2012 Mathematics

Higher

Finalised Marking Instructions

General Comments

These marking instructions are for use with the 2012 Higher Mathematics Examination.

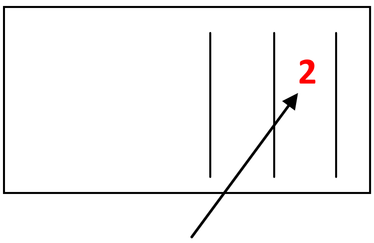
For each question the marking instructions are in two sections, namely **Illustrative Scheme** and **Generic Scheme**. The **Illustrative Scheme** covers methods which are commonly seen throughout the marking. The **Generic Scheme** indicates the rationale for which each mark is awarded. In general markers should use the **Illustrative Scheme** and only use the **Generic Scheme** where a candidate has used a method not covered in the **Illustrative Scheme**.

All markers should apply the following general marking principles throughout their marking:

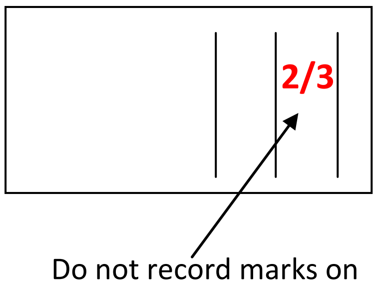
1 Marks must be assigned in accordance with these marking instructions. In principle, marks are awarded for what is correct, rather than deducted for what is wrong.

2 Award one mark for each •. There are no half marks.

3 The mark awarded for **each part** of a question should be entered in the **outer** right hand margin, opposite the end of the working concerned. The marks should correspond to those on the question paper and these marking instructions. Only the mark, **as a whole number**, should be written.



Marks in this column - whole numbers only



Do not record marks on scripts in this manner.

4 Where a candidate has not been awarded any marks for a question, or part of a question, 0 should be written in the right hand margin against their answer. It should not be left blank.

5 Every page of a candidate's script should be checked for working. Unless blank, every page which is devoid of a marking symbol should have a tick placed in the bottom right hand margin.

6 Where the solution to part of a question is fragmented and continues later in the script, the marks should be recorded at the end of the solution. This should be indicated with a down arrow (↓), in the margin, at the earlier stages.

7 Working subsequent to an error must be **followed through**, with possible full marks for the subsequent working, provided that the level of difficulty involved is approximately similar. Where, subsequent to an error, the working for a follow through mark has been eased, the follow through mark cannot be awarded.

8 As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Throughout this paper, unless specifically mentioned in the marking instructions, a correct answer with no working receives no credit.

9 Marking Symbols

No comments or words should be written on scripts. Please use the following and the symbols indicated on the welcome letter and from comment 6 on the previous page.

✓ A tick should be used where a piece of working is correct and gains a mark. Markers must check through the whole of a response, ticking the work only where a mark is awarded.

___ X At the point where an error occurs, the error should be underlined and a cross used to indicate where a mark has not been awarded. If no mark is lost the error should only be underlined, i.e. a cross is only used where a mark is not awarded.

A cross-tick should be used to indicate “correct” working where a mark is awarded as a result of **follow through** from an error.

A double cross-tick should be used to indicate correct working which is irrelevant or insufficient to score any marks. This should also be used for working which has been **eased**.

~ A tilde should be used to indicate a minor error which is not being penalised, e.g. **bad form**.

This should be used where a candidate is given the **benefit of the doubt**.

^ A roof should be used to show that something is missing, such as part of a solution or a crucial step in the working.

These will help markers to maintain consistency in their marking and will assist the examiners in the later stages of SQA procedures.

The examples below illustrate the use of the marking symbols .

Example 1

$y = x^3 - 6x^2$

$\frac{dy}{dx} = 3x^2 - \underline{12}$ X $\qquad \bullet^1$ ✓ $\quad \bullet^2$ X

$3x^2 - 12 = 0$ $\qquad \bullet^3$

$x = 2$ ^ $\qquad \bullet^4$ ^

$y = -16$ $\qquad \bullet^5$

Example 2

$A(4,4,0),\ B(2,2,6),\ C(2,2,0)$

$\overrightarrow{AB} = \underline{\mathbf{b} + \mathbf{a}} = \begin{pmatrix} 6 \\ 6 \\ 6 \end{pmatrix}$ X $\bullet^1$

$\overrightarrow{AC} = \begin{pmatrix} 6 \\ 6 \\ 0 \end{pmatrix}$ $\bullet^2$ (repeated error)

Example 3

$3\sin x - 5\cos x$

$k\sin x\cos a - \cos x\sin a$ ✓ $\bullet^1$

$k\cos a = 3,\ k\sin a = 5$ ✓ $\bullet^2$

Example 4

$$\begin{array}{c|cccc} 4 & 1 & -5 & 2 & 8 \\ & & 4 & -4 & -8 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

✓ $\bullet^1$ ✓ $\bullet^2$

Since the remainder is 0, $x-4$ must be a factor. ✓ $\bullet^3$

$(x^2 - x - 2)$ ✓ $\bullet^4$

$(x-4)(x+1)(x-2)$ ✓ $\bullet^5$

$x = 4$ or $x = -1$ or $x = 2$ ✓ $\bullet^6$

10 In general, as a consequence of an error perceived to be trivial, casual or insignificant, e.g. $6 \times 6 = 12$, candidates lose the opportunity of gaining a mark. But note example 4 in comment 9 and comment 11.

11 Where a transcription error (paper to script or within script) occurs, the candidate should be penalised, e.g.

This is a transcription error and so the mark is not awarded.

Eased as no longer a solution of a quadratic equation.

$$x^2 + 5x + 7 = 9x + 4$$
$$\underline{x} - 4x + 3 = 0$$
$$x = 1$$

Exceptionally this error is not treated as a transcription error as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt.

$$x^2 + 5x + 7 = 9x + 4$$
$$x - 4x + 3 = 0$$
$$(x-3)(x-1) = 0$$
$$x = 1 \text{ or } 3$$

12 Cross marking

Where a question results in two pairs of solutions, this technique should be applied, but only if indicated in the detailed marking instructions for the question.

Example: Point of intersection of line with curve

Illustrative Scheme:		Cross marked:	
$\bullet^5$	$x = 2,\ x = -4$	$\bullet^5$	$x = 2,\ y = 5$
$\bullet^6$	$y = 5,\ y = -7$	$\bullet^6$	$x = -4,\ y = -7$

Markers should choose whichever method benefits the candidate, but **not** a combination of both.

13 In final answers, numerical values should be simplified as far as possible.

Examples: $\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$ $\qquad$ $\frac{43}{1}$ must be simplified to 43

$\frac{15}{0{\cdot}3}$ must be simplified to 50 $\qquad$ $\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8

The square root of perfect squares up to and including 100 must be known.

14 Regularly occurring responses (ROR) are shown in the marking instructions to help mark common and/or non-routine solutions. RORs may also be used as a guide in marking similar non-routine candidate responses.

15 Unless specifically mentioned in the marking instructions, the following should not be penalised:

- Working subsequent to a **correct** answer;
- Correct working in the wrong part of a question;
- Legitimate variations in numerical answers, e.g. angles in degrees rounded to nearest degree;
- Omission of units;
- Bad form;
- Repeated error within a question, but not between questions or papers.

16 In any 'Show that . . .' question, where the candidate has to arrive at a formula, the last mark of that part is not available as a follow through from a previous error.

17 All working should be carefully checked, even where a fundamental misunderstanding is apparent early in the candidate's response. Marks may still be available later in the question so reference must be made continually to the marking instructions. All working must be checked: the appearance of the correct answer does not necessarily indicate that the candidate has gained all the available marks.

18 In the **exceptional** circumstance where you are in doubt whether a mark should or should not be awarded, consult your Team Leader (TL).

19 Scored out working which **has not been replaced** should be marked where still legible. However, if the scored out working **has been replaced**, only the work which has not been scored out should be marked.

20 A valid approach, within Mathematical problem solving, is to try different strategies. Where this occurs, all working should be marked. The mark awarded to the candidate is from the **highest** scoring strategy. This is distinctly different from the candidate who gives two or more solutions to a question/part of a question, deliberately leaving all solutions, hoping to gain some benefit. All such contradictory responses should be marked and the **lowest** mark given.

21 It is of great importance that the utmost care should be exercised in totalling the marks.

The recommended procedure is as follows:

Step 1 Manually calculate the total from the candidate's script.

Step 2 Check this total using the grid issued with these marking instructions.

Step 3 In EMC, enter the marks and obtain a total, which should now be compared to the manual total.

This procedure enables markers to identify and rectify any errors in data entry before submitting each candidate's marks.

22 **The candidate's script for Paper 2 should be placed inside the script for Paper 1, and the candidate's total score (i.e. Paper 1 Section B + Paper 2) written in the space provided on the front cover of the script for Paper 1.**

23 In cases of difficulty, covered neither in detail nor in principle in these instructions, markers should contact their TL in the first instance. A referral to the Principal Assessor (PA) should only be made in consultation with the TL. Further details of PA Referrals can be found in The General Marking Instructions.

