X100/301

NATIONAL QUALIFICATIONS 2007

TUESDAY, 15 MAY
9.00 AM – 10.10 AM

MATHEMATICS
HIGHER
Units 1, 2 and 3
Paper 1
(Non-calculator)

Read Carefully

1 **Calculators may <u>NOT</u> be used in this paper.**

2 Full credit will be given only where the solution contains appropriate working.

3 Answers obtained by readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}| \cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\sin 2A &= 2\sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

ALL questions should be attempted.

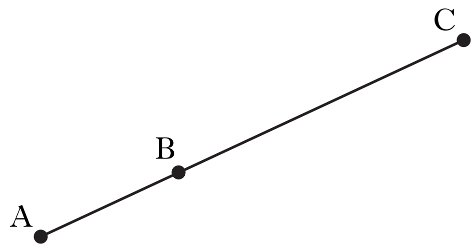
Marks

1. Find the equation of the line through the point (–1, 4) which is parallel to the line with equation $3x - y + 2 = 0$. **3**

2. Relative to a suitable coordinate system A and B are the points (–2, 1, –1) and (1, 3, 2) respectively.

A, B and C are collinear points and C is positioned such that BC = 2AB.

Find the coordinates of C. **4**



3. Functions f and g, defined on suitable domains, are given by $f(x) = x^2 + 1$ and $g(x) = 1 - 2x$.

Find:

(*a*) $g(f(x))$; **2**

(*b*) $g(g(x))$. **2**

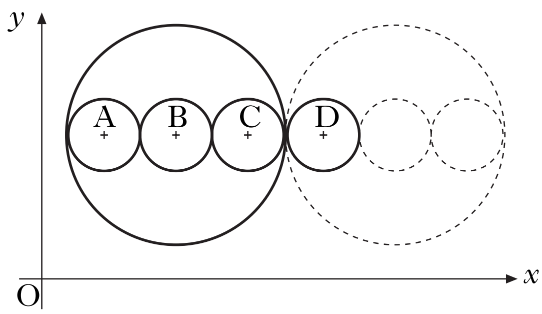
4. Find the range of values of k such that the equation $kx^2 - x - 1 = 0$ has no real roots. **4**

5. The large circle has equation $x^2 + y^2 - 14x - 16y + 77 = 0$.

Three congruent circles with centres A, B and C are drawn inside the large circle with the centres lying on a line parallel to the x-axis.

This pattern is continued, as shown in the diagram.

Find the equation of the circle with centre D. **5**



[Turn over

Marks

6. Solve the equation $\sin 2x° = 6\cos x°$ for $0 \leq x \leq 360$. **4**

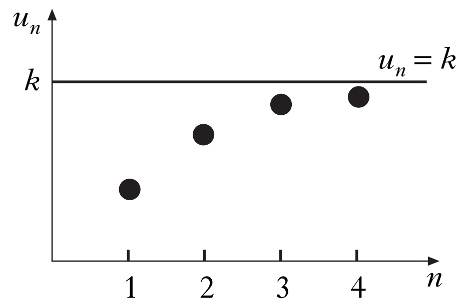
7. A sequence is defined by the recurrence relation

$$u_{n+1} = \frac{1}{4}u_n + 16,\ u_0 = 0.$$

(*a*) Calculate the values of u_1, u_2 and u_3. **3**

Four terms of this sequence, u_1, u_2, u_3 and u_4 are plotted as shown in the graph.

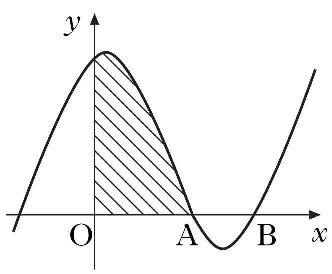
As $n \to \infty$, the points on the graph approach the line $u_n = k$, where k is the limit of this sequence.



(*b*) (i) Give a reason why this sequence has a limit.

