HIGHER MATHS 2025

Paper 1

(1) $y = 2^3 - 2(2)^2 + 5$

$= 8 - 8 + 5$

$= 5 \quad \therefore (2,5)$

$\frac{dy}{dx} = 3x^2 - 4x$

$\therefore m = 3(2)^2 - 4(2)$

$= 12 - 8$

$= 4$

$y - 5 = 4(x - 2)$

$y - 5 = 4x - 8$

$\underline{y = 4x - 3}$

(2) $\text{mid} = \left(\frac{1+9}{2}, \frac{4+10}{2}\right)$

$= (5, 7)$

$m_{AB} = \frac{10-4}{9-1}$

$= \frac{6}{8}$

$= \frac{3}{4}$

$\therefore m_{PB} = -\frac{4}{3}$

$y - 7 = -\frac{4}{3}(x - 5)$

$3y - 21 = -4x + 20$

$\underline{3y = -4x + 41}$

(3) $\int (12x^{-2} + x^{1/2})\,dx$

$= \frac{12x^{-1}}{-1} + \frac{x^{3/2}}{\frac{3}{2}} + c$

$\underline{= -12x^{-1} + \frac{2}{3}x^{3/2} + c}$

(4) $3\log_3 2 + \log_3 \frac{1}{24}$

$= \log_3 2^3 + \log_3 \frac{1}{24}$

$= \log_3 8 + \log_3 \frac{1}{24}$

$= \log_3 \frac{8}{24}$

$= \log_3 \frac{1}{3}$

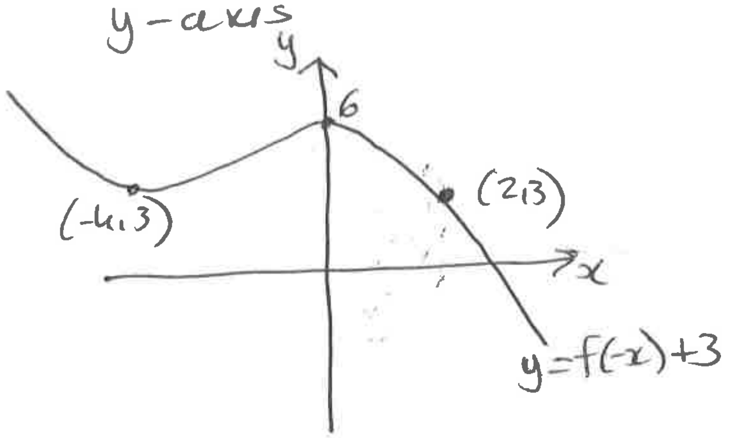
$= \log_3 (3^{-1})$

$= \underline{\underline{-1}}$

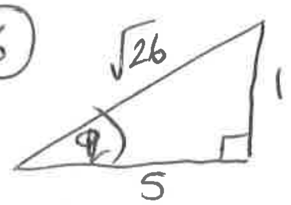
(5) $y = f(-x) + 3$

Flip in y-axis

+3 vertically



(6)



$h^2 = 5^2 + 1^2$

$= 26$

$h = \sqrt{26}$

a) (i) $\sin 2q = 2\sin q \cos q$

$= 2\left(\frac{1}{\sqrt{26}}\right)\left(\frac{5}{\sqrt{26}}\right)$

$= \frac{10}{26}$

$= \underline{\frac{5}{13}}$

(ii) $\cos 2q = \cos^2 q - \sin^2 q$

$= \left(\frac{5}{\sqrt{26}}\right)^2 - \left(\frac{1}{\sqrt{26}}\right)^2$

$= \frac{25}{26} - \frac{1}{26}$

$= \frac{24}{26} = \underline{\underline{\frac{12}{13}}}$

b) $\sin(2q - r)$

$= \sin 2q \cos r - \cos 2q \sin r$

$= \left(\frac{5}{13} \times \frac{4}{\sqrt{17}}\right) - \left(\frac{12}{13} \times \frac{1}{\sqrt{17}}\right)$

$= \frac{20}{13\sqrt{17}} - \frac{12}{13\sqrt{17}}$

$= \frac{8}{13\sqrt{17}}$

7 a) let $f(x) = 5x^3 + 16x^2 - x - 12$

$f(-3) = 5(-3)^3 + 16(-3)^2 - (-3) - 12$

$= 5(-27) + 16(9) + 3 - 12$

$= -135 + 144 - 9$

$= 0$

$\therefore x = -3$ is a root

$(x+3)$ is a factor

b)

