## X100/301

NATIONAL QUALIFICATIONS 2002

## MONDAY, 27 MAY 9.00 AM - 10.10 AM

# MATHEMATICS HIGHER 

Units 1, 2 and 3
Paper 1
(Non-calculator)

## Read Carefully

1 Calculators may NOT be used in this paper.
2 Full credit will be given only where the solution contains appropriate working.
3 Answers obtained by readings from scale drawings will not receive any credit.

## ALL questions should be attempted.

1. The point $\mathrm{P}(2,3)$ lies on the circle $(x+1)^{2}+(y-1)^{2}=13$. Find the equation of the tangent at $P$.
2. The point Q divides the line joining $\mathrm{P}(-1,-1,0)$ to $\mathrm{R}(5,2,-3)$ in the ratio $2: 1$. Find the coordinates of Q .
3. Functions $f$ and $g$ are defined on suitable domains by $f(x)=\sin \left(x^{\circ}\right)$ and $g(x)=2 x$.
(a) Find expressions for:
(i) $f(g(x))$;
(ii) $g(f(x))$.
(b) Solve $2 f(g(x))=g(f(x))$ for $0 \leq x \leq 360$.
4. Find the coordinates of the point on the curve $y=2 x^{2}-7 x+10$ where the tangent to the curve makes an angle of $45^{\circ}$ with the positive direction of the $x$-axis.
5. In triangle $A B C$, show that the exact value of $\sin (a+b)$ is $\frac{2}{\sqrt{5}}$.

6. The graph of a function $f$ intersects the $x$-axis at $(-a, 0)$ and $(e, 0)$ as shown.
There is a point of inflexion at $(0, b)$ and a maximum turning point at $(c, d)$.
Sketch the graph of the derived function $f^{\prime}$.

7. (a) Express $f(x)=x^{2}-4 x+5$ in the form $f(x)=(x-a)^{2}+b$.
(b) On the same diagram sketch:
(i) the graph of $y=f(x)$;
(ii) the graph of $y=10-f(x)$.
(c) Find the range of values of $x$ for which $10-f(x)$ is positive.
8. The diagram shows the graph of a cosine function from 0 to $\pi$.
(a) State the equation of the graph.
(b) The line with equation $y=-\sqrt{3}$ intersects this graph at points A and B.
Find the coordinates of B.

9. (a) Write $\sin (x)-\cos (x)$ in the form $k \sin (x-a)$ stating the values of $k$ and $a$ where $k>0$ and $0 \leq a \leq 2 \pi$.
(b) Sketch the graph of $y=\sin (x)-\cos (x)$ for $0 \leq x \leq 2 \pi$, showing clearly the graph's maximum and minimum values and where it cuts the $x$-axis and the $y$-axis.
10. (a) Find the derivative of the function $f(x)=\left(8-x^{3}\right)^{\frac{1}{2}}, x<2$.
(b) Hence write down $\int \frac{x^{2}}{\left(8-x^{3}\right)^{\frac{1}{2}}} d x$.
11. The graph illustrates the law $y=k x^{n}$. If the straight line passes through $\mathrm{A}(0 \cdot 5,0)$ and $\mathrm{B}(0,1)$, find the values of $k$ and $n$.

