Kilsyth Academy

Higher Maths

Homework

Booklet Number

**Ex 1 The Straight Line**

**Only use your calculator if you need to!**

1. Find the equation of the line –

* 1. with gradient 2/3. passing through (4 , −1)
  2. passing through (7 , 3) and (9 , −1)

x

y

0

40°

2. A line makes an angle of 40° with the positive direction

of the *x* axis, as shown in the diagram.

Find the gradient of the line (correct to 2 decimal places).

3. a) Write down the gradient of any line parallel to *y* = 1/4*x* + 5

b) Write down the gradient of any line perpendicular to 2*x* − *y* + 3 = 0

4. Find the equation of the straight line which is parallel to the line with equation 3*x* + 2*y* = 4 and which passes through (3 , −2).

5. P is the point (1 , 3) and Q (5 , −7).

Find the equation of the perpendicular bisector of PQ.

[HINT: Find the eqn of line which goes through the midpoint of PQ and has a perpendicular gradient to PQ]

6. Calculate the distance between the points (1 , 5) and (2 , −2).

[HINT: Use the distance formula. Simplify your answer as far as possible]

7. Find the size of the angle *x*° that the line joining the points A(2 , 3√3) and B(−1 , 0) makes with the positive direction of the *x* axis.

[HINT: Find gradient then use m = tanθ]

8. A triangle KLF has vertices K(−4 , 1), L(12 , 3) and F(7 , −7).

1. Find the equation of the median from F.
2. Find the equation of the altitude from K.
3. Find the co-ordinates of the point of intersection of the median and the altitude.

**Ex 2 Recurrence Relations**

1. For each of the following recurrence relations, find u3.

a) un+1 = 3un – 2 u0 = 4

b) un+1 = −0⋅8un + 5 u0 = 100

2. A sequence is defined by the recurrence relation un = 0⋅7un−1 + 3 , u1 = 2

1. Calculate the value of u2.
2. What is the smallest value of n for which un > 7
3. Explain how you know that a limit exists.
4. Find the limit of this sequence as n → ∞.

3. In a hare colony one quarter of the existing hares are eaten by predators each summer, however during the winter 84 hares are born.

There are un hares at the start of a particular summer.

1. Write down a recurrence relation for un+1, the number of hares at the start of the next summer.
2. Find the limit of the sequence generated by this recurrence relation and explain what the limit means in the context of this question.

4. Two sequences are generated by the recurrence relations -

un+1 = aun + 10 and vn+1 = a2vn + 8

The two sequences approach the same limit as n → ∞.

Determine the value of a and evaluate the limit.

5. On 1st February, a bank loans a man £2600 at a fixed rate of interest of 2% per month.

This interest is added on the last day of each month and is calculated on the amount due on the first day of the month. He agrees to make repayments on the first day of each subsequent month. Each repayment is £420 except for the smaller final amount which will pay off the loan.

1. The amount that he owes at the start of each month is taken to be the amount still owing just after the monthly repayment has been made. Let un and un+1 represent the amount he owes at the start of two successive months. Write down a recurrence relation involving un+1 and un.
2. Find the date and the amount of the final payment.

*Revision*

7. A line *L* has equation 2*y* + 4*x* = 5. What is the gradient of any line parallel to *L*?

A. −4 B. −2 C. 2 D. ½

**Ex 3 The Circle**

**Only use your calculator if you need to!**

1. Write down the equation of the circle -

1. with centre at the origin and radius 4 units.
2. With centre at the origin and passing through the point P(−3 , 4)
3. With centre (7 , −2) and radius 7
4. With centre (4 , 0) and passing through (−1 ,2)

2. a) Write down the centre and radius of the circle x2 + y2 − 6x − 2y − 15 = 0

b) Say whether each of these points lies inside, outside or on the circle -

A (3 , 5) B (−2 , 1) C (9 , 0)

3. The point P (2 , 3) lies on the circle x2 + y2 + 2x − 2y − 11 = 0.

Find the equation of the tangent at P.

