Paper 1

Marking instructions for each question

Question	Generic scheme	Illustrative scheme	Max mark
1.	$\bullet^1$ differentiate $\bullet^2$ calculate gradient $\bullet^3$ find the value of y $\bullet^4$ find equation of tangent	$\bullet^1$ $2x - 4$ $\bullet^2$ 6 $\bullet^3$ 12 $\bullet^4$ $y = 6x - 18$	4
2.	$\bullet^1$ find the centre $\bullet^2$ calculate the radius $\bullet^3$ state equation of circle	$\bullet^1$ $(-3, 4)$ $\bullet^2$ $\sqrt{17}$ $\bullet^3$ $(x+3)^2 + (y-4)^2 = 17$ or equivalent	3
3. (a)	$\bullet^1$ find gradient l_1 $\bullet^2$ state gradient l_2	$\bullet^1$ $\dfrac{1}{\sqrt{3}}$ $\bullet^2$ $-\sqrt{3}$	2
3. (b)	$\bullet^3$ using $m = \tan\theta$ $\bullet^4$ calculating angle	$\bullet^3$ $\tan\theta = -\sqrt{3}$ $\bullet^4$ $\theta = \dfrac{2\pi}{3}$ or $120°$	2
4.	$\bullet^1$ complete integration $\bullet^2$ substitute limits $\bullet^3$ evaluate	$\bullet^1$ $-\dfrac{1}{6}x^{-1}$ $\bullet^2$ $\left(-\dfrac{1}{6\times2}\right) - \left(-\dfrac{1}{6\times1}\right)$ $\bullet^3$ $\dfrac{1}{12}$	3

Question	Generic scheme	Illustrative scheme	Max mark
5.	$\bullet^1$ find $\overrightarrow{CD}$	$\bullet^1$ $\begin{pmatrix} x-4 \\ -3 \\ -1 \end{pmatrix}$	4
	$\bullet^2$ find $\overrightarrow{AB}$	$\bullet^2$ $\begin{pmatrix} 5 \\ -10 \\ -5 \end{pmatrix}$	
	$\bullet^3$ equate scalar product to zero	$\bullet^3$ $5(x-4)+(-10)(-3)+(-5)(-1)=0$	
	$\bullet^4$ calculate value of x	$\bullet^4$ $x=-3$	
6.	$\bullet^1$ substitute into discriminant	$\bullet^1$ $(p+1)^2 - 4 \times 1 \times 9$	4
	$\bullet^2$ apply condition for no real roots	$\bullet^2$ $... < 0$	
	$\bullet^3$ determine zeroes of quadratic expression	$\bullet^3$ $-7, 5$	
	$\bullet^4$ state range with justification	$\bullet^4$ $-7 < p < 5$ with eg sketch or table of signs	
7.	$\bullet^1$ substitute for y in equation of circle	$\bullet^1$ $x^2 + (3x-5)^2 + 2x - 4(3x-5) - 5 = 0$	5
	$\bullet^2$ express in standard quadratic form	$\bullet^2$ $10x^2 - 40x + 40 = 0$	
	$\bullet^3$ demonstrate tangency	$\bullet^3$ $10(x-2)^2 = 0$ only one solution implies tangency	
	$\bullet^4$ find x-coordinate	$\bullet^4$ $x = 2$	
	$\bullet^5$ find y-coordinate	$\bullet^5$ $y = 1$	

Question	Generic scheme	Illustrative scheme	Max mark
8. (a)	$\bullet^1$ use appropriate strategy	$\bullet^1$ $(1)^3 - 4(1)^2 + a(1) + b = 0$	5
	$\bullet^2$ obtain an expression for a and b	$\bullet^2$ $a + b = 3$	
	$\bullet^3$ obtain a second expression for a and b	$\bullet^3$ $2a + b = -4$	
	$\bullet^4$ find the value of a or b	$\bullet^4$ $a = -7$ or $b = 10$	
	$\bullet^5$ find the second value	$\bullet^5$ $b = 10$ or $a = -7$	
8. (b)	$\bullet^6$ obtain quadratic factor	$\bullet^6$ $\left(x^2 - 3x - 10\right)$	3
	$\bullet^7$ complete factorisation	$\bullet^7$ $(x-1)(x-5)(x+2)$	
	$\bullet^8$ state solutions	$\bullet^8$ $x = 1$, $x = 5$, $x = -2$	
9. (a)	$\bullet^1$ interpret information	$\bullet^1$ $13 = 28m + 6$	2
	$\bullet^2$ solve to find m	$\bullet^2$ $m = \dfrac{1}{4}$	
9. (b) (i)	$\bullet^3$ state condition	$\bullet^3$ a limit exists as $-1 < \dfrac{1}{4} < 1$	1
9. (b) (ii)	$\bullet^4$ know how to calculate limit	$\bullet^4$ $L = \dfrac{1}{4}L + 6$	2
	$\bullet^5$ calculate limit	$\bullet^5$ $L = 8$	

