X056/301

NATIONAL QUALIFICATIONS 2000

THURSDAY, 25 MAY
9.00 AM – 10.10 AM

MATHEMATICS
HIGHER
Paper 1
(Non-calculator)

Read Carefully

1 **Calculators may NOT be used in this paper.**

2 There are three Sections in this paper.

Section A assesses the compulsory units Mathematics 1 and 2.
Section B assesses the optional unit Mathematics 3.
Section C assesses the optional unit Statistics.

Candidates must attempt **all** questions in Section A (Mathematics 1 and 2) **and**

either Section B (Mathematics 3)

or Section C (Statistics).

3 Full credit will be given only where the solution contains appropriate working.

4 Answers obtained by readings from scale drawings will not receive any credit.

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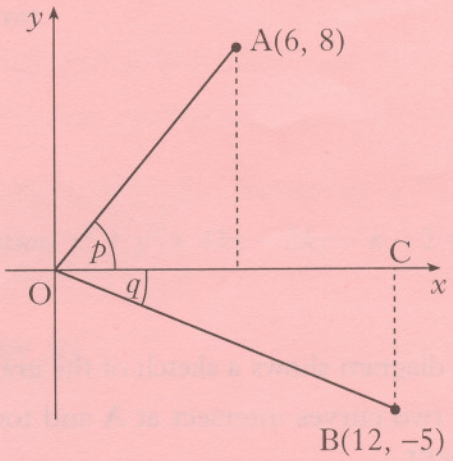
SECTION A (Mathematics 1 and 2)

Marks

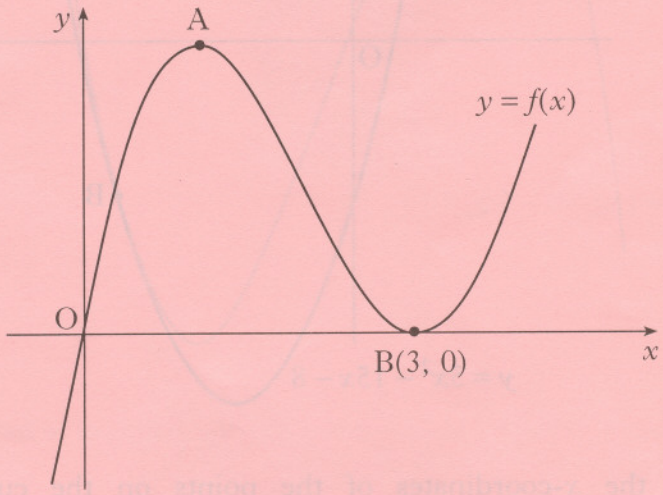
ALL candidates should attempt this Section.

A1. On the coordinate diagram shown, A is the point (6, 8) and B is the point (12, –5). Angle AOC = p and angle COB = q.

Find the exact value of $\sin(p + q)$. 4



A2. A sketch of the graph of $y = f(x)$ where $f(x) = x^3 - 6x^2 + 9x$ is shown below. The graph has a maximum at A and a minimum at B(3, 0).



(*a*) Find the coordinates of the turning point at A. 4

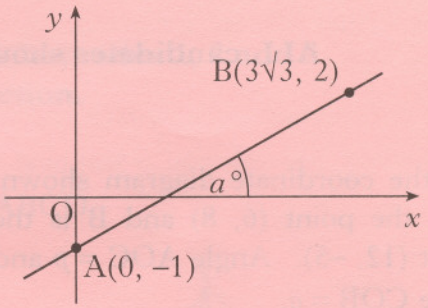
(*b*) Hence sketch the graph of $y = g(x)$ where $g(x) = f(x + 2) + 4$.

Indicate the coordinates of the turning points. There is no need to calculate the coordinates of the points of intersection with the axes. 2

(*c*) Write down the range of values of k for which $g(x) = k$ has 3 real roots. 1