4. For what range of values of *t* does the equation x2 + y2 + 4*t*x − 2*t*y − *t* + 4 = 0 represent a circle?

5. Prove that the line y = 3x + 10 is a tangent to the circle x2 + y2 − 8x − 4y − 20 = 0 and find the point of contact.

6. a) Show that the point *A* (−1 , −3) lies on the circle with equation x2 + y2 − 6x + 8y + 8 = 0.

b) Find the equation of the tangent at *A* to this circle.

c) Show that this line is also a tangent to the parabola y = x2 + 5 , stating the co-ordinates of the point of contact.

x

y

─6

─8

0

7. Alex has designed a pendant made from gold

wire, in the shape of two touching circles

with two tangents to the outer circle as shown

in the diagram opposite.

The circles touch at (0 , 0).

The equation of the inner circle is x2 + y2 + 5y = 0.

The outer circle intersects the y axis at (0 , ─6).

The tangents meet the y axis at (0 , ─8).

Find the total length of gold wire needed to make this pendant.

*Revision*

8. Find the equation of the line which passes through (−4 , 1) and is at an angle of 135° with the positive direction of the x axis.

**Ex 4 Differentiation 1**

1. Differentiate the following -

a)  b) 

1.  d) 

e)  f) 

**Only use your calculator if you need to!**

2. Find *f* ’(4) where *f*(*x*) = 

3. Find the equation of the tangent to the curve *y* = *x2* − 6*x* + 8 at the point with *x* = 5.

4. Find the co-ordinates of the point on the curve *y* = 2*x*2 − 11*x* + 13 where the tangent to the curve makes an angle of 45° with the positive direction of the *x* axis.

5. Find the rate of change of *g* , where *g*(*x*) =  when *x* = 2

6. Find the stationary points of the function f(*x*) = 2*x*3 − 6*x* and determine their nature.

# Revision

7. A sequence is defined by the recurrence relation un+1 = *a*un + 8

1. If *a* = 0⋅2, find the limit of the sequence as n → ∞.
2. Find the value of *a* for which the limit of the sequence is 24.

**Ex 5 Differentiation 2**

1. Differentiate the following -

a)  b) 

c)  d) 

e)  f) 

2. A tangent to the curve with equation  is drawn at the point (-2 , -4).

What is the gradient of this tangent?

A 2 B 3 C 4 D 10

3. Sketch the curve *y* = *x*(3 − *x*)2

HINT: Find where the curve crosses the x and y axes and the stationary points.

4. Show that the function *y* = *x*3 + 2*x* − 5 is never decreasing.

(12,0)

x

y

0

5. A company spends *x* thousand pounds a year

on advertising and this results in a profit of

*P* thousand pounds. A mathematical model,

illustrated in the diagram, suggests that *P* and

*x* are related by  for .

Find the value of *x* which gives the maximum

profit. [SQA 2001]

*Revision*

6. Find the equation of the line which passes through the points (3 , -5) and (-7 , 5).

7. Prove that the line 4x − y + 7 = 0 is a tangent to the circle x2 + y2 − 6x − 4y − 4 = 0 and find the point of contact.

**Ex 6 Integration**

**Only use your calculator if you need to!**

1. Find –

a)  b) 

c)  d) 

2. Evaluate –

a)  b)  c) 

4. Calculate the shaded area enclosed between the parabolas with equations

*y* = 1 + 10*x* − 2*x*2 and *y* = 1 + 5*x* − *x*2.

3. Calculate the area enclosed by

the parabola *y* = 2 + *x* − *x*2 and

the line *y* = 3*x* − 1.

*y* = 1 + 10*x* − 2*x*2

*y*

0

x

*y*

0

*y* = 2 + *x* − *x*2

*y* = 3*x* − 1

2

*y* = 1 + 5*x* − *x*2

*x*

*Revision*

4. Find the equation of a circle centre (7 , ─4), radius 8 units.

5. For what value of *t* does the equation *x2* ─ 5*x* + (*t* + 6) = 0 have equal roots?

**Ex 7 Exponential and Logarithmic Functions**

1. On the same diagram show the graphs of *y* = 4*x* and  **

(the co-ordinates of 2 points should be indicated on each.)