Paper 1 Section A

Question	Answer
1	C
2	D
3	B
4	B
5	A
6	C
7	A
8	C
9	A
10	B
11	D
12	B
13	D
14	A
15	D
16	C
17	D
18	B
19	B
20	A

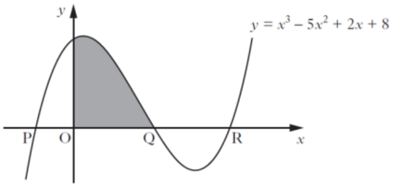
Summary		
	A	5
	B	6
	C	4
	D	5

21	(a)	(i)	Show that $(x-4)$ is a factor of x^3-5x^2+2x+8.	
		(ii)	Factorise x^3-5x^2+2x+8 fully.	
		(iii)	Solve $x^3-5x^2+2x+8=0$.	6

Generic Scheme			Illustrative Scheme	
21 (a)				
			Method 1 : Using synthetic division	
$\bullet^1$	ss	know to use $x=4$	$\bullet^1$ $\begin{array}{c\|cccc} 4 & 1 & -5 & 2 & 8 \\ \hline & & & & \end{array}$	
$\bullet^2$	pd	complete evaluation	$\bullet^2$ $\begin{array}{c\|cccc} 4 & 1 & -5 & 2 & 8 \\ & & 4 & -4 & -8 \\ \hline & 1 & -1 & -2 & 0 \end{array}$	
$\bullet^3$	ic	state conclusion	$\bullet^3$ 'remainder is zero so $(x-4)$ is a factor'	
$\bullet^4$	ic	find quadratic factor	$\bullet^4$ x^2-x-2	**stated, or implied by $\bullet^5$**
$\bullet^5$	pd	factorise completely	$\bullet^5$ $(x-4)(x-2)(x+1)$	**stated explicitly in any order**
$\bullet^6$	ic	state solutions	$\bullet^6$ $-1,\ 2,\ 4$	
			Method 2 : Using substitution and inspection	
			$\bullet^1$ know to use $x=4$	
			$\bullet^2$ $64-80+8+8=0$	
			$\bullet^3$ $(x-4)$ is a factor	
			$\bullet^4$ $(x-4)(x^2-x-2)$	**stated, or implied by $\bullet^5$**
			$\bullet^5$ $(x-4)(x-2)(x+1)$	**stated explicitly in any order**
			$\bullet^6$ $-1,\ 2,\ 4$	6

Notes

1. $\bullet^3$ is only available as a consequence of the evidence for $\bullet^1$ and $\bullet^2$.
2. Communication at $\bullet^3$ must be consistent with working at $\bullet^2$.
   i.e. candidate's working must arrive legitimately at zero before $\bullet^3$ is awarded.
   If the remainder is not 0 then an appropriate statement would be '$(x-4)$ is not a factor'.
3. Accept any of the following for $\bullet^3$:
   - '$f(4)=0$ so $(x-4)$ is a factor'
   - 'since remainder is 0, it is a factor'
   - the 0 from table linked to word 'factor' by e.g. 'so', 'hence', '$\therefore$', '$\rightarrow$', '$\Rightarrow$'.
4. Do not accept any of the following for $\bullet^3$:
   - double underlining the zero or boxing in the zero, without a comment
   - '$x=4$ is a factor', '$(x+4)$ is a factor', '$x=4$ is a root', '$(x-4)$ is a root'
   - the word 'factor' **only**, with no link.
5. To gain $\bullet^6$, $4, -1, 2$ **must** appear together in (a).
6. $(x-4)(x-2)(x+1)$ leading to $(4,\ 0)$, $(2,\ 0)$ and $(-1,\ 0)$ **only** does not gain $\bullet^6$.
7. $(x-2)(x+1)$ only, leading to $x=2,\ x=-1$ does not gain $\bullet^6$ as equation solved is not a cubic.
8. Candidates who attempt to solve the cubic equation subsequent to $x=-1, 2, 4$ and obtain different solutions, or no solutions, cannot gain $\bullet^6$.

21 (b)	The diagram shows the curve with equation $y = x^3 - 5x^2 + 2x + 8$. The curve crosses the x-axis at P, Q and R. Determine the shaded area. 	6

Generic Scheme | **Illustrative Scheme**

21 (b)

•7	ic	identify x_Q from working in (a)	•7	2
•8	ic	interpret appropriate limits	•8	0, 2
•9	ss	know and start to integrate	•9	integrate one term correctly (but see Note 10)
•10	pd	complete integration	•10	$\frac{1}{4}x^4 - \frac{5}{3}x^3 + \frac{2}{2}x^2 + 8x$ or equivalent
•11	ic	substitute limits	•11	$\left(\frac{1}{4}(2)^4 - \frac{5}{3}(2)^3 + 2^2 + 8\times 2\right) - 0$
•12	pd	state area	•12	$\frac{32}{3}$ or $10\frac{2}{3}$ but not a decimal approximation

6

Notes

9. Evidence for •7 and •8 may not appear until •11 stage.
10. Where a candidate differentiates one or more terms at •9, then •9, •10, •11 and •12 are not available.
11. Candidates who substitute at •11, without integrating at •9, do not gain •9, •10, •11 and •12.
12. For candidates who make an error in (a), •8 is only available if 0 is the lower limit and a positive integer value is used for the upper limit.
13. •11 is only available where both limits are numerical values.
14. Candidates must show evidence that they have considered the lower limit 0 in their substitution at •11 stage.

Regularly occurring responses

Response 1

Candidates who use Q throughout

Candidate A

$$\int_0^Q (x^3 - 5x^2 + 2x + 8)\,dx$$ •7 ✗

•8 ✗

$$= \left[\frac{1}{4}x^4 - \frac{5}{3}x^3 + \frac{2}{2}x^2 + 8x\right]_0^Q$$ •9 ✓

•10 ✓

$$= \frac{1}{4}Q^4 - \frac{5}{3}Q^3 + Q^2 + 8Q - 0$$ •11 ✗

•12 ✗

However, if Q is replaced by 2 at this stage, and working continues, all 6 marks may still be available .

Response 2

Dealing with negatives

Candidate B

Q(−1, 0) ✗ •7

$$\int_0^{-1} (x^3 - 5x^2 + 2x + 8)\,dx$$ ✗ •8

$$= \left[\frac{1}{4}x^4 - \frac{5}{3}x^3 + x^2 + 8x\right]_0^{-1}$$ ✓ •9 ✓ •10

$$= \frac{1}{4}(-1)^4 - \frac{5}{3}(-1)^3 + (-1)^2 + 8(-1) - 0$$ ✗ •11

$$= -\frac{61}{12}$$

cannot be negative so $\frac{61}{12}$ ✗ •12

but

$$= -\frac{61}{12}$$

$A = \frac{61}{12}$ ✗ •12

22 (a) The expression $\cos x - \sqrt{3}\sin x$ can be written in the form $k\cos(x+a)$ where $k>0$ and $0 \le a < 2\pi$. Calculate the values of k and a.	4

Generic Scheme | **Illustrative Scheme**

22 (a)

		Generic Scheme		Illustrative Scheme	
•¹	ss	use compound angle formula	•¹	$k\cos x\cos a - k\sin x\sin a$	**stated explicitly**
•²	ic	compare coefficients	•²	$k\cos a = 1$ and $k\sin a = \sqrt{3}$	**stated explicitly**
•³	pd	process k	•³	2 (do not accept $\sqrt{4}$)	
•⁴	pd	process a	•⁴	$\frac{\pi}{3}$ **but** must be consistent with •²	4

Notes

1. Treat $k\cos x\cos a - \sin x\sin a$ as bad form only if the equations at the •² stage both contain k.
2. $2\cos x\cos a - 2\sin x\sin a$ or $2(\cos x\cos a - \sin x\sin a)$ is acceptable for •¹ and •³.
3. Accept $k\cos a = 1$ and $-k\sin a = -\sqrt{3}$ for •².
4. •² is not available for $k\cos x = 1$ and $k\sin x = \sqrt{3}$, however, •⁴ is still available.
5. •⁴ is only available for a single value of a.
6. Candidates who work in degrees and do not convert to radian measure in (a) do not gain •⁴.
7. Candidates may use any form of the wave equation for •¹, •² and •³, however, •⁴ is only available if the value of a is interpreted for the form $k\cos(x+a)$.

Regularly occurring responses

Response 1 : Missing information in working

Candidate A

$2\cos a = 1$

$-2\sin a = -\sqrt{3}$ ✓

$\tan a = \frac{\sqrt{3}}{1}$

$a = \frac{\pi}{3}$ ✓

•¹ ✗ •² ✓ •³ ✓ •⁴ ✓

3 marks out of 4

Candidate B

$\cos a = 1$

$\sin a = \sqrt{3}$

$\tan a = \frac{\sqrt{3}}{1}$

$a = \frac{\pi}{3}$

•¹ ✗ •² ✗ •³ ✗ •⁴ ✗

0 marks out of 4

Not consistent with evidence at •².

