2010 Mathematics

Higher

Finalised Marking Instructions

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Exam date: 21 May 2010

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Strictly Confidential

These instructions are **strictly confidential** and, in common with the scripts you will view and mark, they must never form the subject of remark of any kind, except to Scottish Qualifications Authority staff.

Marking

The utmost care must be taken when entering Item level marks into Appointees Online.

It is of particular importance that you enter a zero (0) when the candidate has attempted a question but has not gained a mark and a dash (-) when the candidate has not attempted a question.

Part One : General Marking Principles for Mathematics Higher

This information is provided to help you understand the general principles you must apply when marking candidate responses to questions in this Paper. These principles must be read in conjunction with the specific Marking Instructions for each question.

For each question the marking instructions are split into two sections, namely Generic Scheme and Illustrative Scheme. The Generic Scheme indicates what each mark is being awarded for. The Illustrative Scheme cover methods which you will commonly see throughout your marking. In general you should use the Illustrative Scheme for marking and revert to the Generic Scheme only where a candidate has used a method not covered in the Illustrative Scheme or you are unsure of whether you should award a mark or not.

All markers should apply the following general marking principles throughout their marking:

1. Marks for each candidate response must **always** be assigned in line with these general marking principles and the specific Marking Instructions for the relevant question. If a specific candidate response does not seem to be covered by either the principles or detailed Marking Instructions, and you are uncertain how to assess it, you must seek guidance from your Team Leader. You can do this by e-mailing/phoning your team leader. Alternatively, you can refer the issue directly to your Team Leader by checking the 'Referral' box on the marking screen.

2. Marking should always be positive, i.e. marks should be awarded for what is correct and not deducted for errors or omissions.

3. Award one mark for each •. Each error should be underlined in **red** at the point where it first occurs, and not at any subsequent stage of the working.

4 The total mark for each question should be entered in **red** in the **outer** right hand margin, opposite the end of the working concerned. Only the mark, as a whole number, should be written; do not use fractions. The marks should correspond to those on the question paper and these marking instructions.

5 Where a candidate has scored zero for any question, or part of a question, 0 should be written in the right hand margin against their answer.

6 Working subsequent to an error must be followed through; with possible full marks for the subsequent working, provided that the level of difficulty involved is approximately similar. Where, subsequent to an error, the working is eased, a deduction of mark(s) should be made.

7 There is no such thing as a transcription error, a trivial error, a casual error or an insignificant error. In general, as a consequence of one of these errors, candidates lose the opportunity of gaining the appropriate *ic* mark or *pd* mark.

8 As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Throughout this paper, unless specifically mentioned in the marking scheme, a correct answer with no working receives no credit.

9 Normally, do not penalise:

- Working subsequent to a correct answer;
- Legitimate variations in numerical answers;
- Correct working in the wrong part of a question;
- Omission of units;
- Bad form;

unless specifically mentioned in the marking scheme.

10 No piece of working should be ignored without careful checking – even where a fundamental misunderstanding is apparent early in the answer. Reference should always be made to the marking scheme. Answers which are widely off-beam are unlikely to include anything of relevance but in the vast majority of cases candidates still have the opportunity of gaining the odd mark or two; provided it satisfies the criteria for the marks.

11 If in doubt between two marks, give an intermediate mark, but without fractions. When in doubt between consecutive numbers, give the higher mark.

12 No marks at this stage should be deducted for careless or badly arranged work.

13 It is of great importance that the utmost care should be exercised in adding up the marks. Using the Electronic Marks Capture (EMC) screen to tally marks for you is not recommended. A manual check of the total, using the grid issued with this marking scheme, can be confirmed by the EMC system.

14 In cases of difficulty, covered neither in detail nor in principle in these instructions, attention may be directed to the assessment of particular answers by making a referral to the Principal Assessor (P.A.) Please see the general Instructions for P.A. referrals.

15 If a candidate presents multiple solutions for a question <u>and</u> it is not clear which is intended as their final attempt, mark each one and award the lowest mark.

Marking Scripts

No comments, words or acronyms should be written on scripts.

Please use the following and **nothing else**.

✓	Tick when a piece of working is correct and gains a mark. You are not expected to tick every line of working but you must check through the whole of a response.
___ ✗	Underline each error and place a cross at the end of the line where it occurs.
	A cross-tick should be used to indicate 'correct working' where a mark is awarded as a result of follow through from an error.
	A double cross-tick should be used to indicate correct working which is inadequate to score any marks e.g. incorrect method which is mathematically correct or eased working.
~~	A tilde should be used to indicate a minor transgression which is not being penalised, e.g. bad form.
∧	Use a roof to show that something is missing such as a crucial step in the working or part of a solution.

These are essential for later stages of the SQA procedures.

N.B. Bullets may be used along with the signs opposite to indicate which marks are being awarded.

Margins

Example 1

$\frac{dy}{dx} = 4x - 7$ ✓ $\bullet^1$

$4x - \underline{7 = 0}$ ✗

$x = \frac{7}{4}$

$y = \frac{31}{8}$ $\bullet^3$ — **2**

Example 2

$x^2 - 3x = 28$ ✓ $\bullet^6$

$x = 7$ ∧ — **1**

Example 3

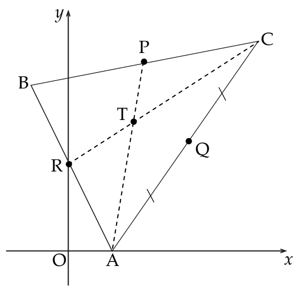
$\sin x = \underset{\sim\sim\sim}{0 \cdot 75 = \sin^{-1}} 0 \cdot 75 = 48 \cdot 6^\circ$ ✓ $\bullet^6$ — **1**

marks go in this column as whole numbers only

Question	Answer
1	A
2	C
3	D
4	A
5	B
6	D
7	C
8	B
9	C
10	B
11	D
12	A
13	B
14	C
15	C
16	A
17	B
18	B
19	C
20	A

Summary		
	A	5
	B	6
	C	6
	D	3

21 Triangle ABC has vertices A (4, 0), B (–4, 16) and C (18, 20), as shown in the diagram opposite.



Medians AP and CR intersect at the point T (6, 12).

(*a*) Find the equation of median BQ. **3**

(*b*) Verify that T lies on BQ. **1**

	Generic Scheme		Illustrative Scheme
21 (a)			
$\bullet^1$	ss	know and find midpoint of AC	$\bullet^1$ (11, 10)
$\bullet^2$	pd	calculate gradient of BQ	$\bullet^2$ $-\frac{6}{15}$ or equivalent
$\bullet^3$	ic	state equation	$\bullet^3$ $y-16=-\frac{2}{5}(x-(-4))$ **or** $y-10=-\frac{2}{5}(x-11)$

Notes

1. Candidates who do not use a midpoint lose $\bullet^2$ and $\bullet^3$.
2. There is no need to simplify m_{BQ} for $\bullet^2$. It must, however, be simplified before $\bullet^3$ can be awarded.
   Do not award $\bullet^3$ for $6x+15y-216=0$, although $\bullet^3$ would be awarded for $6x+15y-216=0$ then $2x+5y-72=0$.
3. If m_{BQ} cannot be simplified, due to an error, then $\bullet^3$ is still available.
4. $\bullet^3$ is available for using $y=mx+c$ where $m=-\frac{2}{5}$ **and** $c=\frac{72}{5}$.
5. Accept $y=-0\cdot4x+14\cdot4$.
6. Candidates who find the equations of AP or CR can only gain 1 mark.

   AP : $y-0=6(x-4)$ or $y-12=6(x-6)$ CR : $y-20=\frac{2}{3}(x-18)$ or $y-12=\frac{2}{3}(x-6)$

	Generic Scheme		Illustrative Scheme
21 (b)			
$\bullet^4$	ic	substitute in for T and complete	$\bullet^4$ e.g. Substitution : $2(6)+5(12)=12+60=72$ Gradients : $m_{BT}=-\frac{4}{10}=-\frac{2}{5}=m_{BQ}$ Vectors : $\overrightarrow{BT}=\begin{pmatrix}10\\-4\end{pmatrix}$, $\overrightarrow{TQ}=\begin{pmatrix}5\\-2\end{pmatrix}$ and $\overrightarrow{BT}=2\overrightarrow{TQ}$

Notes

7. $\bullet^4$ is available as follow through with an appropriate communication statement, e.g. 'T does not lie on BQ'.
8. Statements such as 'PA, RC and BQ are all medians and therefore all share the same point T' do not gain $\bullet^4$.
9. Since only 1 mark is available here, do not penalise the omission of any reference to a "common point" or "parallel".

