X100/301

NATIONAL QUALIFICATIONS 2006

FRIDAY, 19 MAY
9.00 AM – 10.10 AM

MATHEMATICS
HIGHER
Units 1, 2 and 3
Paper 1
(Non-calculator)

Read Carefully

1 **Calculators may NOT be used in this paper.**

2 Full credit will be given only where the solution contains appropriate working.

3 Answers obtained by readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}| \cos \theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\begin{aligned} \sin(\mathrm{A} \pm \mathrm{B}) &= \sin \mathrm{A} \cos \mathrm{B} \pm \cos \mathrm{A} \sin \mathrm{B} \\ \cos(\mathrm{A} \pm \mathrm{B}) &= \cos \mathrm{A} \cos \mathrm{B} \mp \sin \mathrm{A} \sin \mathrm{B} \\ \sin 2\mathrm{A} &= 2\sin \mathrm{A} \cos \mathrm{A} \\ \cos 2\mathrm{A} &= \cos^2 \mathrm{A} - \sin^2 \mathrm{A} \\ &= 2\cos^2 \mathrm{A} - 1 \\ &= 1 - 2\sin^2 \mathrm{A} \end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

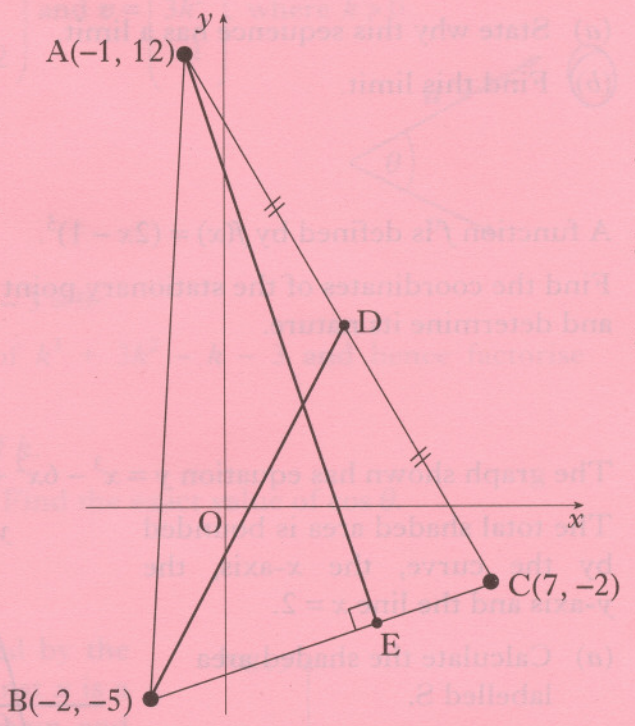
Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

ALL questions should be attempted.

Marks

1. Triangle ABC has vertices A(–1, 12), B(–2, –5) and C(7, –2).

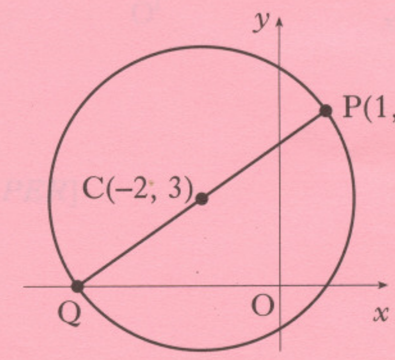


(*a*) Find the equation of the median BD. **3**

(*b*) Find the equation of the altitude AE. **3**

(*c*) Find the coordinates of the point of intersection of BD and AE. **3**

2. A circle has centre C(–2, 3) and passes through P(1, 6).



(*a*) Find the equation of the circle. **2**

(*b*) PQ is a diameter of the circle. Find the equation of the tangent to this circle at Q. **4**

3. Two functions f and g are defined by $f(x) = 2x + 3$ and $g(x) = 2x - 3$, where x is a real number.

(*a*) Find expressions for:

(i) $f(g(x))$;