Response 2 : Correct expansion of $k\cos(x+a)$ and possible errors for •² and •⁴

Candidate C

$k\cos a = 1$

$k\sin a = \sqrt{3}$ ✓ •²

$\tan a = \frac{1}{\sqrt{3}}$ so $a = \frac{\pi}{6}$ ✗ •⁴

Candidate D

$k\cos a = \sqrt{3}$ ✗ •²

$k\sin a = 1$

$\tan a = \frac{1}{\sqrt{3}}$ so $a = \frac{\pi}{6}$ ✗✓ •⁴

Candidate E

$k\cos a = 1$

$k\sin a = -\sqrt{3}$ ✗ •²

$\tan a = -\sqrt{3}$ so $a = \frac{5\pi}{3}$ ✗✓ •⁴

Response 3 : Labelling incorrect using $\cos(A+B) = \cos A\cos B - \sin A\sin B$ from formula list

Candidate F

$k\cos A\cos B - k\sin A\sin B$ ✗ •¹

$k\cos a = 1$

$k\sin a = \sqrt{3}$ ✓ •²

$\tan a = \sqrt{3}$ so $a = \frac{\pi}{3}$ ✓ •⁴

Candidate G

$k\cos A\cos B - k\sin A\sin B$ ✗ •¹

$k\cos x = 1$ ✗ •²

$k\sin x = \sqrt{3}$

$\tan x = \sqrt{3}$ so $x = \frac{\pi}{3}$ ✗✓ •⁴

Candidate H

$k\cos A\cos B - k\sin A\sin B$ ✗ •¹

$k\cos B = 1$

$k\sin B = \sqrt{3}$ ✗✓ •²

$\tan B = \sqrt{3}$ so $B = \frac{\pi}{3}$ ✗✓ •⁴

22 (b)	Find the points of intersection of the graph of $y=\cos x-\sqrt{3}\sin x$ with the x and y axes, in the interval $0\le x\le 2\pi$.	3

Generic Scheme | **Illustrative Scheme**

22 (b)

$\bullet^5$	ic	interpret y-intercept	$\bullet^5$ 1
$\bullet^6$	ss	strategy for finding roots	$\bullet^6$ e.g. $2\cos\left(x+\frac{\pi}{3}\right)=0$ **or** $\sqrt{3}\sin x=\cos x$
$\bullet^7$	ic	state both roots	$\bullet^7$ $\frac{\pi}{6},\ \frac{7\pi}{6}$

3

Notes

8. Candidates should only be penalised once for leaving their answer in degrees in (a) and (b).
9. If the expression used in (b) is not consistent with (a) then only $\bullet^5$ and $\bullet^7$ are available.
10. Correct roots without working cannot gain $\bullet^6$ but will gain $\bullet^7$.
11. Candidates should only be penalised once for not simplifying $\sqrt{4}$ in (a) and (b).

Regularly occurring responses

Response 4 : Communication for $\bullet^5$

Candidate I

(1, 0) without working. ✗ $\bullet^5$

Candidate J

$\cos 0-\sqrt{3}\sin 0=1$ ✓ $\bullet^5$

so (1, 0).

Response 5 : Follow through from a wrong value of a

Candidate K

From (a) $a=\frac{\pi}{6}$ $\bullet^6$ ✗

then in (b) $x=\frac{\pi}{3},\ \frac{4\pi}{3}$ only $\bullet^7$ ✓

Candidate L

From (a) $a=60^\circ$ ✗ $\bullet^4$ $\bullet^6$ ✗

then in (b) $x=30^\circ,\ 210^\circ$ only $\bullet^7$ ✓

Note 10

Response 6 : Root or graphical approach

Candidate M

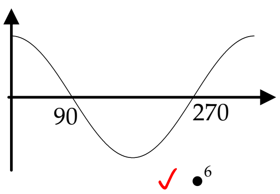
$\frac{\pi}{2}-\frac{\pi}{3}$ and $\frac{3\pi}{2}-\frac{\pi}{3}$ ✓ $\bullet^6$

$=\frac{\pi}{6}$ and $\frac{7\pi}{6}$ ✓ $\bullet^7$

Candidate N

(a) 60° ✗ $\bullet^4$

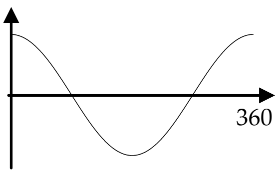
(b)



✓ $\bullet^6$

When $x=30^\circ,\ 210^\circ$ ✗ $\bullet^7$

Candidate O



moved 60° to left ✓ $\bullet^6$

cuts x-axis at $\frac{\pi}{6},\ \frac{2\pi}{3}$ ✗ $\bullet^7$

Response 7 : Circular argument not leading anywhere

Candidate P

$2\cos x\times\frac{1}{2}-2\sin x\times\frac{\sqrt{3}}{2}=0$ $\bullet^6$ ✗

$\cos x-\sqrt{3}\sin x=0$ $\bullet^7$ ✗

Response 8 : Transcription error in (b)

Candidate Q

(a) correct

(b) $2\cos\left(x-\frac{\pi}{3}\right)=0$ so $x=\frac{5\pi}{6},\ \frac{11\pi}{6}$ ✗ $\bullet^6$ ✓ $\bullet^7$

$y=2\cos\left(0-\frac{\pi}{3}\right)=2\cos\left(-\frac{\pi}{3}\right)=1$ ✓ $\bullet^5$

$x-\frac{\pi}{3}$ is penalised as $x+\frac{\pi}{3}$ obtained in (a).
However $\bullet^5$ and $\bullet^7$ are still available as follow through. See Note 9.

23 (a) Find the equation of ℓ_1, the perpendicular bisector of the line joining $P(3,-3)$ to $Q(-1,9)$.	4

Generic Scheme | **Illustrative Scheme**

23 (a)

•1	ss	find midpoint of PQ	•1	$(1, 3)$
•2	ss	find gradient of PQ	•2	-3
•3	ic	interpret perpendicular gradient	•3	$\frac{1}{3}$
•4	ic	state equation of perp. bisector	•4	$y-3=\frac{1}{3}(x-1)$

4

Notes

1. •4 is only available if a midpoint **and** a perpendicular gradient are used.
2. Candidates who use $y=mx+c$ must obtain a numerical value for c before •4 is available.

Regularly occurring responses

Response 1 : Candidates who use wrong midpoint or no midpoint

Candidate A

midpoint $M(2,-6)$ ✗	✗	•1
$m_{MQ}=-5$ ✓✗	✓✗	•2
$m_{\perp}=\frac{1}{5}$ ✓✗	✓✗	•3
$y-(-6)=\frac{1}{5}(x-2)$ ✓✗	✓✗	•4

Candidate B

$m_{PQ}=-3$ ✓	✗	•1
$m_{\perp}=\frac{1}{3}$ ✓	✓	•2
using R, $y-(-2)=\frac{1}{3}(x-1)$ ✗	✓	•3
	✗	•4

23 (b) Find the equation of ℓ_2 which is parallel to PQ and passes through $R(1,-2)$.	2

Generic Scheme | **Illustrative Scheme**

23 (b)

•5	ic	use parallel gradients	•5	-3	**stated, or implied by** •6
•6	ic	state equation of line	•6	$y-(-2)=-3(x-1)$	

2

Notes

3. •6 is only available to candidates who use R and their gradient of PQ from (a).

Regularly occurring responses

Response 2 : Not using parallel gradient for equation

Candidate C

$y-(-2)=\frac{1}{3}(x-1)$ ✗

•5 ✗
•6 ✗

Candidate D

Parallel so same gradients

so $m=\frac{1}{3}$ ✗

$y-(-2)=\frac{1}{3}(x-1)$

•5 ✗
•6 ✓✗

Candidate E

$m=-3$ ✓

$y-(-2)=\frac{1}{3}(x-1)$ ✗

•5 ✓
•6 ✗

If $m_{PQ}=-3$ only do not award •5

23 (c)	Find the point of intersection of ℓ_1 and ℓ_2.	3

Generic Scheme			Illustrative Scheme
23 (c)			
$\bullet^7$	ss	use valid approach	$\bullet^7$ e.g. $x-3y=-8$ **and** $9x+3y=3$ **or** $-3x+1=\frac{1}{3}x+\frac{8}{3}$ **or** $3(3y-8)+y=1$
$\bullet^8$	pd	solve for one variable	$\bullet^8$ e.g. $x=-\frac{1}{2}$
$\bullet^9$	pd	solve for other variable	$\bullet^9$ e.g. $y=\frac{5}{2}$
			3

Notes

4. **Neither** $x-3y=-8$ and $3x+y=1$ **nor** $y=-3x+1$ and $3y=x+8$ are sufficient to gain $\bullet^7$.

5. $\bullet^7$, $\bullet^8$ and $\bullet^9$ are not available to candidates who:
   - Equate zeros
   - Give answers only, without working
   - Use R for equations in both (a) and (b)
   - Use the same gradient for the lines in (a) and (b).

23 (d)	Hence find the shortest distance between PQ and ℓ_2.	2

Generic Scheme			Illustrative Scheme
23 (d)			
$\bullet^{10}$	ss	identify appropriate points	$\bullet^{10}$ (1, 3) and $\left(-\frac{1}{2},\frac{5}{2}\right)$
$\bullet^{11}$	pd	calculate distance	$\bullet^{11}$ $\sqrt{\frac{5}{2}}$ accept $\frac{\sqrt{10}}{2}$ or $\sqrt{2\cdot5}$
			2

Notes

6. $\bullet^{10}$ and $\bullet^{11}$ are only available for considering the distance between the midpoint of PQ and the candidate's answer from (c) **or** for considering the perpendicular distance from P or Q to ℓ_2.