Question	Generic scheme	Illustrative scheme	Max mark
10. (a)	$\bullet^1$ state value	$\bullet^1$ 2	1
10. (b)	$\bullet^1$ use laws of logarithms	$\bullet^1$ $\log_4 x(x-6)$	5
	$\bullet^2$ link to part (a)	$\bullet^2$ $\log_4 x(x-6)=2$	
	$\bullet^3$ use laws of logarithms	$\bullet^3$ $x(x-6)=4^2$	
	$\bullet^4$ write in standard quadratic form	$\bullet^4$ $x^2-6x-16=0$	
	$\bullet^5$ solve for x and identify appropriate solution	$\bullet^5$ 8	
11.	$\bullet^1$ start to differentiate	$\bullet^1$ $3\times 4\sin^2 x...$	3
	$\bullet^2$ complete differentiation	$\bullet^2$ $...\times\cos x$	
	$\bullet^3$ evaluate derivative	$\bullet^3$ $\dfrac{-3\sqrt{3}}{2}$	
12.	$\bullet^1$ calculate lengths AC and AD	$\bullet^1$ $AC=\sqrt{17}$ and $AD=5$ stated or implied by $\bullet^3$	5
	$\bullet^2$ select appropriate formula and express in terms of p and q	$\bullet^2$ $\cos q \cos p + \sin q \sin p$ stated or implied by $\bullet^4$	
	$\bullet^3$ calculate two of $\cos p,\ \cos q,\ \sin p,\ \sin q$	$\bullet^3$ $\cos p = \dfrac{4}{\sqrt{17}}$, $\cos q = \dfrac{4}{5}$ $\sin p = \dfrac{1}{\sqrt{17}}$, $\sin q = \dfrac{3}{5}$	
	$\bullet^4$ calculate other two and substitute into formula	$\bullet^4$ $\dfrac{4}{5}\times\dfrac{4}{\sqrt{17}}+\dfrac{3}{5}\times\dfrac{1}{\sqrt{17}}$	
	$\bullet^5$ arrange into required form	$\bullet^5$ $\dfrac{19}{5\sqrt{17}}\times\dfrac{\sqrt{17}}{\sqrt{17}}=\dfrac{19\sqrt{17}}{85}$ or $\dfrac{19}{5\sqrt{17}}=\dfrac{19\sqrt{17}}{5\times 17}=\dfrac{19\sqrt{17}}{85}$	

Question	Generic scheme	Illustrative scheme	Max mark
13.	$\bullet^1$ know to and start to integrate	$\bullet^1$ eg $y = \dfrac{4}{2}x^2...$	4
	$\bullet^2$ complete integration	$\bullet^2$ $y = \dfrac{4}{2}x^2 - \dfrac{6}{3}x^3 + c$	
	$\bullet^3$ substitute for x and y	$\bullet^3$ $9 = 2(-1)^2 - 2(-1)^3 + c$	
	$\bullet^4$ state expression for y	$\bullet^4$ $y = 2x^2 - 2x^3 + 5$	
14. (a)		**Method 1: Using factorisation**	5
	$\bullet^1$ use double angle formula	$\bullet^1$ $2\cos^2 x° - 1...$ **stated or implied by $\bullet^2$**	
	$\bullet^2$ express as a quadratic in $\cos x°$	$\bullet^2$ $2\cos^2 x° - 3\cos x° + 1 = 0$ $\left.\begin{array}{}\end{array}\right\}$ = 0 must appear at either of these lines to gain $\bullet^2$	
	$\bullet^3$ start to solve	$\bullet^3$ $(2\cos x° - 1)(\cos x° - 1)$	
		Method 2: Using quadratic formula	
		$\bullet^1$ $2\cos^2 x° - 1...$ **stated or implied by $\bullet^2$**	
		$\bullet^2$ $2\cos^2 x° - 3\cos x° + 1 = 0$ **stated explicitly**	
		$\bullet^3$ $\dfrac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 2 \times 1}}{2 \times 2}$	
	$\bullet^4$ reduce to equations in $\cos x°$ only	**In both methods:**	
		$\bullet^4$ $\cos x° = \dfrac{1}{2}$ and $\cos x° = 1$	
	$\bullet^5$ process solutions in given domain	$\bullet^5$ $0, 60, 300$	
		Candidates who include 360 lose $\bullet^5$.	
		or	
		$\bullet^4$ $\cos x = 1$ and $x = 0$	
		$\bullet^5$ $\cos x° = \dfrac{1}{2}$ and $x = 60$ or 300	
		Candidates who include 360 lose $\bullet^5$.	
14. (b)	$\bullet^6$ interpret relationship with (a)	$\bullet^6$ $2x = 0$ and 60 and 300	2
	$\bullet^7$ state valid values	$\bullet^7$ 0, 30, 150, 180, 210 and 330	