2. Sketch the graphs -

1. 

**Only use your calculator if you need to!**

1. 
2. 

3. Find -

a) log327 b) log91 c) log162 d) log5

4. Simplify -

a) log432 − log42 b) `log5 + log

c) log34 + log36 − log38 d) 2log63 + 2log62

5. Solve for x :  = 2

6. Solve (to 3 decimal places) -

a) 7x = 65 b) *e3x* = 45 c) *lnx* = 5

7. A radioactive element decays according to the law *mt = m0 e−0⋅01t* , where *m0* is the initial mass and *mt* is the mass after *t* years.

Calculate the half-life of this element ie. the time required for the radioactive mass to reduce to half of its original mass.

8. Simplify  , where x > 0 and y > 0 to get:

A 2 B 3y C  D 

*Revision*

9. Find the equation of the line through the point (3 , −1) and perpendicular to the line with equation 3*x* − 2*y* + 5 = 0.

**Ex 8 Addition Formulae**

1. By expanding the brackets, simplify as far as possible-

a) *sin*(90 − *x*) b) √2*cos*(*x* − π/4) c) *sin*(A + B) − *sin*(A − B)

2. Without using a calculator, find the value of - *cos*137°*cos*133° − *sin*137°*sin*133°

3. In triangle PQR, show that the **exact value** of

**NO CALCULATOR!**

*sin*(*p* + *q*) is  

4. Using triangle ABC, as shown,

1

1

3

P

R

Q

*q*

*p*

2

√7

A

B

C

*a*

find the **exact value** of *cos*2*a*.

5. Prove that -

1. (*cosθ* + *sinθ*)(*cosθ* − *sinθ*) = *cos2θ*
2. 

*Revision*

6. ∆XYZ has vertices X(−1, 2) Y(5 , 4) and Z (7 , 2).

**Only use your calculator if you need to!**

Find the equation of the median from Y.

7. Prove that *y* = 4*x* − 9 is a tangent to the curve y = 4*x2* − 8*x* and find the co-ordinates of the point of contact.

8. My dog gets put out every night, and I put his food (400g of *chump*) in his dish every morning before I let him in.

Every day he eats only 85% of whatever amount of food is originally in his dish in the morning.

If the dish holds 500g of *chump* when full, will it ever overflow if this feeding pattern is maintained indefinitely?

**Ex 9 Sets and Functions**

1. a) Two functions f and g are given by f(*x*) = 3*x* + 5 and g(*x*) = *x*2 .

Obtain an expression for - (i) f(g(3)) (ii) f(g(*x*)) (iii) g(f(*x*))

b) Functions h and k are given by h(*x*) = 3*x* and k(*x*) = sin*x*.

Find expressions for - (i) h(k(*x*)) (ii) k(h(*x*))

2. f(*x*) = 2*x* + 1 and g(*x*) = *x*2 + 6

1. Find g(f(*x*))
2. Find f(g(*x*))
3. Show that the equation g(f(*x*)) − f(g(*x*)) = 0 simplifies to 2*x*2 + 4*x* − 6 = 0.
4. Solve the equation 2*x*2 + 4*x* − 6 = 0.

(1,3)

x

y

0

y = 3x

1

3. Make a copy of the graph of *y* = 3*x* as

shown opposite.

1. On the same diagram show –
   1. The line *y* = *x*
   2. the graph of the inverse function of *y* = 3*x*.

b) What is the equation of the inverse function?

4. A function is given by f(*x*) = . Which of the following is a suitable domain of f?

A. *x* ≥ 4 B. *x* ≤ 4 C. −4 ≤ *x* ≤ 4 D. −16 ≤ *x* ≤ 16

*Revision*

5. Find ****

6. Find the rate of change of  at x = 4.

7. For what values of t does the equation x2 − 2tx + (t + 2) = 0 have real roots?

**Ex 10 Graphs of functions**

0

x

y

−4

2

(−1 , −9)

1. The graph of a quadratic function is shown.

On separate diagrams sketch the graphs of –

1. f(*x*) + 5

1. f(*x* − 2)
2. 4 − f(*x*) {note: same as –f(*x*) + 4}

[HINT: The co-ordinates of 3 points

should be indicated on each graph.]