	$5x^2$	$+x$	-4
x	$5x^3$	$+x^2$	$-4x$
$+3$	$+15x^2$	$+3x$	-12

$\therefore f(x) = (x+3)(5x^2 + x - 4)$

	$5x$	-4
x	$5x^2$	$-4x$
$+1$	$+5x$	-4

20	
1	20
2	10
4	5

$\therefore F(x) = (x+3)(x+1)(5x-4)$

8 $\log_a 75 = 2 + \log_a 3$

$\log_a 75 - \log_a 3 = 2$

$\log_a 25 = 2$

$25 = a^2$

$a = \sqrt{25}$

$a = \cancel{\pm}\, 5$

$\therefore a = 5$

9

$x^2 + (x+1)^2 - 2x + 6(x+1) - 15 = 0$

$x^2 + x^2 + 2x + 1 - 2x + 6x + 6 - 15 = 0$

$2x^2 + 6x - 8 = 0$

$2(x^2 + 3x - 4) = 0$

$2(x+4)(x-1) = 0$

$x = -4 \quad x = 1$

$y = -3 \quad y = 2$

$\therefore (-4, -3) \quad (1, 2)$

10 $\underline{u} \cdot \underline{v}$

$= (1 \times 1) + (1 \times 3) + (0 \times k)$

$= 1 + 3 + 0$

$= 4$

$|\underline{u}| = \sqrt{1^2 + 1^2 + 0^2}$

$= \sqrt{2}$

$|\underline{v}| = \sqrt{1^2 + 3^2 + k^2}$

$= \sqrt{k^2 + 10}$

(11) $a = 9 \quad b = 3k \quad c = k$

For real, distinct roots

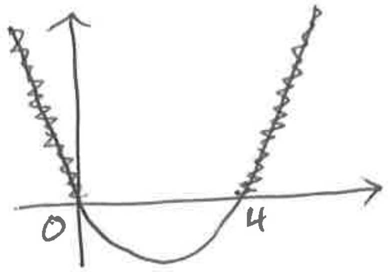
$$b^2 - 4ac > 0$$

$$\therefore (3k)^2 - 4(9)(k) > 0$$

$$9k^2 - 36k > 0$$

$$9k(k-4) > 0$$

roots @ $k = 0$, $k = 4$



$$\therefore \underline{k < 0, \; k > 4}$$

(12) $y = \int (6\cos x + 8\sin 2x)\,dx$

$$= 6\sin x - \frac{8}{2}\cos 2x + C$$

$$= 6\sin x - 4\cos 2x + C$$

$$\therefore 6\sin\left(\frac{\pi}{6}\right) - 4\cos 2\left(\frac{\pi}{6}\right) + C = 4$$

$$6\sin 30° - 4\cos 60° + C = 4$$

$$6\left(\tfrac{1}{2}\right) - 4\left(\tfrac{1}{2}\right) + C = 4$$

$$3 - 2 + C = 4$$

$$C + 1 = 4$$

$$\underline{C = 3}$$

$$\therefore \underline{y = 6\sin x - 4\cos 2x + 3}$$

(13) a) SP's @ $f'(x) = 0$

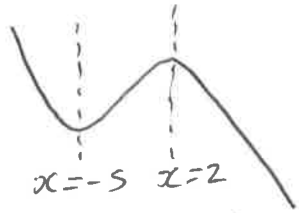
$$\therefore (x+5)(2-x) = 0$$

$$x = -5 \quad x = 2$$

x	$\xrightarrow{-6}$	-5	$\xrightarrow{0}$	$\xrightarrow{0}$	2	$\xrightarrow{3}$
$f'(x)$	$-$	0	$+$	$+$	0	$-$
shape	\	—	/	/	—	\