Question	Generic scheme	Illustrative scheme	Max mark
15. (a)	$\bullet^1$ interpret notation	$\bullet^1$ $g\left(x^3-1\right)$	2
	$\bullet^2$ complete process	$\bullet^2$ $3x^3-2$	
15. (b)	$\bullet^3$ start to rearrange for x	$\bullet^3$ $3x^3=y+2$	3
	$\bullet^4$ rearrange	$\bullet^4$ $x=\sqrt[3]{\dfrac{y+2}{3}}$	
	$\bullet^5$ state expression for $h(x)$	$\bullet^5$ $h(x)=\sqrt[3]{\dfrac{x+2}{3}}$	

[END OF SPECIMEN MARKING INSTRUCTIONS]

Paper 2
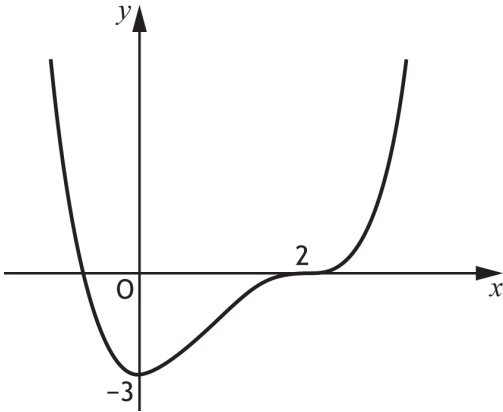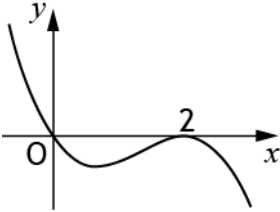
Marking instructions for each question

Question	Generic scheme	Illustrative scheme	Max mark
1. (a)	$\bullet^1$ calculate gradient of AB $\bullet^2$ use property of perpendicular lines $\bullet^3$ determine equation of altitude	$\bullet^1$ $m_{AB} = -3$ $\bullet^2$ $m_{alt} = \dfrac{1}{3}$ $\bullet^3$ $x - 3y = 4$	3
1. (b)	$\bullet^4$ calculate midpoint of AC $\bullet^5$ calculate gradient of median $\bullet^6$ determine equation of median	$\bullet^4$ $(4, 5)$ $\bullet^5$ $m_{BM} = 2$ $\bullet^6$ $y = 2x - 3$	3
1. (c)	$\bullet^7$ find x or y coordinate $\bullet^8$ find remaining coordinate	$\bullet^7$ $x = 1$ or $y = -1$ $\bullet^8$ $y = -1$ or $x = 1$	2
2.	$\bullet^1$ write in integrable form $\bullet^2$ integrate one term $\bullet^3$ integrate other term $\bullet^4$ complete integration and simplify	$\bullet^1$ $4x + x^{-2}$ $\bullet^2$ eg $\dfrac{4}{2}x^2 + \ldots$ $\bullet^3$ $\ldots \dfrac{x^{-1}}{-1}$ $\bullet^4$ $2x^2 - x^{-1} + c$	4
3.	$\bullet^1$ value of a $\bullet^2$ value of b $\bullet^3$ calculate k	$\bullet^1$ 1 $\bullet^2$ -2 $\bullet^3$ -1	3

