

**X056/301**

NATIONAL  
QUALIFICATIONS  
2001

THURSDAY, 17 MAY  
9.00 AM – 10.10 AM

**MATHEMATICS  
HIGHER**

Units 1, 2 and 3

Paper 1

(Non-calculator)

**Read Carefully**

- 1 Calculators may **NOT** be used in this paper.
- 2 Full credit will be given only where the solution contains appropriate working.
- 3 Answers obtained by readings from scale drawings will not receive any credit.

ALL questions should be attempted.

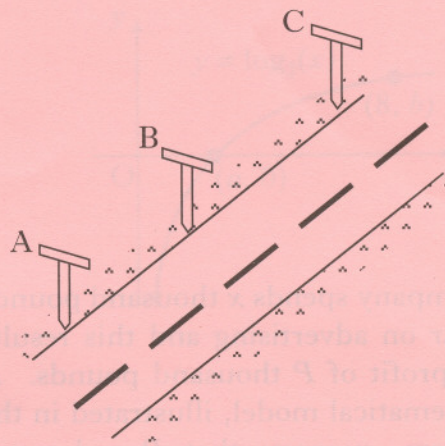
Marks

1. Find the equation of the straight line which is parallel to the line with equation  $2x + 3y = 5$  and which passes through the point  $(2, -1)$ . 3

2. For what value of  $k$  does the equation  $x^2 - 5x + (k + 6) = 0$  have equal roots? 3

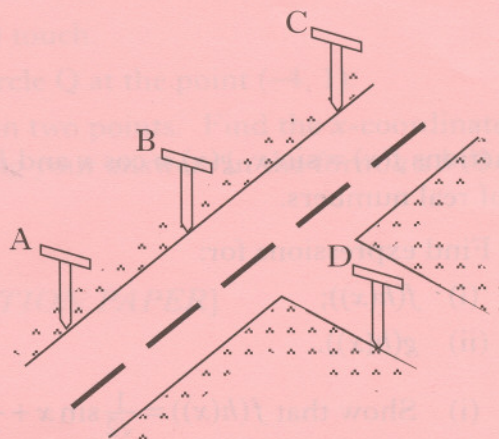
3. (a) Roadmakers look along the tops of a set of T-rods to ensure that straight sections of road are being created. Relative to suitable axes the top left corners of the T-rods are the points  $A(-8, -10, -2)$ ,  $B(-2, -1, 1)$  and  $C(6, 11, 5)$ .

Determine whether or not the section of road ABC has been built in a straight line.



(b) A further T-rod is placed such that D has coordinates  $(1, -4, 4)$ .

Show that DB is perpendicular to AB.



4. Given  $f(x) = x^2 + 2x - 8$ , express  $f(x)$  in the form  $(x + a)^2 - b$ . 2

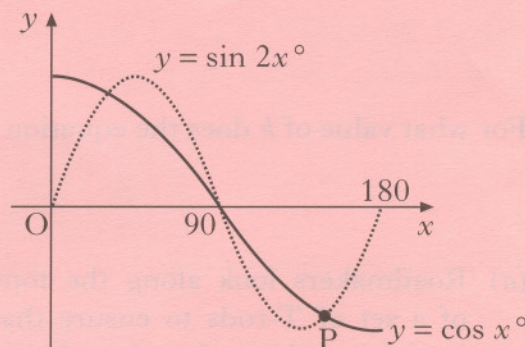
[Turn over

5. (a) Solve the equation  $\sin 2x^\circ - \cos x^\circ = 0$  in the interval  $0 \leq x \leq 180$ .

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(b) The diagram shows parts of two trigonometric graphs,  $y = \sin 2x^\circ$  and  $y = \cos x^\circ$ .

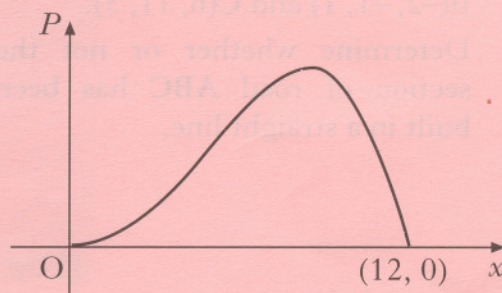
Use your solutions in (a) to write down the coordinates of the point P.



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6. A company spends  $x$  thousand pounds a year on advertising and this results in a profit of  $P$  thousand pounds. A mathematical model, illustrated in the diagram, suggests that  $P$  and  $x$  are related by  $P = 12x^3 - x^4$  for  $0 \leq x \leq 12$ .

Find the value of  $x$  which gives the maximum profit.



5

7. Functions  $f(x) = \sin x$ ,  $g(x) = \cos x$  and  $h(x) = x + \frac{\pi}{4}$  are defined on a suitable set of real numbers.

(a) Find expressions for:

(i)  $f(h(x))$ ;

(ii)  $g(h(x))$ .

2

(b) (i) Show that  $f(h(x)) = \frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x$ .

(ii) Find a similar expression for  $g(h(x))$  and hence solve the equation  $f(h(x)) - g(h(x)) = 1$  for  $0 \leq x \leq 2\pi$ .

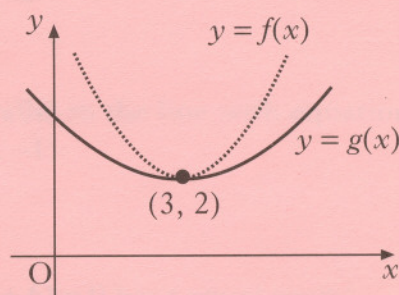
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8. Find  $x$  if  $4 \log_x 6 - 2 \log_x 4 = 1$ .

3

9. The diagram shows the graphs of two quadratic functions  $y = f(x)$  and  $y = g(x)$ . Both graphs have a minimum turning point at  $(3, 2)$ .

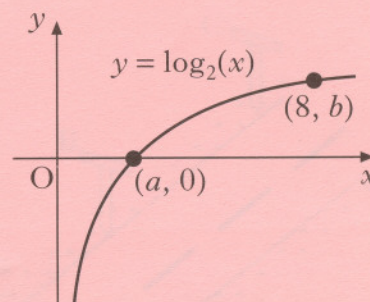
Sketch the graph of  $y = f'(x)$  and on the same diagram sketch the graph of  $y = g'(x)$ .



2

10. The diagram shows a sketch of part of the graph of  $y = \log_2(x)$ .

- (a) State the values of  $a$  and  $b$ .  
 (b) Sketch the graph of  $y = \log_2(x + 1) - 3$ .



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11. Circle P has equation  $x^2 + y^2 - 8x - 10y + 9 = 0$ . Circle Q has centre  $(-2, -1)$  and radius  $2\sqrt{2}$ .

- (a) (i) Show that the radius of circle P is  $4\sqrt{2}$ .  
 (ii) Hence show that circles P and Q touch. (4)  
 (b) Find the equation of the tangent to circle Q at the point  $(-4, 1)$ . (3)  
 (c) The tangent in (b) intersects circle P in two points. Find the  $x$ -coordinates of the points of intersection, expressing your answers in the form  $a \pm b\sqrt{3}$ . (3)

[END OF QUESTION PAPER]