

X100/301

NATIONAL
QUALIFICATIONS
2003

WEDNESDAY, 21 MAY
9.00 AM – 10.10 AM

MATHEMATICS HIGHER

Units 1, 2 and 3

Paper 1

(Non-calculator)

Read Carefully

- 1 Calculators may **NOT** be used in this paper.
- 2 Full credit will be given only where the solution contains appropriate working.
- 3 Answers obtained by readings from scale drawings will not receive any credit.



ALL questions should be attempted.

Marks

1. Find the equation of the line which passes through the point $(-1, 3)$ and is perpendicular to the line with equation $4x + y - 1 = 0$. 3

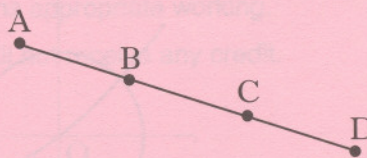
2. (a) Write $f(x) = x^2 + 6x + 11$ in the form $(x + a)^2 + b$. 2
 (b) Hence or otherwise sketch the graph of $y = f(x)$. 2

3. Vectors \mathbf{u} and \mathbf{v} are defined by $\mathbf{u} = 3\mathbf{i} + 2\mathbf{j}$ and $\mathbf{v} = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$.
 Determine whether or not \mathbf{u} and \mathbf{v} are perpendicular to each other. 2

4. A recurrence relation is defined by $u_{n+1} = pu_n + q$, where $-1 < p < 1$ and $u_0 = 12$.
 (a) If $u_1 = 15$ and $u_2 = 16$, find the values of p and q . 2
 (b) Find the limit of this recurrence relation as $n \rightarrow \infty$. 2

5. Given that $f(x) = \sqrt{x} + \frac{2}{x^2}$, find $f'(4)$. 5

6. A and B are the points $(-1, -3, 2)$ and $(2, -1, 1)$ respectively.
 B and C are the points of trisection of AD, that is $AB = BC = CD$.
 Find the coordinates of D. 3



7. Show that the line with equation $y = 2x + 1$ does not intersect the parabola with equation $y = x^2 + 3x + 4$. 5

8. Find $\int_0^1 \frac{dx}{(3x+1)^{\frac{1}{2}}}$. 4

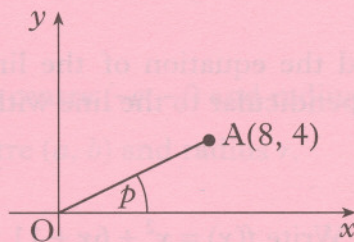
9. Functions $f(x) = \frac{1}{x-4}$ and $g(x) = 2x + 3$ are defined on suitable domains.
 (a) Find an expression for $h(x)$ where $h(x) = f(g(x))$. 2
 (b) Write down any restriction on the domain of h . 1

[Turn over for Questions 10 to 12 on Page four

10. A is the point (8, 4). The line OA is inclined at an angle p radians to the x -axis.

(a) Find the exact values of:

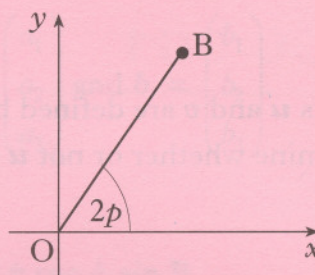
- (i) $\sin(2p)$;
- (ii) $\cos(2p)$.



5

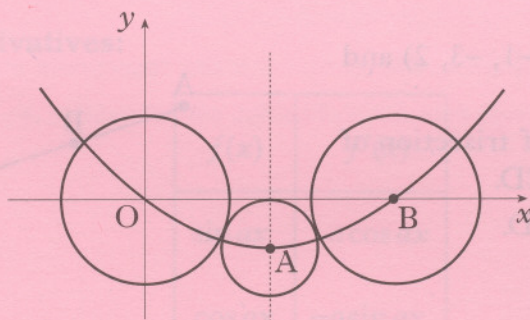
The line OB is inclined at an angle $2p$ radians to the x -axis.

(b) Write down the exact value of the gradient of OB.



1

11. • O, A and B are the centres of the three circles shown in the diagram below.
- The two outer circles are congruent and each touches the smallest circle.
 - Circle centre A has equation $(x - 12)^2 + (y + 5)^2 = 25$.
 - The three centres lie on a parabola whose axis of symmetry is shown by the broken line through A.



- (a) (i) State the coordinates of A and find the length of the line OA. 2
- (ii) Hence find the equation of the circle with centre B. 3
- (b) The equation of the parabola can be written in the form $y = px(x + q)$.
Find the values of p and q . 2

12. Simplify $3 \log_e(2e) - 2 \log_e(3e)$ expressing your answer in the form $A + \log_e B - \log_e C$ where A, B and C are whole numbers. 4

[END OF QUESTION PAPER]