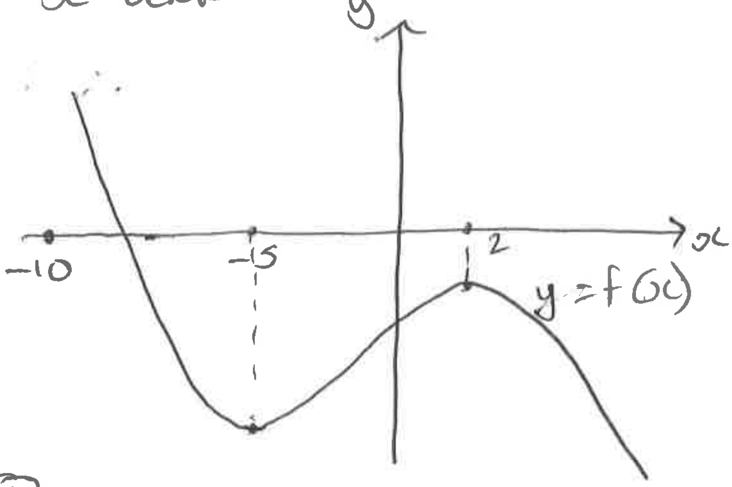
$\therefore$ Min TP @ $x = -5$
Max TP @ $x = 2$

b) From a): shape is



- $f(0) < 0 \rightarrow$ y-intercept is negative.
- $f(x) = 0$ one solution $\rightarrow$ one root, between -10 and 10.

y-axis must be between the min and max TP.
Since only one root, the Max TP must be below the x-axis.



HIGHER MATHS 2025

Paper 2

1) a) $M_{AC} = \frac{-14+24}{-9-21}$

$= \frac{10}{-30}$

$= -\frac{1}{3}$

$\therefore M_{alt} = 3$

$y - 20 = 3(x-9)$

$y - 20 = 3x - 27$

$y = 3x - 7$

b) mid $= (15, -2)$

$M_{med} = \frac{-2+14}{15+9}$

$= \frac{12}{24}$

$= \frac{1}{2}$

$y + 2 = \frac{1}{2}(x - 15)$

$2y + 4 = x - 15$

$2y = x - 19$

c) $3x - y = 7$ (1)

$x - 2y = 19$ (2)

(1) ×2 $6x - 2y = 14$ (3)

(3) − (2) $5x = -5$

$x = -1$

$y = 3(-1) - 7$

$= -10$

$\therefore (-1, -10)$

2) $p(x+q)^2 + r$

$= p(x^2 + 2qx + q^2) + r$

$= px^2 + 2pqx + pq^2 + r$

$2x^2 + 16x + 5$

$\therefore p = 2$

$2pq = 16$

$2(2)q = 16$

$4q = 16$

$q = 4$

$pq^2 + r = 5$

$2(4)^2 + r = 5$

$2(16) + r = 5$

$32 + r = 5$

$r = -27$

$\therefore 2(x+4)^2 - 27$

3) $\int_2^4 (x^2 - 2x + 3)\,dx$

$= \left[\frac{x^3}{3} - \frac{2x^2}{2} + 3x\right]_2^4$

$= \left[\frac{x^3}{3} - x^2 + 3x\right]_2^4$

$= \left(\frac{4^3}{3} - 4^2 + 3(4)\right) - \left(\frac{2^3}{3} - 2^2 + 3(2)\right)$

$= \left(\frac{64}{3} - 16 + 12\right) - \left(\frac{8}{3} - 4 + 6\right)$

$= \left(\frac{64}{3} - 4\right) - \left(\frac{8}{3} + 2\right)$

$= \left(\frac{64}{3} - \frac{12}{3}\right) - \left(\frac{8}{3} + \frac{6}{3}\right)$

$= \frac{52}{3} - \frac{14}{3}$

$= \frac{38}{3}u^2$

⑤ a) $\vec{AB} = \underline{b} - \underline{a}$

$= \begin{pmatrix} 6 \\ -1 \\ 5 \end{pmatrix} - \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}$

$= \begin{pmatrix} 9 \\ -3 \\ 6 \end{pmatrix} = 3\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$

$\vec{BC} = \underline{c} - \underline{b}$

$= \begin{pmatrix} 12 \\ -3 \\ 9 \end{pmatrix} - \begin{pmatrix} 6 \\ -1 \\ 5 \end{pmatrix}$

$= \begin{pmatrix} 6 \\ -2 \\ 4 \end{pmatrix} = 2\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$