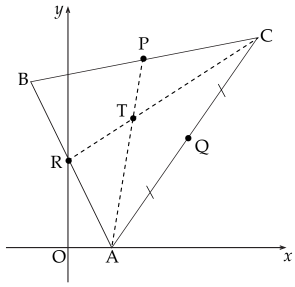
Regularly occurring responses

Gradient approach : (b) $m_{BT}=-\frac{4}{10}=-\frac{2}{5}=m_{BQ}$ leading to $2:1$ in (c), without further working, gains $\bullet^4$ and $\bullet^6$ but loses $\bullet^5$.

but (b) $m_{BQ}=-\frac{6}{15}$ and $m_{TQ}=-\frac{2}{5}$ leading to $m_{BQ}=3m_{TQ}$ so T lies on BQ leading to $2:1$ in (c), without further working, loses $\bullet^4$ and $\bullet^5$ but gains $\bullet^6$.

21 Triangle ABC has vertices A (4, 0), B (–4, 16) and C (18, 20), as shown in the diagram opposite.

Medians AP and CR intersect at the point T (6, 12).



(*c*) Find the ratio in which T divides BQ. **2**

Generic Scheme	Illustrative Scheme

21 (c)

$\bullet^5$ ss valid method for finding the ratio

$\bullet^6$ ic complete to simplified ratio

For $2:1$ **without** working, only $\bullet^6$ is awarded.
Be aware that the working may appear in (b).
Some candidates obtain $2:1$ from erroneous working thus losing $\bullet^6$.

Method 1 : Vector approach

$\bullet^5$ e.g. $\overrightarrow{BT} = \begin{pmatrix} 10 \\ -4 \end{pmatrix}$ and $\overrightarrow{TQ} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$

$\bullet^6$ $2:1$

Method 2 : "Stepping out" approach

$\bullet^5$ e.g. B —10— T —5— Q or

B	T	Q
– 4	6	11
10	5	

$\bullet^6$ $2:1$

Method 3 : Distance Formula approach

$\bullet^5$ e.g. $d_{BT} = \sqrt{116}$ and $d_{TQ} = \sqrt{29}$

$\bullet^6$ $2:1$

Notes

10. Any evidence of appropriate steps, e.g. 10 and 5 or 4 and 2 but not 2 and 1, can be awarded $\bullet^5$ leading to $\bullet^6$,
e.g. B —2— T —1— Q is not sufficient on its own and so loses $\bullet^5$ but gains $\bullet^6$.

11. $\sqrt{116} : \sqrt{29}$ with no further simplification may be awarded $\bullet^5$ but not $\bullet^6$.

12. In this question working for (c) may appear in (b), where the working appears for $\bullet^4$.

Regularly occurring responses

Response 1

(b) $\overrightarrow{BT} = \begin{pmatrix} 10 \\ -4 \end{pmatrix}$ $\overrightarrow{TQ} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$ ✓ $\bullet^5$

$\overrightarrow{BT} = 2\overrightarrow{TQ}$ ✓ $\bullet^4$

(c) $2:1$ ✓ $\bullet^6$

3 marks out of 3

Response 2

(b) $\overrightarrow{QT} = \begin{pmatrix} -5 \\ 2 \end{pmatrix}$ $\overrightarrow{BQ} = \begin{pmatrix} 15 \\ -6 \end{pmatrix}$ ✓ $\bullet^5$ $= -3\overrightarrow{QT}$ ✓ $\bullet^4$

(c) $2:1$ ✓ $\bullet^6$

3 marks out of 3

Response 3

(b) $\overrightarrow{QT} = \begin{pmatrix} -5 \\ 2 \end{pmatrix}$ $\overrightarrow{TB} = \begin{pmatrix} -10 \\ 4 \end{pmatrix}$ ✓ $\bullet^5$

✓ $\bullet^4$

(c) $\begin{pmatrix} -10 \\ 4 \end{pmatrix} = 2\begin{pmatrix} -5 \\ 2 \end{pmatrix}$ so $2:1$ ✓ $\bullet^6$

3 marks out of 3

Response 4

(b) $\overrightarrow{BT} = \begin{pmatrix} 10 \\ -4 \end{pmatrix}$ $\overrightarrow{TQ} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$ ✓ $\bullet^5$ so $2\overrightarrow{BT} = \overrightarrow{TQ}$ ✗ $\bullet^4$ (c) $2:1$ ✗ $\bullet^6$ but $1:2$ would have gained $\bullet^6$

22	(*a*) (i) Show that $(x-1)$ is a factor of $f(x)=2x^3+x^2-8x+5$.	
	(ii) Hence factorise $f(x)$ fully.	**5**
	(*b*) Solve $2x^3+x^2-8x+5=0$.	**1**

Generic Scheme			Illustrative Scheme	
22 (a)			**Method 1** : Using synthetic division	
$\bullet^1$	ss	know to use $x=1$	$\bullet^1$ $\begin{array}{r\|rrrr} 1 & 2 & 1 & -8 & 5 \\ \hline & & & & \end{array}$	
$\bullet^2$	ic	complete evaluation	$\bullet^2$ $\begin{array}{r\|rrrr} 1 & 2 & 1 & -8 & 5 \\ & & 2 & 3 & -5 \\ \hline & 2 & 3 & -5 & 0 \end{array}$	
$\bullet^3$	ic	state conclusion	$\bullet^3$ $(x-1)$ is a factor	*see note 2*
$\bullet^4$	pd	find quadratic factor	$\bullet^4$ $2x^2+3x-5$	*stated, or implied by* $\bullet^5$
$\bullet^5$	pd	factorise completely	$\bullet^5$ $(x-1)(x-1)(2x+5)$	*stated explicitly*
			Method 2 : Using substitution and inspection	
			$\bullet^1$ know to use $x=1$	
			$\bullet^2$ $2+1-8+5=0$	
			$\bullet^3$ $(x-1)$ is a factor	*see note 2*
			$\bullet^4$ $(x-1)(2x^2+3x-5)$	
			$\bullet^5$ $(x-1)(x-1)(2x+5)$	*stated explicitly*

Notes

1. Communication at $\bullet^3$ must be consistent with working at $\bullet^2$.
   i.e. candidate's working must arrive legitimately at zero before $\bullet^3$ is awarded.
   If the remainder is not 0 then an appropriate statement would be '$(x-1)$ is not a factor'.
2. For $\bullet^3$, minimum acceptable statement is 'factor'.
   Unacceptable statements : $x=1$ is a factor, $(x+1)$ is a factor, $x=1$ is a root, $(x-1)$ is a root etc.
3. At $\bullet^5$ the expression may be written as $(x-1)^2(2x+5)$.

Generic Scheme			Illustrative Scheme	
22 (b)				These may appear in the working at (a).
$\bullet^6$	ic	state solutions	$\bullet^6$ $x=1$ **and** $x=-\frac{5}{2}$ or $-2\cdot5$ or $-2\frac{1}{2}$	

Notes

4. From (a) $(x-1)(x-1)(2x+5)$ leading to $x=1, x=-\frac{5}{2}$ then $(1, 0)$ and $\left(-\frac{5}{2}, 0\right)$ gains $\bullet^6$.

   However, $(x-1)(x-1)(2x+5)$ leading to $(1, 0)$ and $\left(-\frac{5}{2}, 0\right)$ **only** does not gain $\bullet^6$.

5. From (a) $(x-1)(2x+5)$ only leading to $x=1, x=-\frac{5}{2}$ does not gain $\bullet^6$ as equation solved is not a cubic, but $(x-1)(x+1)(2x-5)$ leading to $x=1, x=-1$ and $x=\frac{5}{2}$ gains $\bullet^6$ as follow through from a cubic equation.

22	(*c*)	The line with equation $y = 2x - 3$ is a tangent to the curve with equation $y = 2x^3 + x^2 - 6x + 2$ at the point G. Find the coordinates of G.	**5**
	(*d*)	This tangent meets the curve again at the point H. Write down the coordinates of H.	**1**

Generic Scheme			Illustrative Scheme
22 (c)			
Method 1 : Equating curve and line			**Method 1** : Equating curve and line
•7	ss	set $y_{\text{CURVE}} = y_{\text{LINE}}$	•7 $2x^3 + x^2 - 6x + 2 = 2x - 3$ *stated explicitly*
•8	ic	express in standard form	•8 $2x^3 + x^2 - 8x + 5$ $\}= 0$ *see note 6*
•9	ss	compare with (a) or factorise	•9 $(x-1)(x-1)(2x+5)$ $\}= 0$
•10	ic	identify x_G	•10 $x = 1$
•11	pd	evaluate y_G	•11 $y = -1$
Method 2 : Differentiation			**Method 2** : Differentiation
•7	ss	know to and differentiate curve	•7 $6x^2 + 2x - 6$
•8	ic	set derivative to gradient of line	•8 $6x^2 + 2x - 6 = 2$
•9	pd	solve quadratic equation	•9 $x = -\frac{4}{3}$ and 1
•10	ss	process to identify x_G	•10 at $x = 1$ evaluate y_{CURVE} **and** y_{LINE}
•11	ic	complete to $y_{\text{CURVE}} = y_{\text{LINE}}$	•11 $y = -1$ from both curve and line

Notes

In method 1:

6. •8 is only available if '= 0' appears at either the •8 or •9 stage.
7. •9 , •10 and •11 are only available via the working from •7 and •8.
8. If $(x-1)(x-1)(2x+5)$ does not appear at •9 stage, it can be implied by •5 **and** •10.
9. At •9 a quadratic used from (a) may gain •9, •11 and •12 but a quadratic from •8 may gain •11 and •12 only.
10. If G and H are interchanged then •10 is lost but •11 and •12 are still available.
11. Candidates who obtain three distinct factors at •9 can gain •11 for evaluating **all** y values, but lose •10 and •12.
12. A repeated factor at •5 **or** •9 stage is required for •10 to be awarded without justification.

