X100/12/02

NATIONAL QUALIFICATIONS 2012

MONDAY, 21 MAY
1.00 PM – 2.30 PM

MATHEMATICS
HIGHER
Paper 1
(Non-calculator)

Read carefully

Calculators may NOT be used in this paper.

Section A – Questions 1–20 (40 marks)

Instructions for completion of **Section A** are given on Page two.

For this section of the examination you must use an **HB pencil**.

Section B (30 marks)

1 Full credit will be given only where the solution contains appropriate working.

2 Answers obtained by readings from scale drawings will not receive any credit.

Read carefully

1 Check that the answer sheet provided is for **Mathematics Higher (Section A)**.

2 For this section of the examination you must use an **HB pencil** and, where necessary, an eraser.

3 Check that the answer sheet you have been given has **your name**, **date of birth**, **SCN** (Scottish Candidate Number) and **Centre Name** printed on it.

Do not change any of these details.

4 If any of this information is wrong, tell the Invigilator immediately.

5 If this information is correct, **print** your name and seat number in the boxes provided.

6 The answer to each question is **either** A, B, C or D. Decide what your answer is, then, using your pencil, put a horizontal line in the space provided (see sample question below).

7 There is **only one correct** answer to each question.

8 Rough working should **not** be done on your answer sheet.

9 At the end of the exam, put the **answer sheet for Section A inside the front cover of your answer book**.

Sample Question

A curve has equation $y = x^3 - 4x$.

What is the gradient at the point where x = 2?

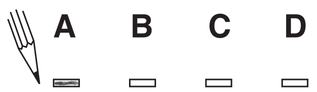
A 8

B 1

C 0

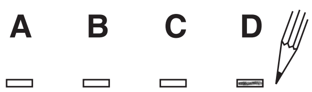
D -4

The correct answer is **A—**8. The answer **A** has been clearly marked in **pencil** with a horizontal line (see below).



Changing an answer

If you decide to change your answer, carefully erase your first answer and, using your pencil, fill in the answer you want. The answer below has been changed to **D**.



FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\mathbf{a.b} = |\mathbf{a}||\mathbf{b}| \cos\theta$, where θ is the angle between **a** and **b**

or $\mathbf{a.b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\sin 2A &= 2\sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

[Turn over

SECTION A

ALL questions should be attempted.

1. A sequence is defined by the recurrence relation $u_{n+1} = 3u_n + 4$, with $u_0 = 1$.

Find the value of u_2.

A 7

B 10

C 25

D 35

2. What is the gradient of the tangent to the curve with equation $y = x^3 - 6x + 1$ at the point where $x = -2$?

A –24

B 3

C 5

D 6

3. If $x^2 - 6x + 14$ is written in the form $(x - p)^2 + q$, what is the value of q?

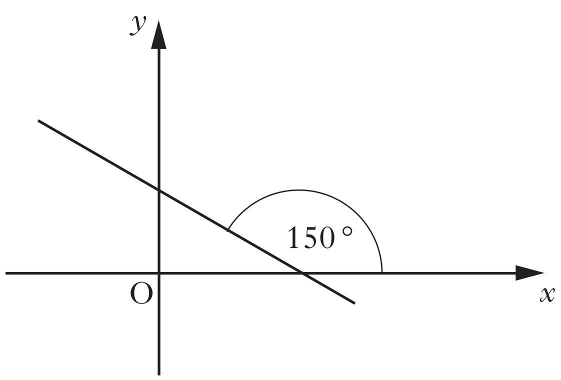
A –22

B 5

C 14

D 50

4. What is the gradient of the line shown in the diagram?



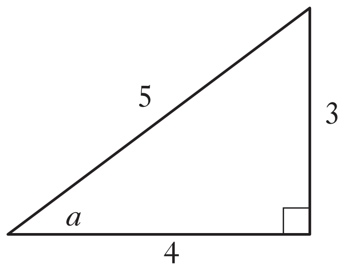
A $-\sqrt{3}$

B $-\frac{1}{\sqrt{3}}$

C $-\frac{1}{2}$

D $-\frac{\sqrt{3}}{2}$

5. The diagram shows a right-angled triangle with sides and angles as marked.



What is the value of $\cos 2a$?

