2001 Advanced Higher

Section A (Mathematics 1 and 2)

All candidates should attempt this Section.

Answer all the questions.

Marks

A1. Use Gaussian elimination to solve the following system of equations

$$\begin{aligned} x + y + z &= 10 \\ 2x - y + 3z &= 4 \\ x + 2z &= 20. \end{aligned}$$ 5

A2. Differentiate with respect to x

(a) $f(x) = (2+x)\tan^{-1}\sqrt{x-1}, \quad x > 1.$ 4

(b) $g(x) = e^{\cot 2x}, \quad 0 < x < \frac{\pi}{2}.$ 2

A3. Find the value of

$$\int_0^{\pi/4} 2x \sin 4x \, dx.$$ 5

A4. Prove by induction that, for all integers $n \geq 1$,

$$2 + 5 + 8 + \ldots + (3n-1) = \tfrac{1}{2}n(3n+1).$$ 5

A5. (a) Obtain partial fractions for

$$\frac{x}{x^2-1}, \qquad x > 1.$$ 2

(b) Use the result of (a) to find

$$\int \frac{x^3}{x^2-1}\,dx, \qquad x > 1.$$ 4

A6. Expand

$$\left(x^2 - \frac{2}{x}\right)^4, \qquad x \neq 0$$

and simplify as far as possible. 5

Marks

A7. A curve has equation $xy + y^2 = 2$.

(a) Use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y. 3

(b) Hence find an equation of the tangent to the curve at the point (1, 1). 2

A8. A function f is defined by $f(x) = \frac{x^2 + 6x + 12}{x + 2}$, $x \neq -2$.

(a) Express $f(x)$ in the form $ax + b + \frac{b}{x+2}$ stating the values of a and b. 2

(b) Write down an equation for each of the two asymptotes. 2

(c) Show that $f(x)$ has two stationary points.
Determine the coordinates and the nature of the stationary points. 4

(d) Sketch the graph of f. 1

(e) State the range of values of k such that the equation $f(x) = k$ has no solution. 1

A9. (a) Given that $-1 = \cos\theta + i\sin\theta$, $-\pi < \theta \leq \pi$, state the value of θ. 1

(b) Use de Moivre's Theorem to find the non-real solutions, z_1 and z_2, of the equation $z^3 + 1 = 0$. 5
Hence show that $z_1^2 = -z_2$ and $z_2^2 = -z_1$. 2

(c) Plot all the solutions of $z^3 + 1 = 0$ on an Argand diagram and state their geometrical significance. 3

[Turn over

Marks

A10. A chemical plant food loses effectiveness at a rate proportional to the amount present in the soil. The amount M grams of plant food effective after t days satisfies the differential equation

$$\frac{dM}{dt} = kM, \text{ where } k \text{ is a constant.}$$

(*a*) Find the general solution for M in terms of t where the initial amount of plant food is M_0 grams. 3

(*b*) Find the value of k if, after 30 days, only half the initial amount of plant food is effective. 3

(*c*) What percentage of the original amount of plant food is effective after 35 days? 2

(*d*) The plant food has to be renewed when its effectiveness falls below 25%. Is the manufacturer of the plant food justified in calling its product "sixty day super food"? 2

[*END OF SECTION A*]

Candidates should now attempt ONE of the following

Section B (Mathematics 3) on *Page five*
Section C (Statistics 1) on *Page six*
Section D (Numerical Analysis 1) on *Page eight*
Section E (Mechanics 1) on *Page ten*.

Section B (Mathematics 3)

Marks

ONLY candidates doing the course Mathematics 1, 2 and 3 should attempt this Section.

Answer all the questions.

Answer these questions in a separate answer book, showing clearly the section chosen.

B1. Use the Euclidean algorithm to find integers x and y such that

$$149x + 139y = 1.$$ 4

B2. Find the general solution of the following differential equation:

$$\frac{dy}{dx} + \frac{y}{x} = x, \qquad x > 0.$$ 4

B3. Let

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & -1 & -1 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 0 & 1 \\ 4 & -2 & -2 \\ -3 & 2 & 1 \end{pmatrix}.$$

Show that $AB = kI$ for some constant k, where I is the 3×3 identity matrix. Hence obtain (i) the inverse matrix A^{-1}, and (ii) the matrix A^2B. 4

B4. Find the first four terms in the Maclaurin series for $(2 + x)\ln(2 + x)$. 4

B5. Find the general solution of the following differential equation:

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = 6x - 1.$$ 5

B6. Let L_1 and L_2 be the lines

$$L_1: \quad x = 8 - 2t,\ y = -4 + 2t,\ z = 3 + t$$

$$L_2: \quad \frac{x}{-2} = \frac{y+2}{-1} = \frac{z-9}{2}.$$

(a) (i) Show that L_1 and L_2 intersect and find their point of intersection. 4

(ii) Verify that the acute angle between them is

$$\cos^{-1}\left(\frac{4}{9}\right).$$ 2

(b) (i) Obtain an equation of the plane Π that is perpendicular to L_2 and passes through the point $(1, -4, 2)$. 3

(ii) Find the coordinates of the point of intersection of the plane Π and the line L_1. 2

[END OF SECTION B]