7. At least one coordinate at $\bullet^{10}$ stage must be a fraction for $\bullet^{11}$ to be available.

8. There should only be one calculation of a distance to gain $\bullet^{11}$.

Regularly occurring responses

Response 3 : Following through from correct (a), (b) and (c)

Candidate F

$(1,3),\ (1,-2)$ ✗ $\bullet^{10}$

$d=5$ ✗ $\bullet^{11}$

Response 4 : Following through from correct (a), (b) and (c)

Candidate G

$(1,3),\ \left(-\frac{1}{2},\frac{5}{2}\right)$ ✓ $\bullet^{10}$

$PR=\sqrt{5},\ QR=\sqrt{125},\ d=\sqrt{2\cdot5}$

so $\sqrt{2\cdot5}$ is shortest distance. ✗ $\bullet^{11}$

If reference was made to this being the perpendicular distance then $\bullet^{11}$ would be available.

1	Functions f and g are defined on the set of real numbers by • $f(x)=x^2+3$ • $g(x)=x+4$ (a) Find expressions for: (i) $f(g(x))$; (ii) $g(f(x))$.	3

Generic Scheme | **Illustrative Scheme**

1 (a)

	Generic Scheme		Illustrative Scheme
$\bullet^1$	ic start composite process	$\bullet^1$	e.g. $f(x+4)$ **stated, or implied by** $\bullet^2$
$\bullet^2$	ic correct substitution into expression	$\bullet^2$	$(x+4)^2+3$
$\bullet^3$	ic complete second composite	$\bullet^3$	x^2+3+4

3

Notes

1. Candidates must clearly identify which of their answers are $f(g(x))$ and $g(f(x))$; the minimum evidence for this could be as little as using (i) and (ii) as labels.
2. Candidates who interpret the composite functions as either $f(x)\times g(x)$ or $f(x)+g(x)$, do not gain any marks.

Regularly occurring responses

Response 1 : The first two marks are for **either** $f(g(x))$ **or** $g(f(x))$ correct. The third mark is for the other composite function.

Candidate A

$f(g(x))$
$=(x+4)^2+3$ ✓ $\bullet^1$ ✓ $\bullet^2$

$g(f(x))$
$=x^2+12$ ✗ $\bullet^3$

2 marks out of 3

Candidate B

$f(g(x))$
$=(x+7)^2$ ✗ $\bullet^3$

$g(f(x))$
$=x^2+7$ ✓ $\bullet^1$ ✓ $\bullet^2$

2 marks out of 3

Response 2 : Interpreting $f(g(x))$ as $g(f(x))$ and vice versa. A maximum of 2 marks are available.

Candidate C

$f(g(x))$
$=x^2+7$ ✗ $\bullet^1$ ✓ $\bullet^2$

$g(f(x))$
$=(x+4)^2+3$ ✓ $\bullet^3$

2 marks out of 3

Candidate D

$f(g(x))$
$=x^2+7$ ✗ $\bullet^1$ ✓ $\bullet^2$

1 mark out of 3

Response 3 : Identifying $f(g(x))$ and $g(f(x))$

Candidate E

$(x+4)^2+3$ ✗ $\bullet^1$ ✓ $\bullet^2$
x^2+7 ✓ $\bullet^3$

2 marks out of 3

Candidate F

x^2+7 ✗ $\bullet^1$ ✗ $\bullet^2$
$(x+4)^2+3$ ✓ $\bullet^3$

1 mark out of 3

Candidate G

x^2+7 ONLY
or $(x+4)^2+3$ ONLY

0 marks out of 3

Candidate H

(i) $(x+4)^2+3$ ✓ $\bullet^1$ ✓ $\bullet^2$
(ii) x^2+7 ✓ $\bullet^3$

3 marks out of 3

1	(b)	Show that $f\left(g(x)\right)+g\left(f(x)\right)=0$ has no real roots.	3

Generic Scheme | **Illustrative Scheme**

1 (b)

Generic Scheme			Illustrative Scheme	
Method 1 : Discriminant			Method 1 : Discriminant	
•4	pd	obtain a quadratic expression	•4	$2x^2+8x+26$
•5	ss	know to and use discriminant	•5	$8^2-4\times2\times26$ or $4^2-4\times1\times13$ **stated, or implied by •6**
•6	ic	interpret result	•6	$-144<0$ or $-36<0$ so no real roots
Method 2 : Quadratic Formula			Method 2 : Quadratic Formula	
•4	pd	obtain a quadratic expression	•4	$2x^2+8x+26$
•5	ss	know to and use quadratic formula	•5	$\dfrac{-8\pm\sqrt{8^2-4\times2\times26}}{2\times2}$ **stated, or implied by •6**
•6	ic	interpret result	•6	$\sqrt{-144}$ not possible so no real roots

3

Notes

3. Candidates who use $f(x)\times g(x)$ can gain no marks in (b) as a cubic will be obtained.

4. Candidates who use $f(x)+g(x)$ do not gain •4 (eased) but •5 and •6 are available as follow through marks.

5. In method 1, any other formula masquerading as a discriminant cannot gain •5 and •6.

6. •4, •5 and •6 are only available if $f(g(x))+g(f(x))$ simplifies to a quadratic expression of the form ax^2+bx+c, with b and c both non-zero.

7. •6 is only available for a numerical value, calculated correctly from the candidate's response at •4, and leading to no real roots.

8. Do not accept for •6:
   - 'no roots' in lieu of 'no real roots'
   - 'maths error' or 'ma error'.

9. Candidates who use the word derivative instead of discriminant should not be penalised.

Regularly occurring responses

Response 4 : Candidates who do not simplify the value of their discriminant

Candidate I

$8^2-4\times2\times26$ ✓ •5 ✓

$=64-208<0$ so no real roots •6 ✗

Response 5 : Acceptable communication marks

Method 1

Candidate J

$\sqrt{8^2-4\times2\times26}$ ✓ •5

$=\sqrt{-144}$

not valid

so no real roots ✓ •6

Candidate K

Discriminant $=\sqrt{8^2-4\times2\times26}$ ✓ •5

$=\sqrt{-144}$

can't find root of negative

so no real roots ✓ •6

Candidate L

no real roots if $b^2-4ac<0$

$64-208=-144$ ✓ •6

✓ •5

Method 2

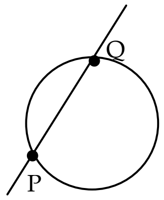
Candidate M

$\dfrac{-(-4)\pm\sqrt{8^2-4\times2\times26}}{2\times2}$ ✓ •5

$=\dfrac{4\pm\sqrt{-144}}{4}$

no $\sqrt{-\text{ve}}$

so no real roots ✓ •6

2	(a)	Relative to a suitable set of coordinate axes, diagram 1 shows the line $2x-y+5=0$ intersecting the circle $x^2+y^2-6x-2y-30=0$ at the points P and Q. Find the coordinates of P and Q.  Diagram 1	6

Generic Scheme	Illustrative Scheme

2 (a)

$\bullet^1$ ss rearrange linear equation	**Substituting for y** $\bullet^1$ $y=2x+5$	**stated, or implied by $\bullet^2$**
$\bullet^2$ ss substitute into circle	$\bullet^2$ $\ldots(2x+5)^2\ldots-2(2x+5)\ldots$	
$\bullet^3$ pd express in standard form	$\bullet^3$ $5x^2+10x-15$	= 0 must appear at the $\bullet^3$
$\bullet^4$ pd start to solve	$\bullet^4$ e.g. $5(x+3)(x-1)$	or $\bullet^4$ stage to gain $\bullet^3$.
$\bullet^5$ ic state roots	$\bullet^5$ $x=-3$ and $x=1$	
$\bullet^6$ pd determine corresponding y-coordinates	$\bullet^6$ $y=-1$ and $y=7$	
	Substituting for x $\bullet^1$ $x=\dfrac{y-5}{2}$	**stated, or implied by $\bullet^2$**
	$\bullet^2$ $\left(\dfrac{y-5}{2}\right)^2\ldots-6\left(\dfrac{y-5}{2}\right)\ldots$	
	$\bullet^3$ $5y^2-30y-35$	= 0 must appear at the $\bullet^3$
	$\bullet^4$ e.g. $5(y+1)(y-7)$	or $\bullet^4$ stage to gain $\bullet^3$.
	$\bullet^5$ $y=-1$ and $y=7$	
	$\bullet^6$ $x=-3$ and $x=1$	6

Notes

1. At $\bullet^4$ the quadratic must lead to two real distinct roots for $\bullet^5$ and $\bullet^6$ to be available.
2. Cross marking is available here for $\bullet^5$ and $\bullet^6$.
3. Candidates do not need to distinguish between points P and Q.

Regularly occurring responses

Response 1 : Solving quadratic equation

Candidate A

✓ $\bullet^1$ ✓ $\bullet^2$

$5x^2+10\underline{x+5}=0$ ✗ $\bullet^3$

$5(x+1)(x+1)$ ✗ $\bullet^4$

$x=-1$ ✗ $\bullet^5$

$y=3$ ✗ $\bullet^6$

Cross marking is **not** available here for $\bullet^5$ and $\bullet^6$, as there are no distinct roots. See Note 1.