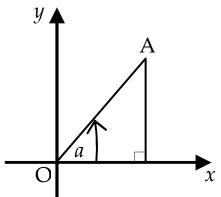
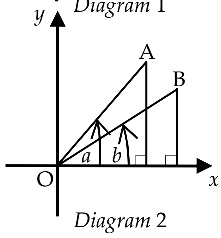
In both methods:

13. All marks in (c) are available as a result of differentiating $2x^3 + x^2 - 6x + 2$ and solving this equal to 2 (from method 2).
    Only marks •7 and •8 (from method 1) are available to those candidates who choose to differentiate $2x^3 + x^2 - 8x + 5$ and solve this equal to 0.
14. Candidates may choose a combination of making equations equal and differentiation.

Generic Scheme			Illustrative Scheme
22 (d)			
•12	pd	state solution	•12 $\left(-\frac{5}{2}, -8\right)$ *may appear in (c)*

Notes

15. Method 2 from (c) would not yield a value for H and so •12 is not available.

23 (*a*) Diagram 1 shows a right angled triangle, where the line OA has equation $3x-2y=0$. (i) Show that $\tan a=\frac{3}{2}$. (ii) Find the value of $\sin a$.	 *Diagram 1*	**4**
(*b*) A second right angled triangle is added as shown in Diagram 2. The line OB has equation $3x-4y=0$. Find the values of $\sin b$ and $\cos b$.	 *Diagram 2*	**4**

Generic Scheme | **Illustrative Scheme**

23 (a)

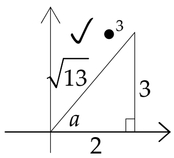
Generic Scheme			Illustrative Scheme		
$\bullet^1$	ss	write in slope/intercept form	$\bullet^1$	$y=\frac{3}{2}x$ or $y=1\cdot 5x$	*stated explicitly*
$\bullet^2$	ic	connect gradient and $\tan a$	$\bullet^2$	$m=\frac{3}{2}$ and $\tan a=\frac{3}{2}$ **or** $m=\tan a$ and $\tan a=\frac{3}{2}$	
$\bullet^3$	pd	calculate hypotenuse	$\bullet^3$	$\sqrt{13}$	*stated, or implied by* $\bullet^4$
$\bullet^4$	ic	state value of sine ratio	$\bullet^4$	$\frac{3}{\sqrt{13}}$ or $\frac{3\sqrt{13}}{13}$	*may not appear until* (c)

Notes

1. $\bullet^4$ is only available if $-1\le \sin a\le 1$.
2. Only numerical answers are acceptable for $\bullet^3$ and $\bullet^4$.

Regularly occurring responses

Response 1



$3x-2y=0$

$3\times(2)-2\times(3)=0$ ∧ $\bullet^1$

$\tan a=\frac{3}{2}$ $\cancel{\checkmark}$ $\bullet^2$

$\sin a=\frac{3}{\sqrt{13}}$ ✓ $\bullet^4$

2 marks out of 4

23 (b)

Generic Scheme			Illustrative Scheme		
$\bullet^5$	ss	determine $\tan b$	$\bullet^5$	$\tan b=\frac{3}{4}$	*stated, or implied by* $\bullet^6$
$\bullet^6$	ss	know to complete triangle	$\bullet^6$	right angled triangle with 3 and 4 correctly shown	
$\bullet^7$	pd	determine hypotenuse	$\bullet^7$	5	*stated, or implied by* $\bullet^8$
$\bullet^8$	ic	state values of sine and cosine ratios	$\bullet^8$	$\sin b=\frac{3}{5}$ and $\cos b=\frac{4}{5}$	*may not appear until* (c)

Notes

3. $\bullet^8$ is only available if $-1\le \sin b\le 1$ **and** $-1\le \cos b\le 1$.
4. $\sin b=\frac{3}{5}$ and $\cos b=\frac{4}{5}$ without working is awarded 3 marks only.
5. Only numerical answers are acceptable for $\bullet^7$ and $\bullet^8$.

23 (*c*) (i) Find the value of $\sin(a-b)$.
(ii) State the value of $\sin(b-a)$. **4**

Generic Scheme			**Illustrative Scheme**
23 (c)			
$\bullet^9$	ss	know to use addition formula	$\bullet^9$ $\sin a\cos b-\cos a\sin b$
$\bullet^{10}$	ic	substitute into expansion	$\bullet^{10}$ $\frac{3}{\sqrt{13}}\times\frac{4}{5}-\frac{2}{\sqrt{13}}\times\frac{3}{5}$
$\bullet^{11}$	pd	evaluate sine of compound angle	$\bullet^{11}$ $\frac{6}{5\sqrt{13}}$
$\bullet^{12}$	ss	use $\sin(-x)=-\sin x$	$\bullet^{12}$ $-\frac{6}{5\sqrt{13}}$

Notes

6. $\sin(A-B)=\sin A\cos B-\cos A\sin B$, or just $\sin A\cos B-\cos A\sin B$, with no further working does not gain $\bullet^9$.
7. Candidates should not be penalised further at $\bullet^{10}$, $\bullet^{11}$ and $\bullet^{12}$ for values of sine and cosine outside the range -1 to 1.
8. Candidates who use $\sin(a-b)=\sin a-\sin b$ lose $\bullet^9$, $\bullet^{10}$ and $\bullet^{11}$ but can gain $\bullet^{12}$, as follow through, only for a non-zero answer which is obtained from the result $\sin(-x)=-\sin x$.
9. Treat $\sin\frac{3}{\sqrt{13}}\cos\frac{4}{5}-\cos\frac{2}{\sqrt{3}}\sin\frac{3}{5}$ as bad form only if 'sin' and 'cos' subsequently disappear.
10. It is acceptable to work through the whole expansion again for $\bullet^{12}$.

Regularly occurring responses

Response 1

$\sin(a-b)=\sin a-\sin b$ ✗ $\bullet^9$
$=6-6$ ✗ $\bullet^{10}$
$=0$ ✗ $\bullet^{11}$

$\sin(b-a)=0$ ✗ $\bullet^{12}$

0 marks out of 4

Response 2

$\sin a=3 \quad \cos a=2$ — Marks lost in (a) or (b)
$\sin b=3 \quad \cos b=4$
$\sin(a-b)=\sin a\cos b-\cos a\sin b$ ✓ $\bullet^9$
$=3\times4-2\times3$ ✓ $\bullet^{10}$
$=6$ ✓̶ $\bullet^{11}$ — Eased - not dealing with fraction containing a surd.

$\sin(b-a)=-6$ ✓ $\bullet^{12}$

3 marks out of 4

Response 3

From (a) and (b) $\sin a=\frac{2}{3}$ $\cos a=\frac{1}{3}$
$\sin b=\frac{2}{3}$ $\cos b=\frac{3}{5}$

(c) (i) $\sin(a-b)=\sin a\cos b-\cos a\sin b$ ✓ $\bullet^9$
$=\frac{2}{3}\times\frac{3}{5}-\frac{1}{3}\times\frac{2}{5}$ ✓ $\bullet^{10}$
$=\frac{4}{15}$ ✓̶ $\bullet^{11}$
(ii) $\sin(b-a)=\sin b\cos a-\cos b\sin a$
$=\frac{2}{5}\times\frac{1}{3}-\frac{3}{5}\times\frac{2}{3}$
$=-\frac{4}{15}$ ✓ $\bullet^{12}$ — Here the working was not necessary; the answer would gain $\bullet^{12}$, provided it is non zero.