(ii) $g(f(x))$. **3**

(*b*) Determine the least possible value of the product $f(g(x)) \times g(f(x))$. **2**

[Turn over

Marks

4. A sequence is defined by the recurrence relation $u_{n+1} = 0{\cdot}8u_n + 12$, $u_0 = 4$.

(a) State why this sequence has a limit. 1

(b) Find this limit. 2

5. A function f is defined by $f(x) = (2x - 1)^5$.

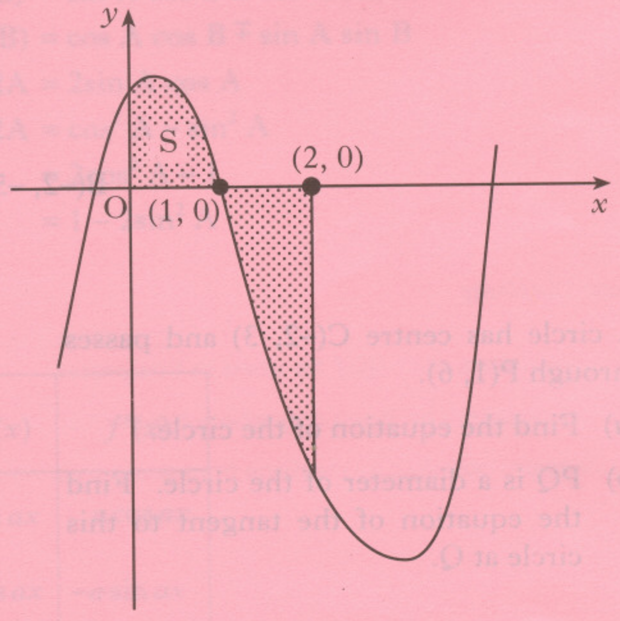
Find the coordinates of the stationary point on the graph with equation $y = f(x)$ and determine its nature. 7

6. The graph shown has equation $y = x^3 - 6x^2 + 4x + 1$.

The total shaded area is bounded by the curve, the x-axis, the y-axis and the line $x = 2$.

(a) Calculate the shaded area labelled S. 4

(b) Hence find the total shaded area. 3



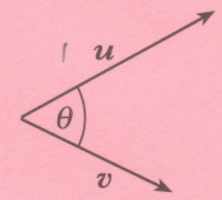
7. Solve the equation $\sin x° - \sin 2x° = 0$ in the interval $0 \leq x \leq 360$. 4

8. (a) Express $2x^2 + 4x - 3$ in the form $a(x + b)^2 + c$. 3

(b) Write down the coordinates of the turning point on the parabola with equation $y = 2x^2 + 4x - 3$. 1

Marks

9. $\boldsymbol{u}$ and $\boldsymbol{v}$ are vectors given by $\boldsymbol{u} = \begin{pmatrix} k^3 \\ 1 \\ k+2 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 1 \\ 3k^2 \\ -1 \end{pmatrix}$, where $k > 0$.



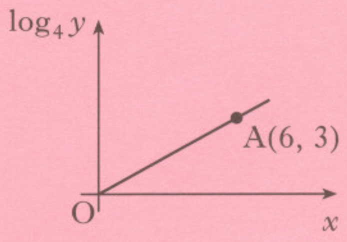
(*a*) If $\boldsymbol{u}.\boldsymbol{v} = 1$, show that $k^3 + 3k^2 - k - 3 = 0$. **2**

(*b*) Show that $(k + 3)$ is a factor of $k^3 + 3k^2 - k - 3$ and hence factorise $k^3 + 3k^2 - k - 3$ fully. **5**

(*c*) Deduce the only possible value of k. **1**

(*d*) The angle between $\boldsymbol{u}$ and $\boldsymbol{v}$ is θ. Find the exact value of $\cos\theta$. **3**

10. Two variables, x and y, are connected by the law $y = a^x$. The graph of $\log_4 y$ against x is a straight line passing through the origin and the point A(6, 3). Find the value of a. **4**



[*END OF QUESTION PAPER*]