



National
Qualifications
SPECIMEN ONLY

S847/76/11

**Mathematics
Paper 1 (Non-calculator)**

Date — Not applicable

Duration — 1 hour 30 minutes

Total marks — 70

Attempt ALL questions.

You may NOT use a calculator.

To earn full marks you must show your working in your answers.

State the units for your answer where appropriate.

You will not earn marks for answers obtained by readings from scale drawings.

Write your answers clearly in the spaces provided in the answer booklet. The size of the space provided for an answer is not an indication of how much to write. You do not need to use all the space.

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Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.



* S 8 4 7 7 6 1 1 *

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Scalar product:

$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$, where θ is the angle between \mathbf{a} and \mathbf{b}

or $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + c$
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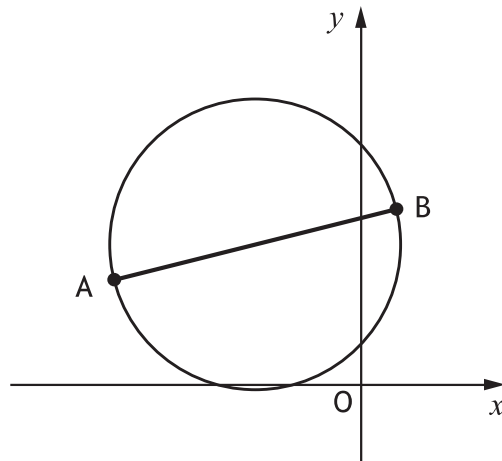
1. A curve has equation $y = x^2 - 4x + 7$.

Find the equation of the tangent to this curve at the point where $x = 5$.

4

2. A and B are the points $(-7, 3)$ and $(1, 5)$.

AB is a diameter of a circle.



Find the equation of this circle.

3

3. Line l_1 has equation $\sqrt{3}y - x = 0$.

(a) Line l_2 is perpendicular to l_1 . Find the gradient of l_2 .

2

(b) Calculate the angle l_2 makes with the positive direction of the x -axis.

2

4. Evaluate $\int_1^2 \frac{1}{6}x^{-2} dx$.

3

5. The points $A(0, 9, 7)$, $B(5, -1, 2)$, $C(4, 1, 3)$ and $D(x, -2, 2)$ are such that \vec{AB} is perpendicular to \vec{CD} .

Determine the value of x .

4

6. Determine the range of values of p such that the equation $x^2 + (p+1)x + 9 = 0$ has no real roots.

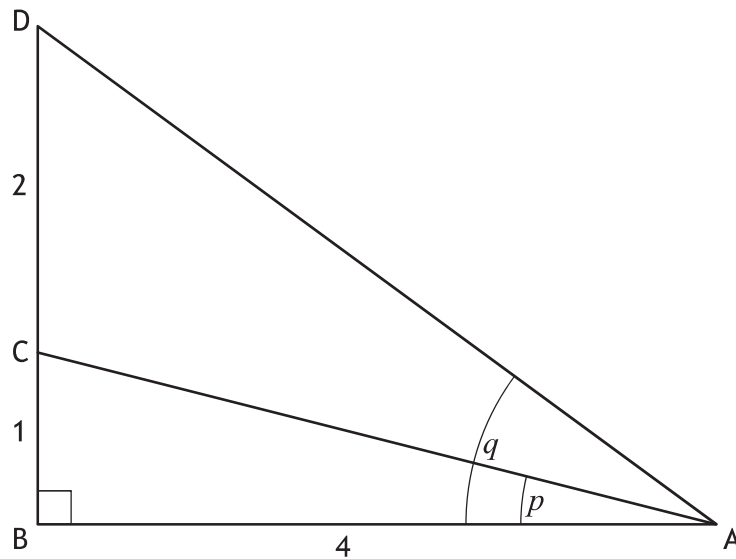
4

7. Show that the line with equation $y = 3x - 5$ is a tangent to the circle with equation $x^2 + y^2 + 2x - 4y - 5 = 0$ and find the coordinates of the point of contact.