Question	Generic scheme	Illustrative scheme	Max mark
4. (a)	$\bullet^1$ state components of $\overrightarrow{DB}$ $\bullet^2$ state coordinates of M $\bullet^3$ state components of $\overrightarrow{DM}$	$\bullet^1 \begin{pmatrix} 2 \\ 2 \\ -6 \end{pmatrix}$ $\bullet^2 \; (2,0,0)$ stated or implied by $\bullet^3$ $\bullet^3 \begin{pmatrix} 0 \\ -2 \\ -6 \end{pmatrix}$	3
4. (b)	$\bullet^4$ evaluate $\overrightarrow{DB}.\overrightarrow{DM}$ $\bullet^5$ evaluate $\left|\overrightarrow{DB}\right|$ $\bullet^6$ evaluate $\left|\overrightarrow{DM}\right|$ $\bullet^7$ use scalar product $\bullet^8$ calculate angle	$\bullet^4 \; 32$ $\bullet^5 \; \sqrt{44}$ $\bullet^6 \; \sqrt{40}$ $\bullet^7 \; \cos BDM = \dfrac{32}{\sqrt{44}\sqrt{40}}$ $\bullet^8 \; 40\cdot3°$ or 0.703 rads	5

Question	Generic scheme	Illustrative scheme	Max mark
5.	$\bullet^1$ know to integrate and interpret limits $\bullet^2$ use 'upper $-$ lower' $\bullet^3$ integrate $\bullet^4$ substitute limits $\bullet^5$ evaluate area	$\bullet^1$ $\displaystyle\int_{-3}^{0} \ldots dx$ $\bullet^2$ $\displaystyle\int_{-3}^{0}\left(x^3+3x^2+2x+3\right)-\left(2x+3\right)dx$ $\bullet^3$ $\dfrac{1}{4}x^4+x^3$ $\bullet^4$ $0-\left(\dfrac{1}{4}(-3)^4+(-3)^3\right)$ $\bullet^5$ $\dfrac{27}{4}$ units^2	**5**

Question	Generic scheme	Illustrative scheme	Max mark
6. (a)	**Method 1**	**Method 1**	3
	$\bullet^1$ identify common factor	$\bullet^1$ $3(x^2+8x\ldots\ldots$ stated or implied by $\bullet^2$	
	$\bullet^2$ complete the square	$\bullet^2$ $3(x+4)^2\ldots\ldots$	
	$\bullet^3$ process for c and write in required form	$\bullet^3$ $3(x+4)^2+2$	
	Method 2	**Method 2**	3
	$\bullet^1$ expand completed square form	$\bullet^1$ $ax^2+2abx+ab^2+c$	
	$\bullet^2$ equate coefficients	$\bullet^2$ $a=3,\ 2ab=24,\ ab^2+c=50$	
	$\bullet^3$ process for b and c and write in required form	$\bullet^3$ $3(x+4)^2+2$	
6. (b)			2
	$\bullet^4$ differentiate two terms	$\bullet^4$ $3x^2+24x\ldots$	
	$\bullet^5$ complete differentiation	$\bullet^5$ $\ldots+50$	
6. (c)	**Method 1**	**Method 1**	2
	$\bullet^6$ link with (a) and identify sign of $(x+4)^2$	$\bullet^6$ $f'(x)=3(x+4)^2+2$ and $(x+4)^2\geq0\ \forall x$	
	$\bullet^7$ communicate reason	$\bullet^7$ $\therefore 3(x+4)^2+2>0\ \Rightarrow$ always strictly increasing	
	Method 2	**Method 2**	2
	$\bullet^6$ identify minimum value of $f'(x)$	$\bullet^6$ eg minimum value $=2$ or annotated sketch	
	$\bullet^7$ communicate reason	$\bullet^7$ $2>0 \therefore (f'(x)>0)\Rightarrow$ always strictly increasing	

Question	Generic scheme	Illustrative scheme	Max mark
7. (a)	$\bullet^1$ evidence of reflecting in x-axis $\bullet^2$ vertical translation of 2 units identifiable from graph	$\bullet^1$ reflection of graph in x-axis $\bullet^2$ graph moves parallel to y-axis by 2 units upwards 	2
7. (b)	$\bullet^3$ identify roots $\bullet^4$ interpret point of inflexion $\bullet^5$ complete cubic curve	$\bullet^3$ 0 **and** 2 only $\bullet^4$ turning point at $(2,0)$ $\bullet^5$ cubic passing through origin with negative gradient 	3

