

**X100/303**

NATIONAL  
QUALIFICATIONS  
2003

WEDNESDAY, 21 MAY  
10.30 AM – 12.00 NOON

**MATHEMATICS  
HIGHER**

Units 1, 2 and 3  
Paper 2

**Read Carefully**

- 1 Calculators may be used in this paper.
- 2 Full credit will be given only where the solution contains appropriate working.
- 3 Answers obtained by readings from scale drawings will not receive any credit.



ALL questions should be attempted.

Marks

1.  $f(x) = 6x^3 - 5x^2 - 17x + 6$ .

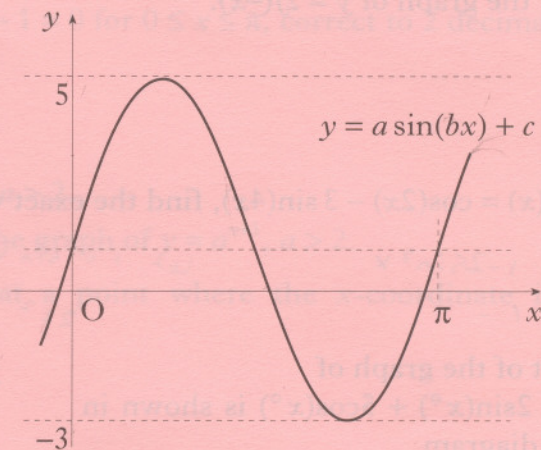
(a) Show that  $(x - 2)$  is a factor of  $f(x)$ .

(b) Express  $f(x)$  in its fully factorised form.

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2. The diagram shows a sketch of part of the graph of a trigonometric function whose equation is of the form  $y = a \sin(bx) + c$ .

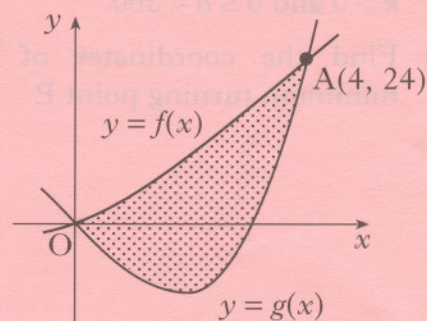
Determine the values of  $a$ ,  $b$  and  $c$ .



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3. The incomplete graphs of  $f(x) = x^2 + 2x$  and  $g(x) = x^3 - x^2 - 6x$  are shown in the diagram. The graphs intersect at  $A(4, 24)$  and the origin.

Find the shaded area enclosed between the curves.



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4. (a) Find the equation of the tangent to the curve with equation  $y = x^3 + 2x^2 - 3x + 2$  at the point where  $x = 1$ .

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(b) Show that this line is also a tangent to the circle with equation  $x^2 + y^2 - 12x - 10y + 44 = 0$  and state the coordinates of the point of contact.

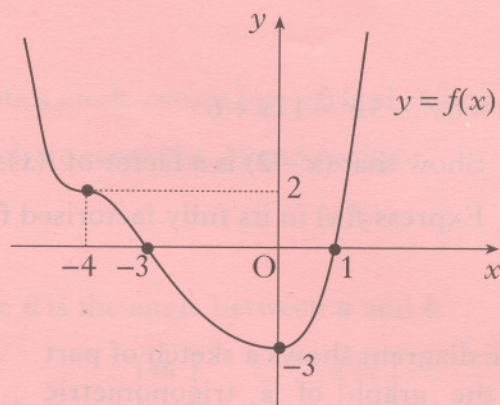
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[Turn over

5. The diagram shows the graph of a function  $f$ .

$f$  has a minimum turning point at  $(0, -3)$  and a point of inflexion at  $(-4, 2)$ .

- (a) Sketch the graph of  $y = f(-x)$ .  
 (b) On the same diagram, sketch the graph of  $y = 2f(-x)$ .



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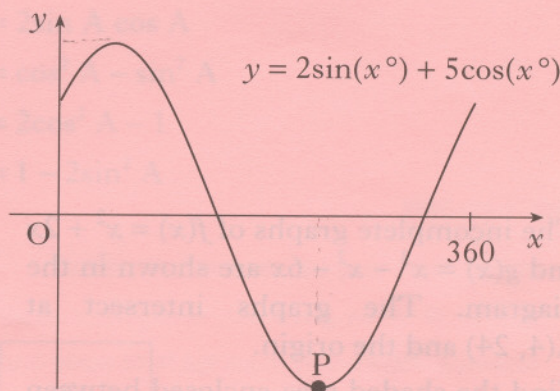
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6. If  $f(x) = \cos(2x) - 3 \sin(4x)$ , find the exact value of  $f'\left(\frac{\pi}{6}\right)$ .

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7. Part of the graph of  $y = 2\sin(x^\circ) + 5\cos(x^\circ)$  is shown in the diagram.

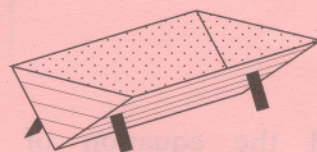
- (a) Express  $y = 2\sin(x^\circ) + 5\cos(x^\circ)$  in the form  $k\sin(x^\circ + a^\circ)$  where  $k > 0$  and  $0 \leq a < 360$ .  
 (b) Find the coordinates of the minimum turning point P.



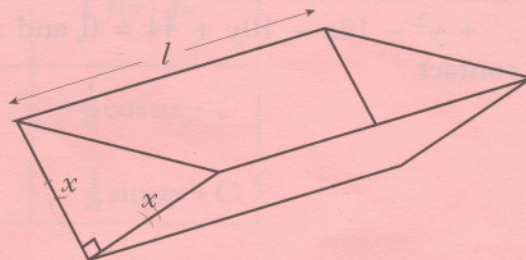
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8. An open water tank, in the shape of a triangular prism, has a capacity of 108 litres. The tank is to be lined on the inside in order to make it watertight.



The triangular cross-section of the tank is right-angled and isosceles, with equal sides of length  $x$  cm. The tank has a length of  $l$  cm.

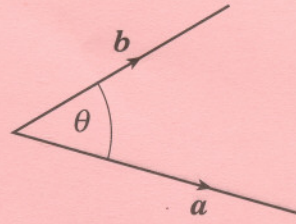


- (a) Show that the surface area to be lined,  $A \text{ cm}^2$ , is given by  $A(x) = x^2 + \frac{432000}{x}$ .  
 (b) Find the value of  $x$  which minimises this surface area.

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9. The diagram shows vectors  $\mathbf{a}$  and  $\mathbf{b}$ .  
If  $|\mathbf{a}| = 5$ ,  $|\mathbf{b}| = 4$  and  $\mathbf{a} \cdot (\mathbf{a} + \mathbf{b}) = 36$ , find the size of the acute angle  $\theta$  between  $\mathbf{a}$  and  $\mathbf{b}$ .



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10. Solve the equation  $3\cos(2x) + 10\cos(x) - 1 = 0$  for  $0 \leq x \leq \pi$ , correct to 2 decimal places.

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11. (a) (i) Sketch the graph of  $y = a^x + 1$ ,  $a > 2$ .

(ii) On the same diagram, sketch the graph of  $y = a^{x+1}$ ,  $a > 2$ .

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- (b) Prove that the graphs intersect at a point where the  $x$ -coordinate is

$$\log_a \left( \frac{1}{a-1} \right).$$

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[END OF QUESTION PAPER]

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