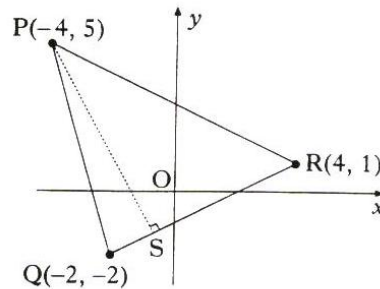


All questions should be attempted

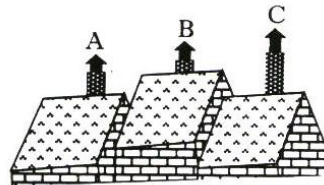
Marks

- NC 1. $P(-4, 5)$, $Q(-2, -2)$ and $R(4, 1)$ are the vertices of triangle PQR as shown in the diagram. Find the equation of PS, the altitude from P.



(3)

- NC 2. Relative to a suitable set of axes, the tops of three chimneys have coordinates given by $A(1, 3, 2)$, $B(2, -1, 4)$ and $C(4, -9, 8)$. Show that A, B and C are collinear.



(3)

- NC 3. Functions f and g , defined on suitable domains, are given by $f(x) = 2x$ and $g(x) = \sin x + \cos x$. Find $f(g(x))$ and $g(f(x))$.

(4)

- NC 4. The position vectors of the points P and Q are $\mathbf{p} = -\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ and $\mathbf{q} = 7\mathbf{i} - \mathbf{j} + 5\mathbf{k}$ respectively.

(a) Express \vec{PQ} in component form.

(2)

(b) Find the length of PQ.

(1)

- NC 5. (a) Find a real root of the equation $2x^3 - 3x^2 + 2x - 8 = 0$.

(2)

(b) Show algebraically that there are no other real roots.

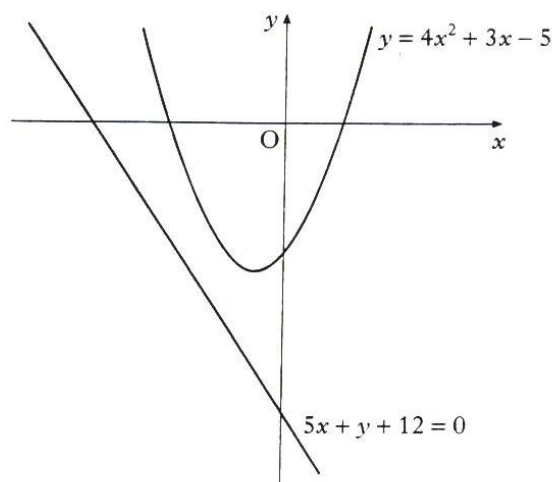
(3)

6. The diagram below shows a parabola with equation $y = 4x^2 + 3x - 5$ and a straight line with equation $5x + y + 12 = 0$.

A tangent to the parabola is drawn parallel to the given straight line.

Find the x -coordinate of the point of contact of this tangent.

(5)



7. If x° is an acute angle such that $\tan x^\circ = \frac{4}{3}$, show that the exact value of $\sin(x + 30)^\circ$ is $\frac{4\sqrt{3} + 3}{10}$.

(3)

8. Given that $y = 2x^2 + x$, find $\frac{dy}{dx}$ and hence show that $x\left(1 + \frac{dy}{dx}\right) = 2y$.

(3)

9. (a) Show that the function $f(x) = 2x^2 + 8x - 3$ can be written in the form $f(x) = a(x + b)^2 + c$ where a , b and c are constants.

(3)

(b) Hence, or otherwise, find the coordinates of the turning point of the function f .

(1)

10. Find the value of $\int_1^4 \sqrt{x} dx$.

(4)

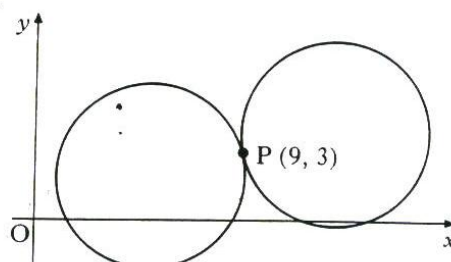
11. Express $2 \sin x^\circ - 5 \cos x^\circ$ in the form $k \sin(x - \alpha)^\circ$, $0 \leq \alpha < 360$ and $k > 0$.