	Question	Generic scheme	Illustrative scheme	Max mark
8.	(a)	$\bullet^1$ use compound angle formula	$\bullet^1$ $k\cos x\cos a - k\sin x\sin a$ **stated explicitly**	4
		$\bullet^2$ compare coefficients	$\bullet^2$ $k\cos a = 5, k\sin a = 2$ **stated explicitly**	
		$\bullet^3$ process for k	$\bullet^3$ $k = \sqrt{29}$	
		$\bullet^4$ process for a and express in required form	$\bullet^4$ $\sqrt{29}\cos(x + 0\cdot 38)$	
8.	(b)	$\bullet^5$ equate to 12 and simplify constant terms	$\bullet^5$ $5\cos x - 2\sin x = 2$ or $5\cos x - 2\sin x - 2 = 0$	4
		$\bullet^6$ use result of part (a) and rearrange	$\bullet^6$ $\cos(x + 0\cdot 3805\ldots) = \dfrac{2}{\sqrt{29}}$	
		$\bullet^7$ solve for $x + a$	$\bullet^7$ $\begin{array}{c\|c} \bullet^7 & \bullet^8 \\ 1\cdot 1902\ldots, & 5\cdot 0928\ldots \end{array}$	
		$\bullet^8$ solve for x	$\bullet^8$ $0\cdot 8097\ldots, \quad 4\cdot 712\ldots$	

Question	Generic scheme	Illustrative scheme	Max mark
9. (a)	•^{1} equate volume to 100 •^{2} obtain an expression for h •^{3} demonstrate result	•^{1} $V = \pi r^2 h = 100$ •^{2} $h = \dfrac{100}{\pi r^2}$ •^{3} $A = \pi r^2 + 2\pi r^2 + 2\pi r \times \dfrac{100}{\pi r^2}$ leading to $A = \dfrac{200}{r} + 3\pi r^2$	3
9. (b)	•^{4} start to differentiate •^{5} complete differentiation •^{6} set derivative to zero •^{7} obtain r •^{8} verify nature of stationary point •^{9} interpret and communicate result	•^{4} $A'(r) = 6\pi r ...$ •^{5} $A'(r) = 6\pi r - \dfrac{200}{r^2}$ •^{6} $6\pi r - \dfrac{200}{r^2} = 0$ •^{7} $r = \sqrt[3]{\dfrac{100}{3\pi}} \ (\approx 2 \cdot 20)$ metres •^{8} table of signs for a derivative when $r = 2 \cdot 1974...$ •^{9} minimum when $r \approx 2 \cdot 20$ (m) or •^{8} $A''(r) = 6\pi + \dfrac{400}{r^3}$ •^{9} $A''(2 \cdot 1974...) > 0 \therefore$ minimum when $r \approx 2 \cdot 20$ (m)	6

Question	Generic scheme	Illustrative scheme	Max mark
10.	$\bullet^1$ start to integrate	$\bullet^1$ $-\dfrac{1}{4}\cos\ldots$	6
	$\bullet^2$ complete integration	$\bullet^2$ $-\dfrac{1}{4}\cos\left(4x-\dfrac{\pi}{2}\right)$	
	$\bullet^3$ process limits	$\bullet^3$ $-\dfrac{1}{4}\cos\left(4a-\dfrac{\pi}{2}\right)+\dfrac{1}{4}\cos\left(\dfrac{4\pi}{8}-\dfrac{\pi}{2}\right)$	
	$\bullet^4$ simplify numeric term and equate to $\dfrac{1}{2}$	$\bullet^4$ $-\dfrac{1}{4}\cos\left(4a-\dfrac{\pi}{2}\right)+\dfrac{1}{4}=\dfrac{1}{2}$	
	$\bullet^5$ start to solve equation	$\bullet^5$ $\cos\left(4a-\dfrac{\pi}{2}\right)=-1$	
	$\bullet^6$ solve for a	$\bullet^6$ $a=\dfrac{3\pi}{8}$	
11.	**Method 1**	**Method 1**	**3**
	$\bullet^1$ substitute for $\sin 2x$	$\bullet^1$ $\dfrac{2\sin x\cos x}{2\cos x}-\sin x\cos^2 x$ stated explicitly as above or in a simplified form of the above	
	$\bullet^2$ simplify and factorise	$\bullet^2$ $\sin x\left(1-\cos^2 x\right)$	
	$\bullet^3$ substitute for $1-\cos^2 x$ and simplify	$\bullet^3$ $\sin x\times\sin^2 x$ leading to $\sin^3 x$	
	Method 2	**Method 2**	**3**
	$\bullet^1$ substitute for $\sin 2x$	$\bullet^1$ $\dfrac{2\sin x\cos x}{2\cos x}-\sin x\cos^2 x$ stated explicitly as above or in a simplified form of the above	
	$\bullet^2$ simplify and substitute for $\cos^2 x$	$\bullet^2$ $\sin x-\sin x\left(1-\sin^2 x\right)$	
	$\bullet^3$ expand and simplify	$\bullet^3$ $\sin x-\sin x+\sin^3 x$ leading to $\sin^3 x$	

