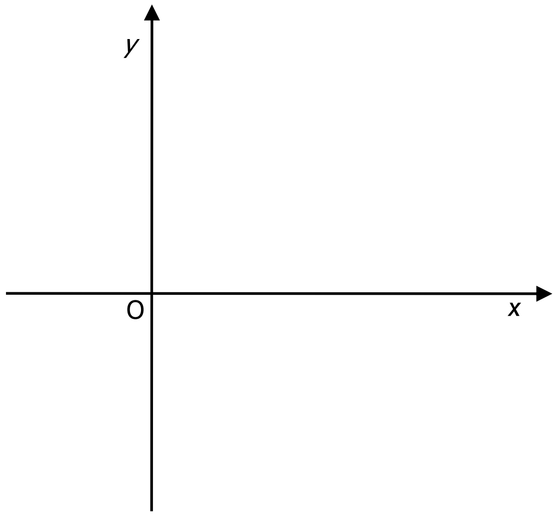
Graphs of Functions

Sketching a Quadratic Graph (Revision)

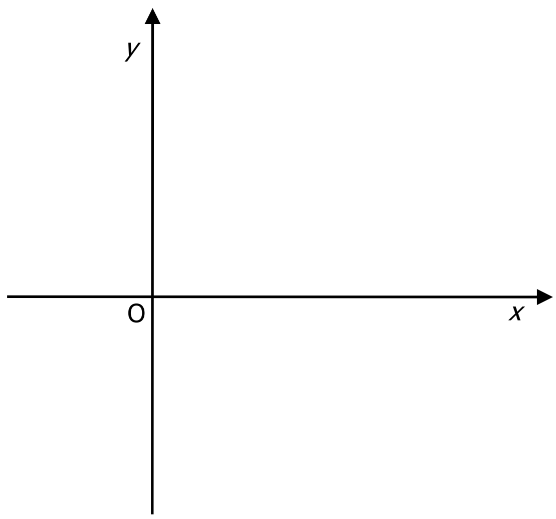
To sketch a quadratic graph:

- **Find the roots (set $y = 0$)**
- **Find the y – intercepts (set $x = 0$)**
- **Find the turning point (x value is halfway between roots; sub. into formula to find y)**

Example 1: Sketch and annotate the graph of $y = x^2 - 2x - 15$



Example 2: Sketch and annotate the graph of $y = x^2 - 4x + 4$



Note: when quickly sketching a quadratic graph, the roots and shape ("happy" or "sad" face) are enough.

Graphs of Functions

Sketching Graphs (Revision)

In the exam, diagrams are provided whenever the question involves a graph. However, this is not the case when working from the textbook: it is therefore important that we are able to sketch basic graphs where necessary, as often the question becomes simpler when you can see it.

Example 3: in the spaces provided, make a **basic** sketch of the graph(s) of the function(s) stated.