2. a) Sketch the graph of the function y = log4*x*, showing the co-ordinates of two points that lie on the curve.

b) On the same diagram sketch y = log4*x* + 1.

c) On the same diagram sketch y = log4(*x* − 3).

**Only use your calculator if you need to!**

# Revision

3. The functions f, g and h are given by -

f(x) = 3*x*2 + 2 g(x) =  h(x) = 3*x* + 1

a)Find, in simplest form, formulae for –

1. f(h(x))

ii) g(h(x))

b) One of your answers to part (a) should suggest a relationship between two of these three given functions. Describe this relationship.

45º

x

y

0

L

4. The diagram shows a line L. The angle between L and the

positive direction of the x axis is 45º, as shown.

What is the gradient of line L:

A √3 B 0 C −1 D 1

**Ex 11 Trig Graphs**

1. Express in radians -

a) 60° b) 90° c) 150° d) 315°

2. Express in degrees -

a)  radians b)  radians c)  radians d) radians

**NO CALCULATOR!**

3. Write down the **exact** value of -

a) *sin30°*  b) *cosπ/6* c) *cos45°*

d) *sinπ/2* e) *tan60°* f) *tan150°*

4. Sketch the graphs of the following trigonometric functions -

a) *y* = *2sinx* 0 ≤ *x* ≤ 360 b) *y* = *cos2x* 0 ≤ *x* ≤ 2π

c) *y* = *−3sin2x* 0 ≤ *x* ≤ 2π d) *y* = *2cosx* − 1 0 ≤ *x* ≤ 360

Indicate on your graphs all important points and features.

5. Write down the equations of the following trig graphs.

a) b)

360

1

−1

0

π

0⋅5

0

−0⋅5

−2

# Revision

**Only use your calculator if you need to!**

6. Find the equation of the line through the point (2 , −1) and perpendicular to the line with equation 2*x* – *y* + 8 = 0.

**Ex 12 Quadratic Functions**

1. Solve these quadratic equations -

a) *t*2 + 2*t* − 3 = 0 b) *x*2 − 7*x* + 12 = 0 c) *y*2 − 2*y* − 4 = 0

2. For each of the following equations -

**Only use your calculator if you need to!**

1. calculate the discriminant

ii) determine the nature of the roots.

a) 2*p*2 + 3*p* + 1 = 0 b) 4*y*2 − 12*y* + 9 = 0

3. Sketch the curve *y* = *x*2 + *x* − 6

4. Solve - *x*2 + *x* ─ 2 < 0

[HINT: Remember quadratic **inequalities** can only be solved from a graph!!]

x

y

0

−1

(1,−8)

5. Find the equations of the quadratic functions represented by -

1. b)

x

y

0

6

(3,3)

6. For what value of *p* does the equation *x*2 − 5*x* + (*p* + 6) = 0 have equal roots.

7. If *x*2 − 6*x* + 14 is written in the form (*x* − *p*)2 + *q*, what is the value of *q*

**A** −22 **B** 5 **C** 14 **D** 50

8. The discriminant of a quadratic equation is 23.

Here are two statements about this quadratic equation:

1. the roots are real
2. the roots are rational.

Which of the following is true?

**A** Neither statement is correct **B** Only statement (1) is correct

**C** Only statement (2) is correct **D** Both statements are correct.

9. Show that the roots of the equation (*x* ─ 2)(*x* ─ 3) = *p2* are always real.

10. State the minimum value of f(*x*) = .

11. Show that y = 2x + 1 is a tangent to the parabola y = 4x − x2 and find the point of contact.

**Ex 13 Vectors**

**1**. For the vectors **u** =  and **v** = , work out 3**u** − 2**v**.

2. In this figure  = **u** and  = **v**.

DC = 3/2AB, and DC is parallel to AB.

A

B

C

D

M

**u**

**v**

M is the midpoint of BC.

