1995

S.C.E. REVISED HIGHER – ANSWERS

1995 I

1. 4 units.

2. (a) Verify that $f(3)=0$ $\therefore (x-3)$ is a factor of $f(x)$.
   (b) $(x-3)(x+4)(2x+1)$

3. $2x^3 - \frac{1}{2}x^2 + \sin x + C$

4. $k=3$

5. $4x+3y=1$

6. Prove that A_1A_2 has equation $3y=2x+5$. Then verify that $(5,4)$ does not satisfy this equation $\therefore$ ACHILLES DOES NOT PASS OVER THE SUBMARINE.

   also, prove that B_1B_2 has equation $4y=5x-9$. Then verify that $(5,4)$ DOES satisfy this equation $\therefore$ BELLIGERENT DOES PASS OVER THE SUBMARINE.

7. $-\frac{8}{x^3}+\frac{3}{2}x^{1/2}$

8. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

9. $(x-1)^2+(y-2)^2=18$

10. (a) $b=-10$ (b) Prove that $f'(-2)=33$. Since $f'(-2)>0$, f is increasing at P

11. (a) $\frac{x^2-2}{x^2-4}$ (b) $\{x : x \in R, x \neq 2 \text{ or } -2\}$

12. $\sin 2\alpha = \frac{3\sqrt{11}}{10}$

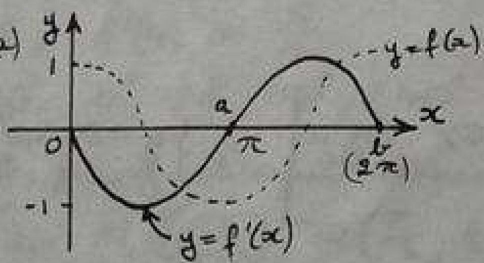
13. $k=\sqrt{29}$, $x=68{\cdot}2^\circ$

14. $f'(a)$ = gradient of line $= -\frac{1}{3}$.

15. $x=60^\circ, 180^\circ, 300^\circ$

16. 9

17. (a)



(b) $f(x)=\cos x$
$f'(x)=-\sin x$

19.8.

18. (a) $k=0{\cdot}035$ (b) $t=8{\cdot}6$ minutes

19. [Replace $(3,0)$ and $(4,3)$ into equation]
    $a=3$, $b=2$

20. $k=-5$ or 3

21. (a) Speed $=20-10t$
    Speed when thrown $=20$ m/s
    (b) 0 m/s. Ball has reached its maximum height.

1995 II

1. (a) Verify that $AB=AC=3\sqrt{5}$ $\therefore \triangle ABC$ is isosceles.
   (b)(i) [AD: $x=4$, BE: $2y-x=3$] $\therefore H(4, 3\frac{1}{2})$
   (ii) Check that $HD=1\frac{1}{2}$ units, $DA=6$ units $\therefore$ H is $\frac{1}{4}$ the way up DA

2. (a) — (b) Tangent meets curve AGAIN at $(-2,-18)$

3. KILLPEST [$u_{n+1}=0{\cdot}35u_n+500$] – ie 769 pests in the long term
   PESTKILL [$u_{n+1}=0{\cdot}15u_n+650$] – ie 765 pests in the long term
   $\therefore$ PESTKILL is more effective in the long term

4. (a)(i) $f(x)=2\sin 3x$ $\therefore a=3, b=2$ (ii) $g(x)=3\cos 3x$ $\therefore c=3, d=3$
   (b) [Let $h(x)=2\sin 3x+3\cos 3x = q\sin(3x+r)$]
   $\therefore q=\sqrt{13}$, $r=56{\cdot}3^\circ$ $\therefore h(x)=\sqrt{13}\sin(3x+56{\cdot}3^\circ)$.

$\left[\vec{QP} = \begin{pmatrix}0\\2\\-2\end{pmatrix},\ \vec{QR} = \begin{pmatrix}-2\\2\\0\end{pmatrix}\right] \longrightarrow P\hat{Q}R = 60°.$

(b)(i) $T(2\frac{1}{3}, 3\frac{1}{3}, 1\frac{1}{3})$ (ii) Verify that $PT = QT = RT = \frac{2}{3}\sqrt{6}$
$\therefore$ P, Q, R are equidistant from T.

(c)(i) Verify that $PA = QA = RA = \sqrt{3}$. $\therefore$ P, Q, R are equidistant from A.
(ii) By construction, T lies within $\triangle PQR$ – ie T is coplanar with P, Q, R.
Also $PA[\sqrt{3}] > PT[\frac{2}{3}\sqrt{6}]$ $\therefore$ A is not coplanar with P, Q, R.
$\therefore$ T is the centre of the circle passing through P, Q, R (AND NOT A).

6. [THE GAUSSIAN ELIMINATION METHOD AS DESCRIBED MUST BE USED.]
[1ST obtain $\left[\begin{array}{ccc|c} 1 & -2 & 1 & 6 \\ 0 & 7 & -4 & -11 \\ 0 & 0 & 2 & 2 \end{array}\right]$ $\therefore$ $x = 3,\ y = -1,\ z = 1$

7. $a = -\frac{1}{3},\ b = \frac{1}{3},\ c = 3$

8. [YOU MUST OBTAIN: $A(-12, -15)$ $r_1 = 5$ $C(24, 12)$ $r_3 = 10$
$AC = 45$ units
$r_2 = 15$ (radius of central circle) $B(4, -3)$ [QUITE HARD]]
$(x-4)^2 + (y+3)^2 = 225$

9. (a) — (b) $-\frac{3}{4}\cos x + \frac{1}{12}\cos 3x + C$

10. (a) 48 units^2 (b) —
Other 2 solutions are $6+6\sqrt{3},\ 6-6\sqrt{3}$ ($\cong$ 16.39, −4.39 APPROX.)
(c)(i) — (ii) Other 2 solutions are $6+6\sqrt{3}$, $6-6\sqrt{3}$ which the other 2 solutions do NOT satisfy.
(iii) p must lie between 0 and 12
$\therefore p = 6$ is the only valid solution to this problem.

11. (a) — (b) —
(c) $x = 4$ (not −4 since A is in the 1ST quadrant)
[Table of values required to verify MAX.]
$\therefore$ MAX. LENGTH = <u>10 units</u>.