a) $y = 2x + 1$

b) $3x + 4y - 12 = 0$

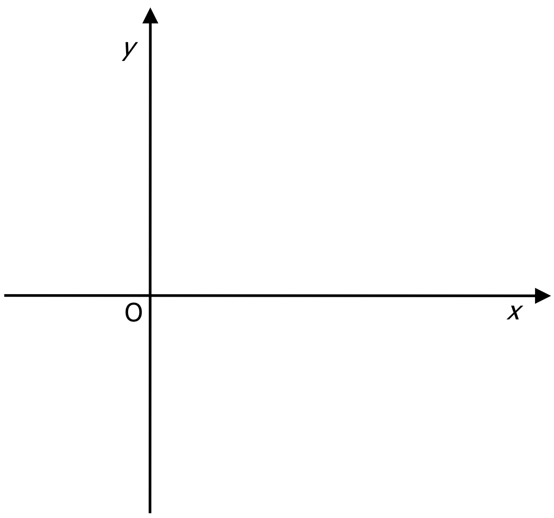
c) $y = -1$ **and** $x = 5$

d) $y = x^2$ **and** $y = 4$

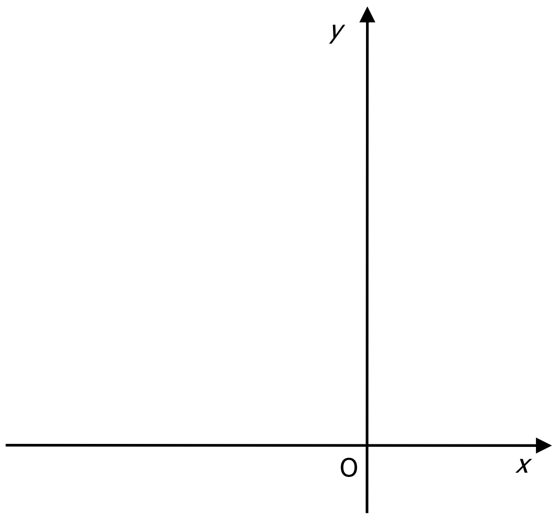
e) $y = x^2 - 4$

f) $y = (x - 2)^2$ **and** $y = 2x - x^2$

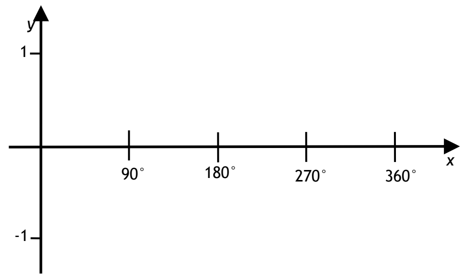
Example 4: Sketch and annotate the graph of $y = x^2 - 2x - 8$

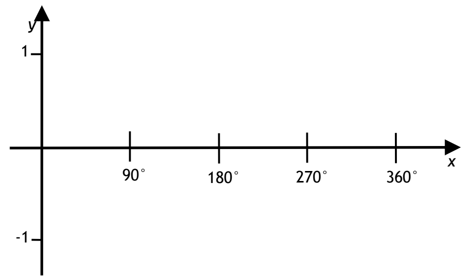


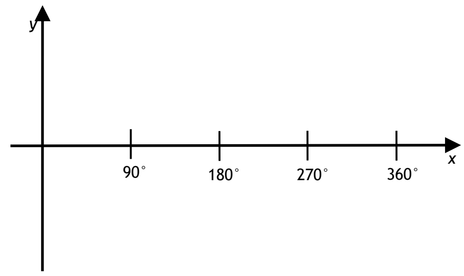
Example 5: Sketch and annotate the graph of $y = (x + 3)^2 + 1$



Example 6: Sketch the graphs of $y = \sin x°$, $y = \cos x°$ and $y = \tan x°$ below.







$y = \sin x°$	**$y = \cos x°$**	**$y = \tan x°$**

For trig graphs, how soon the graph repeats itself horizontally is known as the **period**, and half of the vertical height is known as the **amplitude.**

Function	Period	Amplitude
$y = \sin x°$		
$y = \cos x°$		
$y = \tan x°$		

For the graphs of:

$y = a\sin bx° + c$ and $y = a\cos bx° + c$:

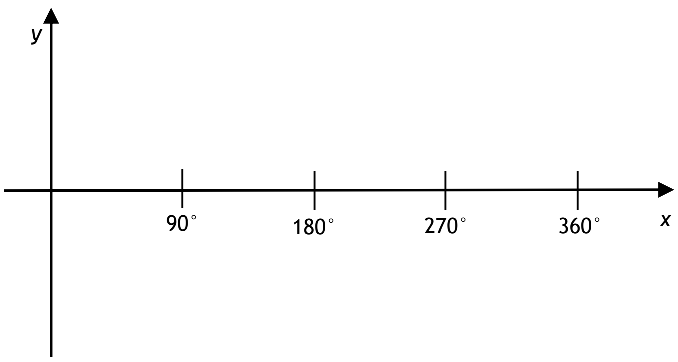
a = amplitude
b = waves in 360°
c = vertical shift

$y = a\tan bx° + c$:

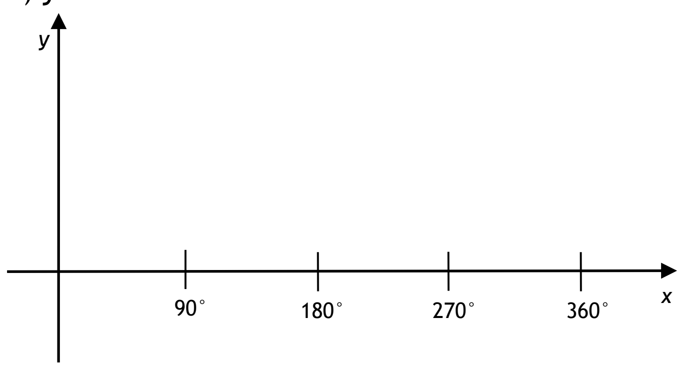
b = "waves" in 180°
c = vertical shift

Example 7: Sketch the graphs of:

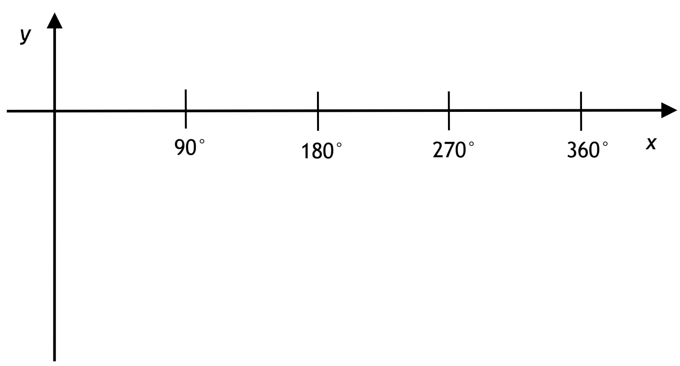
a) $y = \sin 2x°$



b) $y = 5\cos 2x° + 3$

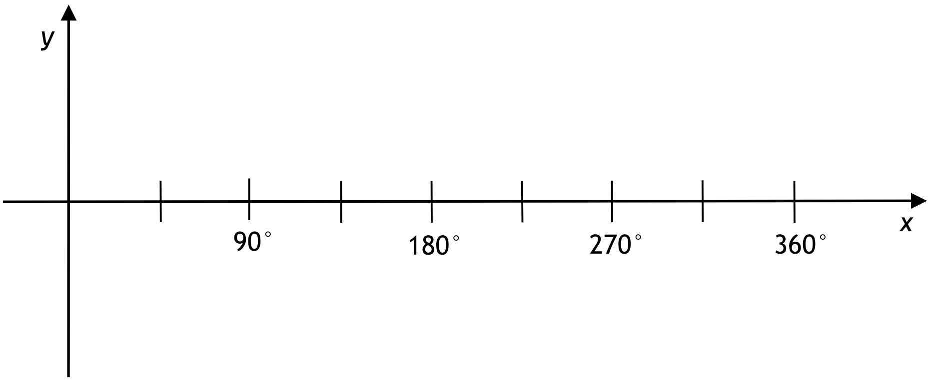


c) $y = -3\sin 3x° - 2$



Compound Angles

A compound angle is one containing two parts, e.g. $(x - 60)^\circ$. The graphs of compound angles can be thought of as the trig version of $y = f(x - a)$, i.e. shifted left or right by a units.



Example 8: On the axes opposite, sketch:

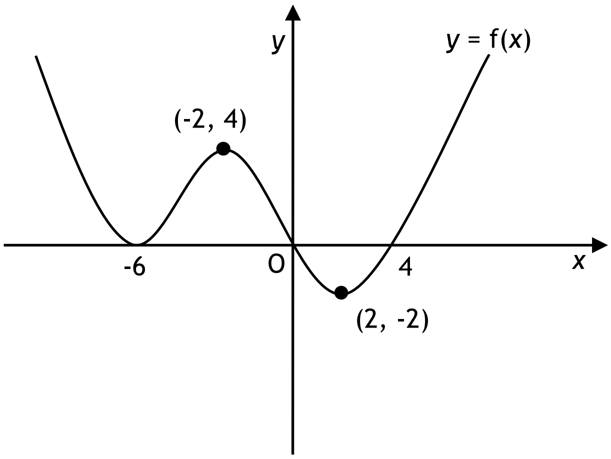
a) $y = \sin x^\circ$

b) $y = \sin (x - 45)^\circ$

Transformation of Graphs

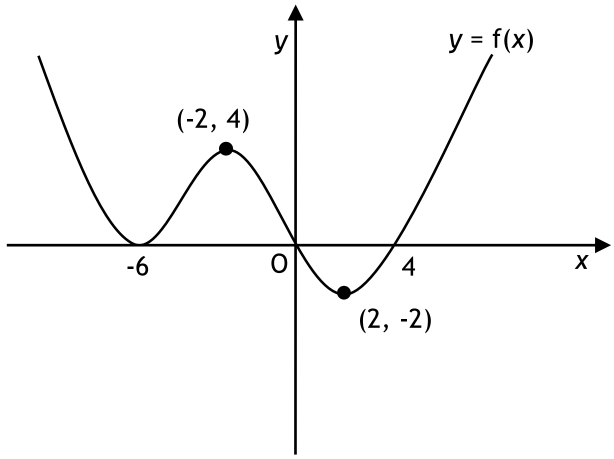
We have seen how the graph of y = sin(x) is different to that of y = sin(2x), and how $y = x^2$ differs from $y = (x - 1)^2$. The six operations below are used to transform the graph of a function:

y = f(*x*) + a



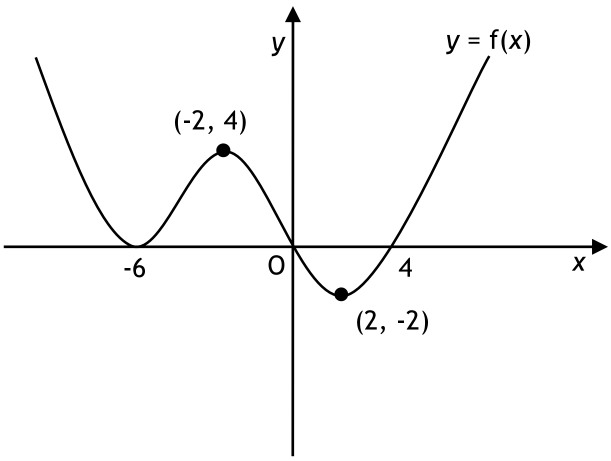
y = f(*x*) + a is obtained by **sliding** y = f(*x*):
Vertically upwards if a > 0
Vertically downwards if a < 0

y = f(*x* + a)



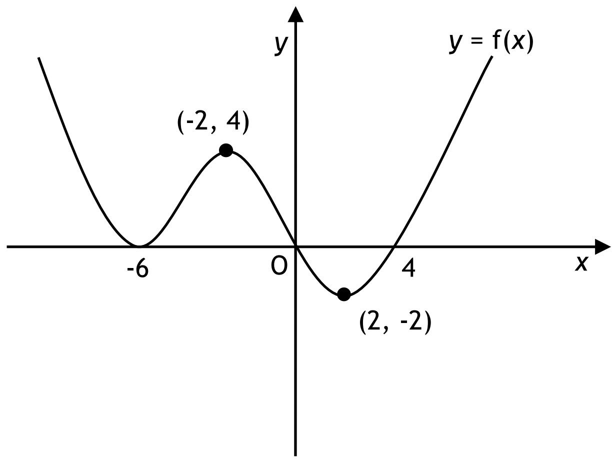
y = f(*x* + a) is obtained by **sliding** y = f(*x*):
Horizontally left if a > 0
Horizontally right if a < 0

y = k f(*x*)



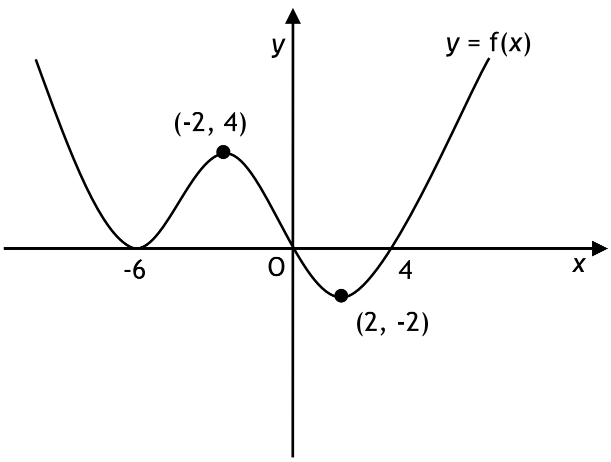
y = k f(*x*) is obtained by **vertically:**
stretching *y* = f(*x*) if k > 1
compressing *y* = f(*x*) if 0 < k < 1

y = f(k*x*)