A $\frac{7}{25}$

B $\frac{3}{5}$

C $\frac{24}{25}$

D $\frac{6}{5}$

6. If $y = 3x^{-2} + 2x^{\frac{3}{2}}$, $x > 0$, determine $\frac{dy}{dx}$.

A $-6x^{-3} + \frac{4}{5}x^{\frac{5}{2}}$

B $-3x^{-1} + 3x^{\frac{1}{2}}$

C $-6x^{-3} + 3x^{\frac{1}{2}}$

D $-3x^{-1} + \frac{4}{5}x^{\frac{5}{2}}$

7. If $\mathbf{u} = \begin{pmatrix} -3 \\ 1 \\ 2t \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 1 \\ t \\ -1 \end{pmatrix}$ are perpendicular, what is the value of t?

A -3

B -2

C $\frac{2}{3}$

D 1

[Turn over

8. The volume of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$.

What is the rate of change of V with respect to r, at $r = 2$?

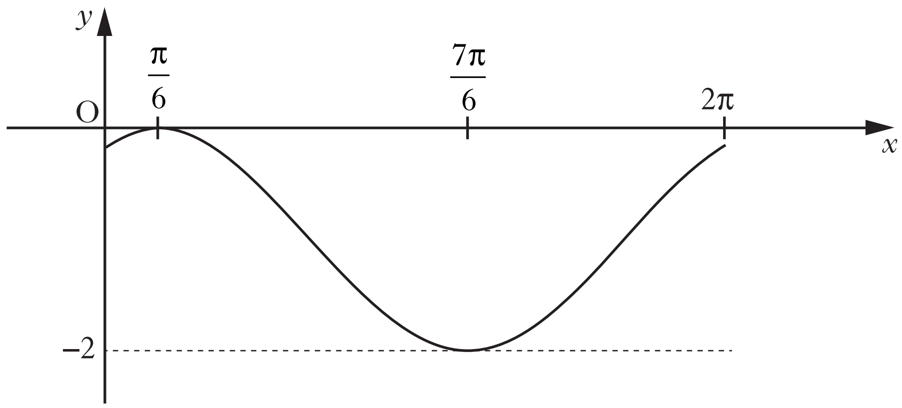
A $\frac{16\pi}{3}$

B $\frac{32\pi}{3}$

C 16π

D 32π

9. The diagram shows the curve with equation of the form $y = \cos(x + a) + b$ for $0 \le x \le 2\pi$.



What is the equation of this curve?

A $y = \cos\left(x - \frac{\pi}{6}\right) - 1$

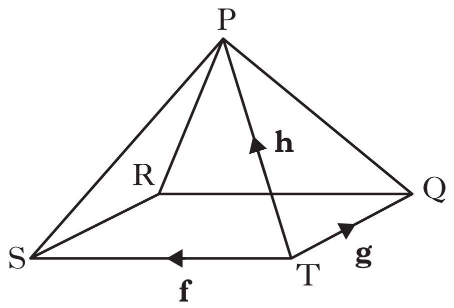
B $y = \cos\left(x - \frac{\pi}{6}\right) + 1$

C $y = \cos\left(x + \frac{\pi}{6}\right) - 1$

D $y = \cos\left(x + \frac{\pi}{6}\right) + 1$

10. The diagram shows a square-based pyramid P,QRST.

$\overrightarrow{TS}$, $\overrightarrow{TQ}$ and $\overrightarrow{TP}$ represent **f**, **g** and **h** respectively.



Express $\overrightarrow{RP}$ in terms of **f**, **g** and **h**.