(4)

12. Two identical circles touch at the point $P(9, 3)$ as shown in the diagram. One of the circles has equation $x^2 + y^2 - 10x - 4y + 12 = 0$.

Find the equation of the other circle.

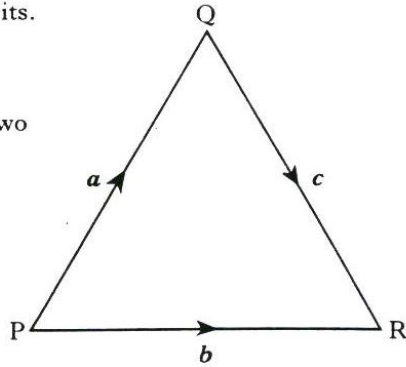
(5)



13. PQR is an equilateral triangle of side 2 units.

$$\vec{PQ} = \mathbf{a}, \vec{PR} = \mathbf{b} \text{ and } \vec{QR} = \mathbf{c}.$$

Evaluate $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c})$ and hence identify two vectors which are perpendicular.



Marks

(4)

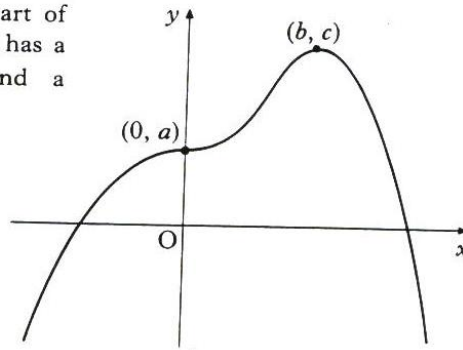
14. For what range of values of c does the equation $x^2 + y^2 - 6x + 4y + c = 0$ represent a circle?

(3)

15. The curve $y = f(x)$ passes through the point $(\frac{\pi}{12}, 1)$ and $f'(x) = \cos 2x$. Find $f(x)$.

(3)

16. The diagram shows a sketch of part of the graph of $y = f(x)$. The graph has a point of inflection at $(0, a)$ and a maximum turning point at (b, c) .



- (a) Make a copy of this diagram and on it sketch the graph of $y = g(x)$ where $g(x) = f(x) + 1$.

(2)

- (b) On a separate diagram, sketch the graph of $y = f'(x)$.

(2)

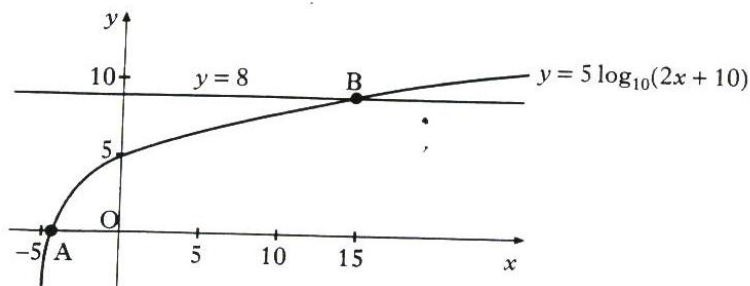
- (c) Describe how the graph of $y = g'(x)$ is related to the graph of $y = f'(x)$.

(1)

17. Part of the graph of $y = 5 \log_{10}(2x + 10)$ is shown in the diagram. This graph crosses the x -axis at the point A and the straight line $y = 8$ at the point B.

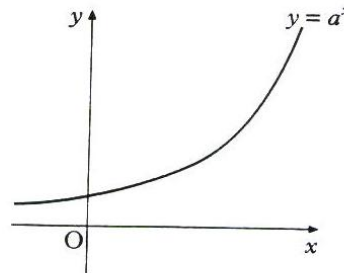
Find algebraically the x -coordinates of A and B.

(4)



18. (a) Show that $2 \cos 2x^\circ - \cos^2 x^\circ = 1 - 3 \sin^2 x^\circ$. (2)
 (b) Hence solve the equation $2 \cos 2x^\circ - \cos^2 x^\circ = 2 \sin x^\circ$ in the interval $0 \leq x < 360$. (4)

19. The diagram shows a sketch of part of the graph of $y = a^x$, $a > 1$.



- (a) If $(1, t)$ and $(u, 1)$ lie on this curve, write down the values of t and u . (2)
 (b) Make a copy of this diagram and on it sketch the graph of $y = a^{2x}$. (2)
 (c) Find the coordinates of the point of intersection of $y = a^{2x}$ with the line $x = 1$. (1)

20. Diagram 1 shows 5 cars travelling up an incline on a roller-coaster. Part of the roller-coaster rail follows the curve with equation $y = 8 + 5 \cos \frac{1}{2}x$.