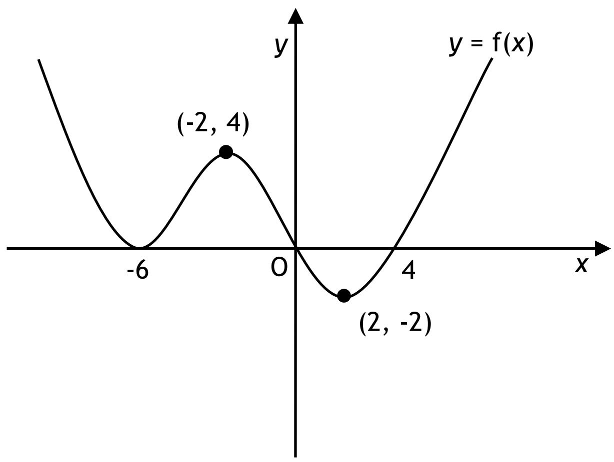
y = f(k*x*) is obtained by **horizontally:**
compressing *y* = f(*x*) if k > 1
stretching *y* = f(*x*) if 0 < k < 1

y = - f(*x*)



y = - f(*x*) is obtained by **reflecting** y = f(*x*):
in the *x* - axis

y = f(-*x*)



y = f(-*x*) is obtained by **reflecting** y = f(*x*):
in the *y* - axis

Multiple Transformations

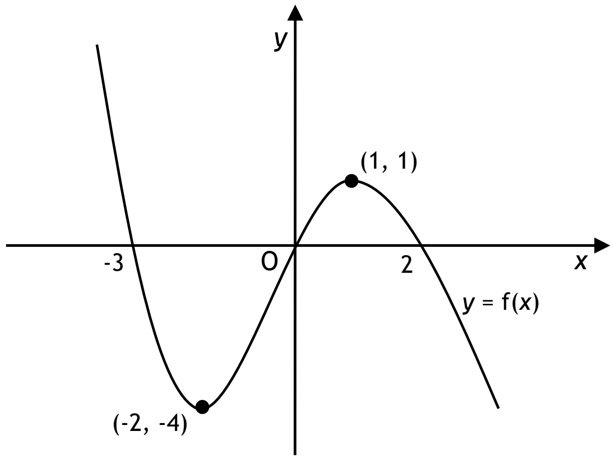
Often, are asked to perform more than one transformation on a graph.
Where appropriate, always leave sliding vertically to last.

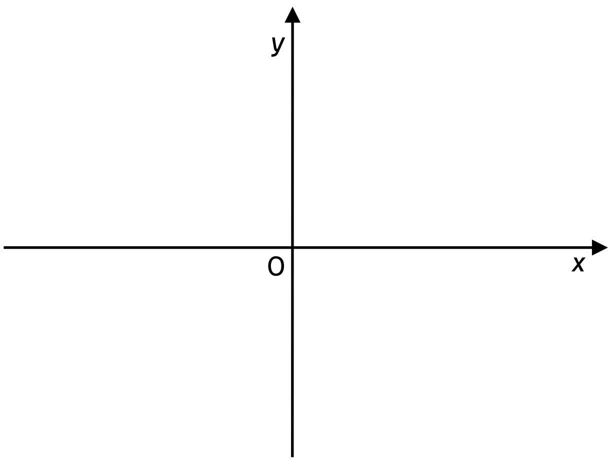
Example 9: Part of the graph of $y = f(x)$ is shown.

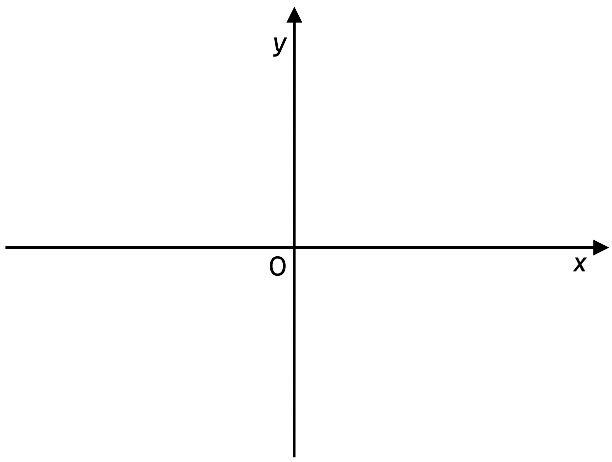
On separate diagrams, sketch:

a) $y = f(-x) + 2$

b) $y = -\frac{1}{2} f(x + 1)$







Past Paper Example: The diagram shows a sketch of the function $y = f(x)$.

To the diagram, add the graphs of:

a) $y = f(2x)$

b) $y = 1 - f(2x)$.