1994 I

1. $\frac{3}{4}x^4 + 2x^2 + c$ 2. $k = 3$ 3. $D(-11, 2, 3)$

4. $\vec{RS} = \begin{pmatrix}3\\3\\6\end{pmatrix}$ $\vec{ST} = \begin{pmatrix}1\\1\\2\end{pmatrix}$ $\therefore \vec{RS} = 3\vec{ST}$ $\therefore$ RS // to ST
$\therefore$ R, S, T are collinear ($\because$ S is a common point)

5. Centre $(1,1)$ [C] $m_{BC} = \frac{1}{2}$, $m_{AC} = -2$ Since $m_{BC} m_{AC} = -1$, BC $\perp^r$ to AC.
But also $C\hat{B}D = C\hat{A}D = 90°$ (angle between TGT and radius)
$\therefore B\hat{D}A = 90°$ (angle sum in ABCD is 360°)
$\therefore$ Tangents at B and A are perpendicular.

6. 0 [Use $\sin(\alpha+\beta)$ TWICE and also special values]

7. $\underset{\sim}{u} + \underset{\sim}{v} = \begin{pmatrix}-2\\8\\2\end{pmatrix}$ $\underset{\sim}{u} - \underset{\sim}{v} = \begin{pmatrix}-4\\-2\\4\end{pmatrix}$. Finally verify that $(\underset{\sim}{u} + \underset{\sim}{v}).(\underset{\sim}{u} - \underset{\sim}{v}) = 0$.

8. (a) A(6,6) B(−2,−2) (b) $(x-2)^2 + (y-2)^2 = 32$

9. (a) $u_2 = 4.7$ (b) [Continue sequence $\rightarrow u_7 = 10.96\ldots$] $\therefore n = 7$ (b) 20

10. $-\frac{3}{x^4} - 3\sin 3x$ 11. $(x+4)^2 + 2$ S.P. is $(-4, 2)$

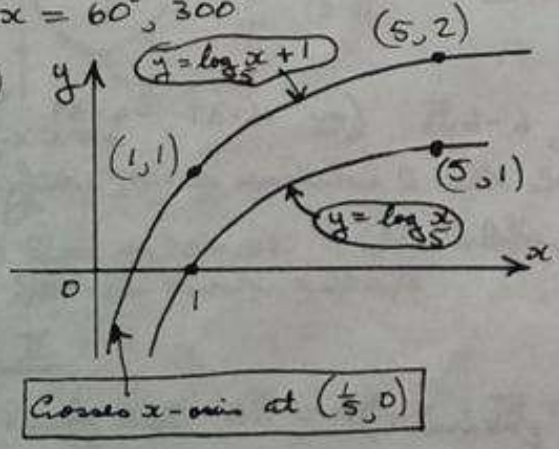
12. (a) $a = 2$, $b = 1$, $c = 2$ (b) [Solve $2 + \sin 2x = 2.5$] $x = \frac{\pi}{12}, \frac{5\pi}{12}$.

13. (a) $\sin 2\theta = \frac{24}{25}$ (b) $\sin 4\theta = \frac{336}{625}$

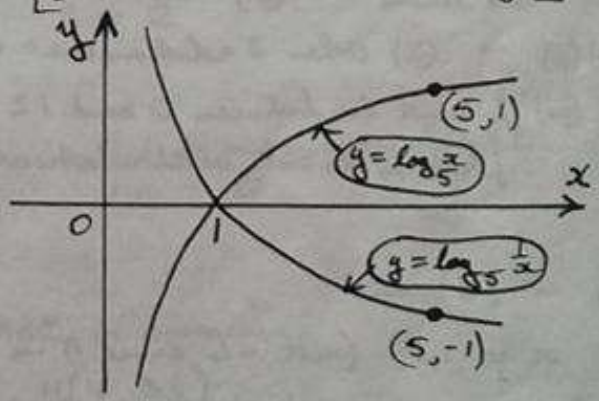
14. $m_{TGT} = 4$ [Required angle is $75.96° - 45°$] $= 30.96°$ ($\simeq 31°$)

15. $x = 60°, 300°$

16. (a)



(b) [NB $\log \frac{1}{x} = \log x^{-1} = -\log x$]



17. $\frac{dy}{dx} = 3\sin^2 x \cos x$; $\int \sin^2 x \cos x\, dx = \frac{1}{3}\sin^3 x + c$

18. [Let S/T be $(x, 0, 0)$] S(1,0,0) T(7,0,0)