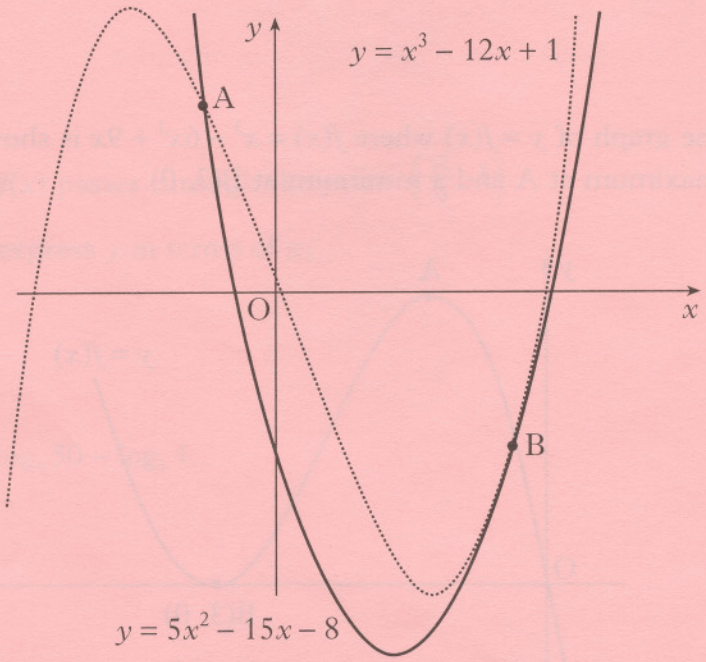
[Turn over

A3. Find the size of the angle $a°$ that the line joining the points A(0, –1) and B($3\sqrt{3}$, 2) makes with the positive direction of the x-axis. **3**



A4. The diagram shows a sketch of the graphs of $y = 5x^2 - 15x - 8$ and $y = x^3 - 12x + 1$.

The two curves intersect at A and touch at B, ie at B the curves have a common tangent.



(*a*) (i) Find the x-coordinates of the points on the curves where the gradients are equal. **4**

(ii) By considering the corresponding y-coordinates, or otherwise, distinguish geometrically between the two cases found in part (i). **1**

(*b*) The point A is (–1, 12) and B is (3, –8).

Find the area enclosed between the two curves. **5**

Marks

A5. Two sequences are generated by the recurrence relations $u_{n+1} = au_n + 10$ and $v_{n+1} = a^2v_n + 16$.

The two sequences approach the same limit as $n \to \infty$.

Determine the value of a and evaluate the limit. **5**

A6. For what range of values of k does the equation $x^2 + y^2 + 4kx - 2ky - k - 2 = 0$ represent a circle? **5**

[END OF SECTION A]

Candidates should now attempt
EITHER Section B (Mathematics 3) on *Page six*
OR Section C (Statistics) on *Pages seven and eight*

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SECTION B (Mathematics 3)

Marks

ONLY candidates doing the course Mathematics 1, 2 and 3 should attempt this Section.

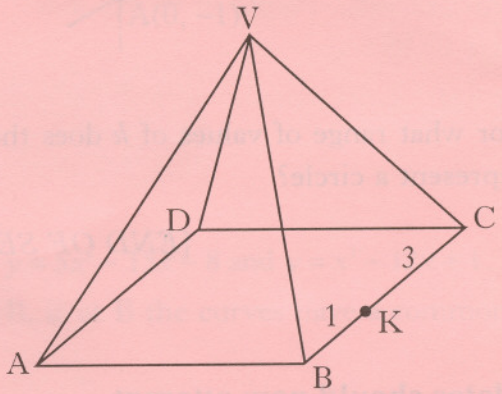
B7. VABCD is a pyramid with a rectangular base ABCD.

Relative to some appropriate axes,

$\overrightarrow{VA}$ represents $-7\boldsymbol{i} - 13\boldsymbol{j} - 11\boldsymbol{k}$

$\overrightarrow{AB}$ represents $6\boldsymbol{i} + 6\boldsymbol{j} - 6\boldsymbol{k}$

$\overrightarrow{AD}$ represents $8\boldsymbol{i} - 4\boldsymbol{j} + 4\boldsymbol{k}$.



K divides BC in the ratio 1:3.

Find $\overrightarrow{VK}$ in component form. **3**

B8. The graph of $y = f(x)$ passes through the point $\left(\frac{\pi}{9}, 1\right)$.

If $f'(x) = \sin(3x)$, express y in terms of x. **4**

B9. Evaluate $\log_5 2 + \log_5 50 - \log_5 4$. **3**

B10. Find the maximum value of $\cos x - \sin x$ and the value of x for which it occurs in the interval $0 \leq x \leq 2\pi$. **6**

[END OF SECTION B]