Candidate B

$y=2x+5$ ✓ $\bullet^1$

$x^2+(2x+5)^2-6x-2(\underline{7}x+5)-30=0$ ✗ $\bullet^2$

$5x^2-15=0$ ✗ $\bullet^3$

$x^2=3$ ✗ $\bullet^4$

$x=\pm\sqrt{3}$ ✗ $\bullet^5$

$y=8\cdot5,\ 1\cdot5$ ✗ $\bullet^6$

Candidate C

✓ $\bullet^1$ ✓ $\bullet^2$

$5x^2+10x-15=0$ ✓ $\bullet^3$

$5x^2+10x=15$

$5x(x+2)=15$ ✗ $\bullet^4$

$x(x+2)=3$

$x=3 \quad x=1$ ✗ $\bullet^5$

$y=11 \quad y=7$ ✗ $\bullet^6$

2 (b) Diagram 2 shows the circle from (a) and a second congruent circle, which also passes through P and Q.

Determine the equation of this second circle. **6**

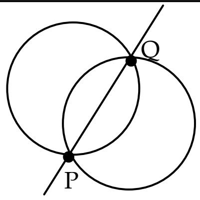


Diagram 2

Generic Scheme	**Illustrative Scheme**
2 (b)	
•7 ic centre of original circle	•7 $(3, 1)$
•8 pd radius of original circle	•8 $\sqrt{40}$ Accept $r^2 = 40$
Method 1 : Using midpoint	Method 1 : Using midpoint
•9 ss midpoint of chord	•9 $(-1, 3)$
•10 ss evidence for finding new centre	•10 e.g. stepping out or midpoint formula
•11 ic centre of new circle	•11 $(-5, 5)$
•12 ic equation of new circle	•12 $(x+5)^2 + (y-5)^2 = 40$
Method 2 : Stepping out using P and Q	Method 2 : Stepping out using P and Q
•9 ss evidence of C_1 to P **or** C_1 to Q	•9 e.g. stepping out or vector approach
•10 ss evidence of Q to C_2 **or** P to C_2	•10 e.g. stepping out or vector approach
•11 ic centre of new circle	•11 $(-5, 5)$
•12 ic equation of new circle	•12 $(x+5)^2 + (y-5)^2 = 40$ **6**

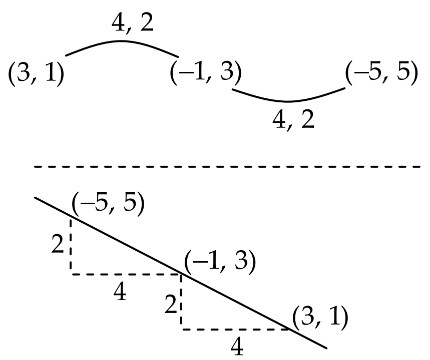
Notes

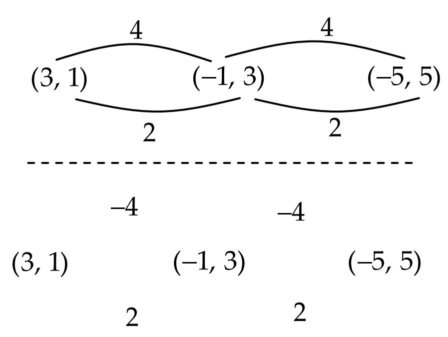
4. The evidence for •7 and •8 may appear in (a).
5. Centre $(-5, 5)$ **without working** in method 1 may still gain •12 but not •10 or •11, in method 2 may still gain •12 but not •9, •10 or •11.

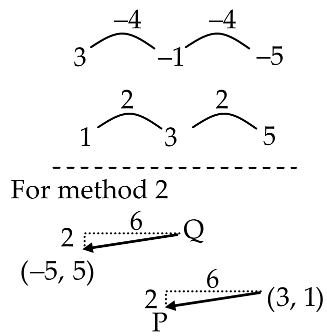
   Any other centre **without working** in method 1 does not gain •10, •11 or •12, in method 2 does not gain •9, •10, •11 or •12.
6. The centre must have been clearly indicated before it is used at the •12 stage.
7. Do not accept e.g. $\sqrt{40}^2$ or $39{\cdot}69$, or any other decimal approximations for •12.
8. The evidence for •8 may not appear until the candidate states the radius or equation of the second circle.

Regularly occurring responses

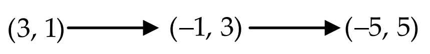
Response 2 : Examples of evidence for stepping out for •10 in method 1 or •9 or •10 in method 2

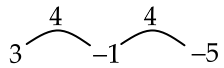


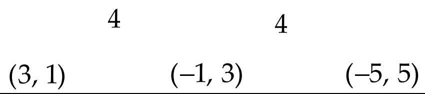




Response 3 : Examples of evidence which do not gain •10 in method 1 for stepping out







3	A function f is defined on the domain $0 \le x \le 3$ by $f(x) = x^3 - 2x^2 - 4x + 6$.	
	Determine the maximum and minimum values of f.	7

Generic Scheme | **Illustrative Scheme**

3

	Generic Scheme		Illustrative Scheme
$\bullet^1$	ss	start to differentiate	$\bullet^1$ differentiate x^3 or $-2x^2$ correctly
$\bullet^2$	ss	complete derivative and set to 0	$\bullet^2$ $3x^2 - 4x - 4$
$\bullet^3$	pd	start to solve $f'(x) = 0$	$\bullet^3$ e.g. $(3x+2)(x-2)$
$\bullet^4$	pd	solve $f'(x) = 0$	$\bullet^4$ $-\frac{2}{3}, 2$
$\bullet^5$	ic	evaluate f at relevant stationary point	$\bullet^5$ $f(2) = -2$
$\bullet^6$	ss	consider end-points	$\bullet^6$ $f(0) = 6$ **and** $f(3) = 3$
$\bullet^7$	ic	state max. and min. values	$\bullet^7$ max. 6 **and** min. -2

$= 0$ must appear at $\bullet^2$ or $\bullet^3$ to gain $\bullet^2$.

7

Notes

1. The only valid approach is via differentiation. A numerical approach can only gain $\bullet^6$.
2. Candidates who consider stationary points only cannot gain $\bullet^6$ or $\bullet^7$.
3. Treat maximum $(0, 6)$ and minimum $(2, -2)$ as bad form.
4. Cross marking is **not** applicable to $\bullet^6$ or $\bullet^7$.
5. Ignore any nature table which may appear in a candidate's solution, however $(2, -2)$ at table is sufficient for $\bullet^5$.

Regularly occurring responses

Response 1 : Algebraic issues in working

Candidate A

$y' = 3x^2 - 4x - 4$ ✓ ^	$\bullet^1$ ✓
$(3x-2)(x+2)$ ✗	$\bullet^2$ ✗
$x = \frac{2}{3}, \quad x = -2$ ✓̸	$\bullet^3$ ✗
When $x = \frac{2}{3}$, $y = \frac{74}{27}$ ✓̸	$\bullet^4$ ✓̸
$f(0) = 6$ and $f(3) = 3$ ✓	$\bullet^5$ ✓̸
$\max = 6, \quad \min = 2\frac{20}{27}$ ✓̸	$\bullet^6$ ✓
	$\bullet^7$ ✓̸

Candidate B

$3x^2 - 4x - 4 = 0$ ✓ ✓	$\bullet^1$ ✓
$(3x-2)(x-2)$ ✗	$\bullet^2$ ✓
$x = \frac{2}{3}$ or $x = 2$ ✓̸	$\bullet^3$ ✗
so $f(2) = -2$ ✓̸	$\bullet^4$ ✓̸
	$\bullet^5$ ✓̸

Since $\frac{2}{3}$ is within the domain, $f\left(\frac{2}{3}\right)$ must also be calculated to gain $\bullet^5$.

Candidate C

$3x^2 - 4x - 4$ ✓ ^	$\bullet^1$ ✓
$(3x+2)(x-2)$ ✓	$\bullet^2$ ✗
$3x + 2 = 0 \quad x - 2 = 0$	$\bullet^3$ ✓
$x = -\frac{2}{3} \quad x = 2$ ✓	$\bullet^4$ ✓
$f(2) = -2$ ✓	$\bullet^5$ ✓

Ignore the value of $f\left(-\frac{2}{3}\right)$ here, if it is included.

Response 2 : Derivative not explicitly set to zero

Candidate D

$f'(x) = 3x^2 - 4x - 4$ ✓ $\bullet^1$

$f'(x) = 0$ ✓ $\bullet^2$

Candidate E

$f'(x) = 0$

$f'(x) = 3x^2 - 4x - 4$ ✓ $\bullet^1$ ✓ $\bullet^2$

$= (3x+2)(x-2)$ ✓ $\bullet^3$

Candidate F

$f'(x) = 0$

$3x^2 - 4x - 4$ ✓ $\bullet^1$ ✗ $\bullet^2$ ^

$= (3x+2)(x-2)$ ✓ $\bullet^3$

Candidate G

^ $f'(x) = 0$ only ✗ $\bullet^1$ ✗ $\bullet^2$

3 continued

Regularly occurring responses

Response 3 : Solving quadratic equation

Ignore omission of negative sign at square here.