3 marks out of 4

Response 4

(i) $\sin(a-b)=\sin a\sin b-\cos a\cos b$ ✗ $\bullet^9$
$=\frac{3}{\sqrt{13}}\times\frac{3}{5}-\frac{2}{\sqrt{13}}\times\frac{4}{5}$ ✓ $\bullet^{10}$
$=\frac{1}{5\sqrt{13}}$ ✓ $\bullet^{11}$
(ii) $\sin(b-a)=-\frac{1}{5\sqrt{13}}$ ✓ $\bullet^{12}$

3 marks out of 4

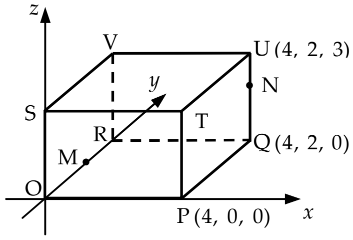
1 The diagram shows a cuboid OPQR,STUV relative to the coordinate axes.

P is the point $(4, 0, 0)$,

Q is $(4, 2, 0)$ and U is $(4, 2, 3)$.

M is the midpoint of OR.

N is the point on UQ such that $UN = \frac{1}{3}UQ$.



(*a*) State the coordinates of M and N. **2**

(*b*) Express the vectors $\overrightarrow{VM}$ and $\overrightarrow{VN}$ in component form. **2**

Treat as bad form, coordinates written as components and vice versa, throughout this question.

	Generic Scheme		Illustrative Scheme
1 (a)			
$\bullet^1$	ic	interpret midpoint for M	$\bullet^1$ (0, 1, 0)
$\bullet^2$	ic	interpret ratio for N	$\bullet^2$ (4, 2, 2)

	Generic Scheme		Illustrative Scheme
1 (b)			
$\bullet^3$	ic	intepret diagram	$\bullet^3$ $\overrightarrow{VM} = \begin{pmatrix} 0 \\ -1 \\ -3 \end{pmatrix}$
$\bullet^4$	pd	process vectors	$\bullet^4$ $\overrightarrow{VN} = \begin{pmatrix} 4 \\ 0 \\ -1 \end{pmatrix}$

Using evidence from (a) or may have been taken directly from diagram.

Notes

1. V is the point $(0, 2, 3)$, which may or may not appear in the working to (b).

Regularly occurring responses

Response 1

(a) M(2, 0, 0) ✗ $\bullet^1$ N(4, 2, −1) ✗ $\bullet^2$

0 marks out of 2

(b) $\overrightarrow{VM} = \begin{pmatrix} 2 \\ -2 \\ -3 \end{pmatrix}$ ✗ $\bullet^3$

Consistent with (a)

$\overrightarrow{VN} = \begin{pmatrix} 4 \\ 0 \\ -1 \end{pmatrix}$ ✓ $\bullet^4$

From diagram

2 marks out of 2

Response 2

Incorrect V stated

(b) V(0, 3, 2)

$\overrightarrow{VM} = \begin{pmatrix} 0 \\ -2 \\ -2 \end{pmatrix}$ ✗ $\bullet^3$

$\overrightarrow{VN} = \begin{pmatrix} 4 \\ -1 \\ 0 \end{pmatrix}$ ✗ $\bullet^4$

1 mark out of 2

Response 3

(a) M(0, 2, 0) ✗ $\bullet^1$ N(4, 2, 2) ✓ $\bullet^2$

1 mark out of 2

(b) $\overrightarrow{VM} = \begin{pmatrix} -4 \\ 0 \\ -3 \end{pmatrix}$ ✗ $\bullet^3$

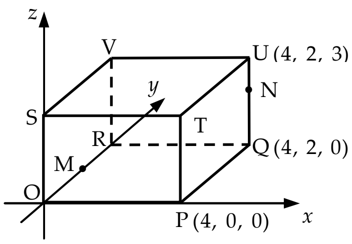
$\overrightarrow{VN} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$ ✗ $\bullet^4$

V (4, 2, 3) used in both but not stated

0 marks out of 2

1 The diagram shows a cuboid OPQR,STUV relative to the coordinate axes.
P is the point $(4, 0, 0)$,
Q is $(4, 2, 0)$ and U is $(4, 2, 3)$.
M is the midpoint of OR.

N is the point on UQ such that $UN = \frac{1}{3}UQ$.



(*c*) Calculate the size of angle MVN. **5**

Treat as bad form, coordinates written as components and vice versa, throughout this question.

Generic Scheme			Illustrative Scheme	
1 (c)				
Method 1 : Vector Approach			**Method 1** : Vector Approach	
•5	ss	know to use scalar product	•5 $\cos M\hat{V}N = \dfrac{\overrightarrow{VM}.\overrightarrow{VN}}{\lvert\overrightarrow{VM}\rvert\lvert\overrightarrow{VN}\rvert}$	*stated, or implied by* •9
•6	pd	find scalar product	•6 $\overrightarrow{VM}.\overrightarrow{VN} = 3$	
•7	pd	find magnitude of a vector	•7 $\lvert\overrightarrow{VM}\rvert = \sqrt{10}$	
•8	pd	find magnitude of a vector	•8 $\lvert\overrightarrow{VN}\rvert = \sqrt{17}$	
•9	pd	evaluate angle	•9 $76{\cdot}7^\circ$ or $1{\cdot}339$ rads or $85{\cdot}2$ grads	
Method 2 : Cosine Rule Approach			**Method 2** : Cosine Rule Approach	
•5	ss	know to use cosine rule	•5 $\cos M\hat{V}N = \dfrac{VM^2 + VN^2 - MN^2}{2 \times VM \times VN}$	*stated, or implied by* •9
•6	pd	find magnitude of a side	•6 $VM = \sqrt{10}$	
•7	pd	find magnitude of a side	•7 $VN = \sqrt{17}$	
•8	pd	find magnitude of a side	•8 $MN = \sqrt{21}$	
•9	pd	evaluate angle	•9 $76{\cdot}7^\circ$ or $1{\cdot}339$ rads or $85{\cdot}2$ grads	

Notes

2. •5 is not available to candidates who choose to evaluate an incorrect angle.
3. For candidates who do not attempt •9, then •5 is only available if the formula quoted relates to the labelling in the question.
4. •9 should be awarded for any answer that rounds to 77° or $1{\cdot}3$ rads or 85 grads (i.e. correct to two significant figures.)

Regularly occurring responses

Response 1

$\cos M\hat{O}N = \dfrac{\overrightarrow{OM}.\overrightarrow{ON}}{\lvert\overrightarrow{OM}\rvert\lvert\overrightarrow{ON}\rvert}$ ✗ •5 — Wrong angle

$\overrightarrow{OM}.\overrightarrow{ON} = 2$ ✗ •6

$\lvert\overrightarrow{OM}\rvert = 1$ ✗ •7 — Eased because only one non-zero component.

$\lvert\overrightarrow{ON}\rvert = \sqrt{24}$ ✗ •8

$65{\cdot}9^\circ$ or $1{\cdot}150$ rads or $73{\cdot}2$ grads ✗ •9

3 marks out of 5

Response 2

$\cos M\hat{V}N = \dfrac{\overrightarrow{VM}.\overrightarrow{VN}}{\lvert\overrightarrow{VM}\rvert\lvert\overrightarrow{VN}\rvert}$ ✓ •5

$\overrightarrow{VM}.\overrightarrow{VN} = 0$ ✗ •6

$\lvert\overrightarrow{VM}\rvert = \sqrt{17}$ ✗ •7

$\lvert\overrightarrow{VN}\rvert = 2$ ✗ •8

90° or equivalent ✗ •9

Going directly to 90° from •6 would lose •7 and •8.

4 marks out of 5

2 (*a*) $12\cos x^\circ - 5\sin x^\circ$ can be expressed in the form $k\cos(x+a)^\circ$, where $k>0$ and $0 \le a < 360$.
Calculate the values of k and a. **4**

Generic Scheme			**Illustrative Scheme**	
(a)				
$\bullet^1$	ss	use addition formula	$\bullet^1$ $k\cos x^\circ \cos a^\circ - k\sin x^\circ \sin a^\circ$ or $k(\cos x^\circ \cos a^\circ - \sin x^\circ \cos a^\circ)$	*stated explicitly*
$\bullet^2$	ic	compare coefficients	$\bullet^2$ $k\cos a^\circ = 12$ and $k\sin a^\circ = 5$ or $-k\sin a^\circ = -5$	*stated explicitly*
$\bullet^3$	pd	process k	$\bullet^3$ 13	*no justification required, but do not accept* $\sqrt{169}$
$\bullet^4$	pd	process a	$\bullet^4$ $22\cdot 6$	*accept any answer which rounds to* 23

Notes

1. Do not penalise the omission of the degree symbol.
2. Treat $k\cos x^\circ \cos a^\circ - \sin x^\circ \sin a^\circ$ as bad form only if the equations at the $\bullet^2$ stage both contain k.
3. $13\cos x^\circ \cos a^\circ - 13\sin x^\circ \sin a^\circ$ or $13(\cos x^\circ \cos a^\circ - \sin x^\circ \sin a^\circ)$ is acceptable for $\bullet^1$ and $\bullet^3$.
4. $\bullet^2$ is not available for $k\cos x^\circ = 12$ and $k\sin x^\circ = 5$ or $-k\sin x^\circ = -5$, however, $\bullet^4$ is still available.
5. $\bullet^4$ is lost to candidates who give a in radians only.
6. $\bullet^4$ may be gained only as a consequence of using evidence at $\bullet^2$ stage.
7. Candidates may use any form of the wave equation for $\bullet^1$, $\bullet^2$ and $\bullet^3$, however $\bullet^4$ is only available if the value of a is interpreted for the form $k\cos(x+a)^\circ$.