A $-\mathbf{f} + \mathbf{g} - \mathbf{h}$

B $-\mathbf{f} - \mathbf{g} + \mathbf{h}$

C $\mathbf{f} - \mathbf{g} - \mathbf{h}$

D $\mathbf{f} + \mathbf{g} + \mathbf{h}$

11. Find $\int \left(\frac{1}{6x^2}\right) dx$, $x \neq 0$.

A $-12x^{-3} + c$

B $-6x^{-1} + c$

C $-\frac{1}{3}x^{-3} + c$

D $-\frac{1}{6}x^{-1} + c$

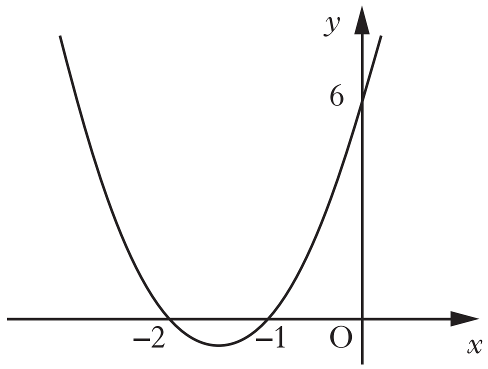
12. Find the maximum value of

$$2 - 3\sin\left(x - \frac{\pi}{3}\right)$$

and the value of x where this occurs in the interval $0 \leq x \leq 2\pi$.

	max value	x
A	-1	$\frac{11\pi}{6}$
B	5	$\frac{11\pi}{6}$
C	-1	$\frac{5\pi}{6}$
D	5	$\frac{5\pi}{6}$

13. A parabola intersects the axes at $x = -2$, $x = -1$ and $y = 6$, as shown in the diagram.



What is the equation of the parabola?

A $y = 6(x - 1)(x - 2)$

B $y = 6(x + 1)(x + 2)$

C $y = 3(x - 1)(x - 2)$

D $y = 3(x + 1)(x + 2)$

14. Find $\int (2x-1)^{\frac{1}{2}}\,dx$ where $x > \frac{1}{2}$.

A $\frac{1}{3}(2x-1)^{\frac{3}{2}} + c$

B $\frac{1}{2}(2x-1)^{-\frac{1}{2}} + c$

C $\frac{1}{2}(2x-1)^{\frac{3}{2}} + c$

D $\frac{1}{3}(2x-1)^{-\frac{1}{2}} + c$

15. If $\mathbf{u} = k\begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix}$, where $k>0$ and $\mathbf{u}$ is a unit vector, determine the value of k.

A $\frac{1}{2}$

B $\frac{1}{8}$

C $\frac{1}{\sqrt{2}}$

D $\frac{1}{\sqrt{10}}$

16. If $y = 3\cos^4 x$, find $\dfrac{dy}{dx}$.

A $12\cos^3 x \sin x$

B $12\cos^3 x$

C $-12\cos^3 x \sin x$

D $-12\sin^3 x$

17. Given that $\mathbf{a} = \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}$ and $\mathbf{a}.(\mathbf{a}+\mathbf{b}) = 7$, what is the value of $\mathbf{a}.\mathbf{b}$?

A $\dfrac{7}{25}$

B $-\dfrac{18}{5}$

C -6

D -18

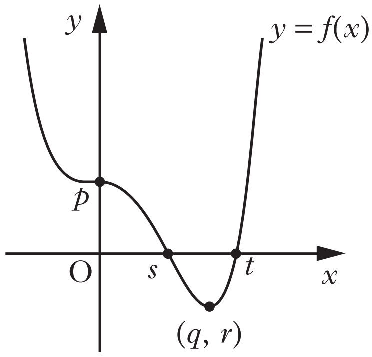
18. The graph of $y = f(x)$ shown has stationary points at $(0, p)$ and (q, r).

Here are two statements about $f(x)$:

(1) $f(x) < 0$ for $s < x < t$;

(2) $f'(x) < 0$ for $x < q$.

Which of the following is true?



A Neither statement is correct.

B Only statement (1) is correct.

C Only statement (2) is correct.

D Both statements are correct.