(ii) Find the exact value of k. **3**

8. The diagram shows a sketch of the graph of $y = x^3 - 4x^2 + x + 6$.



(*a*) Show that the graph cuts the x-axis at (3, 0). **1**

(*b*) Hence or otherwise find the coordinates of A. **3**

(*c*) Find the shaded area. **5**

9. A function f is defined by the formula $f(x) = 3x - x^3$.

(*a*) Find the exact values where the graph of $y = f(x)$ meets the x- and y-axes. **2**

(*b*) Find the coordinates of the stationary points of the function and determine their nature. **7**

(*c*) Sketch the graph of $y = f(x)$. **1**

Marks

10. Given that $y = \sqrt{3x^2 + 2}$, find $\dfrac{dy}{dx}$. **3**

11. (*a*) Express $f(x) = \sqrt{3}\cos x + \sin x$ in the form $k\cos(x - a)$, where $k > 0$ and $0 < a < \dfrac{\pi}{2}$. **4**

(*b*) Hence or otherwise sketch the graph of $y = f(x)$ in the interval $0 \leq x \leq 2\pi$. **4**

[*END OF QUESTION PAPER*]

X100/303

NATIONAL
QUALIFICATIONS
2007

TUESDAY, 15 MAY
10.30 AM – 12.00 NOON

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HIGHER
Units 1, 2 and 3
Paper 2

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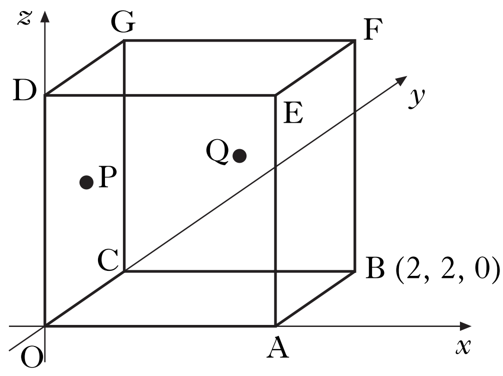
1. OABCDEFG is a cube with side 2 units, as shown in the diagram.

B has coordinates (2, 2, 0).

P is the centre of face OCGD and Q is the centre of face CBFG.

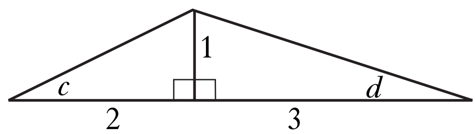


(*a*) Write down the coordinates of G. **1**

(*b*) Find $\boldsymbol{p}$ and $\boldsymbol{q}$, the position vectors of points P and Q. **2**

(*c*) Find the size of angle POQ. **5**

2. The diagram shows two right-angled triangles with angles c and d marked as shown.



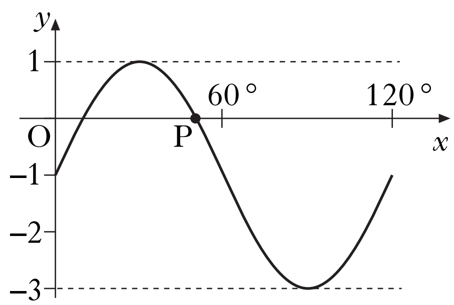
(*a*) Find the exact value of $\sin(c + d)$. **4**

(*b*) (i) Find the exact value of $\sin 2c$.

(ii) Show that $\cos 2d$ has the same exact value. **4**

3. Show that the line with equation $y = 6 - 2x$ is a tangent to the circle with equation $x^2 + y^2 + 6x - 4y - 7 = 0$ and find the coordinates of the point of contact of the tangent and the circle. **6**

4. The diagram shows part of the graph of a function whose equation is of the form $y = a\sin(bx°) + c$.



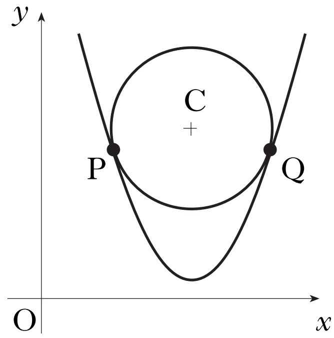
(*a*) Write down the values of a, b and c. **3**

(*b*) Determine the exact value of the x-coordinate of P, the point where the graph intersects the x-axis as shown in the diagram. **3**

[Turn over

Marks

5. A circle centre C is situated so that it touches the parabola with equation $y = \frac{1}{2}x^2 - 8x + 34$ at P and Q.