Regularly occurring responses

Response 1A

$k(\cos x^\circ \cos a^\circ - \sin x^\circ \sin a^\circ)$ ✓ $\bullet^1$

$\sin a = 5$ ✗ $\bullet^2$

$\cos a = 12$

$\tan a^\circ = \frac{5}{12}$

$a = 22\cdot 6$ ✓̸ $\bullet^4$

$13\cos(x+22\cdot 6)$ ✓ $\bullet^3$

2 marks out of 4

Response 1B

$k\cos x\cos a - k\sin x \sin a$ ✓ $\bullet^1$

$k = 13$ ✓ $\bullet^3$ ∧ $\bullet^2$

$\tan a^\circ = \frac{5}{12}$

$a = 22\cdot 6$ ✓̸ $\bullet^4$

2 marks out of 4

Response 2

$k\cos(x-a)$

$= k\cos x\cos a + k\sin x\sin a$ ✓̸ $\bullet^1$

$= 13\cos x\cos a + 13\sin x\sin a$ ✓ $\bullet^3$

$13\cos a = 12$ $13\sin a = -5$ ✓̸ $\bullet^2$

then $a = 22\cdot 6$ ✗ $\bullet^4$ See note 6

or $a = 337\cdot 4$ ✗ $\bullet^4$ See note 7

3 marks out of 4

Response 3A

$k\cos x^\circ \cos a^\circ - \sin x^\circ \sin a^\circ$

$k\sin a = 5$ ✓ $\bullet^1$ ✓ $\bullet^2$

$k\cos a = 12$

$k = 13$ ✓ $\bullet^3$ $\tan a^\circ = \frac{12}{5}$ ✗ $\bullet^4$

$a = 67\cdot 4$

3 marks out of 4

Response 3B

$k\cos x^\circ \cos a^\circ - \sin x^\circ \sin a^\circ$

$k\sin a = 12$ ✓ $\bullet^1$ ✗ $\bullet^2$

$k\cos a = 5$

$k = 13$ ✓ $\bullet^3$ $\tan a^\circ = \frac{12}{5}$

$a = 67\cdot 4$ ✓̸ $\bullet^4$

3 marks out of 4

Response 4

$k\cos x^\circ \cos a^\circ - k\sin x^\circ \sin a^\circ$ ✓ $\bullet^1$

$k\cos a = 12$ ✗ $\bullet^2$

$k\sin a = -5$

See note 6

$k = 13$ ✓ $\bullet^3$ $\tan a^\circ = -\frac{5}{12}$

$a = 337\cdot 4$ ✓̸ $\bullet^4$

3 marks out of 4

2 (*b*) (i)	Hence state the maximum and minimum values of $12\cos x^\circ - 5\sin x^\circ$.	
(ii)	Determine the values of x, in the interval $0 \le x < 360$, at which these maximum and minimum values occur.	**3**

Generic Scheme			**Illustrative Scheme**
(b)			
$\bullet^5$	ss	state maximum and minimum	$\bullet^5$ 13, -13
$\bullet^6$	ic	find x corresponding to max. value	$\bullet^6$ maximum at $337\cdot4$ and no others
$\bullet^7$	pd	find x corresponding to min. value	$\bullet^7$ minimum at $157\cdot4$ and no others
			or
			$\bullet^6$ $337\cdot4$ and $157\cdot4$ and no others
			$\bullet^7$ maximum at $337\cdot4$ and minimum at $157\cdot4$

Notes

8. $\bullet^5$ is available for $\sqrt{169}$ and $-\sqrt{169}$ only if $\sqrt{169}$ has been penalised at $\bullet^3$.
9. Accept answers which round to 337 and 157 for $\bullet^6$ and $\bullet^7$.
10. Candidates who continue to work in radian measure should not be penalised further.
11. Extra solutions, correct or incorrect, should be penalised at $\bullet^6$ or $\bullet^7$ but not both.
12. $\bullet^6$ and $\bullet^7$ are not available to candidates who work with $13\cos(x+22\cdot6)^\circ = 0$ or $13\cos(x+22\cdot6)^\circ = 1$.
13. Candidates who use $13\cos(x-22\cdot6)^\circ$ from a correct (a) lose $\bullet^6$ but $\bullet^7$ is still available.

Regularly occurring responses

Response 1

From (a) $a = 67\cdot4$

max/ min $= \pm13$ ✓ $\bullet^5$

max at $292\cdot6$ ✗ $\bullet^6$

min at $112\cdot6$ ✗ $\bullet^7$

3 marks out of 3

Response 2

From (a) $\sqrt{169}\cos(x+22\cdot6)^\circ$

max $= \sqrt{169}$ min $= -\sqrt{169}$ ✗ $\bullet^5$

max at $22\cdot6$ ✗ $\bullet^6$

min at $202\cdot6$ ✗ $\bullet^7$

2 marks out of 3

$\sqrt{169}$ already penalised at (a)

Response 3A

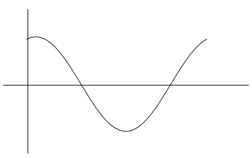
$13\cos(x+22\cdot6)^\circ$ max at $22\cdot6$ ✗ $\bullet^6$

$\times13$ $\quad 22\cdot6$ min at $202\cdot6$ ✗ $\bullet^7$

1 mark out of 3

Insufficient evidence for $\bullet^5$

Response 3B



$\xrightarrow{22\cdot6}$ $13\cos(x+22\cdot6)^\circ$ max at $22\cdot6$ ✗ $\bullet^6$

min at $202\cdot6$ ✗ $\bullet^7$

1 mark out of 3

N.B. Candidates who use differentiation in (b) can gain $\bullet^5$ only, as a direct result of their response in (a).
This question is in degrees and so calculus is not appropriate for $\bullet^6$ and $\bullet^7$.

3	(a)	(i) Show that the line with equation $y = 3 - x$ is a tangent to the circle with equation $x^2 + y^2 + 14x + 4y - 19 = 0$.	
		(ii) Find the coordinates of the point of contact, P.	**5**

Generic Scheme			**Illustrative Scheme**
(a)			
•[1]	ss	substitute	•[1] $x^2 + (3-x)^2 + 14x + 4(3-x) - 19 = 0$
			Method 1 : Factorising
•[2]	pd	express in standard form	•[2] $2x^2 + 4x + 2$ $= 0$ *see note* 1
•[3]	ic	start proof	•[3] $2(x+1)(x+1)$ $= 0$ *see note* 1
•[4]	ic	complete proof	•[4] equal roots so line is a tangent
			Method 2 : Discriminant
			•[2] $2x^2 + 4x + 2 = 0$ *stated explicitly*
			•[3] $4^2 - 4\times2\times2$
			•[4] $b^2 - 4ac = 0$ so line is a tangent
•[5]	pd	coordinates of P	•[5] $x = -1,\ y = 4$

Notes

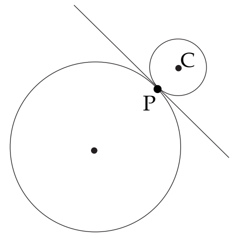
For method 1 :

1. •[2] is only available if "= 0" appears at either •[2] or •[3] stage.
2. Alternative wording for •[4] could be e.g. 'repeated roots', 'repeated factor', ' only one solution', 'only one point of contact' **along with** 'line is a tangent'.

For both methods :

3. Candidates must work with a quadratic equation at the •[3] and •[4] stages.
4. Simply stating the tangency condition without supporting working cannot gain •[4].
5. For candidates who obtain two distinct roots, •[4] is still available for 'not equal roots so not a tangent' or '$b^2 - 4ac \neq 0$ so line is not a tangent', but •[5] is not available.

3 (*b*) Relative to a suitable set of coordinate axes, the diagram below shows the circle from (*a*) and a second smaller circle with centre C.



The line $y = 3 - x$ is a common tangent at the point P.

The radius of the larger circle is three times the radius of the smaller circle.