Write down, in terms of **u** and **v**, the vectors

a)  b) 

c)  d) 

3. Work out the magnitude of the vector 2**i** − 3**j** + 4**k** .

4. A (3 , −1 , 0) B(5 , 2 , 5) and C (9 , 8 , 15) are three points in space.

Prove that A, B and C are collinear and state the ratio in which B divides AC.

5. G is the point (3 , −2 , 6) and T is the point (−2 , 3 , −4). B divides GT in the ratio 4:1

Find the co-ordinates of B.

6.  = 4**i** + 3**j** + **k** and = 3**i** − 5**j** + 9**k**.

1. Calculate **.** 
2. Find angle PQR.

7. If **m** = *k*  where *k* > 0 and **m** is a unit vector, which of these is the value of **m:**

A  B  C  D 

8. The vectors *x***i** + 3**j** − **k** and 4**i** − 2**j** + 2**k** are perpendicular. What is the value of *x*?

9. P (1 , 3 , −2) , Q (5 , 4 , 1) and R (−2 , 6 , 3) are three vertices of a parallelogram PQRS.

Find the co-ordinates of the fourth point S.

**Ex 14 Polynomials**

1. Show that (*x* + 3) is a factor of 2*x*3 + 3*x*2 − 11*x* − 6 and find the other factors.

2. Use synthetic division to find the quotient and the remainder when 2*x*4 + 5*x*3 + *x* + 7 is divided by *x* + 2.

**Only use your calculator if you need to!**

3. Factorise 2*x*4 − 5*x*2 − 12

4. Solve the equation 3*x*3 − 7*x*2 − 7*x* + 3 = 0

5. (*x* − 4) is a factor of *x*3 + *kx*2 − *x* + 4. Find the value of *k*.

6. a) Given that *x* + 2 is a factor of 2*x*3 + *x*2 + *kx* + 2 , find the value of k.

b) Hence solve the equation 2*x*3 + *x*2 + *kx* + 2 = 0 when *k* takes this value.

7. (*x* − 2) is a factor of *x*3 + *x*2 + *ax* + *b* , and when *x*3 + *x*2 + *ax* + *b* is divided by (*x* + 2) the remainder is 12. Find the values of *a* and *b*.

8. f(x) = x3 − x2 − 8x + 12

1. Find where f(x) cuts the x and y axes.
2. Find the stationary points of the function and determine their nature.
3. Make a neat sketch of the graph, showing all relevant points.

9. a) Show that the equation x3 − x2 + x + 1 = 0 has a root between −1 and 0.

b) Use iteration to find this root correct to 1 decimal place.

x

y

0

(b,c)

*Revision*

10. The graph shows a sketch of y = f(x), a quartic function.

(0,a)

The graph has a point of inflection at (0,a) and a

Maximum turning point at (b,c).

1. Sketch the graph of y = f’(x).
2. Sketch the graph of y = f(x) + 2

11. The point G divides the line joining F(−1, −1, 0) to H(5, 2, −3) in the ratio 2 : 1.

Find the coordinates of G.

12. Find the equation of the line through the point (2,−1) and perpendicular to the line with equation 2x − y + 6 = 0.

**Ex 15 Trig Equations**

**Only use your calculator if you need to!**

1. Solve the equations-

a) √2*cosx°* + 1 = 0 (0 ≤ *x* ≤ 360) b) 3*tan2z* + 3 = 0 (0 ≤ *z* ≤ π)

c) 4*sin*(2*x* − 10)° = 2 (0 ≤ *x* ≤ 360) d) *sin2x* + *sinx* = 0 0 ≤ *x* ≤ 2π

e) *sin2x°* − *cosx°* = 0 0 ≤ *x* ≤ 180 f) 2*cos2x* + 3*sinx* = 1 0 ≤ *x* ≤ 2π Round this answer to 2 decimal places

y

0

6

1 2

*Revision*

1. A parabola intersects the axes at *x* = 1, *x* = 2 and

*y* = 6, as shown in the diagram.

What is the equation of the parabola?

A *y* = 6(*x* – 1)(*x* – 2)

B *y* = 6(*x* + 1)(*x* + 2)

C *y* = 3(*x* – 1)(*x* – 2)

x

D *y* = 3(*x* + 1)(*x* + 2)

2. A curve has equation *y* = 5*x*3 – 12*x*.

What is the equation of the tangent at the point (1 , –7)?