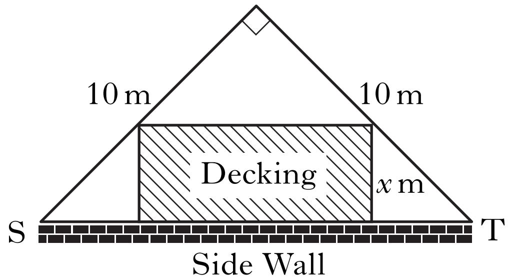
(*a*) The gradient of the tangent to the parabola at Q is 4. Find the coordinates of Q. **5**

(*b*) Find the coordinates of P. **2**

(*c*) Find the coordinates of C, the centre of the circle. **2**

6. A householder has a garden in the shape of a right-angled isosceles triangle.

It is intended to put down a section of rectangular wooden decking at the side of the house, as shown in the diagram.



(*a*) (i) Find the exact value of ST.

(ii) Given that the breadth of the decking is x metres, show that the area of the decking, A square metres, is given by

$$A = \left(10\sqrt{2}\right)x - 2x^2.$$ **3**

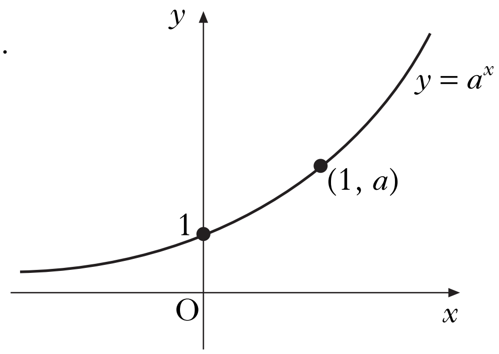
(*b*) Find the dimensions of the decking which maximises its area. **5**

7. Find the value of $\int_0^2 \sin(4x+1)\,dx$. **4**

8. The curve with equation $y = \log_3(x - 1) - 2{\cdot}2$, where $x > 1$, cuts the x-axis at the point $(a, 0)$.

Find the value of a. **4**

9. The diagram shows the graph of $y = a^x$, $a > 1$.

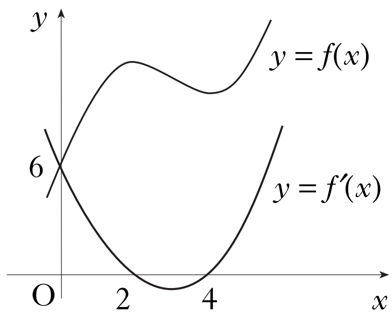


On separate diagrams, sketch the graphs of:

(a) $y = a^{-x}$; **2**

(b) $y = a^{1-x}$. **2**

10. The diagram shows the graphs of a cubic function $y = f(x)$ and its derived function $y = f'(x)$.



Both graphs pass through the point (0, 6).

The graph of $y = f'(x)$ also passes through the points (2, 0) and (4, 0).

(a) Given that $f'(x)$ is of the form $k(x - a)(x - b)$:

(i) write down the values of a and b;

(ii) find the value of k. **3**

(b) Find the equation of the graph of the cubic function $y = f(x)$. **4**

11. Two variables x and y satisfy the equation $y = 3 \times 4^x$.

(a) Find the value of a if (a, 6) lies on the graph with equation $y = 3 \times 4^x$. **1**

(b) If $(-\frac{1}{2}, b)$ also lies on the graph, find b. **1**

(c) A graph is drawn of $\log_{10}y$ against x. Show that its equation will be of the form $\log_{10}y = Px + Q$ and state the gradient of this line. **4**

[*END OF QUESTION PAPER*]