Find the equation of the smaller circle. **6**

$x^2 + y^2 + 14x + 4y - 19 = 0$

Generic Scheme	Illustrative Scheme
(b)	
Method 1 : via centre and radius	**Method 1** : via centre and radius
•6 ic state centre of larger circle	•6 $(-7, -2)$ *see note* 11
•7 ss find radius of larger circle	•7 $\sqrt{72}$ *see note* 6 *stated, or implied by* •8
•8 pd find radius of smaller circle	•8 $\sqrt{8}$ *see note* 7
•9 ss strategy for finding centre	•9 e.g "Stepping out"
•10 ic interpret centre of smaller circle	•10 (1, 6)
•11 ic state equation	•11 $(x-1)^2 + (y-6)^2 = 8$ or $x^2 + y^2 - 2x - 12y + 29 = 0$
Method 2 : via ratios	**Method 2** : via ratios
•6 ic state centre of larger circle	•6 $(-7, -2)$ *see note* 11
•7 ss strategy for finding centre	•7 e.g. "Stepping out"
•8 ic state centre of smaller circle	•8 (1, 6)
•9 ss strategy for finding radius	•9 $\sqrt{2^2 + 2^2}$
•10 pd find radius of smaller circle	•10 $\sqrt{8}$ *see note* 10
•11 ic state equation	•11 $(x-1)^2 + (y-6)^2 = 8$ or $x^2 + y^2 - 2x - 12y + 29 = 0$

Notes

For method 1:

6. Acceptable alternatives for •7 are $6\sqrt{2}$ or decimal equivalent which rounds to $8 \cdot 5$ i.e. to two significant figures.
7. Acceptable alternatives for •8 are $\frac{\sqrt{72}}{3}$ or $2\sqrt{2}$ or decimal equivalent which rounds to $2 \cdot 8$.
8. (1, 6) without working gains •10 but loses •9.

For method 2:

9. (1, 6) without working gains •8 but loses •7.
10. Acceptable alternatives for •10 are $2\sqrt{2}$ or decimal equivalent which rounds to $2 \cdot 8$.

In both methods:

11. If $m = 1$ is used in a 'stepping out' method the centre of the larger circle need not be stated explicitly for •6.
12. For the smaller circle, candidates who 'guess' values for either the centre or radius cannot be awarded •11.
13. At •11 e.g. $\sqrt{8}^2$, $2 \cdot 8^2$ are unacceptable, but any decimal which rounds to $7 \cdot 8$ is acceptable.
14. •11 is not available to candidates who divide the coordinates of the centre of the larger circle by 3.

4	Solve $2\cos 2x - 5\cos x - 4 = 0$ for $0 \le x < 2\pi$.	**5**

Generic Scheme	Illustrative Scheme
4	**Method 1** : Using factorisation
$\bullet^1$ ss know to use double angle formula	$\bullet^1$ $2\times(2\cos^2 x - 1\,)\ldots$
$\bullet^2$ ic express as quadratic in $\cos x$	$\bullet^2$ $4\cos^2 x - 5\cos x - 6$
$\bullet^3$ ss start to solve	$\bullet^3$ $(4\cos x + 3)(\cos x - 2)$
	$\left.\right\}$ = 0 must appear at either of these lines to gain $\bullet^2$.
	Method 2 : Using quadratic formula
	$\bullet^1$ $2\times(2\cos^2 x - 1)\ldots$
	$\bullet^2$ $4\cos^2 x - 5\cos x - 6 = 0$
	$\bullet^3$ $\cos x = \dfrac{-(-5) \pm \sqrt{(-5)^2 - 4\times 4\times(-6)}}{2\times 4}$
	In both methods :
$\bullet^4$ pd reduce to equations in $\cos x$ only	$\bullet^4$ $\cos x = -\frac{3}{4}$ and $\cos x = 2$
$\bullet^5$ pd complete solutions to include only one where $\cos x = k$ with $\lvert k\rvert > 1$	$\bullet^5$ $2\cdot419,\ 3\cdot864$ and no solution
	or
	$\bullet^4$ $\cos x = 2$ and no solution
	$\bullet^5$ $\cos x = -\frac{3}{4}$ and $2\cdot419,\ 3\cdot864$

Notes

1. $\bullet^1$ is not available for simply stating that $\cos 2\mathrm{A} = 2\cos^2\mathrm{A} - 1$ with no further working.
2. Substituting $\cos 2\mathrm{A} = 2\cos^2\mathrm{A} - 1$ or $\cos 2a = 2\cos^2 a - 1$ etc. should be treated as bad form throughout.
3. In the event of $\cos^2 x - \sin^2 x$ or $1 - 2\sin^2 x$ being substituted for $\cos 2x$, $\bullet^1$ cannot be given until the equation reduces to a quadratic in $\cos x$.
4. Candidates may express the quadratic equation obtained at the $\bullet^2$ stage in the form $4c^2 - 5c + 6 = 0$, $4x^2 - 5x + 6 = 0$ etc. For candidates who do not solve a trig. equation at $\bullet^5$, $\cos x$ must appear explicitly to gain $\bullet^4$.
5. $\bullet^4$ and $\bullet^5$ are only available as a consequence of solving a quadratic equation subsequent to a substitution.
6. Any attempt to solve $a\cos^2 x + b\cos x = c$ loses $\bullet^3$, $\bullet^4$ and $\bullet^5$.
7. Accept answers given as decimals which round to $2\cdot4$ and $3\cdot9$.
8. There must be an indication after $\cos x = 2$ that there are no solutions to this equation.

   Acceptable evidence : e.g. "$\cancel{\cos x = 2}$" , "NA", "out of range", "invalid" and "$\cos x = 2$ no", "$\cos x = 2$ ✗"

   Unacceptable evidence : e.g. "$\underline{\underline{\cos x = 2}}$", "$\cos x = 2$???", "Maths Error".

9. $\bullet^5$ is not available to candidates who work throughout in degrees and do not convert their answer into radian measure.
10. Do not accept e.g. $221\cdot4,\ 138\cdot6,\ \dfrac{221\cdot4\pi}{180},\ \dfrac{221\pi}{180},\ 1\cdot23\pi$.
11. Ignore correct solution outside the interval $0 \le x < 2\pi$.

4

Regularly occurring responses

Response 1

$2\times 2\cos^2 x - 1\ldots$ ✓ $\bullet^1$

$4\cos^2 x - 5\cos x - 5 = 0$ ✗ $\bullet^2$

$\cos x = \dfrac{-(-5) \pm \sqrt{(-5)^2 - 4\times 4\times(-5)}}{2\times 4}$ ✓ $\bullet^3$

$\cos x = \frac{5-\sqrt{105}}{8}$ and $\cos x = \frac{5+\sqrt{105}}{8}$ ✓ $\bullet^4$

$2\cdot 286,\ 3\cdot 997$ and no solution ✓ $\bullet^5$

$\bullet^5$ is only available to candidates where one, but not both, of their equations has no solution for $\cos x$.

4 marks out of 5

Response 2

$4\cos^2 x - 1\ldots$ ✓ $\bullet^1$

$4\cos^2 x - 5\cos x - 5 = 0$ ✗ $\bullet^2$

$(2\cos x + 1)(2\cos x - 5) = 0$ ✗ $\bullet^3$

$\cos x = -\frac{1}{2}$ and $\cancel{\cos x = \frac{5}{2}}$ ✓ $\bullet^4$

$x = \frac{2\pi}{3},\ \frac{4\pi}{3}$ ✓ $\bullet^5$

$4\cos^2 x - 1$ with no further working cannot gain $\bullet^1$; however if a quadratic in $\cos x$ subsequently appears then $\bullet^1$ is awarded but $\bullet^2$ is not available.

3 marks out of 5

$\bullet^1$ is lost here as it is not clear whether the candidate has used $2\cos^2 x - 1$ or $\cos^2 x - 1$ as their substitution.

Response 3A

$2\cos^2 x - 1 - 5\cos x - 4 = 0$ ✗ $\bullet^1$

$2\cos^2 x - 5\cos x - 5 = 0$ ✓ $\bullet^2$

$\cos x = \dfrac{5 \pm \sqrt{25+40}}{4}$ ✓ $\bullet^3$

$\cos x = -0\cdot 766$ and $\cos x = 3\cdot 267$ ✓ $\bullet^4$

$x = 2\cdot 44,\ 3\cdot 84$ and undefined ✓ $\bullet^5$

4 marks out of 5

Response 3B

$\cos 2x = 2\cos^2 x - 1$

$2\cos^2 x - 1 - 5\cos x - 4 = 0$ ✓ $\bullet^1$

$2\cos^2 x - 5\cos x - 5 = 0$ ✗ $\bullet^2$

$\cos x = \dfrac{5 \pm \sqrt{25+40}}{4}$ ✓ $\bullet^3$

$\cos x = -0\cdot 766$ and $\cos x = 3\cdot 267$ ✓ $\bullet^4$

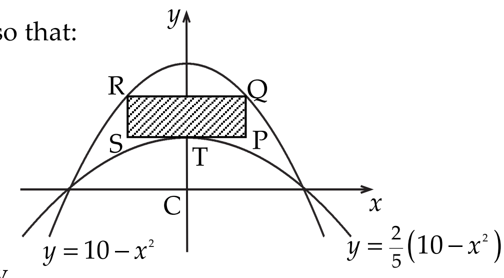
$x = 2\cdot 44,\ 3\cdot 84$ and undefined ✓ $\bullet^5$

4 marks out of 5

5 The parabolas with equations $y = 10 - x^2$ and $y = \frac{2}{5}(10 - x^2)$ are shown in the diagram below.

A rectangle PQRS is placed between the two parabolas as shown, so that:

- Q and R lie on the upper parabola.
- RQ and SP are parallel to the x-axis.
- T , the turning point of the lower parabola, lies on SP.