Candidate H

$f'(x) = 3x^2 - 4x - 4$ $\bullet^1$ ✓

$3x^2 - 4x - 4 = 0$ ✓ $\bullet^2$ ✓

$3x^2 - 4x = 4$ $\bullet^3$ ✗

$x(3x - 4) = 4$ $\bullet^4$ ✗

$\underline{x = 4, \frac{4}{3}}$ $\bullet^5$ ✗

Due to 'method' chosen $\bullet^3$, $\bullet^4$, $\bullet^5$ and $\bullet^7$ are not available.

Candidate I

$3x^2 - 4x - 4 = 0$ ✓ $\bullet^1$ ✓ $\bullet^2$

$x = \dfrac{-(-4) \pm \sqrt{(4)^2 - 4 \times 3 \times (-4)}}{2 \times 3}$ ✓ $\bullet^3$

Response 4 : Numerical approach

Candidate J

$f(0) = 6$

$f(3) = 3$ ✓ $\bullet^6$

This candidate has stayed within the interval $0 \le x \le 3$.

Candidate K

$f(0) = 6$

$f(1) = 1$

$f(2) = -2$ ✗ $\bullet^5$

$f(3) = 3$ ✓ $\bullet^6$

Candidate L

$f(0) = 6$

$f(1) = 1$

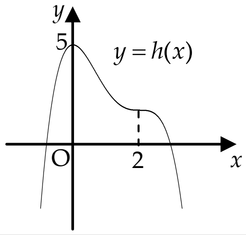
$f(2) = -2$ ✗ $\bullet^5$

$f(3) = 3$ ✗ $\bullet^6$

$f(4) = 22$

This candidate has gone outwith the interval $0 \le x \le 3$.

For $\bullet^5$, $f(2)$ must come from calculus and not from any other approach.

4	The diagram below shows the graph of a quartic $y = h(x)$, with stationary points at $x = 0$ and $x = 2$. On separate diagrams sketch the graphs of:		
	(a) $y = h'(x)$;		3
	(b) $y = 2 - h'(x)$.		3

Generic Scheme | **Illustrative Scheme**

4 (a)

Generic Scheme			Illustrative Scheme
•1	ic	identify roots	•1 0 **and** 2 only
•2	ic	interpret point of inflection	•2 turning point at (2, 0)
•3	ic	complete cubic curve	•3 cubic, passing through O with negative gradient

3

Notes

1. All graphs must include both the x and y axes (labelled or unlabelled), however the origin need not be labelled.
2. No marks are available unless a graph is attempted.
3. No marks are available to a candidate who makes several attempts at a graph on the same diagram, unless it is clear which is the final graph.
4. A linear graph gains no marks in both (a) and (b).

4 (b)

Generic Scheme			Illustrative Scheme
•4	ic	reflection in x-axis	•4 reflection of graph in (a) in x-axis
•5	ic	translation $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$	•5 graph moves parallel to y-axis by 2 units upwards
•6	ic	annotation of 'transformed' graph	•6 two 'transformed' points appropriately annotated (see Note 5)

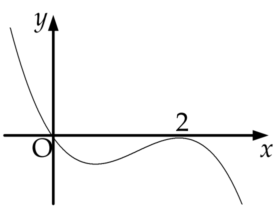
3

Notes

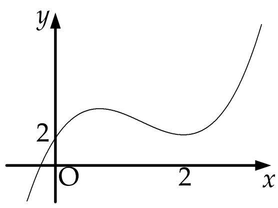
5. 'Transformed' here means a reflection followed by a translation.
6. •4 and •5 apply to the entire curve.
7. In each of the following circumstances :
   - Candidates who transform the original graph
   - Candidates who sketch a parabola in (a)

   mark the candidate's attempt as normal and unless a mark of 0 has been scored, deduct the last mark awarded. Indicate this with ✗ (see Regular occurring response G).
8. A reflection in any line parallel to the y-axis does not gain •4 or •6 .
9. A translation other than $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$ does not gain •5 or •6 .

Graph for (a)



Graph for (b)



•4 ic reflection in $x -$axis

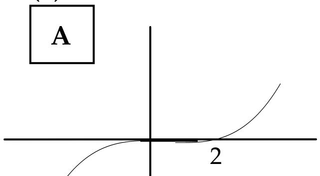
•5 ic translation $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$

4 continued

Regularly occurring responses

In (a)

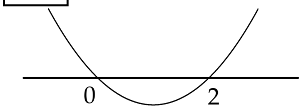
A



Curve does not only pass through 0 and 2 on x-axis.

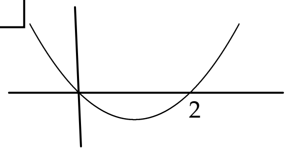
✗ •1
✗ •2
✗ •3

B



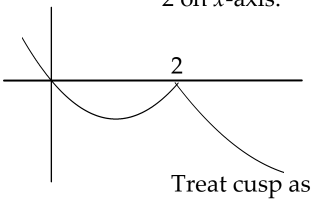
✗ •1
✗ •2
✗ •3

C



✓ •1
✗ •2
✗ •3

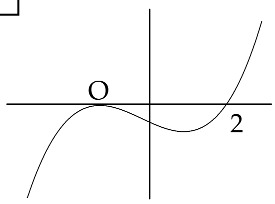
D



Treat cusp as turning point.

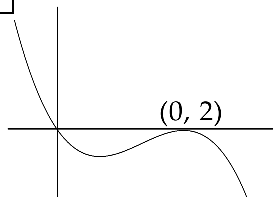
✓ •1
✓ •2
✓ •3

E



✗ •1
✗ •2
✗ •3

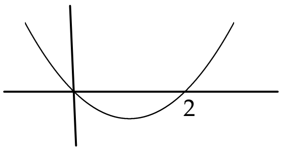
F



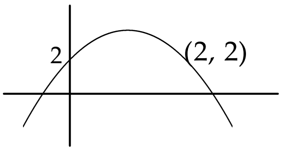
✓ •1
✓ •2
✓ •3

In (a) and (b)

G



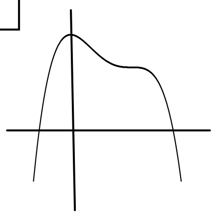
✓ •1
✗ •2
✗ •3



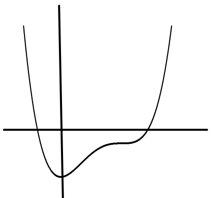
See Note 7

✗ •4
✗ •5
✗ •6

H



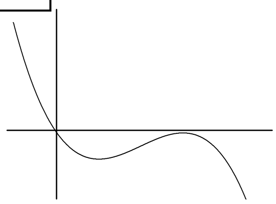
✗ •1
✗ •2
✗ •3



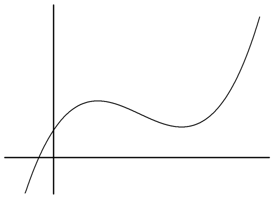
See Note 7

✗ •4
✗ •5
✗ •6

I

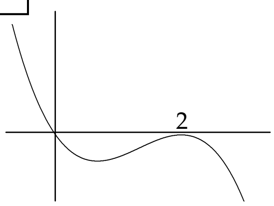


✗ •1
✗ •2
✓ •3

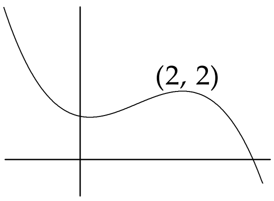


✓ •4
✗ •5
✗ •6

J



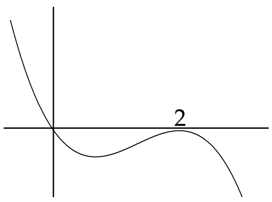
✓ •1
✓ •2
✓ •3



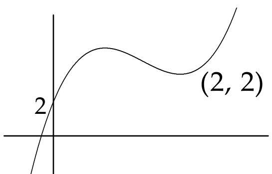
See Note 6

✗ •4
✗ •5
✗ •6

K



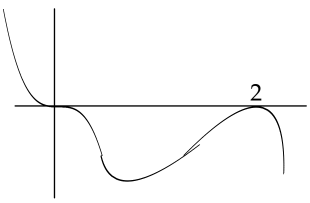
✓ •1
✓ •2
✓ •3



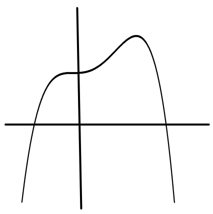
y-values of 2 should be roughly in line – too far apart here

✓ •4
✓ •5
✗ •6

L



✓ •1
✓ •2
✗ •3



✗ •4
✗ •5
✗ •6

5 A is the point $(3, -3, 0)$, B is $(2, -3, 1)$ and C is $(4, k, 0)$.

(a) (i) Express $\overrightarrow{BA}$ and $\overrightarrow{BC}$ in component form.