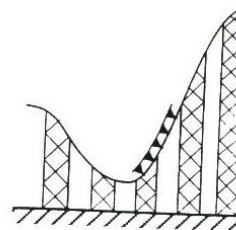


Diagram 1

Diagram 2 shows an enlargement of the last car and its position relative to a suitable set of axes. The floor of the car lies parallel to the tangent at P , the point of contact.

Calculate the acute angle a between the floor of the car and the horizontal when the car is at the point where $x_p = \frac{7\pi}{3}$.

Express your answer in degrees.

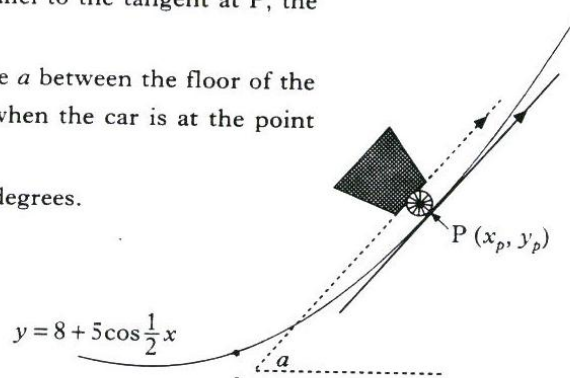


Diagram 2

[END OF QUESTION PAPER]

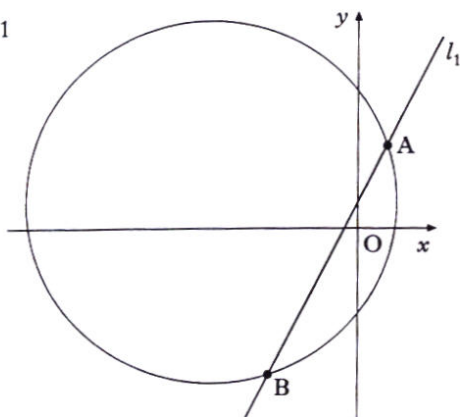
All questions should be attempted

Marks

1. Diagram 1 shows a circle with equation $x^2 + y^2 + 10x - 2y - 14 = 0$ and a straight line, l_1 , with equation $y = 2x + 1$.

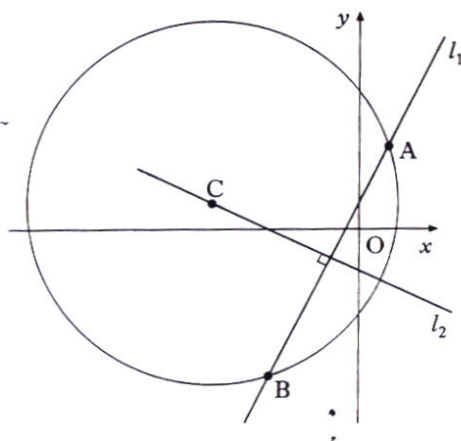
The line intersects the circle at A and B.

Diagram 1



- (a) Find the coordinates of the points A and B. (5)
- (b) Diagram 2 shows a second line, l_2 , which passes through the centre of the circle, C, and is at right angles to line l_1 .

Diagram 2



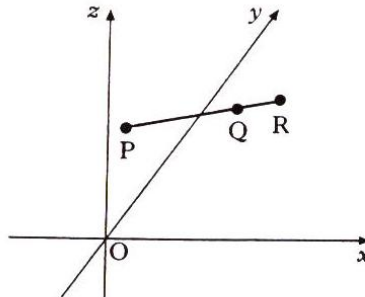
- (i) Write down the coordinates of C. (1)
- (ii) Find the equation of the line l_2 . (3)

Marks

2. Relative to the axes shown and with an appropriate scale, $P(-1, 3, 2)$ and $Q(5, 0, 5)$ represent points on a road. The road is then extended to the point R such that $\vec{PR} = \frac{4}{3}\vec{PQ}$.