Question	Generic scheme	Illustrative scheme	Max mark
12. (a)	**Method 1** $\bullet^1$ calculate m_{AB} $\bullet^2$ calculate m_{BC} $\bullet^3$ interpret result and state conclusion	**Method 1** $\bullet^1$ eg $m_{AB} = \dfrac{3}{9} = \dfrac{1}{3}$ $\bullet^2$ eg $m_{BC} = \dfrac{5}{15} = \dfrac{1}{3}$ $\bullet^3$ ... $\Rightarrow$ AB and BC are parallel (common direction), B is a common point, hence A, B and C are collinear.	3
	Method 2 $\bullet^1$ calculate an appropriate vector, eg $\overrightarrow{AB}$ $\bullet^2$ calculate a second vector, eg $\overrightarrow{BC}$ and compare $\bullet^3$ interpret result and state conclusion	**Method 2** $\bullet^1$ eg $\overrightarrow{AB} = \begin{pmatrix} 9 \\ 3 \end{pmatrix}$ $\bullet^2$ eg $\overrightarrow{BC} = \begin{pmatrix} 15 \\ 5 \end{pmatrix} \therefore \ \overrightarrow{AB} = \dfrac{3}{5}\overrightarrow{BC}$ $\bullet^3$ ... $\Rightarrow$ AB and BC are parallel (common direction), B is a common point, hence A, B and C are collinear.	3
	Method 3 $\bullet^1$ calculate m_{AB} $\bullet^2$ find equation of line and substitute point $\bullet^3$ communication	**Method 3** $\bullet^1$ $m_{AB} = \dfrac{3}{9} = \dfrac{1}{3}$ $\bullet^2$ eg, $y - 1 = \dfrac{1}{3}(x - 2)$ leading to $6 - 1 = \dfrac{1}{3}(17 - 2)$ $\bullet^3$ since C lies on line A, B and C are collinear	3
12. (b)	$\bullet^4$ find radius $\bullet^5$ determine an appropriate ratio $\bullet^6$ find centre $\bullet^7$ state equation of circle	$\bullet^4$ $6\sqrt{10}$ $\bullet^5$ eg $2:3$ or $\dfrac{2}{5}$ (using B and C) or $3:5$ or $\dfrac{8}{5}$ (using A and C) $\bullet^6$ $(8,3)$ $\bullet^7$ $(x-8)^2 + (y-3)^2 = 360$	4

Question	Generic scheme	Illustrative scheme	Max mark
13. (a)	•¹ interpret half-life •² process equation •³ write in logarithmic form •⁴ process for k	•¹ $\frac{1}{2}P_0 = P_0 e^{-25k}$ **stated or implied by •²** •² $e^{-25k} = \frac{1}{2}$ •³ $\log_e \frac{1}{2} = -25k$ •⁴ $k \approx 0 \cdot 028$	4
13. (b)	•⁵ interpret equation •⁶ process •⁷ state percentage decrease	•⁵ $P_t = P_0 e^{-80 \times 0 \cdot 028}$ •⁶ $P_t \approx 0 \cdot 1065 P_0$ •⁷ 89%	3

[END OF SPECIMEN MARKING INSTRUCTIONS]