[Turn over

19. Solve $6 - x - x^2 < 0$.

A $-3 < x < 2$

B $x < -3, x > 2$

C $-2 < x < 3$

D $x < -2, x > 3$

20. Simplify $\dfrac{\log_b 9a^2}{\log_b 3a}$, where $a > 0$ and $b > 0$.

A 2

B $3a$

C $\log_b 3a$

D $\log_b(9a^2 - 3a)$

[*END OF SECTION A*]

SECTION B

Marks

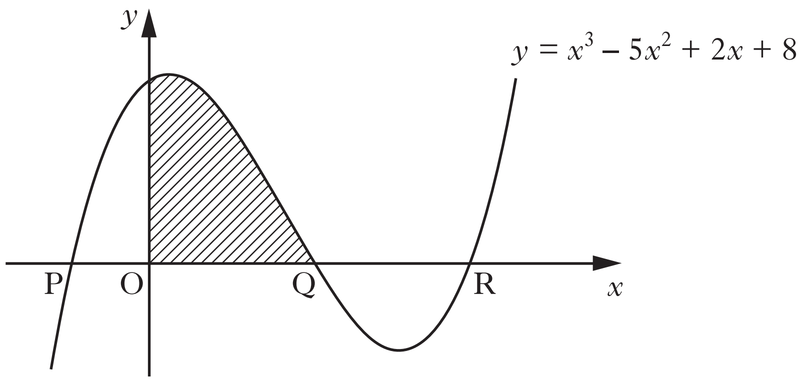
ALL questions should be attempted.

21. (*a*) (i) Show that $(x - 4)$ is a factor of $x^3 - 5x^2 + 2x + 8$.

(ii) Factorise $x^3 - 5x^2 + 2x + 8$ fully.

(iii) Solve $x^3 - 5x^2 + 2x + 8 = 0$. **6**

(*b*) The diagram shows the curve with equation $y = x^3 - 5x^2 + 2x + 8$.



The curve crosses the x-axis at P, Q and R.

Determine the shaded area. **6**

22. (*a*) The expression $\cos x - \sqrt{3}\sin x$ can be written in the form $k\cos(x + a)$ where $k > 0$ and $0 \le a < 2\pi$.

Calculate the values of k and a. **4**

(*b*) Find the points of intersection of the graph of $y = \cos x - \sqrt{3}\sin x$ with the x and y axes, in the interval $0 \le x \le 2\pi$. **3**

[Turn over for Question 23 on *Page twelve*

Marks

23. (*a*) Find the equation of ℓ_1, the perpendicular bisector of the line joining P(3, –3) to Q(–1, 9). **4**

(*b*) Find the equation of ℓ_2 which is parallel to PQ and passes through R(1, –2). **2**

(*c*) Find the point of intersection of ℓ_1 and ℓ_2. **3**

(*d*) Hence find the shortest distance between PQ and ℓ_2. **2**

[*END OF SECTION B*]

[*END OF QUESTION PAPER*]

X100/12/03

NATIONAL QUALIFICATIONS 2012

MONDAY, 21 MAY
2.50 PM – 4.00 PM

MATHEMATICS
HIGHER
Paper 2

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LI X100/12/03 6/28110

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Trigonometric formulae:

$$\begin{aligned} \sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\ \sin 2A &= 2\sin A \cos A \\ \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A \end{aligned}$$

Table of standard derivatives:

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Table of standard integrals:

$f(x)$	$\int f(x)dx$
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ALL questions should be attempted. *Marks*

1. Functions f and g are defined on the set of real numbers by

- $f(x) = x^2 + 3$
- $g(x) = x + 4$.

(*a*) Find expressions for:

(i) $f(g(x))$;

(ii) $g(f(x))$. **3**

(*b*) Show that $f(g(x)) + g(f(x)) = 0$ has no real roots. **3**

2. (*a*) Relative to a suitable set of coordinate axes, Diagram 1 shows the line $2x - y + 5 = 0$ intersecting the circle $x^2 + y^2 - 6x - 2y - 30 = 0$ at the points P and Q.

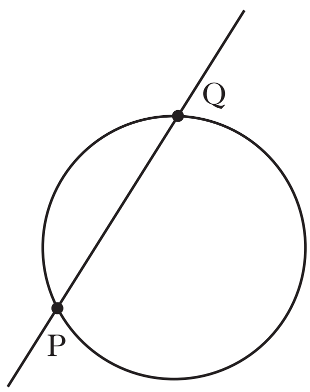


Diagram 1

Find the coordinates of P and Q. **6**

(*b*) Diagram 2 shows the circle from (*a*) and a second congruent circle, which also passes through P and Q.