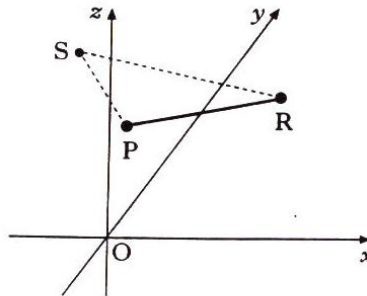
(a) Find the coordinates of R .

(3)



- (b) Roads from P and R are built to meet at the point $S(-2, 2, 5)$. Calculate the size of angle PSR .

(7)



3. The sum of £1000 is placed in an investment account on January 1st and, thereafter, £100 is placed in the account on the first day of each month.

- Interest at the rate of 0.5% per month is credited to the account on the last day of each month.
- This interest is calculated on the amount in the account on the first day of the month.

(a) How much is in the account on June 30th?

(4)

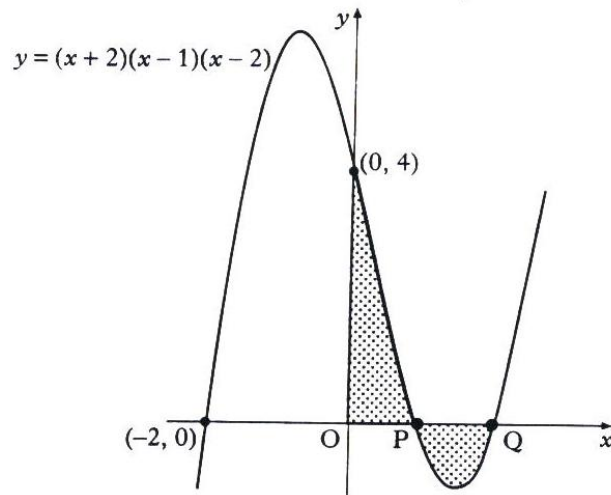
(b) On what date does the account first exceed £2000?

(2)

(c) Find a recurrence relation which describes the amount in the account, explaining your notation carefully.

(3)

4. The diagram shows a sketch of the graph of $y = (x + 2)(x - 1)(x - 2)$. The graph cuts the axes at $(-2, 0)$, $(0, 4)$ and the points P and Q. *Marks*



- (a) Write down the coordinates of P and Q. (2)
- (b) Find the total shaded area. (7)

5. Diagram 1 shows a sketch of part of the graph of $y = f(x)$ where $f(x) = (x - 2)^2 + 1$.
The graph cuts the y -axis at A and has a minimum turning point at B.

(a) Write down the coordinates of A and B.

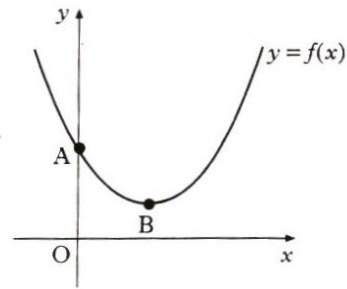


Diagram 1

- (b) Diagram 2 shows the graphs of $y = f(x)$ and $y = g(x)$ where $g(x) = 5 + 4x - x^2$.
Find the area enclosed by the two curves.

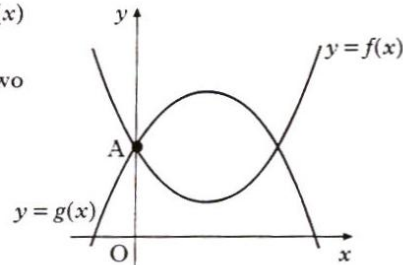
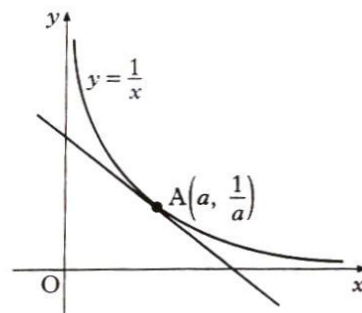


Diagram 2

- (c) $g(x)$ can be written in the form $m + n \times f(x)$ where m and n are constants.
Write down the values of m and n .

6. (a) A sketch of part of the graph of $y = \frac{1}{x}$ is shown in the diagram.
The tangent at $A\left(a, \frac{1}{a}\right)$ has been drawn.
Find the gradient of this tangent.



- (b) Hence show that the equation of this tangent is $x + a^2y = 2a$.

