hline \Rightarrow b & = & 23 \end{array}$$

Pupils are taught to move the “subtract 9” to the other side of the equation and it would become “add 9” on the other side.

**Example 3** Solve the equation  $5p = 60$

$$\begin{aligned} 5p &= 60 \\ p &= 60 \div 5 \\ \Rightarrow p &= 12 \end{aligned}$$

Pupils are taught to move the “multiply by 5” to the other side of the equation and it would become “divide by 5” on the other side.

When Is This Taught?

<b>S1-3</b>	Pupils are introduced to solving equations in S1. The topic is re-visited in S2 and S3.
<b>S4-6</b>	Solving equations is an essential skill for all but the least able pupils.

## Two Step Equations

### MAIN METHOD 1 - all pupils are shown this method

**Example 1** Solve the equation  $3y + 2 = 29$

$$\begin{aligned} 3y + 2 &= 29 \\ -2 &-2 \\ 3y &= 27 \\ \Rightarrow \div 3 &\div 3 \\ y &= 9 \end{aligned}$$

The balance method is used to isolate the letter, or unknown. Here, the pupils would be taught to do the same to both sides, that is firstly subtract 2 then to divide by 3.

**Example 2** Solve the equation  $6t - 1 = 47$

$$\begin{aligned} 6t - 1 &= 47 \\ +1 &+1 \\ \Rightarrow 6t &= 48 \\ \div 6 &\div 6 \\ \Rightarrow t &= 8 \end{aligned}$$

The balance method is used to isolate the letter, or unknown. Here, the pupils would be taught to do the same to both sides, that is firstly to add 1 then to divide by 6.





## When Is This Taught?

<b>S1-3</b>	The more able pupils would tackle equations of this nature in S1, and only pupils who are recommended for National 3 would not tackle equations of this sort by the end of BGE.
<b>S4-6</b>	Solving equations is an essential skill for most pupils. Less able pupils are not assessed in this topic, are not likely to study equations of this nature. (Note: there is no algebra in the Application of Mathematics courses)

## 6. Changing the Subject of a Formula

Pupils should be given the following guidance:

- If the desired variable is on the right hand side, reverse the formula
- Remove fractions by multiplying through on both sides
- Remove brackets
- Use normal rules for solving equations
- Apply one step at a time and show each step clearly
- If the desired variable lies under a root, reverse the operation

Examples:

a) Change the subject of the formula to e

$$\begin{aligned}f &= 2e + 5 \\2e + 5 &= f && \text{(reverse)} \\2e &= f - 5 \\e &= \frac{f-5}{2} && \text{(2 changes side from } \times \text{ to } \div)\end{aligned}$$

b) Change the subject of the formula to m:

$$\begin{aligned}k &= \sqrt{mn} \\ \sqrt{mn} &= k && \text{(reverse)} \\ mn &= k^2 && \text{(square root changes to square)} \\ m &= \frac{k^2}{n} && \text{(changes side from } \times \text{ to } \div)\end{aligned}$$

## When Is This Taught?

<b>S4-6</b>	National 4 and National 5 Mathematics. (Note: Techniques for inverse operations should be emphasised)
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# 7. Ratio & Proportion

Ratios are used to compare different quantities. Pupils are taught in maths to write, simplify and use ratios.

## Writing Ratios

### Example 1

To make a fruit drink 4 parts water is mixed with 1 part cordial.

The ratio of water to cordial is 4 : 1

The ratio of cordial to water is 1 : 4.

The order is important when writing ratios

## Simplifying Ratios

### Example 2

Pupils are shown that ratios can be simplified in much the same way as fractions.

10 : 6 divide both sides by 2

5 : 3

To simplify a ratio pupils are shown to divide each figure in the ratio by a common factor.

## Using Ratio

Ratio calculations involve working out a missing value using ratio or sharing an amount in the given ratio.

### Example 3

The ratio of fruit to nuts in a chocolate bar is 3 : 2.

If a bar contains 15g of fruit, what weight of nuts will it contain?

	Fruit	Nuts	
	3	2	
×5	15	10	×5

Pupils are encouraged to use a table to help find missing values using ratios.

