Sets and Functions

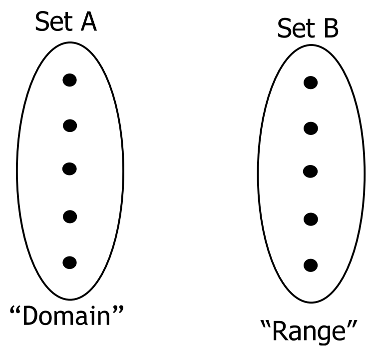
A **set** is a group of numbers which share common properties. Some common sets are:

Natural Numbers	$N = \{1, 2, 3, 4, 5, \ldots\ldots\}$
Whole Numbers	$W = \{0, 1, 2, 3, 4, 5, \ldots\ldots\}$
Integers	$Z = \{\ldots\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\ldots\}$
Rational Numbers	Q = all integers **and** fractions of them (e.g. ¾, -⅝, etc)
Real Numbers	R = all rational **and** irrational numbers (e.g. $\sqrt{2}, \pi$, etc.)

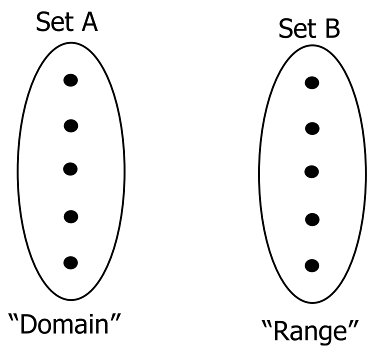
Sets are written inside curly brackets. The set with no members "{ }" is called the **empty set.**

$\in$ means "is a member of", e.g. $5 \in \{3, 4, 5, 6, 7\}$ $\notin$ means "is not a member of", e.g. $5 \notin \{6, 7, 8\}$

A **function** is a rule which links an element in Set A to **one and only one** element in Set B.



This shows a function



This does **not** show a function

The set that the function works on is called the **domain**; the values produced are called the **range.** For graphs of functions, we can think of the **domain** as the ***x* - values**, and the **range** as the ***y* - values**.

This means that any operation which produces **more than one answer** is **not** considered a function. For example, since $\sqrt{4} = 2$ **and** -2, "$f(x) = \sqrt{x}$" is **not** considered a function.

Example 1: Each function below is defined on the set of real numbers. State the **range** of each.

a) $f(x) = \sin x^\circ$ b) $g(x) = x^2$ c) $h(x) = 1 - x^2$

When choosing the domain, two cases MUST be avoided:	a) Denominators can't be zero b) Can't find the square root of a negative value

e.g. For $f(x) = \dfrac{1}{x+5}$, $x \neq -5$, i.e. $\{x \in R : x \neq -5\}$ e.g. For $g(x) = \sqrt{x-3}$, $x \geq 3$, i.e. $\{x \in R : x \geq 3\}$

Example 2: For each function, state a suitable domain.

a) $g(x) = \sqrt{3x-2}$ b) $p(\theta) = \dfrac{2}{5-\theta}$ c) $f(y) = \dfrac{y^2}{\sqrt{y-1}}$

Composite Functions

In the linear function $y = 3x - 5$, we get y by doing **two** acts: (i) multiply x by 3; (ii) then subtract 5. This is called a **composite function**, where we "do" a function to the range of another function.

e.g. If h(x) is the composite function obtained by performing f(x) on g(x), then we say

h(x) = f(g(x)) ("f of g of x")

Example 3: $f(x) = 5x + 1$ and $g(x) = 3x^2 + 2x$.

a) Find f(g(-1))

b) Find f(g(x))

c) Find f(f(x))

d) Find g(f(x))

NOTE: Usually, f(g(x)) and g(f(x)) are NOT the same!

Example 4: $f(x) = 2x + 1$, $g(x) = x^2 + 6$

a) Find formulae for:

(i) f(g(x))

(ii) g(f(x))

b) Solve the equation f(g(x)) = g(f(x))

Example 5: $f(x) = \frac{3}{x+1}$, $x \neq -1$. Find an expression for f(f(x)), as a fraction in its simplest form.

Past Paper Example: Functions f and g are defined on a set of real numbers by

$$f(x) = x^2 + 3 \qquad\qquad g(x) = x + 4$$

a) Find expressions for:

(i) f(g(x))

(ii) g(f(x))

b) Show that f(g(x)) + g(f(x)) = 0 has no real roots