3. Solve 6 – *x* – *x*2 > 0

4. Find the equation of the tangent to the curve *y* = 4√*x* at the point where *x* = 1.

5. Sketch the graph of *y* = 3cos2*x*°, , indicating all important points.

6. Find 

(4,8)

(-6,8)

y

7. The graph of *y* = *f*(*x*) is shown. Make a sketch of:

1. *y* = –*f*(*x*)
2. *y* = *f*(2*x*)

0

**Ex 16 The Wave Function**

1. Express each of the following in the given form.

a) for 0 ≤ *x* ≤ 360 - 3*cosx*° + 4*sinx*° , *kcos*(*x* ─ *α*)°

b) for 0 ≤ *x* ≤ 2π - 3*sinx* − 3*cosx* , *ksin*(*x* + *α*)

2. a) Write *sinx − cosx* in the form *ksin(x − α)* stating the values of *k* and *α*

where *k* > 0 and 0 ≤ *α* ≤ 2π.

b) Sketch the graph of y = *sinx − cosx* for 0 ≤ *x* ≤ 2π, showing clearly the graphs maximum and minimum values and where it cuts the x axis and y axis.

3. a) What is the minimum value of *g(x)* = 2 + 3*cosx°* ─ 4*sinx°* ?

b) For what value of *x*, between 0 and 360, does this minimum occur?

4. Express 3*cos*2*θ* ─ 4*sin*2*θ* in the form *kcos*(2*θ* ─ *α*) 0 ≤ *θ* ≤ 2π .

[Round α to 3 decimal places].

5. Solve these equations:

**Only use your calculator if you need to!**

a) 8*sinx* ─ 6*cosx* = 10 0 ≤ *x* ≤ 360

b) 3*cosx* + *sinx* = 2 0 ≤ *x* ≤ 2π

*Revision*

6. Evaluate log52 + log550 ─ log54

7. For what value of *k* are the vectors **a** =  **b** =  perpendicular?

8. Find the range of values of *x* for which 2*x2* + 5*x* ─ 7 ≥ 0 . (*x* ∈ **R**)

9. Find the values of *p* and *q* if *(x ─ 2)* and *(x + 4)* are both factors of *x4* + *px3* ─ *x2* + *qx* ─ 8.

10. The functions f and g are defined on suitable domains by *f(x)* = *sinx* and *g(x)* = 1 − *x2*.

Find and simplify the formulae for – a) *g(f(x))* b) *g(g(x))*

11. Solve the equation 3*cos2x*° + 2*cosx*° = 1 for 0 ≤ *x* ≤ 360.

**Ex 17 Further Calculus**

1. Differentiate –

a) f(*x*) = 5sin*x* ─ 2cos*x* b) f(*x*) = cos3*x*

c) f(*x*) = cos5*x* + 2sin3*x* d) f(*x*) = sin(3*x* + 2)

2. Integrate –

a)  b)  c) 

3. 

4. Differentiate –

a) f(*x*) = (2*x* + 1)4 b) f(*x*) = (8 ─ *x*3)½ c) f(*x*) = 

5. Integrate –

a)  b)  c) 

6. 

7. Show that the function *f(x)* = *sinx* + 2*x* is never decreasing.

8. The graph of *y* = *f(x)* passes through the point (π/9 , 1).

If *f’(x)* = sin*(3x)* express *y* in terms of *x*.

**Only use your calculator if you need to!**

9. Differentiate  with respect to *x*.

*Revision*

**NO CALCULATOR!**

10. The point D divides the line joining A (─1 , ─1 , 0) and B (5 , 2 , ─3) in the ratio 1:2.

Find the co-ordinates of D.

11. Express 8*cosx*° ─ 6*sinx*° in the form *kcos*(*x* + *α*)° where *k* > 0 and 0 < *α* < 360.

12. Given that and  , where *P* and *Q* are acute angles,

find the **exact** value of sin(2*P* + *Q*).