## Proportional Division

If sharing a quantity in a given ratio pupils must:

- 1) Calculate the 'number of parts'
- 2) Divide the quantity by the number of parts (this gives you the value of 1 part)
- 3) Multiply each number in the ratio by the value of 1 part

Example 4

£35 is split between Jack and Jill in the ratio 3 : 2. How much does Jack receive and how much does Jill receive

Number of shares: 3 parts + 2 parts = 5 parts  
Value of 1 share: £35 ÷ 5 = £7

Jack's share: 3 x £7 = £21  
Jill's share: 2 x £7 = £14 (check that £21 + £14 = £35)

Direct Proportion

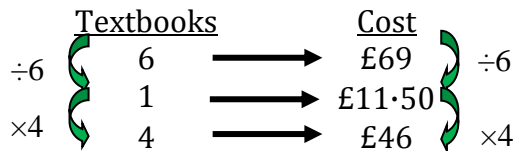
Pupils will be shown to:

- Set working out in a table or with clear headings
- **Divide** by the given amount to find the unitary value
- **Multiply** to find the required amount
- Write the final answer under the table

With direct proportion, as one quantity increases so does the other and as one quantity decreases so does the other.

Example 5

6 copies of a textbook cost £69. Find the cost of 4 textbooks.



**The cost of 4 textbooks is £46**

Indirect (or Inverse) Proportion

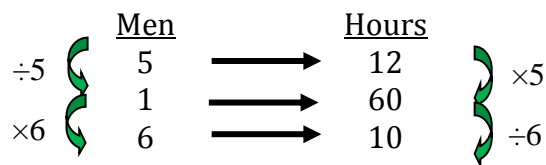
Pupils will be shown to:

- Set working out in a table or with clear headings
- **Multiply** to find out how long/much the unitary value will be
- **Divide** to find the required amount
- Write the final answer under the table

With indirect proportion, as **one quantity increases the other decreases**.

Example

If it takes 5 men 12 hours to paint a fence, how long will it take 6 men?



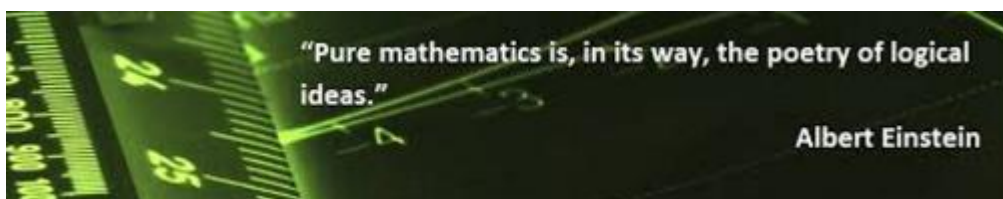
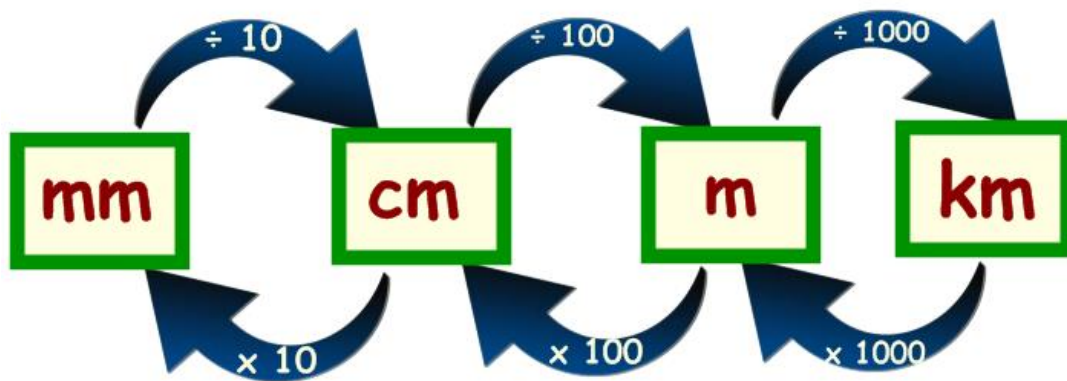
**It takes 6 men 10 hours to paint the fence.**

When Is This Taught?