(*a*) (i) If TP $= x$ units, find an expression for the length of PQ.

(ii) Hence show that the area, A, of rectangle PQRS is given by

$$A(x) = 12x - 2x^3$$ **3**

(*b*) Find the maximum area of this rectangle. **6**

Generic Scheme			Illustrative Scheme	
5 (a)				
$\bullet^1$	ss	know to and find OT	$\bullet^1$ 4 or $(0,4)$	*stated, or implied by* $\bullet^2$
$\bullet^2$	ic	obtain an expression for PQ	$\bullet^2$ $10 - x^2 - 4$	
$\bullet^3$	ic	complete area evaluation	$\bullet^3$ $2x \times (6 - x^2) = 12x - 2x^3$	

Notes

1. The evidence for $\bullet^1$ and $\bullet^2$ may appear on a sketch.
2. No marks are available to candidates who work backwards from the area formula.
3. $\bullet^3$ is only available if $\bullet^2$ has been awarded.

5 (b)				
$\bullet^4$	ss	know to and start to differentiate	$\bullet^4$ $A'(x) = 12\ldots$	*stated, or implied by* $\bullet^5$
$\bullet^5$	pd	complete differentiation	$\bullet^5$ $12 - 6x^2$	
$\bullet^6$	ic	set derivative to zero	$\bullet^6$ $12 - 6x^2 = 0$	
$\bullet^7$	pd	obtain x	$\bullet^7$ $\sqrt{2}$ or decimal equivalent $\left(\text{ignore inclusion of } -\sqrt{2}\right)$	
$\bullet^8$	ss	justify nature of stationary point	$\bullet^8$ $\begin{array}{c\|ccc} x & \ldots & \sqrt{2} & \ldots \\ \hline A'(x) & + & 0 & - \end{array}$ (Note : accept $12 - 6x^2$ in lieu of $A'(x)$ in the nature table.)	
$\bullet^9$	ic	interpret result and evaluate area	$\bullet^9$ Max **and** $8\sqrt{2}$ or decimal equivalent N.B. To conclude a maximum the evidence must come from $\bullet^8$.	

Notes

4. At $\bullet^7$ accept any answer which rounds to $1\cdot 4$.
5. Throughout this question treat the use of $f'(x)$ or $\dfrac{dy}{dx}$ as bad form.
6. At $\bullet^8$ the nature can be determined using the second derivative.
7. At $\bullet^9$ accept any answer which rounds to $11\cdot 3$ or $11\cdot 4$.

5

Regularly occurring responses

Response 1

$A(x) = 12x - 2x^3$

$A'(x) = 24x^2 - 6x^3$ ✗ $\bullet^4$ ✗ $\bullet^5$

$24x^2 - 6x^3 = 0$ ✓̸ $\bullet^6$

$A'(x) = 0$ on its own would not be sufficient for $\bullet^6$.

Response 2

At stationary points, $A'(x) = 0$

$12 - 6x^2$ ✓ $\bullet^4$ ✓ $\bullet^5$ ✓ $\bullet^6$

$x = \sqrt{2}$ ✓ $\bullet^7$

Response 3

$A(x) = 12x - 2x^3$

$= 12 - 6x^2$ ✓ $\bullet^4$ ✓ $\bullet^5$

Bad form

Response 4A

	...	$\sqrt{2}$	...
$A'(x)$	+	0	–

✗ $\bullet^8$

x missing

Response 4B

x	1	$\sqrt{2}$	2
$A'(x)$	6	0	–12

✓ $\bullet^8$

Response 4C

x	...	$\sqrt{2}$	...
$A'(x)$	/	—	\

✗ $\bullet^8$

or 'slope' etc.

signs or values are necessary

Response 5

Maximum at $x = \sqrt{2}$

$y = 12\sqrt{2} - 2\sqrt{2}^3 = 8\sqrt{2}$ ✓ $\bullet^9$

Area $= 2\sqrt{2} \times 8\sqrt{2} = 32$

Treat this as an error subsequent to a correct answer.

6 (*a*) A curve has equation $y=(2x-9)^{\frac{1}{2}}$.

Show that the equation of the tangent to this curve at the point where $x=9$ is $y=\frac{1}{3}x$. **5**

(*b*) Diagram 1 shows part of the curve and the tangent.

The curve cuts the x-axis at the point A.

Find the coordinates of point A. **1**

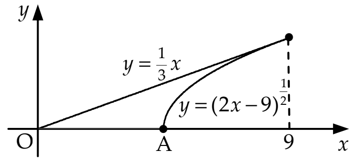


Diagram 1

Generic Scheme			Illustrative Scheme	
6 (a)				
$\bullet^1$	ss	know to and start to differentiate	$\bullet^1$	$\frac{1}{2}(2x-9)^{-\frac{1}{2}}$
$\bullet^2$	pd	complete chain rule derivative	$\bullet^2$	$\ldots\times 2$
$\bullet^3$	pd	gradient via differentiation	$\bullet^3$	$\frac{1}{3}$
$\bullet^4$	pd	obtain y_{CURVE} at $x=9$	$\bullet^4$	3
$\bullet^5$	ic	state equation and complete	$\bullet^5$	$y-3=\frac{1}{3}(x-9)$ and complete to $y=\frac{1}{3}x$

Notes

1. $\bullet^3$ is only available as a consequence of differentiating equation of the curve.
2. Candidates must arrive at the equation of the tangent via the point $(9, 3)$ and not the origin.
3. For $\bullet^3$ accept $9^{-\frac{1}{2}}$.

Regularly occurring responses

Response 1

Candidates who equate derivatives:

$\frac{1}{2}(2x-9)^{-\frac{1}{2}}\times 2=\frac{1}{3}$ ✓ $\bullet^3$

✓ $\bullet^1$ ✓ $\bullet^2$

leading to $x=9$ and $y=3$ from curve ✓ $\bullet^4$

Also obtaining $y=3$ from line and so line is a tangent ✓ $\bullet^5$

$\boxed{\text{5 marks out of 5}}$

Response 2

Candidates who intersect curve and line:

$(2x-9)^{\frac{1}{2}}=\frac{1}{3}x$ ✓ $\bullet^1$

$2x-9=\left(\frac{1}{3}x\right)^2$ ✓ $\bullet^2$

$\frac{1}{9}x^2-2x+9=0$ ✓ $\bullet^3$

$\boxed{\text{5 marks out of 5}}$

Factorising or using discriminant ✓ $\bullet^4$

Equal roots or $b^2-4ac=0$ so line is a tangent ✓ $\bullet^5$

(b)				
$\bullet^6$	ic	obtain coordinates of A	$\bullet^6$	$\left(\frac{9}{2}, 0\right)$

Notes

4. Accept $x=\frac{9}{2}$, $y=0$ where $y=0$ may appear from $(2x-9)^{\frac{1}{2}}=0$.
5. For $\left(\frac{9}{2}, 0\right)$ without working $\bullet^6$ is awarded, but from erroneous working $\bullet^6$ is lost (see response below).

Regularly occurring responses

Response 1

$\sqrt{2x-9}=0 \Rightarrow \sqrt{2x}-3=0 \Rightarrow 2x-9=0 \Rightarrow x=4\cdot 5$

Here $\bullet^6$ cannot be awarded due to the erroneous working.