(ii) Show that $\cos A\hat{B}C = \dfrac{3}{\sqrt{2(k^2+6k+14)}}$. **7**

Generic Scheme			Illustrative Scheme
5(a)			
•1	ic	interpret vector	•1 $\begin{pmatrix}1\\0\\-1\end{pmatrix}$ Treat $\begin{pmatrix}1\\0\\-1\end{pmatrix}$ written as $(1, 0, -1)$ as bad form.
•2	pd	process vector	•2 $\begin{pmatrix}2\\k+3\\-1\end{pmatrix}$
•3	ss	use scalar product	•3 $\cos A\hat{B}C = \dfrac{\overrightarrow{BA}.\overrightarrow{BC}}{\lvert\overrightarrow{BA}\rvert\lvert\overrightarrow{BC}\rvert}$ see Note 1
•4	pd	find scalar product	•4 3
•5	pd	find $\lvert\overrightarrow{BA}\rvert$	•5 $\sqrt{2}$
•6	ic	find expression for $\lvert\overrightarrow{BC}\rvert$	•6 $\sqrt{2^2+(k+3)^2+(-1)^2}$ or equivalent
•7	ic	complete to result	•7 $\dfrac{3}{\sqrt{2}\sqrt{k^2+6k+14}}$ **and** $\dfrac{3}{\sqrt{2(k^2+6k+14)}}$ **or** $\lvert\overrightarrow{BA}\rvert\lvert\overrightarrow{BC}\rvert = \sqrt{2}\times\sqrt{k^2+6k+14}$ **and** $\dfrac{3}{\sqrt{2(k^2+6k+14)}}$

7

Notes

1. If the evidence for •3 does not appear explicitly, then •3 is only awarded if working for •7 is attempted.
2. •7 is dependent on gaining •4, •5 and •6.

Regularly occurring responses

Response 1 : Calculating wrong angle

Candidate A

$\cos AOC = \dfrac{\overrightarrow{OA}.\overrightarrow{OC}}{\lvert\overrightarrow{OA}\rvert\lvert\overrightarrow{OC}\rvert}$ ✗ •3

$\overrightarrow{OA}.\overrightarrow{OC} = 3\times4+(-3)\times k+0\times0 = 12-3k$ ✗ •4

$\lvert\overrightarrow{OA}\rvert = \sqrt{18}$ ✗ •5

$\lvert\overrightarrow{OC}\rvert = \sqrt{16+k^2}$ ✗ •6

$\cos ABC = \dfrac{12-3k}{\sqrt{18}\sqrt{16+k^2}}$ ✓ •7

Candidate B

$\cos AOB = \dfrac{\overrightarrow{OA}.\overrightarrow{OB}}{\lvert\overrightarrow{OA}\rvert\lvert\overrightarrow{OB}\rvert}$ ✗ •3

$\overrightarrow{OA}.\overrightarrow{OB} = 3\times2+(-3)\times(-3)+0\times1 = 15$ ✗ •4

$\lvert\overrightarrow{OA}\rvert = \sqrt{18}$ ✗ •5

$\lvert\overrightarrow{OB}\rvert = \sqrt{14}$ ✓ •6

$\cos ABC = \dfrac{15}{\sqrt{18}\sqrt{14}}$ ✓ •7

5	(b) If angle $ABC = 30^\circ$, find the possible values of k.	5

Generic Scheme | **Illustrative Scheme**

5(b)

Generic Scheme

Method 1 : Squaring first

- •8 ic link with (a)
- •9 ss square both sides
- •10 pd rearrange into 'non-fractional' format
- •11 pd write in standard form
- •12 pd solve for k

Method 2 : Dealing with fractions first

- •8 ic link with (a)
- •9 pd rearrange into 'non-fractional' format
- •10 ss square both sides
- •11 pd write in standard form
- •12 pd solve for k

Illustrative Scheme

Method 1 : Squaring first

- •8 $\dfrac{3}{\sqrt{2(k^2+6k+14)}} = \cos 30^\circ$
- •9 $\left(\dfrac{3}{\sqrt{2(k^2+6k+14)}}\right)^2 = \left(\dfrac{\sqrt{3}}{2}\right)^2$
- •10 $k^2+6k+14=6$ or equivalent
- •11 $k^2+6k+8=0$ or equivalent
- •12 $k=-2$ or -4

= 0 must appear at this stage.

Method 2 : Dealing with fractions first

- •8 $\dfrac{3}{\sqrt{2(k^2+6k+14)}} = \cos 30^\circ$
- •9 $\sqrt{3}\sqrt{2(k^2+6k+14)} = 6$
- •10 $6(k^2+6k+14)=36$
- •11 $k^2+6k+8=0$ or equivalent
- •12 $k=-2$ or -4

= 0 must appear at this stage.

5

Notes

3. The evidence for •9 may appear in the working for •10 in both methods.

4. •9 is the only mark available to candidates who replace cos 30° by 30 in method 1 and •10 in method 2.

5. All 5 marks are available to candidates who use 0·87 for cos 30° but 0·9 can gain a maximum of 4 marks.

Regularly occurring responses

Response 2 : Working with cos 30° throughout the question

Candidate C (Method 1)

$\cos 30^\circ = \dfrac{3}{\sqrt{2(k^2+6k+14)}}$ ✓ •8

$(\cos 30^\circ)^2 = \left(\dfrac{3}{\sqrt{2(k^2+6k+14)}}\right)^2$ ✓ •9

$(\cos 30^\circ)^2 = \dfrac{9}{2(k^2+6k+14)}$

$2(\cos 30^\circ)^2(k^2+6k+14)=9$ ✓ •10

If cos 30° is subsequently evaluated then •11 and •12 may still be available.

Response 3 : Using the wrong value for cos 30°

Candidate D (Method 2)

$\dfrac{3}{\sqrt{2(k^2+6k+14)}} = \dfrac{1}{\underline{2}}$ ✗ •8

$\sqrt{2(k^2+6k+14)}=6$ ✓̸ •9

$2(k^2+6k+14)=36$ ✓̸ •10

$k^2+6k+14=18$

$k^2+6k-4=0$ ✓̸ •11

$k = \dfrac{-6\pm\sqrt{6^2-4\times1\times(-4)}}{2\times1}$

$=0\cdot61, -6\cdot61$ ✓̸ •12

6	For $0 < x < \frac{\pi}{2}$, sequences can be generated using the recurrence relation $$u_{n+1} = (\sin x)u_n + \cos 2x, \text{ with } u_0 = 1.$$ (a) Why do these sequences have a limit?	2

Generic Scheme | **Illustrative Scheme**

6 (a)

	Generic Scheme		Illustrative Scheme
$\bullet^1$	ic condition on u_n coefficient	$\bullet^1$	$-1 < \sin x < 1$
$\bullet^2$	ic connect coefficient with given interval	$\bullet^2$	in interval, $0 < \sin x < 1$

2

Notes

1. For $\bullet^1$ **do not** accept:
   - $\sin x$ lies between -1 and 1
   - $-1 < x < 1$
   - $-1 < \sin < 1$

   However, accept '$\sin x$ greater than -1 and less than 1' for $\bullet^1$.
2. Do not accept $-1 < a < 1$ for $\bullet^1$ unless a is clearly identified as $\sin x$, which may not appear until (b).
3. $0 < \sin x < 1$ and nothing else, does not gain $\bullet^1$ but gains $\bullet^2$.
4. $0 \le \sin x \le 1$ and nothing else, does not gain $\bullet^1$ or $\bullet^2$.

Regularly occurring responses

Response 1 : Attempts at giving a reason for limit

Candidate A

This sequence has a limit because $-1 < a < 1$, $\quad \bullet^1$ ✓

$-1 < \sin x < 1$ within the domain. $\quad \bullet^2$ X

Candidate B

Since $\sin x$ in this domain will always $\quad \bullet^1$ X

be greater than 0 and less than 1. ✓ $\quad \bullet^2$ ✓

Candidate C

$\sin\frac{\pi}{2} = 1$ and $\sin 0 = 0$ so the multiplier $\quad \bullet^1$ X

of u_n is between 0 and 1, so it has a limit. $\quad \bullet^2$ X

Candidate D

$-1 \le \sin x \le 1$, $\quad \bullet^1$ X

for $0 < x < \frac{\pi}{2}$, $0 < \sin x < 1$ ✓ $\quad \bullet^2$ ✓

so limit exists

Response 2 : Minimum response for both marks

Candidate E

for $0 < x < \frac{\pi}{2}$, $0 < \sin x < 1$ $\quad \bullet^2$ ✓

so $-1 < \sin x < 1$ $\quad \bullet^1$ ✓

so limit

Candidate F

if limit, $-1 < \sin x < 1$ $\quad \bullet^1$ ✓

for $0 < x < \frac{\pi}{2}$, $0 < \sin x < 1$ $\quad \bullet^2$ ✓

6	(b)	The limit of one particular sequence generated by this recurrence relation is $\frac{1}{2}\sin x$. Find the value(s) of x.	**7**

Generic Scheme | **Illustrative Scheme**

6 (b)

		Generic Scheme		Illustrative Scheme
$\bullet^3$	ss	appropriate limit method	$\bullet^3$	limit $=\dfrac{\cos 2x}{1-\sin x}$ **or** $l=\sin x\times l+\cos 2x$
$\bullet^4$	ic	substitute for limit	$\bullet^4$	$\frac{1}{2}\sin x=\dfrac{\cos 2x}{1-\sin x}$ **or** $\frac{1}{2}\sin x=\sin x\times\frac{1}{2}\sin x+\cos 2x$ ($\bullet^3$ **may be stated, or implied by** $\bullet^4$ in both methods)
$\bullet^5$	ss	use appropriate double angle formula	$\bullet^5$	$...1-2\sin^2 x...$
$\bullet^6$	pd	express in standard form	$\bullet^6$	e.g. $3\sin^2 x+\sin x-2$ $= 0$ must appear at $\bullet^6$ or $\bullet^7$ to gain $\bullet^6$.
$\bullet^7$	pd	start to solve quadratic equation	$\bullet^7$	e.g. $(3\sin x-2)(\sin x+1)$
$\bullet^8$	pd	reduce to equations in $\sin x$ only	$\bullet^8$	$\sin x=\frac{2}{3}$ or $\sin x=-1$
$\bullet^9$	ic	select valid solution	$\bullet^9$	$x=0\cdot730$ or outwith interval