19. $f[f(x)] = \frac{x}{1-2x}$ 20. $x = 3.402$; $P(3.402, 42)$

1994 II

1. (a) $a = 6$ [$\therefore y = 2x^3 + x^2 - 13x + 6$] Crosses y-axis at (0,6)
(b) OTHER points are $(-3, 0)$, $(\frac{1}{2}, 0)$ [APART FROM (2,0)]

2. (a) $2x + y = 10$ (b) $\begin{bmatrix}2x+y=10\\y=\frac{1}{2}x\end{bmatrix} \rightarrow D(4, 2)$ (c) [$AD = \sqrt{5}$] Area $= 5$ units^2

(a) $B(6,4,2)$ $C(4,3,4)$ $D(6,2,2)$

(b) Calculate mid-pt of AD to be $(4,3,4)$ $= C$.

(c) $\left[\vec{OA} = \begin{pmatrix}2\\4\\6\end{pmatrix} \quad \vec{OB} = \begin{pmatrix}6\\4\\2\end{pmatrix}\right] \rightarrow A\hat{O}B = 44.4^\circ$. (d) $O\hat{A}B = 67.8^\circ$

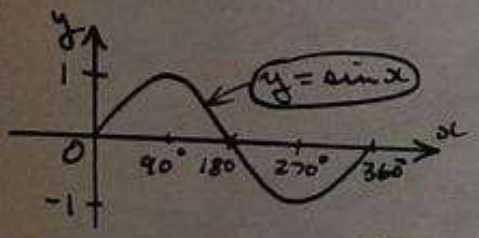
4. (a)(i) $[Q(0,3) \quad P(14,10)] \rightarrow [PQ = 7\sqrt{5} \text{ units}] = 35\sqrt{5}$ cm $= 78.26$ cm

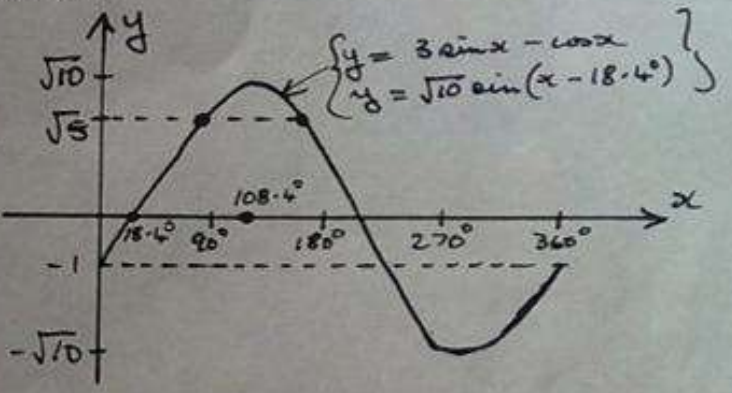
(ii) $[r_1 = 3 \text{ units} = 15 \text{ cm} \; ; \; r_2 = 10 \text{ units} = 50 \text{ cm}][78.26 - 65]$ 13 cm 3 mm (to nearest mm.)

(b)(i) $m_{PB} = 1$ (ii) $[AC: \; x + y = 10]$ $A(8,2)$ $C(6,4)$

5. (a) $\sqrt{10}\sin(x - 18.4^\circ)$ (b) $63.4^\circ, 153.4^\circ$

(c)





$3\sin x - \cos x \leq \sqrt{5} \quad (0^\circ \leq x \leq 360^\circ)$

$\therefore 0^\circ \leq x \leq 63.4^\circ$ or $153.4^\circ \leq x \leq 180^\circ$

6. (a) $f(0) = -1$, $f(0.5) = 0.75$

Since $f(0) < 0$ and $f(0.5) > 0$, there is a root between 0 and 0.5

(b) $[x_0 = 0.25, \; x_2 = 0.32, \; x_3 = 0.312, \; x_4 = 0.313, \; x_5 = 0.313]$ 0.31 (to 2 D.P.)

7. (a) — (b) [MIN. OCCURS WHEN $x = 4$ – TABLE REQUIRED]

MIN. AREA $= 16 \text{ units}^2$ MAX. AREA $= 24 \text{ units}^2$.

8. (a) — (b) $4x + 7$ (c) $6x + 5$

9. (a) $2k + q = -2$ (b) $\left[\text{when } x = 2, \; \frac{dy}{dx} = 1\right] \rightarrow k = -3 \quad [q = 4]$

Equation of parabola is $y = x^2 - 3x + 4$

(c) $\Delta = -7$ $\therefore$ No real roots $\therefore$ Parabola does not cross x-axis.

10. Cross-sectional area $= 69\frac{1}{3} \text{ m}^2$

Volume $= 4160 \text{ m}^3$