5

8. For the polynomial, $x^3 - 4x^2 + ax + b$
- $x - 1$ is a factor
 - -12 is the remainder when it is divided by $x - 2$
- (a) Determine the values of a and b . 5
- (b) Hence solve $x^3 - 4x^2 + ax + b = 0$. 3
9. A sequence is generated by the recurrence relation $u_{n+1} = mu_n + 6$ where m is a constant.
- (a) Given $u_1 = 28$ and $u_2 = 13$, find the value of m . 2
- (b) (i) Explain why this sequence approaches a limit as $n \rightarrow \infty$. 1
- (ii) Calculate this limit. 2
10. (a) Evaluate $\log_5 25$. 1
- (b) Hence solve $\log_4 x + \log_4 (x - 6) = \log_5 25$, where $x > 6$. 5
11. Find the rate of change of the function $f(x) = 4 \sin^3 x$ when $x = \frac{5\pi}{6}$. 3

12. Triangle ABD is right-angled at B with angles $BAC = p$ and $BAD = q$ and lengths as shown in the diagram below.



Show that the exact value of $\cos(q - p)$ is $\frac{19\sqrt{17}}{85}$.

5

13. The curve $y = f(x)$ is such that $\frac{dy}{dx} = 4x - 6x^2$. The curve passes through the point $(-1, 9)$. Express y in terms of x .

4

14. (a) Solve $\cos 2x^\circ - 3 \cos x^\circ + 2 = 0$ for $0 \leq x < 360$.

5

- (b) Hence solve $\cos 4x^\circ - 3 \cos 2x^\circ + 2 = 0$ for $0 \leq x < 360$.

2

15. Functions f and g are defined on suitable domains by $f(x) = x^3 - 1$ and $g(x) = 3x + 1$.

(a) Find an expression for $k(x)$, where $k(x) = g(f(x))$. 2

(b) If $h(k(x)) = x$, find an expression for $h(x)$. 3

[END OF SPECIMEN QUESTION PAPER]



National
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SPECIMEN ONLY

S847/76/12

**Mathematics
Paper 2**

Date — Not applicable

Duration — 1 hour 45 minutes

Total marks — 80

Attempt ALL questions.

You may use a calculator.

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* S 8 4 7 7 6 1 2 *

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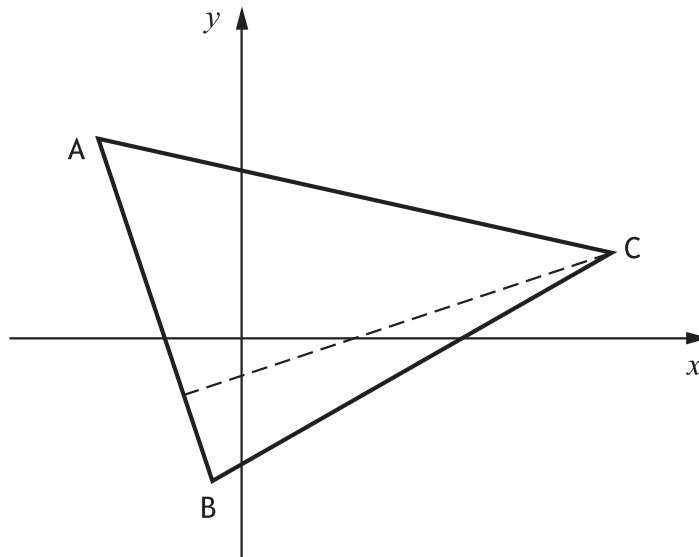
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Attempt ALL questions

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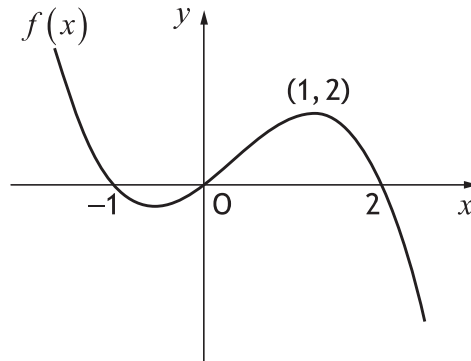
1. The vertices of triangle ABC are $A(-5, 7)$, $B(-1, -5)$ and $C(13, 3)$ as shown in the diagram.

The broken line represents the altitude from C.



- (a) Find the equation of the altitude from C. 3
- (b) Find the equation of the median from B. 3
- (c) Find the coordinates of the point of intersection of the altitude from C and the median from B. 2
2. Find $\int \frac{4x^3 + 1}{x^2} dx, x \neq 0$. 4

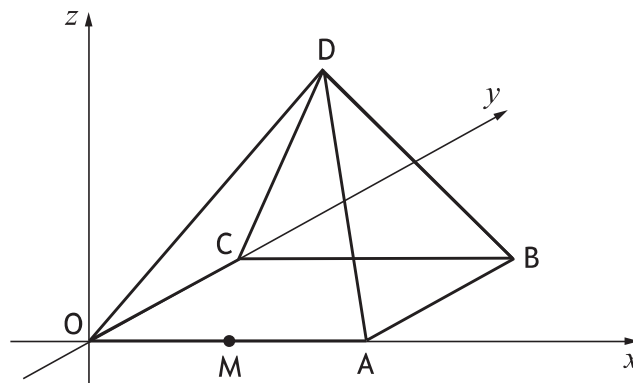
3. The diagram shows the curve with equation $y = f(x)$, where $f(x) = kx(x+a)(x+b)$.
The curve passes through $(-1, 0)$, $(0, 0)$, $(1, 2)$ and $(2, 0)$.