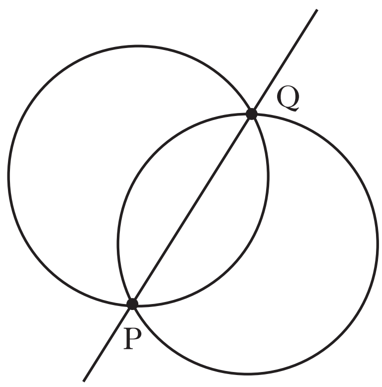


Diagram 2

6

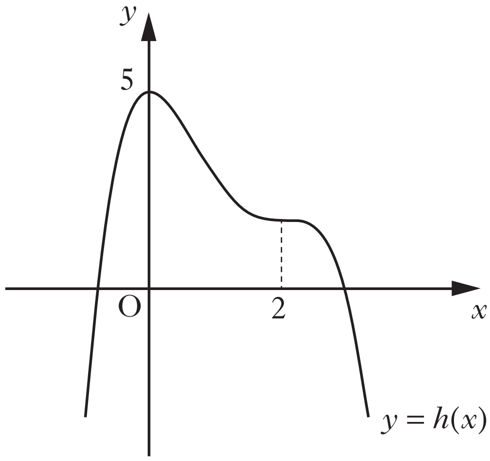
Determine the equation of this second circle.

Marks

3. A function f is defined on the domain $0 \leq x \leq 3$ by $f(x) = x^3 - 2x^2 - 4x + 6$.

Determine the maximum and minimum values of f. **7**

4. The diagram below shows the graph of a quartic $y = h(x)$, with stationary points at $x = 0$ and $x = 2$.



On separate diagrams sketch the graphs of:

(*a*) $y = h'(x)$; **3**

(*b*) $y = 2 - h'(x)$. **3**

5. A is the point (3, –3, 0), B is (2, –3, 1) and C is (4, k, 0).

(*a*) (i) Express $\overrightarrow{BA}$ and $\overrightarrow{BC}$ in component form.

(ii) Show that $\cos A\hat{B}C = \dfrac{3}{\sqrt{2(k^2 + 6k + 14)}}$. **7**

(*b*) If angle ABC = 30°, find the possible values of k. **5**

Marks

6. For $0 < x < \frac{\pi}{2}$, sequences can be generated using the recurrence relation

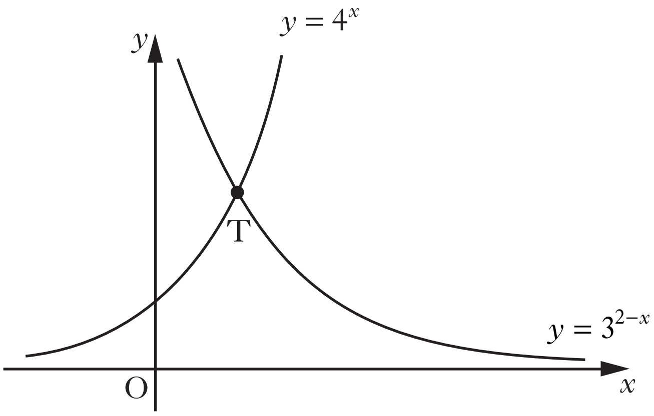
$$u_{n+1} = (\sin x)u_n + \cos 2x, \text{ with } u_0 = 1.$$

(*a*) Why do these sequences have a limit? **2**

(*b*) The limit of one sequence generated by this recurrence relation is $\frac{1}{2}\sin x$.

Find the value(s) of x. **7**

7. The diagram shows the curves with equations $y = 4^x$ and $y = 3^{2-x}$.



The graphs intersect at the point T.

(*a*) Show that the x – coordinate of T can be written in the form $\frac{\log_a p}{\log_a q}$,

for all $a > 1$. **6**

(*b*) Calculate the y – coordinate of T. **2**

[*END OF QUESTION PAPER*]

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