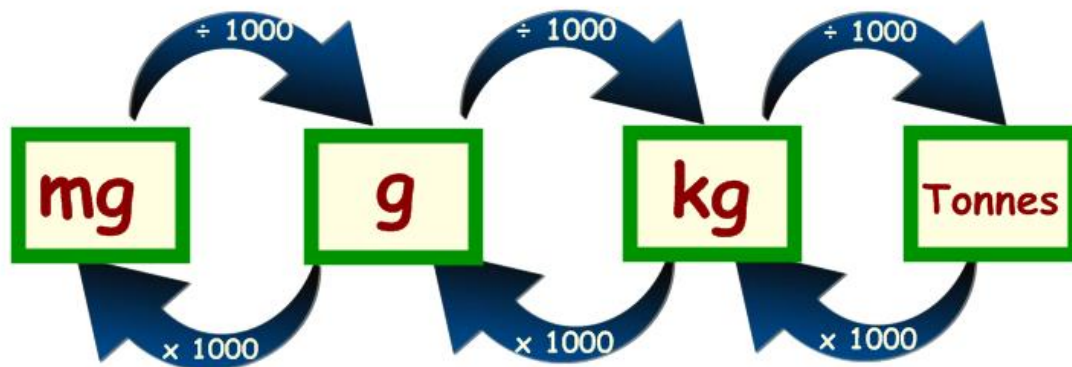
<b>S1-3</b>	All pupils will learn about writing and simplifying ratio in S2. The better pupils will go on and learn to use ratio in calculations.
<b>S4-6</b>	Ratio and proportion are taught within the National 4 Numeracy unit.

## 8. Unit Conversion

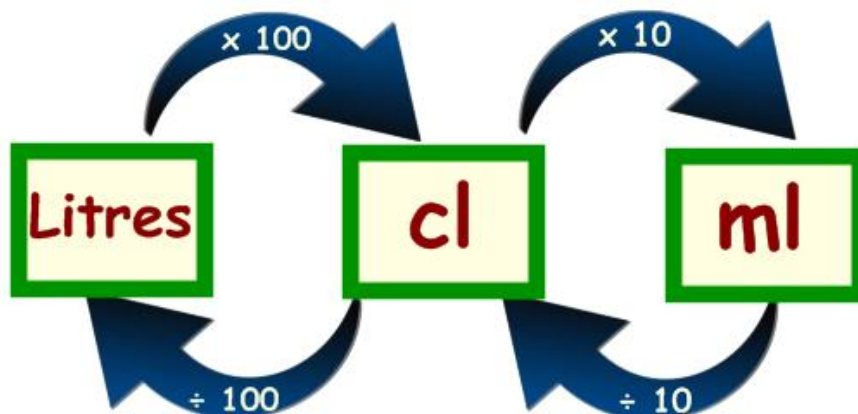
Pupils should be aware of the following ways to convert between units of length (millimetres, centimetres, metres and kilometres):



Pupils should be aware of the following ways to convert between units of mass and capacity (milligrams, grams, kilograms and tonnes):



Pupils should be aware of the following ways to convert between units of mass and capacity (litres, centilitres and millilitres):



When Is This Taught?

<b>S1</b>	Pupils are taught this topic at primary and it is recapped at S1. Converting measurements is an area of focus for most year groups and requires regular consolidation and practice.
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# 9. Estimating Measurement

Pupils will practice estimation of

## Length

- Height and length in cm and m
- Length of a ruler = 30 cm
- Height of a door = 2m

## Area/Weight/Volume

- Small weights, small areas, small volumes
- Weight of a bag of sugar = 1 kg
- Area of an envelope =  $10 \times 8 = 80 \text{ cm}^2$
- Volume of lemonade bottle = 1 litre

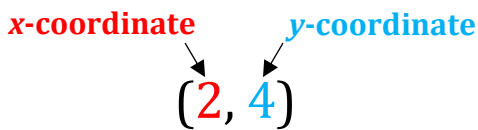
NOTE: In the outside world, measurements of length tend to be stated in mm e.g. worktop heights for kitchen units.

## When Is This Taught?

<b>S1</b>	Pupils are taught this topic at primary and it is recapped in S1.
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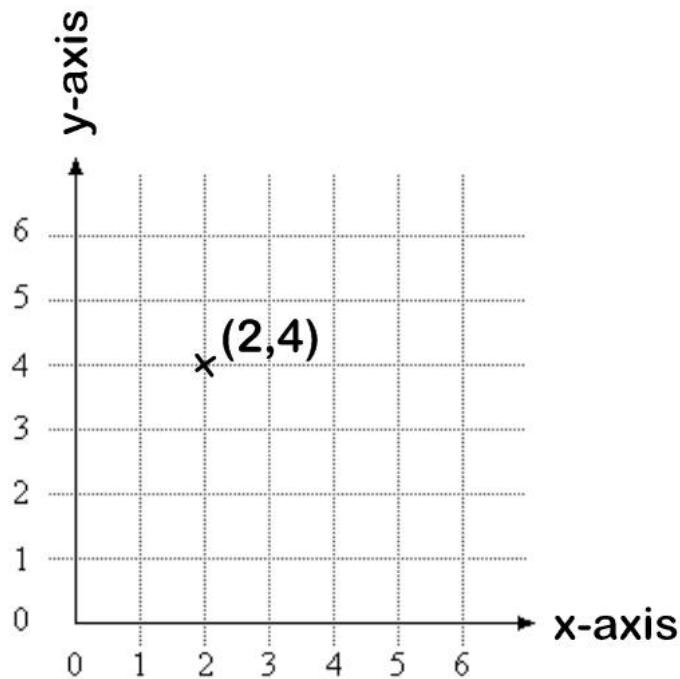
# 10. Coordinates