- (c) This tangent cuts the y -axis at B and the x -axis at C.
(i) Calculate the area of triangle OBC.
(ii) Comment on your answer to c(i).

Marks

(3)

(5)

(2)

(4)

(2)

(3)

(1)

7. In certain topics in Mathematics, such as calculus, we often require to write an expression such as $\frac{8x+1}{(2x+1)(x-1)}$ in the form $\frac{2}{2x+1} + \frac{3}{x-1}$.

$\frac{2}{2x+1} + \frac{3}{x-1}$ are called **Partial Fractions** for $\frac{8x+1}{(2x+1)(x-1)}$.

The worked example shows you how to find partial fractions for the expression $\frac{6x+2}{(x+2)(x-3)}$.

Worked Example

Find partial fractions for $\frac{6x+2}{(x+2)(x-3)}$.

$$\text{Let } \frac{6x+2}{(x+2)(x-3)} = \frac{A}{x+2} + \frac{B}{x-3} \text{ where } A \text{ and } B \text{ are constants}$$

$$= \frac{A(x-3)}{(x+2)(x-3)} + \frac{B(x+2)}{(x-3)(x+2)}$$

$$\text{ie } \frac{6x+2}{(x+2)(x-3)} = \frac{A(x-3) + B(x+2)}{(x+2)(x-3)}$$

Hence $6x+2 = A(x-3) + B(x+2)$ for all values of x .

A and B can be found as follows:

Select a value of x that makes the first bracket zero

Let $x = 3$ (this eliminates A)

$$18+2 = A \times 0 + B \times 5$$

$$20 = 5B$$

$$\underline{B = 4}$$

Select a value of x that makes the second bracket zero

Let $x = -2$ (this eliminates B)

$$-12+2 = A \times (-5) + B \times 0$$

$$-10 = -5A$$

$$\underline{A = 2}$$

$$\text{Therefore } \frac{6x+2}{(x+2)(x-3)} = \frac{2}{x+2} + \frac{4}{x-3}$$

Find partial fractions for $\frac{5x+1}{(x-4)(x+3)}$.

(6)

Marks

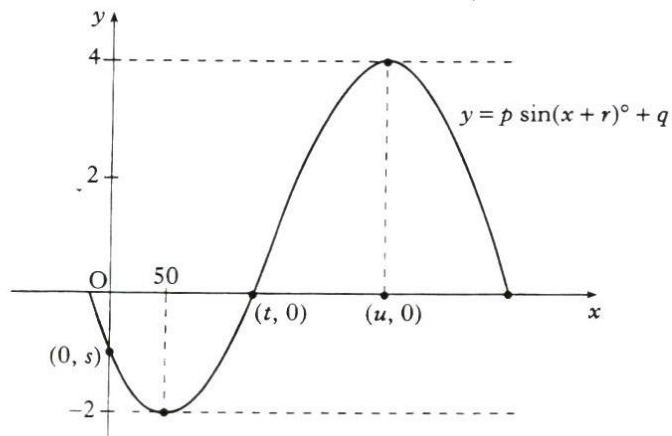
8. The radioactive element carbon-14 is sometimes used to estimate the age of organic remains such as bones, charcoal and seeds.

Carbon-14 decays according to a law of the form $y = y_0 e^{kt}$ where y is the amount of radioactive nuclei present at time t years and y_0 is the initial amount of radioactive nuclei.

- (a) The half-life of carbon-14, ie the time taken for half the radioactive nuclei to decay, is 5700 years. Find the value of the constant k , correct to 3 significant figures. (3)
- (b) What percentage of the carbon-14 in a sample of charcoal will remain after 1000 years? (3)

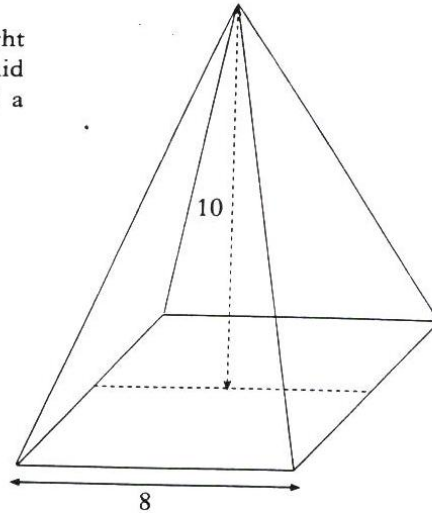
9. The sketch represents part of the graph of a trigonometric function of the form $y = p \sin(x + r)^\circ + q$. It crosses the axes at $(0, s)$ and $(t, 0)$, and has turning points at $(50, -2)$ and $(u, 4)$.