6 *(c)* Calculate the shaded area shown in diagram 2. **7**

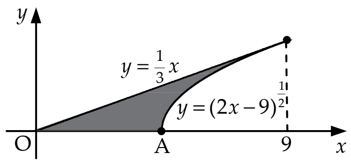


Diagram 2

Generic Scheme	Illustrative Scheme
6 (c)	
Method 1 : Area of triangle – area under curve	**Method 1** : Area of triangle – area under curve
•7 ss strategy for finding shaded area	•7 Shaded area = Area of large $\Delta -$ Area under curve
•8 ss know to integrate $(2x-9)^{\frac{1}{2}}$	•8 $\int (2x-9)^{\frac{1}{2}}\,dx$
•9 pd start integration	•9 $\dfrac{(2x-9)^{\frac{3}{2}}}{\frac{3}{2}}$
•10 pd complete integration	•10 $\ldots \times \frac{1}{2}$
•11 ic limits x_A and 9	•11 $\frac{9}{2}$ and 9
•12 pd substitute limits	•12 $\frac{1}{3}(18-9)^{\frac{3}{2}}-0$
•13 pd evaluate area and complete strategy	•13 $\frac{27}{2}-9=\frac{9}{2}$ or $4\frac{1}{2}$ or $4\cdot5$
Method 2 : Area between line and curve	**Method 2** : Area between line and curve
•7 ss strategy for finding shaded area	•7 Area of small Δ + area between line and curve
•8 ss know to integrate $(2x-9)^{\frac{1}{2}}$	•8 $\int \ldots (2x-9)^{\frac{1}{2}}\,dx$
•9 pd start integration	•9 $\ldots\dfrac{(2x-9)^{\frac{3}{2}}}{\frac{3}{2}}$
•10 pd complete integration	•10 $\ldots\times\frac{1}{2}$
•11 ic limits x_A and 9	•11 $\frac{9}{2}$ and 9
•12 pd 'upper – lower' and substitute limits	•12 $\left(\frac{1}{6}\times9^2-\frac{1}{3}(18-9)^{\frac{3}{2}}\right)-\left(\frac{1}{6}\times\left(\frac{9}{2}\right)^2-\frac{1}{3}(9-9)^{\frac{3}{2}}\right)$
•13 pd evaluate area and complete strategy	•13 $\frac{27}{8}+\frac{9}{8}=\frac{9}{2}$ or $4\frac{1}{2}$ or $4\cdot5$

Notes

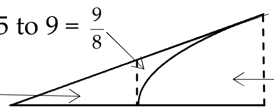
6. •7 may not be obvious until the final line of working and may be implied by final answer or a diagram.
7. At •11 the value of x_A must lie between 0 and 9 exclusively, however, •12 and •13 are only available if $4\cdot5 \le x_A < 9$.
8. Full marks are available to candidates who integrate with respect to y.

You may find the following helpful in marking this question:

Area of $\Delta_{\text{LARGER}} = 13\cdot5$ or $\frac{27}{2}$

Area between curve and line from 4·5 to 9 = $\frac{9}{8}$

Area of $\Delta_{\text{SMALLER}} = \frac{27}{8}$



Area under curve from 4·5 to 9 = 9

7	(*a*) Given that $\log_4 x = \mathrm{P}$, show that $\log_{16} x = \frac{1}{2}\mathrm{P}$.	**3**

Generic Scheme			**Illustrative Scheme**	
(a)				
$\bullet^1$	ss	convert from log to exponential form	$\bullet^1$	$x = 4^{\mathrm{P}}$
$\bullet^2$	ss	know to and convert back to log form	$\bullet^2$	$\log_{16} x = \log_{16} 4^{\mathrm{P}}$
$\bullet^3$	pd	process and complete	$\bullet^3$	$\log_{16} x = \mathrm{P} \times \log_{16} 4$ and complete

Notes

1. No marks are available to candidates who simply substitute in values and verify the result.

   e.g. $\log_4 4 = 1$ and $\log_{16} 4 = \frac{1}{2}$ ✗

   $\log_4 x = \mathrm{P}$ and $\log_{16} x = \frac{1}{2}\mathrm{P}$

Regularly occurring responses

Response 1

$\log_4 x = \mathrm{P}$

$x = 4^{\mathrm{P}}$ ✓ $\bullet^1$

$\log_{16} x = \frac{1}{2}P$ ✗ $\bullet^2$

$x = 16^{\frac{1}{2}\mathrm{P}}$

$= 4^{\mathrm{P}}$ ∧ $\bullet^3$

1 mark out of 3

Response 2

$\log_4 x = P$

$x = 4^P$ ✓ $\bullet^1$

$x^2 = 4^{2P}$

$= 16^P$ ✗ $\bullet^2$

$\log_{16} x^2 = P$

$2\log_{16} x = P$

$\log_{16} x = \frac{1}{2}P$ ✓ $\bullet^3$

2 marks out of 3

Response 3

$x = 4^{\mathrm{P}}$ ✓ $\bullet^1$

$x = \left(16^{\frac{1}{2}}\right)^{\mathrm{P}}$ or $16^{\frac{1}{2}\times\mathrm{P}}$ ✓ $\bullet^2$

$x = 16^{\frac{1}{2}\mathrm{P}}$

$\log_{16} x = \frac{1}{2}\mathrm{P}$ ✓ $\bullet^3$

Without this step $\bullet^2$ would be lost but $\bullet^3$ is still available as follow through.

3 marks out of 3

Response 4

$x = 4^{\mathrm{P}}$ ✓ $\bullet^1$

$\log_4 x = \mathrm{P}$

$4^{\log_4 x} = 4^{\mathrm{P}} = x$

$4^{\log_4 x} = x$

$\log_{16} x = k\mathrm{P}$

$16^{\log_{16} x} = 16^{k\mathrm{P}}$

$x = 16^{k\mathrm{P}}$

$4^{\mathrm{P}} = 16^{k\mathrm{P}}$

$4 = 16^k$ ✗ $\bullet^2$

$k = \frac{1}{2}$

$\therefore \log_{16} x = \frac{1}{2}\mathrm{P}$ as $16^{\frac{1}{2}} = 4$ ✓ $\bullet^3$

2 marks out of 3

Response 5

Beware that some candidates give a circular argument.

This is only worth $\bullet^1$.

$\log_4 x = \mathrm{P}$	then	$\log_{16} x = \frac{1}{2}\mathrm{P}$
$x = 4^{\mathrm{P}}$ ✓ $\bullet^1$		$x = 16^{\frac{1}{2}\mathrm{P}}$
$\log_4 x = \log_4 4^{\mathrm{P}}$		$\log_{16} x = \log_{16} 16^{\frac{1}{2}\mathrm{P}}$
$\log_4 x = \mathrm{P}\log_4 4$		$\log_{16} x = \frac{1}{2}\mathrm{P}\log_{16} 16$
$\log_4 x = \mathrm{P}$		$\log_{16} x = \frac{1}{2}\mathrm{P}$ ✗

1 mark out of 3

Response 6

$\log_{16} x = \dfrac{\log_4 x}{\log_4 16} = \dfrac{\log_4 x}{2} = \frac{1}{2}\mathrm{P}$

✓ $\bullet^1$ ✓ $\bullet^2$ ✓ $\bullet^3$

Using change of base result.

3 mark out of 3

7	(*b*) Solve $\log_3 x + \log_9 x = 12$.	**3**

Generic Scheme	**Illustrative Scheme**	
(b)		
•⁴ ss use appropriate strategy	•⁴ $\log_3 x + \frac{1}{2}\log_3 x = 12$	•⁴ $2\log_9 x + \log_9 x = 12$
•⁵ pd start solving process	•⁵ $\log_3 x = 8$	•⁵ $\log_9 x = 4$
•⁶ pd complete process via log to expo form	•⁶ $x = 3^8$ $(=6561)$	•⁶ $x = 9^4$ $(=6561)$
	or	**or**
	•⁴ $Q + \frac{1}{2}Q = 12$ $Q = 8$	•⁴ $2Q + Q = 12$ $Q = 4$
	•⁵ $\log_3 x = 8$	•⁵ $\log_9 x = 4$
	•⁶ $x = 3^8$ $(=6561)$	•⁶ $x = 9^4$ $(=6561)$

Notes

2. At •⁴ any letter except x may be used in lieu of Q.
3. Candidates who use a trial and improvement technique by substituting values for x gain no marks.
4. The answer with no working gains no marks.

Regularly occurring responses

Response 1

$Q + 2Q = 12$
$3Q = 12$
$Q = 4$
$\log_3 x = 4$
$x = 3^4$ ✗ •⁶
$= 81$

✓ •⁴ ✗ •⁵	or	✗ •⁴ ✗ •⁵

2 marks out of 3

The marks allocated are dependent on what substitution is used for Q.

Response 2

$\log_3 x + 2\log_3 x = 12$ ✗ •⁴
$3\log_3 x = 12$
$\log_3 x = 4$ ✓ •⁵
$x = 3^4$ ✓ •⁶
$= 81$

2 marks out of 3

Response 3

$2\log_9 x + \log_9 x = 12$ ✓ •⁴
$\log_9 x^2 + \log_9 x = 12$
$\log_9 x^3 = 12$ ✓ •⁵
$x^3 = 9^{12}$
$x = \sqrt[3]{9^{12}}$
$x = 9^4$ ✓ •⁶
$x = 3^8$
$= 6561$

3 marks out of 3

Response 4

$\log_3 x = 8$ ✗ •⁴ ✗ •⁵
$x = 3^8$ ✓ •⁶
$= 6561$

Without justification, •⁴ and •⁵ are not available.