Find the values of a , b and k .

3

4. D,OABC is a square-based pyramid as shown.



- O is the origin and $OA = 4$ units.
- M is the mid-point of OA.
- $\vec{OD} = 2\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$

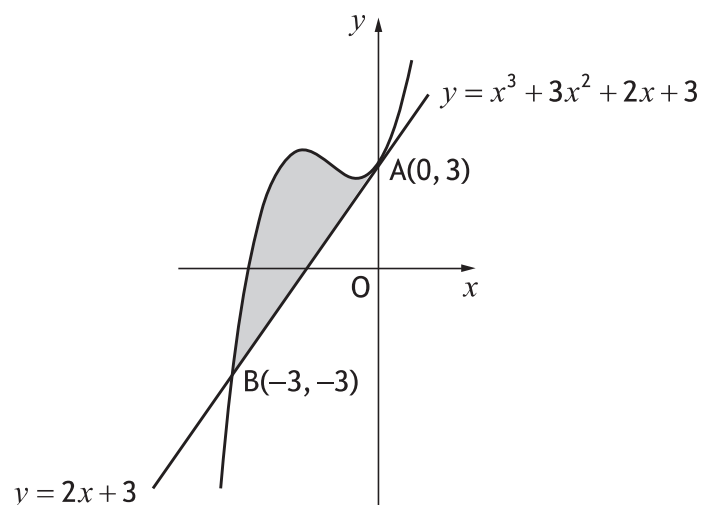
(a) Express \vec{DB} and \vec{DM} in component form.

3

(b) Find the size of angle BDM.

5

5. The line with equation $y = 2x + 3$ is a tangent to the curve with equation $y = x^3 + 3x^2 + 2x + 3$ at $A(0, 3)$, as shown.



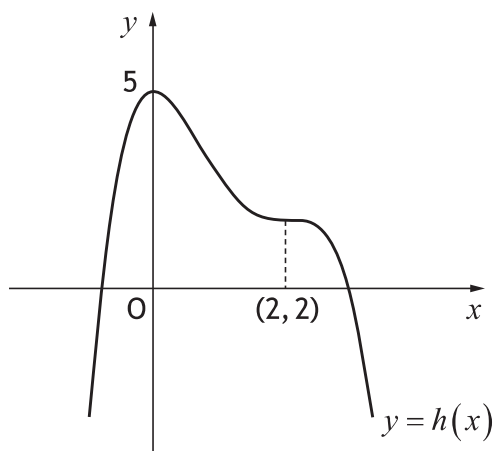
The line meets the curve again at $B(-3, -3)$.

Find the area enclosed by the line and the curve.

5

6. (a) Express $3x^2 + 24x + 50$ in the form $a(x+b)^2 + c$. 3
- (b) Given that $f(x) = x^3 + 12x^2 + 50x - 11$, find $f'(x)$. 2
- (c) Hence, or otherwise, explain why the curve with equation $y = f(x)$ is strictly increasing for all values of x . 2

7. The diagram below shows the graph of a quartic $y = h(x)$, with stationary points at $(0, 5)$ and $(2, 2)$.

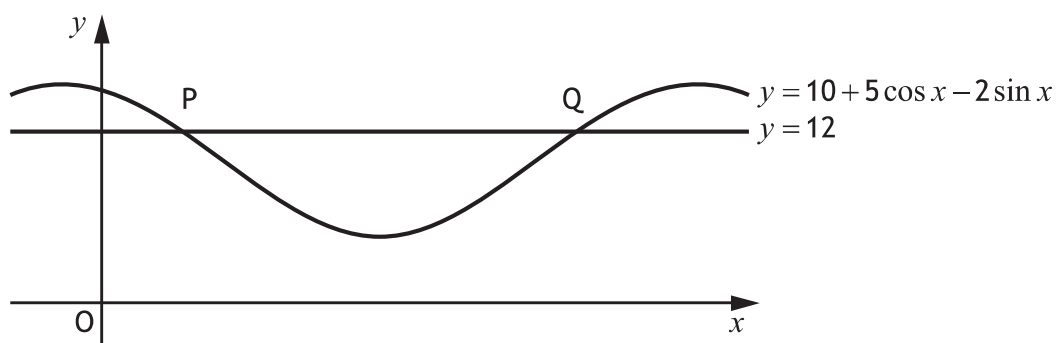


On separate diagrams sketch the graphs of:

- (a) $y = 2 - h(x)$. 2
- (b) $y = h'(x)$. 3
8. (a) Express $5 \cos x - 2 \sin x$ in the form $k \cos(x + a)$, where $k > 0$ and $0 < a < 2\pi$. 4

- (b) The diagram shows a sketch of part of the graph of $y = 10 + 5 \cos x - 2 \sin x$ and the line with equation $y = 12$.

The line cuts the curve at the points P and Q.

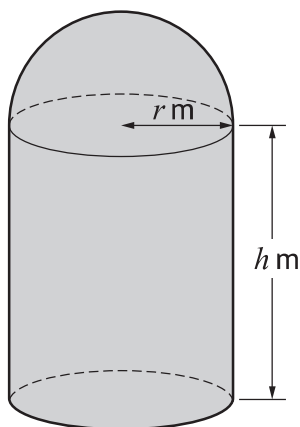


Find the x -coordinates of P and Q.

4

9. A design for a new grain container is in the shape of a cylinder with a hemispherical roof and a flat circular base. The radius of the cylinder is r metres, and the height is h metres.

The volume of the **cylindrical** part of the container needs to be 100 cubic metres.



- (a) Given that the curved surface area of a hemisphere of radius r is $2\pi r^2$ show that the surface area of metal needed to build the grain container is given by:

$$A = \frac{200}{r} + 3\pi r^2 \text{ square metres}$$

3

- (b) Determine the value of r which minimises the amount of metal needed to build the container.

6

10. Given that

$$\int_{\frac{\pi}{8}}^a \sin\left(4x - \frac{\pi}{2}\right) dx = \frac{1}{2}, \quad 0 \leq a < \frac{\pi}{2},$$

calculate the value of a .

6

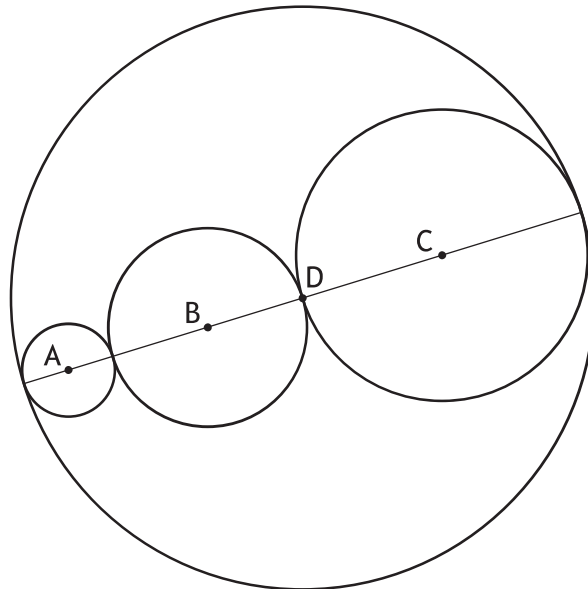
11. Show that $\frac{\sin 2x}{2 \cos x} - \sin x \cos^2 x = \sin^3 x$, where $0 < x < \frac{\pi}{2}$.

3

12. (a) Show that the points $A(-7, -2)$, $B(2, 1)$ and $C(17, 6)$ are collinear.

3

Three circles with centres A , B and C are drawn inside a circle with centre D as shown.



The circles with centres A , B and C have radii r_A , r_B and r_C respectively.

- $r_A = \sqrt{10}$
- $r_B = 2r_A$
- $r_C = r_A + r_B$

(b) Determine the equation of the circle with centre D .

4

13. The concentration of a pesticide in soil can be modelled by the equation

$$P_t = P_0 e^{-kt}$$

where:

- P_0 is the initial concentration;
- P_t is the concentration at time t ;
- t is the time, in days, after the application of the pesticide.

(a) It takes 25 days for the concentration of the pesticide to be reduced to one half of its initial concentration.

Calculate the value of k .

4

(b) Eighty days after the initial application, what is the percentage decrease in concentration of the pesticide?

3

[END OF SPECIMEN QUESTION PAPER]