7

Notes

5. $\bullet^7$, $\bullet^8$ and $\bullet^9$ are only available if a quadratic equation is obtained at $\bullet^6$ stage.
6. Candidates may express the quadratic equation at the $\bullet^6$ stage in the form $3s^2+s-2=0$. For candidates who do not solve a trigonometric quadratic equation at $\bullet^7$ $\sin x$ must appear explicitly to gain $\bullet^8$.
7. $\bullet^7$, $\bullet^8$ and $\bullet^9$ are not available to candidates who 'solve' a quadratic equation in the form $ax^2+bx=c$, $c\neq 0$.
8. For $\bullet^9$ there must be one valid solution, and one solution outwith interval which is rejected.
9. $\bullet^9$ is not available to candidates who leave their answer in degree measure.
10. Cross marking is available for $\bullet^8$ and $\bullet^9$.

Regularly occurring responses

Response 3 : Evidence for identification of a appearing in (b)

Candidate G

(a) $-1<a<1$ $\bullet^1$ ✓

(b) $L=\dfrac{b}{1-a}=\dfrac{\cos 2x}{1-\sin x}$ ✓ $\bullet^3$ ✓ $\bullet^1$ $\bullet^2$ ✗ $\bullet^3$ ✓

Response 4 : Error in algebra and subsequent quadratic equation solution

Candidate H

$L=\dfrac{b}{1-a}=\frac{1}{2}\sin x$

$\dfrac{\cos 2x}{1-\sin x}=\frac{1}{2}\sin x$ ✓ $\bullet^3$ ✓ $\bullet^4$

$\cos 2x=\underline{-\frac{1}{2}\sin^2 x}$ ✗ $\bullet^6$

$\frac{1}{2}\sin^2 x+\cos 2x=0$

$\frac{1}{2}\sin^2 x+(1-2\sin^2 x)=0$ ✓ $\bullet^5$

$-\frac{3}{2}\sin^2 x+1=0$

$\sin^2 x=\frac{2}{3}$ ✓ $\bullet^7$

$\sin x=\sqrt{\frac{2}{3}}$ and $\sin x=-\sqrt{\frac{2}{3}}$ ✓ $\bullet^8$

$x=0\cdot955$, $\cancel{2\cdot186}$ $x=\cancel{4\cdot097}$, $\cancel{5\cdot328}$ ✓ $\bullet^9$

Candidate I

$\dfrac{\cos 2x}{1-\sin x}=\frac{1}{2}\sin x$ ✓ $\bullet^3$ ✓ $\bullet^4$

$\frac{1}{2}\sin x(1-\sin x)=\underline{1-\sin^2 x}$ ✗ $\bullet^5$

$\sin^2 x+\sin x-2=0$ ✓ $\bullet^6$

$(\sin x-1)(\sin x+2)=0$ ✓ $\bullet^7$

$\sin x=1$ and $\sin x=-2$ ✓ $\bullet^8$

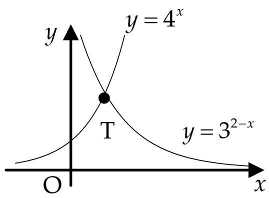
$x=\frac{\pi}{2}$ not possible ✓ $\bullet^9$

See Note 8

7 The diagram shows the curves with equations $y = 4^x$ and $y = 3^{2-x}$.

The graphs intersect at the point T.

(a) Show that the x-coordinate of T can be written in the form $\dfrac{\log_a p}{\log_a q}$, for all $a > 1$. **6**



Generic Scheme | **Illustrative Scheme**

7(a)

Generic Scheme

- •1 ss equate expressions for y
- •2 ss take logarithms of both sides
- •3 ic use law of logs : $\log_a x^n = n\log_a x$
- •4 pd gather like terms
- •5 ic use law of logs : $\log_a p + \log_a q = \log_a pq$
- •6 ic complete to required form

In methods 1 and 2:

If the first line of working is that at the •2 stage, then •1 and •2 are awarded.

If the first line of working is that at the •3 stage, then only •2 and •3 are awarded.

Method 1

- •1 $4^x = 3^{2-x}$
- •2 $\log_a\left(4^x\right) = \log_a\left(3^{2-x}\right)$ **stated, or implied by •3**
- •3 $x\log_a 4 = (2-x)\log_a 3$
- •4 $x(\log_a 4 + \log_a 3) = 2\log_a 3$
- •5 $x\log_a 12 = \log_a 9$
- •6 $\dfrac{\log_a 9}{\log_a 12}$ **stated explicitly**

Method 2

- •1 $4^x = 3^{2-x}$
- •2 $\log_3\left(4^x\right) = 2 - x$
- •3 $x\log_3 4 = 2 - x$
- •4 $x = \dfrac{2}{1+\log_3 4}$
- •5 $\dfrac{2\log_3 3}{\log_3 12}$
- •6 $\dfrac{\log_a 9}{\log_a 12}$ **stated explicitly**

Method 3

- •1 $4^x = 3^{2-x}$
- •2 $4^x = \dfrac{3^2}{3^x}$
- •3 $12^x = 9$
- •4 $\log_a 12^x = \log_a 9$
- •5 $x\log_a 12 = \log_a 9$
- •6 $\dfrac{\log_a 9}{\log_a 12}$ **stated explicitly**

6

Notes

1. In methods 1 and 2, if no base is indicated then •2 is not available, however •3, •4 and •5 are still available.

   In method 3, if no base is indicated then •4 is not available, however •5 is still available.
2. In all methods, if a numerical base is used then •6 is not available.
3. In method 1, the omission of brackets at the •3 stage is treated as bad form, see Response 1.
4. p and q must be numerical values.

Regularly occurring responses

Response 1 : Omission of brackets around $2-x$

Candidate A

$4^x = 3^{2-x}$ ✓ •1

$x\log_a 4 = 2 - x\log_a 3$ ✓ •2 ✓ •3

Candidate B

$4^x = 3^{2-x}$ ✓ •1

$x\log_a 4 = 2 - x\log_a 3$ ✓ •2 ✓ •3

$x(\log_a 4 + \log_a 3) = 2$ ✗ •4

$x\log_a 12 = 2$ ✗ •5

Response 2 : Using different bases

Candidate C

$4^x = 3^{2-x}$ ✓ •1

$\log_3 4^x = \log_4 3^{2-x}$ ✗ •2

$x\log_3 4 = (2-x)\log_4 3$ ✗ •3

Response 3 : Taking logs first

Candidate D

$y = 4^x$ and $y = 3^{2-x}$

$\log_a y = \log_a 4^x$ and $\log_a y = \log_a 3^{2-x}$ ✓ •2

$\log_a y = x\log_a 4$ and $\log_a y = (2-x)\log_a 3$ ✓ •3

$x\log_a 4 = (2-x)\log_a 3$ ✓ •1

$x = \dfrac{2}{\log_a 12}$

$= \dfrac{2\log_a a}{\log_a 12}$

$= \dfrac{\log_a a^2}{\log_a 12}$ ✗ •6

7	(b) Calculate the y-coordinate of T.	2

Generic Scheme | **Illustrative Scheme**

7(b)

Generic Scheme	Illustrative Scheme	
$\bullet^7$ ic substitute in for x	$\bullet^7$ e.g. $y = 4^{\frac{\log_a 9}{\log_a 12}}$	**stated, or implied by $\bullet^8$**
$\bullet^8$ pd process y	$\bullet^8$ e.g. $y \approx 4^{0\cdot8842} \approx 3\cdot4$	2

Notes

5. Candidates must work to at least two significant figures in (b) e.g. $4^{0\cdot9} = 3\cdot5$ does not gain $\bullet^8$, but $\bullet^7$ is available.
6. $\bullet^8$ is only available if the power used comes from $\dfrac{\log_a p}{\log_a q}$ in (a).

Regularly occurring responses

Response 4 : Using p and q as integer values without working

Candidate E

$\left.\begin{array}{l} p = 4 \\ q = 3 \end{array}\right\} y = 4^{1\cdot26} = 5\cdot74 \text{ or } 5\cdot75$ ✗ $\bullet^7$ ✗ $\bullet^8$

Candidate F

$\left.\begin{array}{l} p = 3 \\ q = 4 \end{array}\right\} y = 4^{0\cdot79} = 2\cdot99 \text{ or } 3$ ✗ $\bullet^7$ ✗ $\bullet^8$

Response 5 : Using integer values calculated in (a)

Candidate G

$\left.\begin{array}{l} p = 10 \\ q = 4 \end{array}\right\} y = 4^{2\cdot5} = 32$ ✗ $\bullet^7$ ✗ $\bullet^8$

[END OF MARKING INSTRUCTIONS]