$\vec{AB} = \frac{3}{2}\vec{BC}$ ∴ vectors are parallel

B is common ∴ points are collinear

b) 3 : 2

⑥ a) $k\cos(x+\alpha)$

$= k\cos x\cos\alpha - k\sin x\sin\alpha$

$= k\cos\alpha\cos x \boxed{-k\sin\alpha}\sin x$

$5\cos x \boxed{-9}\sin x$

$-k\sin\alpha = -9$

$\boxed{\begin{matrix} k\sin\alpha = 9 \\ k\cos\alpha = 5 \end{matrix}}$

$k^2 = 9^2 + 5^2$

$= 81 + 25$

$k = \sqrt{106}$

$\tan\alpha = \frac{9}{5}$

$\alpha = \tan^{-1}\left(\frac{9}{5}\right)$

$= 1.06$

$k\sin\alpha = \text{pos}$

$k\cos\alpha = \text{pos}$

S A

T C

∴ Q1

∴ $5\cos x - 9\sin x$

$= \sqrt{106}\cos(x + 1.06)$

b) $\sqrt{106}\cos(x+1.06) = 7$

$\cos(x+1.06) = \frac{7}{\sqrt{106}}$

$x + 1.06 = \cos^{-1}\left(\frac{7}{\sqrt{106}}\right)$

$x + 1.06 = 0.82, 5.46$

$x = -0.24, 1.40$

$x = 1.4, 6.04$

S A

T C

$2\pi - x$

$2\pi - 0.82$

$= 5.46$

⑦ $\int (3x+2)^7 \, dx$

$= \frac{(3x+2)^8}{8 \times 3} + C$

$= \frac{1}{24}(3x+2)^8 + C$

⑧ $\vec{BE} = \vec{BC} + \vec{CD} + \vec{DE}$

$= \vec{AD} - \vec{DC} + \vec{DE}$

$= \begin{pmatrix} 6 \\ 4 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ -4 \\ 2 \end{pmatrix} + \begin{pmatrix} -4 \\ -3 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} -8 \\ 5 \\ 4 \end{pmatrix}$

∴ $-8\underline{i} + 5\underline{j} + 4\underline{k}$

(9) a) $L = \frac{b}{1-a}$

$10 = \frac{4}{1-a}$

$1-a = \frac{4}{10}$

$1-a = 0.4$

$\underline{\underline{a = 0.6}}$

b) $U_{n+1} = 0.6U_n + 4$

$19 = 0.6U_0 + 4$

$0.6U_0 = 15$

$U_0 = \frac{15}{0.6}$

$\underline{\underline{U_0 = 25}}$

(10) a) Triangle $A = \frac{1}{2}bh$

$= \frac{1}{2}(4x)(3x)$

$= \frac{1}{2}(12x^2)$

$= \underline{6x^2}$

Rectangle $A = lb$

$= \underline{5xy}$

$\therefore$ Area $= 150$

$5xy + 6x^2 = 150$

$5y = 150 - 6x^2$

$\underline{y = \frac{30}{x} - \frac{6}{5}x}$

Perimeter $= 4x + 3x + 5x + 2y$

$= 12x + 2\left(\frac{30}{x} - \frac{6}{5}x\right)$

$= 12x + \frac{60}{x} - \frac{12}{5}x$

$= 9.6x + \frac{60}{x}$

(as required)