For plotting coordinates we teach the rule: along then up

When drawing a coordinate grid pupils must make sure:

- Numbers are evenly spaced
- Numbers are drawn 'on the line' and not 'in the boxes'



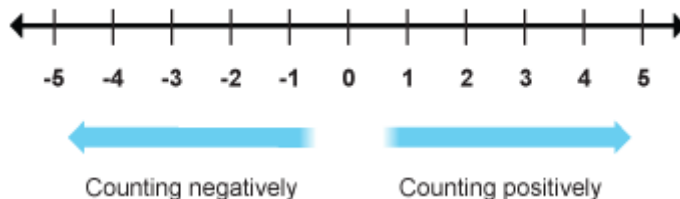
When Is This Taught?

<b>S1-3</b>	Pupils are taught this topic at primary and it is recapped in S1 and will extend this into a 4 quadrant grid where negative numbers are involved.
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# 11. Integers

Integers are positive and negative whole numbers.

Pupils should all be familiar with the number line as shown:



Pupils will learn how to add, subtract, multiply and divide integers in S1.

Adding and Subtracting

Some integer calculations for add and subtract can be done by moving up and down the number line

### Examples

$$\begin{aligned} 1) \quad & 4 - 5 \\ & = -1 \end{aligned}$$

$$\begin{aligned} 2) \quad & -2 + 7 \\ & = 5 \end{aligned}$$

$$\begin{aligned} 3) \quad & 2 + 9 - 10 + 6 \\ & = 3 \end{aligned}$$

However, when adding and subtracting negative numbers pupils must remember that when two signs appear next to each other and are different, then you subtract. When two signs are next to each other and are the same, then you add:



Note: negative numbers are usually written in brackets

### Examples

$$\begin{aligned} 1) \quad & 10 + (-7) \\ & = 10 - 7 \\ & = 3 \end{aligned}$$

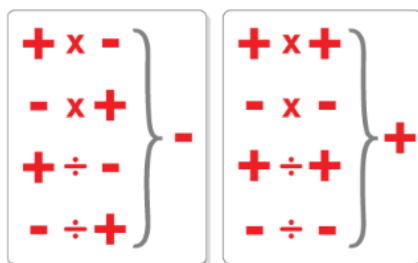
$$\begin{aligned} 2) \quad & 4 - (-3) \\ & = 4 + 3 \\ & = 7 \end{aligned}$$

$$\begin{aligned} 3) \quad & -7 - (-4) \\ & = -7 + 4 \\ & = -3 \end{aligned}$$

## Multiplying and Dividing

The rule for multiplying and dividing is very similar to the rule for adding and subtracting.

- When the signs are different the answer is negative
- When the signs are the same the answer is positive



### Examples

$$\begin{aligned} 1) \quad & 5 \times (-4) \\ & = -20 \end{aligned}$$

$$\begin{aligned} 2) \quad & (-6) \times 3 \\ & = -18 \end{aligned}$$

$$\begin{aligned} 3) \quad & (-3) \times (-4) \\ & = 12 \end{aligned}$$

$$\begin{aligned} 4) \quad & 10 \div (-2) \\ & = -5 \end{aligned}$$

$$\begin{aligned} 5) \quad & (-24) \div 6 \\ & = -4 \end{aligned}$$

$$\begin{aligned} 6) \quad & (-30) \div (-10) \\ & = 3 \end{aligned}$$

### When Is This Taught?

<b>S1</b>	Pupils cover negative numbers and the number line at primary school. This is extended into adding, subtracting, multiplying and dividing negative numbers in S1
<b>S4-6</b>	Negative numbers are revisited as part of National 4.