- (a) Write down values for p, q, r and u . (4)
- (b) Find the values for s and t . (4)

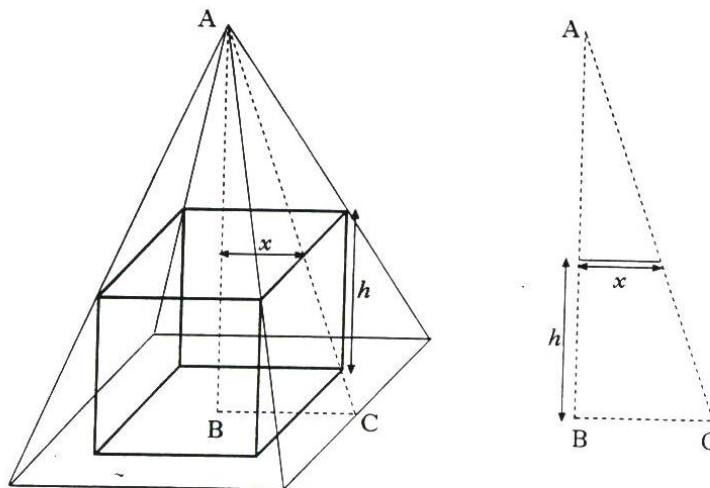


10. A cuboid is to be cut out of a right square-based pyramid. The pyramid has a square base of side 8 cm and a vertical height of 10 cm.

Marks



- (a) The cuboid has a square base of side $2x$ cm and a height of h cm.



If the cuboid is to fit into the pyramid, use the information shown in triangle ABC, or otherwise, to show that:

- (i) $h = 10 - \frac{5}{2}x$; (3)
- (ii) the volume, V , of the cuboid is given by $V = 40x^2 - 10x^3$. (1)
- (b) Hence, find the dimensions of the square-based cuboid with the greatest volume which can be cut from the pyramid. (6)

11. Two identical coins, radius 1 unit, are supported by horizontal and vertical plates at B and C. Diagram 1 shows the coins touching each other and the line of centres is inclined at p radians to the vertical.

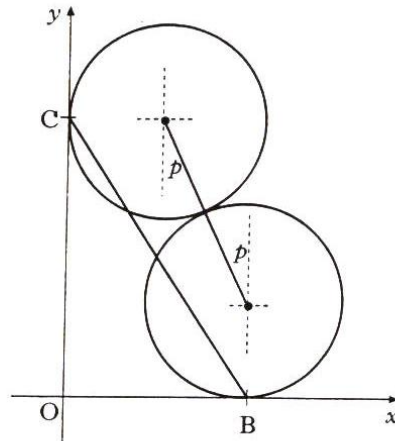


Diagram 1

Let d be the length of BC.

- (a) (i) Show that $OB = 1 + 2\sin p$. (1)
 (ii) Write down a similar expression for OC and hence show that $d^2 = 6 + 4\cos p + 4\sin p$. (2)
- (b) (i) Express d^2 in the form $6 + k\cos(p - \alpha)$. (4)
 (ii) Hence, write down the exact maximum value of d^2 and the value of p for which this occurs. (2)
- (c) Diagram 2 shows the special case where $p = \frac{\pi}{4}$.

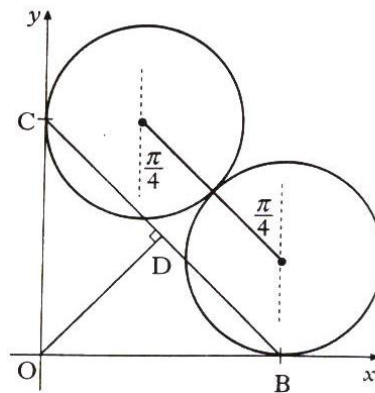


Diagram 2

- (i) Show that $OB = 1 + \sqrt{2}$ and find the exact length of BD. (2)
 (ii) Using your answer to (b) (ii), find the exact value of $\sqrt{6 + 4\sqrt{2}}$. (2)

[END OF QUESTION PAPER]