b) SP's @ $\frac{dP}{dx} = 0$

$P = 9.6x + 60x^{-1}$

$\therefore \frac{dP}{dx} = 9.6 - 60x^{-2}$

$= 9.6 - \frac{60}{x^2}$

$\therefore 9.6 - \frac{60}{x^2} = 0$

$9.6 = \frac{60}{x^2}$

$x^2 = \frac{60}{9.6}$

$x = \sqrt{\frac{60}{9.6}}$

$\underline{\underline{x = 2.5, \cancel{-2.5}}}$

x	$\xrightarrow{2}$	2.5	$\xrightarrow{3}$
$\frac{dP}{dx}$	$-$	0	$+$
shape	\	—	/

$\therefore$ minimum @ $x = 2.5$

$\therefore P = 9.6(2.5) + \frac{60}{2.5}$

$\underline{\underline{= 48 \text{ cm}}}$

(11) $3\sin 2x + 4\cos x = 0$

$3(2\sin x\cos x) + 4\cos x = 0$

$6\sin x\cos x + 4\cos x = 0$

$2\cos x\,(3\sin x + 2) = 0$

$2\cos x = 0 \qquad 3\sin x + 2 = 0$

$\cos x = 0 \qquad \sin x = -\frac{2}{3}$

$x = 90°, 270° \qquad x = \text{"}41.8°\text{"}$

$x = 221.8°, 318.2°$

(6) $\therefore \underline{x = 90°, 221.8°, 270°, 318.2°}$

(12) a) $f(g(x))$

$= f(1-x^3)$

$= (1-x^3)^5 + 3$

b) $h(x) = (1-x^3)^5 + 3$

$h'(x) = 5(1-x^3)^4 \times (-3x^2)$

$= -15x^2(1-x^3)^4$

(13) a) 150 μg

b) $120 = 150e^{-0.0054t}$

$\frac{120}{150} = e^{-0.0054t}$

$\ln\left(\frac{120}{150}\right) = -0.0054t$

$t = \frac{\ln\left(\frac{120}{150}\right)}{-0.0054}$

$t = 41.32$ years

(14) a) $C_1 = (-5, 6)$

$r_1 = 3$

b) $2g = -14 \quad 2f = 6 \quad c = 54$

$g = -7 \quad f = 3$

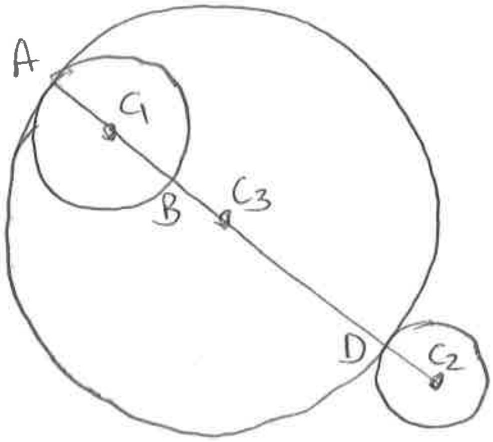
$C_2 = (7, -3)$

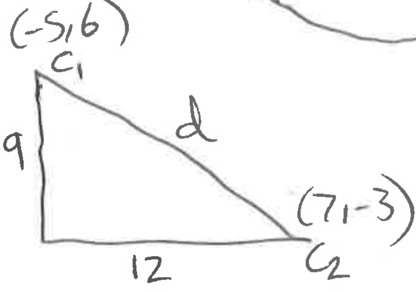
$r_2 = \sqrt{(-7)^2 + 3^2 - 54}$

$= \sqrt{4}$

$= 2$

c)





$C_1 \rightarrow C_2$

$d^2 = 9^2 + 12^2$

$= 225$

$d = 15$

$A \rightarrow C_1 = 3$ so $A \rightarrow C_2 = 3 + 15$

$= 18$

$D \rightarrow C_2 = 2$ so $A \rightarrow D = 18 - 2$

$= 16$

$\therefore$ diameter $= 16$, radius $= 8$

$A \rightarrow B = 6$ and $A \rightarrow C_3 = 8$

so $B \rightarrow C_3 = 2$

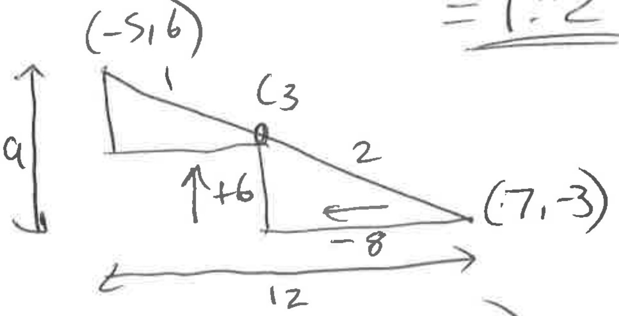
and $\boxed{C_1 \rightarrow C_3 = 5}$

$C_3 \rightarrow D = 8$ and $D \rightarrow C_2 = 2$

so $\boxed{C_3 \rightarrow C_2 = 10}$

$\therefore C_3$ splits $C_1 \rightarrow C_2$ in the ratio $5:10$

$= 1:2$



$\therefore C_3 = (-1, 3)$

$\boxed{\therefore (x+1)^2 + (y-3)^2 = 64}$